

ENG 346

Data Structures and Algorithms for Artificial Intelligence

Recursion

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<https://github.com/mehmetpekmezci/GTU-ENG-346>

Recursion

- When a function calls itself...We have *recursion*.
- Such function/algorithm is called *recursive* function/algorithm.
- Base case(s)
 - Values of the input variables for which we perform no recursive calls are called base cases (there should be at least one base case).
 - Every possible chain of recursive calls must eventually reach a base case.
- Recursive calls
 - Calls to the current method.
 - Each recursive call should be defined so that it makes progress towards a base case.

Example: Factorial

$$f(n) = \begin{cases} 1, & n = 0 \\ n \times f(n - 1), & \text{otherwise} \end{cases}$$

```
def f(n):  
    if n == 0: return 1  
    else: return n * f(n-1)
```

Example: Factorial – continued

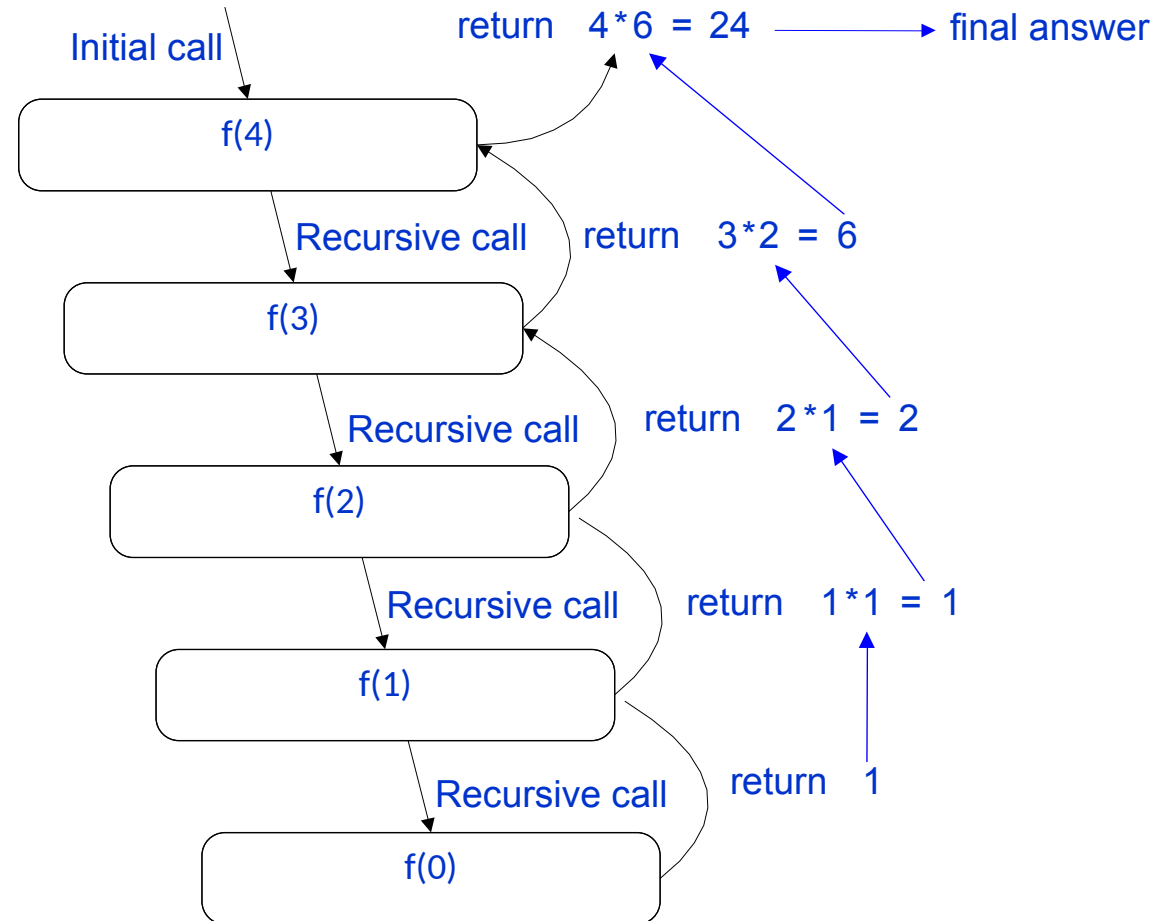
```
def f(n):  
    print ("Called with n=", n)  
    if n == 0: return 1  
    else: return n * f(n-1)
```

```
f(4) # 4!
```

- Output:

Called with n= 4
Called with n= 3
Called with n= 2
Called with n= 1
Called with n= 0
24

Example: Factorial – continued

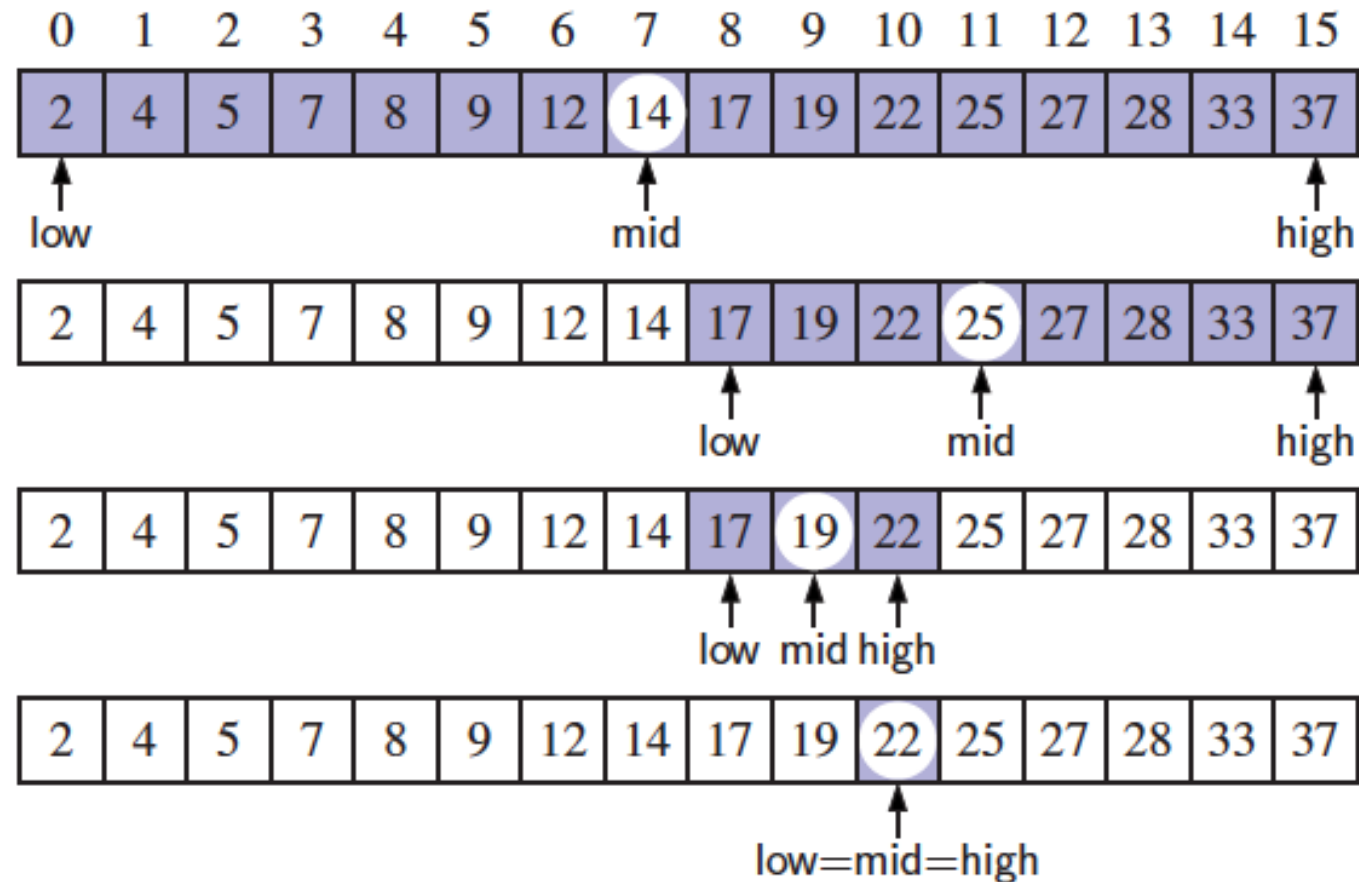


Example: Number Guess

- Pick a number between 1 and 100.

Example: Binary Search

- Search for an integer “target” in an ordered list.



Example: Binary Search – continued

```
1 def binary_search(data, target, low, high):
2     """ Return True if target is found in indicated portion of a Python list.
3
4     The search only considers the portion from data[low] to data[high] inclusive.
5     """
6     if low > high:
7         return False                # interval is empty; no match
8     else:
9         mid = (low + high) // 2
10        if target == data[mid]:      # found a match
11            return True
12        elif target < data[mid]:
13            # recur on the portion left of the middle
14            return binary_search(data, target, low, mid - 1)
15        else:
16            # recur on the portion right of the middle
17            return binary_search(data, target, mid + 1, high)
```


Fibonacci Numbers

$$fib(n) = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ fib(n - 1) + fib(n - 2), & n > 1 \end{cases}$$

- $fib(0) = 0$
- $fib(1) = 1$
- $fib(2) = 1 + 0 = 1$
- $fib(3) = 1 + 1 = 2$
- $fib(4) = 2 + 1 = 3$
- $fib(5) = 3 + 2 = 5$
- $fib(6) = 5 + 3 = 8$
- $fib(7) = 8 + 5 = 13$
- $fib(8) = 13 + 8 = 21$

Fibonacci Numbers – continued

$$fib(n) = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ fib(n-1) + fib(n-2), & n > 1 \end{cases}$$

- Calls for $fib(0) = 1$
- Calls for $fib(1) = 1$
- Calls for $fib(2) = 1 + 1 + 1 = 3$
- Calls for $fib(3) = 1 + 3 + 1 = 5$
- Calls for $fib(4) = 1 + 5 + 3 = 9$
- Calls for $fib(5) = 1 + 9 + 5 = 15$
- Calls for $fib(6) = 1 + 15 + 9 = 25$
- Calls for $fib(7) = 1 + 25 + 15 = 41$
- Calls for $fib(8) = 1 + 41 + 25 = 67$

- Basically:

Call for $fib(n) > 2^{(n/2)}$

- Exponential runtime!

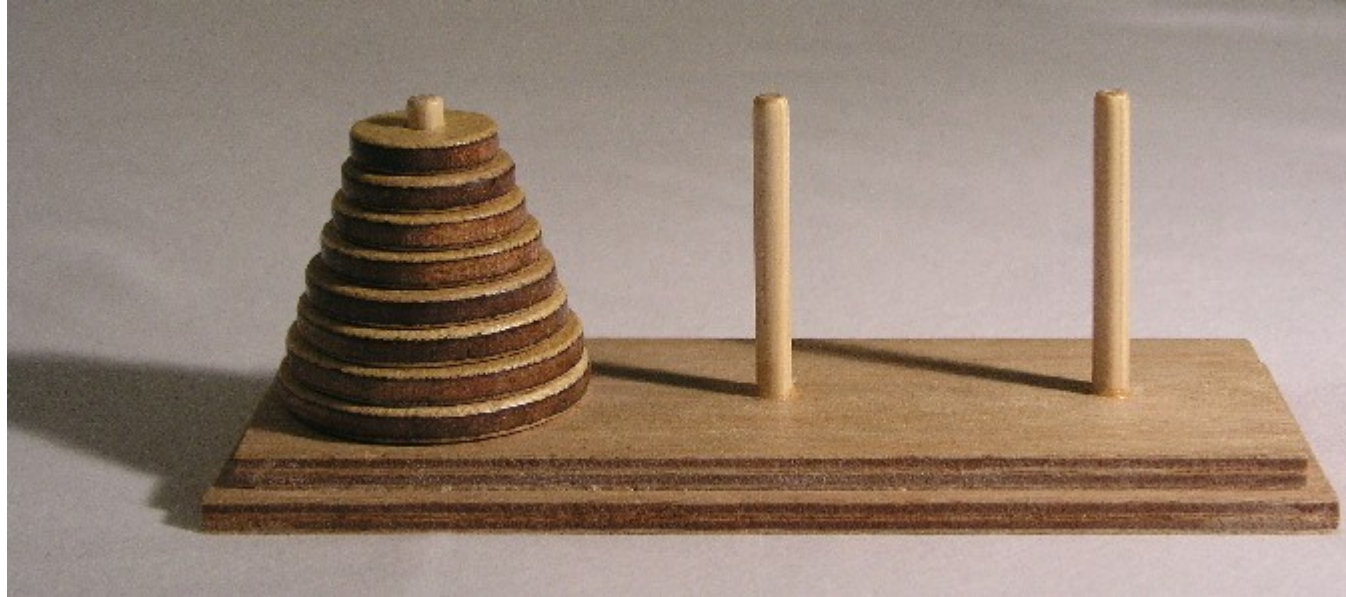
Fibonacci Numbers – continued

```
def fib2(n):  
    """Return the nth Fibonacci  
    number."""  
    if n <= 1: return (n,0)  
    else:  
        (a, b) = fib2(n-1)  
        return (a+b, a)
```

Designing Recursive Algorithms

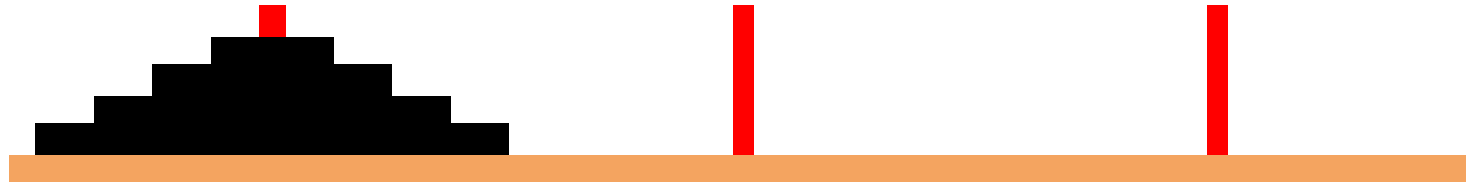
- Test for base cases:
 - There should be at least one.
 - Base case should not contain recursion.
- Recursive step:
 - Perform one or more recursive calls.
 - Input size (or length) should decrease with each recursive call.
 - Chain of recursive calls should reach to base case.

Example: Towers of Hanoi

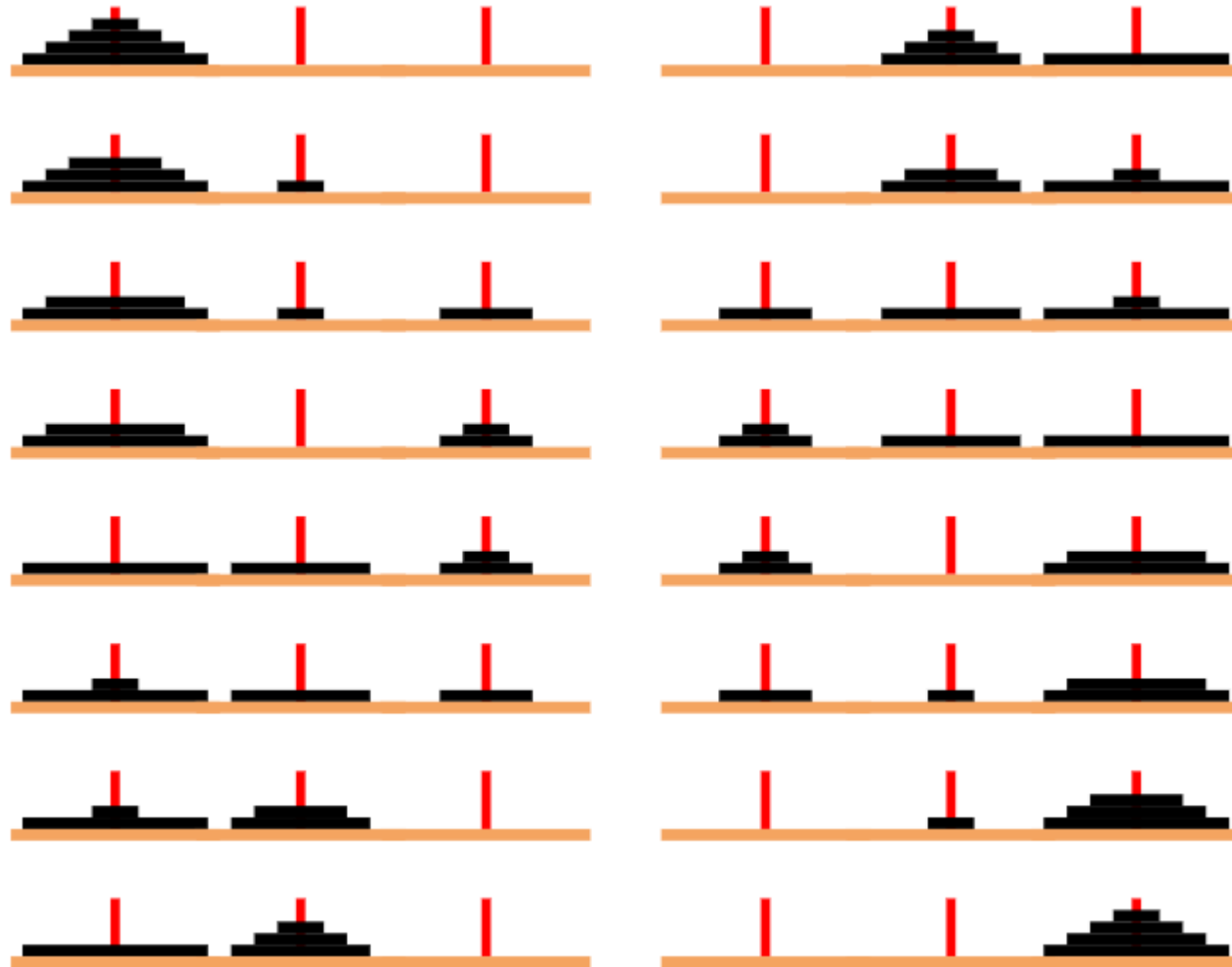


- Game rules:
 - Only one disk may be moved at a time.
 - Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
 - No disk may be placed on top of a disk that is smaller than it.

Example: Towers of Hanoi – continued



Example: Towers of Hanoi – continued



Example: Towers of Hanoi – continued

```
def TowerOfHanoi(n , source, destination, auxiliary):  
    if n==1:  
        print ("Move disk 1 from source",source,"to  
destination",destination)  
        return  
    TowerOfHanoi(n-1, source, auxiliary, destination)  
    print ("Move disk",n,"from source",source,"to  
destination",destination)  
    TowerOfHanoi(n-1, auxiliary, destination, source)
```


Tail Recursion

- Recursive call is the last step of the function.
- Function returns immediately after recursive call.
- Eliminating tail recursion will clear any overhead resulting from recursive function calls.

Iterative Binary Search

```
def binary_search_iterative(data, target):  
    """Return True if target is found in the given Python  
list."""  
    low = 0  
    high = len(data) - 1  
    while low <= high:  
        mid = (low + high) // 2  
        if target == data[mid]: # found a match  
            return True  
        elif target < data[mid]:  
            high = mid - 1 # only consider values left of mid  
        else:  
            low = mid + 1 # only consider values right of mid  
  
    return False # loop ended without success
```

Exercises

- **R-4.1** Describe a recursive algorithm for finding the maximum element in a sequence, S , of n elements. What is your running time and space usage?
- **R-4.7** Describe a recursive function for converting a string of digits into the integer it represents. For example, “13531” represents the integer 13,531.