ENG 346
Data Structures and Algorithms for Artificial Intelligence
Trees

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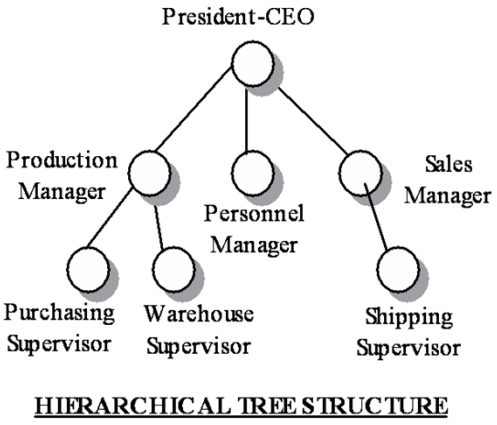
Agenda

- Basic Concepts

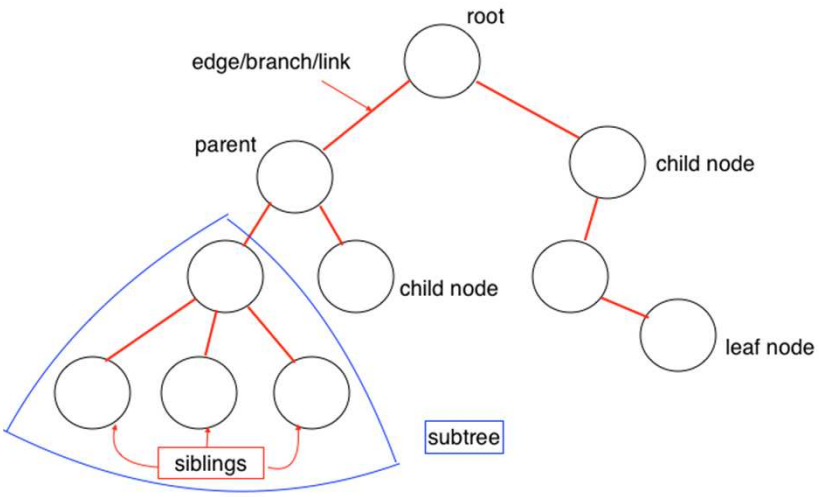
What is a Tree?

- A *tree* is a *hierarchical* data structure consisting of *nodes* connected by *edges*, with a designated *root node* and *branches* that form a directed acyclic graph.



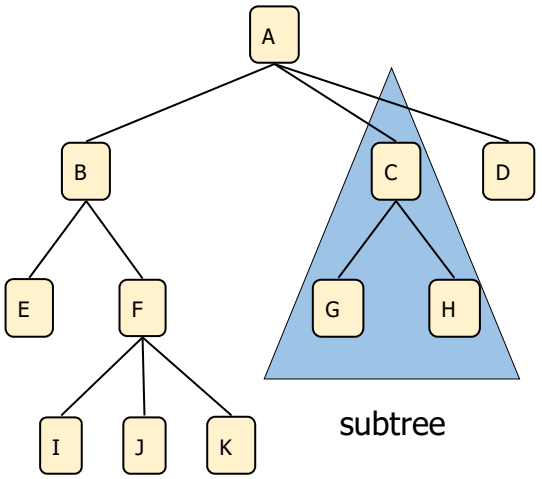
HIERARCHICAL TREE STRUCTURE

Definitions



Tree Terminology

- Root: node without parent (A)
- Internal node: node with at least one child (A, B, C, F)
- External node (a.k.a. leaf): node without children (E, I, J, K, G, H, D)
- Ancestors of a node: parent, grandparent, grand-grandparent, etc.
- Depth of a node: number of ancestors
- Height of a tree: maximum depth of any node (3)
- Descendant of a node: child, grandchild, grand-grandchild, etc.
- Subtree: tree consisting of a node and its descendants



Applications of Trees

- Organization Charts
- Decision Trees in Machine Learning
- File Systems
- Expression Parsing
- DOM Model of Web pages

Tree ADT

- We use positions to abstract nodes
- Generic methods:
  - Integer len()
  - Boolean is_empty()
  - Iterator positions()
  - Iterator iter()
- Accessor methods:
  - position root()
  - position parent(p)
  - Iterator children(p)
  - Integer num_children(p)

- Query methods:
  - Boolean is_leaf(p)
  - Boolean is_root(p)
- Update method:
  - element replace (p, o)
- Additional update methods may be defined by data structures implementing the Tree ADT

Tree ADT

- Assume T is a tree, and p is a position (node of the tree).

Functions/Operations	Description
p.element()	Data stored in p
Accessors	
T.root()	Return the position of the root of the tree. None is T is empty.
T.is_root(p)	True if p is the root, False otherwise.
T.parent(p)	Return the parent of position p. None if p is the root.
T.children(p)	Return list of children of position p.
T.num_children(p)	Number of children of position p.
Queries	
T.is_leaf(p)	True if position p has no children.
T.is_root(p)	True if position p is the root.

Abstract Tree Class

```
class Tree:
  """Abstract base class representing a tree structure."""

  #-------------------------- nested Position class --------------------------
  class Position:
    """An abstraction representing the location of a single element."""

    def element(self):
      """Return the element stored at this Position."""
      raise NotImplementedError('must be implemented by subclass')

    def __eq__(self, other):
      """Return True if other P
      raise NotImplementedErr

    def __ne__(self, other):
      """Return True if other
      return not (self == oth
```

```
  # ---------- abstract methods that concrete subclass must support ----------
  def root(self):
    """Return Position representing the tree's root (or None if empty)."""
    raise NotImplementedError('must be implemented by subclass')

  def parent(self, p):
    """Return Position representing p's parent (or None if p is root)."""
    raise NotImplementedError('must be implemented by subclass')

  def num_children(self, p):
                                                    tion p has."""
                                                    lemented by subclass')

                                                    senting p's children."""
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                                                    the tree."""
                                                    lemented by subclass')
```

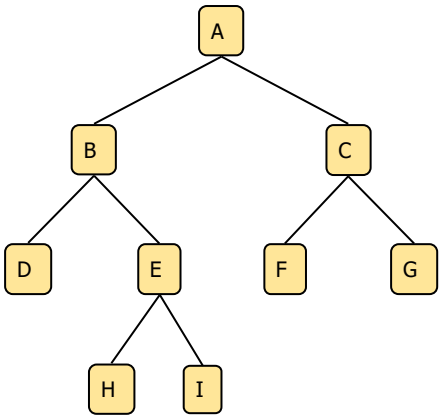
```
  # ---------- concrete methods implemented in this class ----------
  def is_root(self, p):
    """Return True if Position p represents the root of the tree."""
    return self.root( ) == p

  def is_leaf(self, p):
    """Return True if Position p does not have any children."""
    return self.num_children(p) == 0

  def is_empty(self):
    """Return True if the tree is empty."""
    return len(self) == 0
```

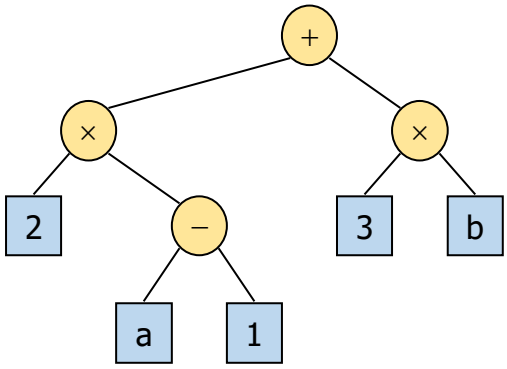

Binary Trees

- A binary tree is a tree with the following properties:
  - Each internal node has at most two children
  - The children of a node are an ordered pair
- The children are called left child and right child
- Applications
  - arithmetic expressions
  - decision processes
  - searching



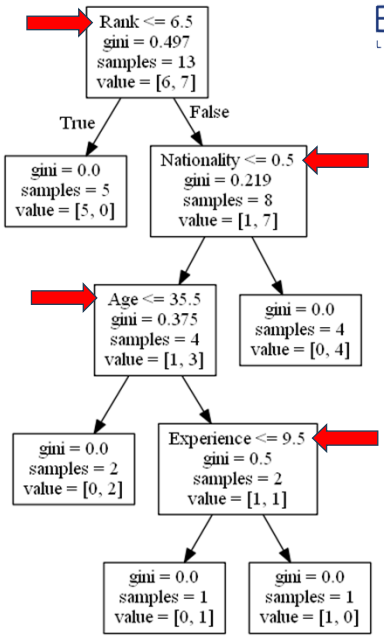
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - Internal nodes: operators
  - Leaf nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



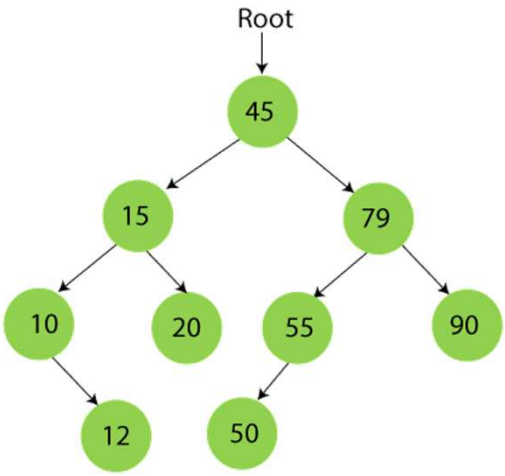
Decision Tree

- Binary tree associated with a decision process
  - Internal nodes: questions with yes/no answer
  - Leaf nodes: decisions



Searching

- Binary search tree



Binary Tree ADT

- The Binary Tree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT

- Additional methods:
  - position left(p)
  - position right(p)
  - position sibling(p)

Abstract BinaryTree Class

```
class BinaryTree(Tree):
  """Abstract base class representing a binary tree structure."""

  # --------------------- additional abstract methods ---------------------
  def left(self, p):
    """Return a Position representing p's left child.

    Return None if p does not have a left child.
    """
    raise NotImplementedError('must be implemented by subclass')

  def right(self, p):
    """Return a Position representing p's right child.

    Return None if p does not have a right child.
    """
    raise NotImplementedError('must be implemented by subclass')
```

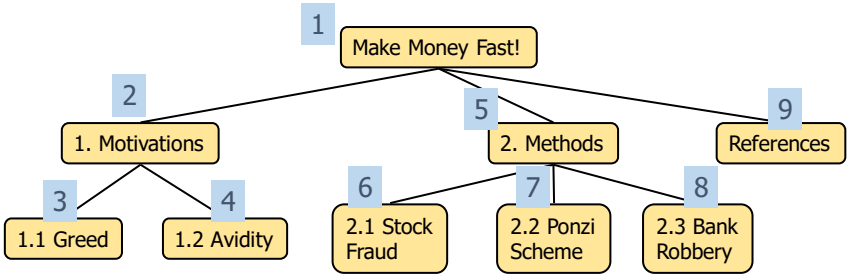
```
  # ---------- concrete methods implemented in this class ----------
  def sibling(self, p):
    """Return a Position representing p's sibling (or None if no sibling)."""
    parent = self.parent(p)
    if parent is None:                  # p must be the root
      return None                       # root has no sibling
    else:
      if p == self.left(parent):
        return self.right(parent)       # possibly None
      else:
        return self.left(parent)        # possibly None

  def children(self, p):
    """Generate an iteration of Positions representing p's children."""
    if self.left(p) is not None:
      yield self.left(p)
    if self.right(p) is not None:
      yield self.right(p)
```


Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document

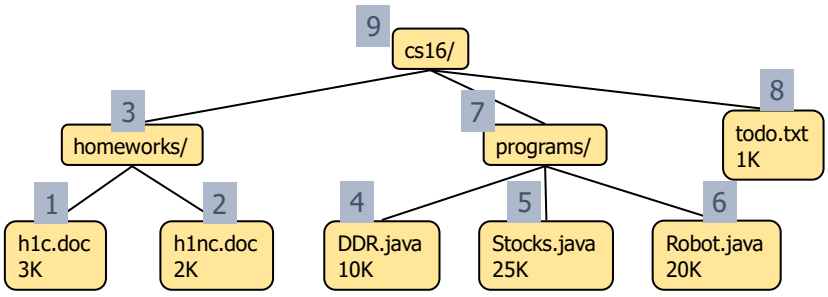
Algorithm *preOrder*(*v*)
  visit(*v*)
  for each child *w* of *v*
    preorder (*w*)



Postorder Traversal

- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories

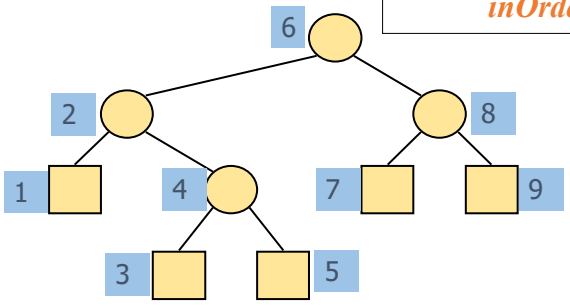
Algorithm *postOrder*(*v*)
  for each child *w* of *v*
    postOrder (*w*)
  visit(*v*)



Inorder Traversal

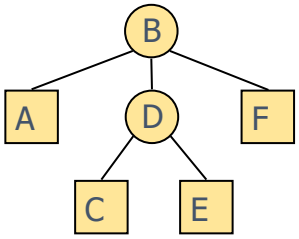
- In an inorder traversal a node is visited after its left subtree and before its right subtree
- Application: draw a binary tree
  - x(v) = inorder rank of v
  - y(v) = depth of v

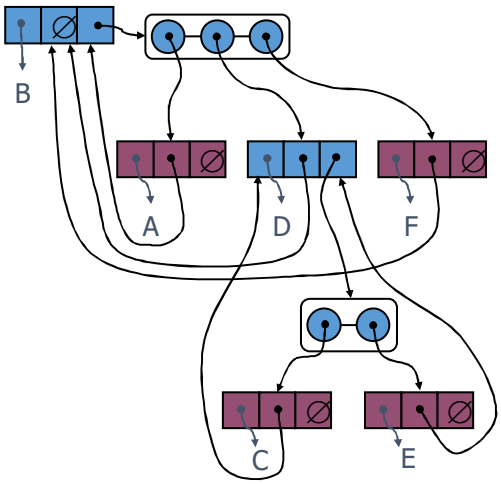
Algorithm *inOrder*(*v*)
  if *v* **has a left child**
    inOrder (*left* (*v*))
  visit(*v*)
  if *v* **has a right child**
    inOrder (*right* (*v*))



Linked Structure for Trees

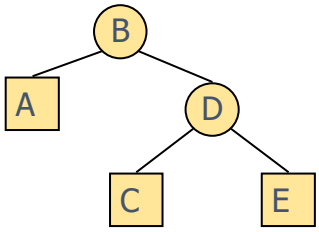
- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes

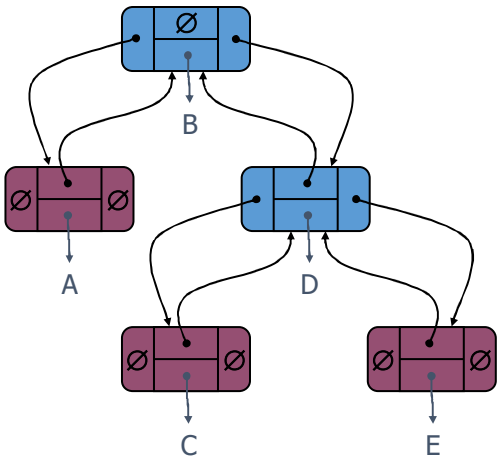




Linked Structure for Binary Trees

- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node





Binary Search Trees

- A Binary Search Tree is a binary tree data structure with the following properties:
  - Binary Tree Structure: It is a binary tree, meaning each node has at most two children: a left child and a right child.
  - Ordering Property: For every node n, all nodes in its left subtree have values less than n, and all nodes in its right subtree have values greater than n.
- This property ensures that the tree maintains a specific ordering, making it suitable for efficient searching.