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Time Delay Estimation Using Windowed Differential Magnitude of Cross Correlation and Its Hilbert Transform

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Abstract

A method is proposed for estimating time delay between MFSK signals received at two spatially separated sensors. Considering the periodical characteristic of the peaks of cross correlation, the proposed method combines window technology and differential magnitude between cross correlation and its Hilbert transform. The time argument at which the differential magnitude achieves the peak in the range of window is the delay estimate. The method is compared with several other methods. Simulation results show that the proposed method can get better performance. Under certain conditions the performance of proposed method can be better than CRLB by using priori information about the transmitter and receivers.

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Keywords: MFSK; time delay; periodical peak; window; Hilbert transform

1. Introduction

TDOA(Time Difference Of Arrival) location is an important method for passive location system, which will play an important role in military and civilian affairs^[1-2]. The key problem of this location method is the estimation of time difference. There are many methods of time difference estimation^[3-4]. These methods mostly are modified on the basis of GCC(Generalized Cross Correlation). When existing periodical peaks of correlation or the signal-to-noise ratio is not high, these methods can't get high accuracy.

MFSK is a widely used modulation method. The cross correlation of *MFSK* signals exists periodical peaks, which will affect the accuracy of time delay estimation. The windowed method can limit the range

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of peak search and eliminate the effect of periodical peaks. Differential magnitude of cross correlation and its Hilbert transform will obtain sharper peak than cross correlation and give higher accuracy of time delay estimation. In this paper, these two methods are combined to estimate the time delay of MFSK signals.

2. MFSK signals model and analysis of correlation

MFSK signals can be mathematically modeled as ^[5]

$$s(t) = \sum_{i=1}^M a_i(t) e^{j2\pi f_i t}, \quad a_i(t) = \sum_n a_{in} g(t - nT_s) \quad (1)$$

In the equation(1), f_i is the frequency of MFSK signal. The variable $a_{in} = 1$ just when the signal frequency of the n th symbol period is f_i , or $a_{in} = 0$. Signal $g(t)$ is rectangular pulse with the width of T_s , which is the symbol period. M is the number of frequencies, such as 2, 4, 8. The magnitude of signal is unitary in the model. A signal emanating from a remote source and monitored in the presence of noise at two spatially separated sensors can be modeled as

$$\begin{cases} x(t) = s(t) + n_1(t) \\ y(t) = s(t - D_0) + n_2(t) \end{cases} \quad (2)$$

Where D_0 is the time delay, and signal $s(t)$ is assumed to be uncorrelated with noise $n_1(t)$ and $n_2(t)$. The signal length is $T = NT_s$, where N is the total number of symbols. So the function of cross correlation is

$$\begin{aligned} R(\tau) &= E[x(t)y^*(t+\tau)] = \frac{1}{2T} \int_{-T}^T x(t)y^*(t+\tau) dt \\ &= \frac{1}{2T} \int_{-T}^T \left[\sum_i \sum_n a_{in} g(t - nT_s) e^{j2\pi f_i t} \right] \left[\sum_i \sum_n a_{in} g(t + \tau - D_0 - nT_s) e^{j2\pi f_i (t + \tau - D_0)} \right] dt \end{aligned} \quad (3)$$

Assume that the calculate τ is limited by that $\tau - D_0$ is in the range of T_s . The total symbols number is N , given that there are M kinds symbol and each kind has the probability $1/M$, so each kind symbol's total number is N/M . The probability of that a symbol is the same kind with its adjacent symbol is $1/M$, then

$$\begin{aligned} R(\tau) &= \frac{1}{2T} \int_{-T}^T \left[\sum_i \sum_n a_{in} g(t - nT_s) e^{j2\pi f_i t} \right] \left[\sum_i \sum_n a_{in} g(t + \tau - D_0 - nT_s) e^{j2\pi f_i (t + \tau - D_0)} \right] dt \\ &= \frac{1}{2T} \sum_{n=0}^{N-1} \int_{nT_s}^{nT_s + T_s} g(t) g(t + \tau - D_0) e^{j2\pi f_i (\tau - D_0)} dt \\ &= \frac{1}{2T} \frac{N}{M} (T_s - |\tau - D_0|) \sum_i e^{j2\pi f_i (\tau - D_0)} + \frac{1}{2T} \frac{N}{M^2} |\tau - D_0| \sum_i e^{j2\pi f_i (\tau - D_0)} \\ &= \frac{1}{2T_s M^2} [MT_s - (M-1)|\tau - D_0|] \sum_i e^{j2\pi f_i (\tau - D_0)} \end{aligned} \quad (4)$$

Assume that the frequencies have the same interval Δf , then

$$\sum_i e^{j2\pi f_i (\tau - D_0)} \sum_i e^{-j2\pi f_i (\tau - D_0)} = M + 2 \sum_i (M-1) \cos[2\pi i \Delta f (\tau - D_0)] \quad (5)$$

The magnitude of correlation is

$$|R(\tau)| = \left| \frac{1}{2T_s M^2} [MT_s - (M-1)|\tau - D_0|] * \left[M + 2 \sum_i (M-1) \cos[2\pi i \Delta f (\tau - D_0)] \right]^{\frac{1}{2}} \right| \quad (6)$$

From equation(6), we can find that the correlation magnitude not only peaks at $\tau - D_0 = 0$, but also has side peaks at $\tau - D_0 = n / \Delta f$. More smaller peaks can be found at $\tau - D_0 = n / i \Delta f$. We just need pay attention to the peaks at $\tau - D_0 = n / \Delta f$. The magnitude ratio of side peaks and the main peak is

$$\frac{|R(D_0 + n / \Delta f)|}{|R(D_0)|} = \frac{MT_s - (M-1) \left| \frac{n}{\Delta f} \right|}{MT_s} = 1 - \left| \frac{(M-1)nR_s}{M\Delta f} \right| \quad (7)$$

Where R_s is the symbol rate. When $R_s = 5K$ and $\Delta f = 40K$, the first side peak to main peak ratio is 93.75% as depicted in Fig. 3.(a) for 2FSK signal. The side peaks may be over the main peak in the environment of noise, leading to that the estimated time delay is far away from the true time delay as depicted in Fig. 1.(a), which will affect the accuracy of estimation. For 2FSK signals, Fig. 1.(b) and Fig. 1.(c) show how the first side peak to main peak ratio change with the data rate and frequency interval.

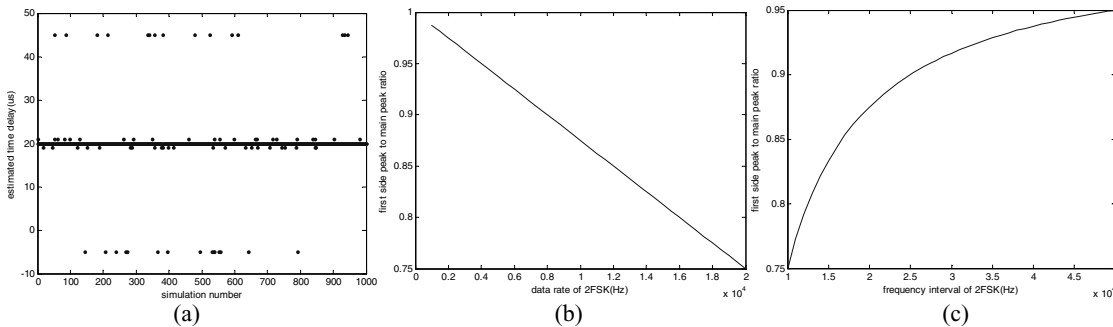


Fig.1 (a) Ambiguous estimation of time delay (SNR=-5dB); (b) Ratio change with data rate; (c) Ratio change with frequency interval

3. Improved algorithms for time delay estimation

3.1. Cross correlation filtering in frequency domain

From frequency domain, the main peak is low frequency components, while the periodical peaks of correlation will make a peak at Δf after FFT. The main peak's width of correlation is $2T_s$, with bandwidth of order $1/2T_s$ ^[10]. So with priori information of the signal such as Δf and T_s , we can filter the result of correlation in the frequency domain to eliminate the effect of periodical peaks. If $h(t)$ is the filter, we can obtain the filtered correlation $\tilde{R}(\tilde{\tau})$ by

$$\tilde{R}(\tilde{\tau}) = R(\tau) * h(\tau) \Leftrightarrow S_{\tilde{R}}(f) = S_R(f)H(f) \quad (8)$$

The simulation result of 2FSK signals shows that this method is effective in the environment of low SNR. But in the environment of higher SNR, the performance isn't as good as GCC. The reason is that the main peak is flattened after filter as depicted in Fig. 3.(c).

3.2. Filtering in time domain by window

We can also eliminate the effect of periodical peaks with a window to filter the correlation result in time domain, and filtering in time domain the main peak won't be flattened. The window can be obtained with priori information of the signal and ambiguous geometry relation between transmitter and receivers. Result after filter in time domain with a window is

$$R_2(\tau) = R(\tau)p(\tau)$$

$$p(\tau) = \begin{cases} 1 & -\frac{1}{2\Delta f} + \hat{D}_0 \leq \tau \leq \frac{1}{2\Delta f} + \hat{D}_0 \\ 0 & \text{else} \end{cases} \quad (9)$$

Where \hat{D}_0 is the priori estimation of time delay by using information in advance, and the window function $p(\tau)$ is the search range of time delay. The window must make sure that there is only one peak in the search range, so there are no effect of periodical peaks.

This method is limited by that we must obtain priori information to confirm the range of the window. To make sure there is only one peak in the search range, the frequency interval Δf and the differential distance between transmitter and receivers D are limited by

$$D_0 = \frac{D}{c} < \frac{1}{\Delta f} \Rightarrow D < \frac{c}{\Delta f} \quad (10)$$

Where c is the velocity of light.

3.3. Windowed differential magnitude of cross correlation and its Hilbert transform

The classical Hilbert transform of $x(t)$ is defined as^[5]

$$\tilde{x}(t) = H\{x(t)\} = x(t) \times h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau \quad (11)$$

Where H is the operator of Hilbert transform, and $h(t)$ is the kernel of Hilbert transform. The classical Hilbert transform possess the following properties: 1), The frequency amplitude of signal $x(t)$ is the same after it's Hilbert transform, $\tilde{R}_{xx}(\tau) = R_{xx}(\tau)$; 2), The Hilbert transform of an autocorrelation function is the cross correlation of the signal and its Hilbert transform; 3), If $R_{xx}(\tau)$ is even, $\tilde{R}_{xx}(\tau)$ is odd.

The peak of cross correlation is not sharp enough for high accuracy estimation. As to get better performance, the method based on Hilbert transform was described by Richard C.Cabot^[7]. The Hilbert transform converts the maximum finding task of a time delay estimator to one of finding a zero crossing. In the environment of noise, the finding of zero crossing is badly effected and the accuracy of estimation is decreased. In this matter, the method of differential magnitude of cross correlation and its Hilbert transform was proposed in [8] as described as follows:

- 1) Cross correlate received signals $x(t)$ and $y(t)$ to get $R_{xy}(\tau)$;
- 2) Cross correlate $y(t)$ and the Hilbert transform of $x(t)$ to get $R_{xy}(\tau) = \tilde{R}_{xy}(\tau)$;
- 3) Difference the results of the two steps above, $R'_{xy}(\tau) = R_{xy}(\tau) - |\tilde{R}_{xy}(\tau)|$;
- 4) Find the peak of result of step 3) to get the time delay estimation τ .

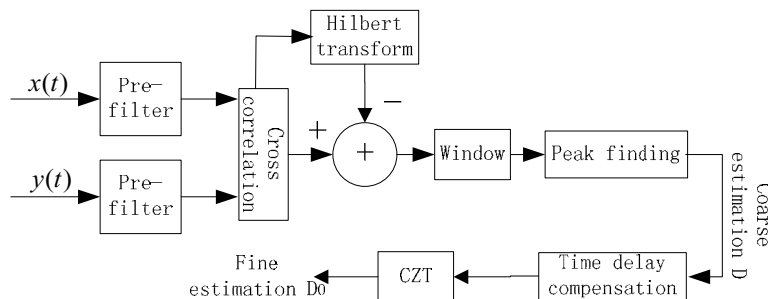


Fig. 2 Using window and differential magnitude of generalized correlation and Hilbert transform

A hybrid method of combining window and differential magnitude of cross correlation and its Hilbert transform has been depicted in Fig. 2. Given the periodicity of correlation of *MFSK* signals, the proposed method can both eliminate the effect of periodical peaks and sharpen the main peak of correlation.

The calculate accuracy is restricted by the sample rate in the methods described above. As to improve the accuracy, the steps of time delay compensation and CZT(Chirp Z Transform) ^[9] are introduced into the proposed method. CZT is a method of frequency analysis in non-unit circle in the Z plane, it can choose the interval neatly according to the claimed resolving ability.

4. Simulation results

Computer simulations have been done to demonstrate the performance of the proposed method for *2FSK* and *4FSK* signals.

4.1. Simulation for *2FSK*

The simulation conditions are as follows: the frequencies of *2FSK* is 371.98MHz and 372.0MHz, the baud rate is 5Kbps and $\Delta f = 40K$, the signal length is 11.2ms, the sample rate is 1MHz, the time delay is 20us and the simulation times is 1000.

Fig.3.(a) shows a part of the correlation result of *2FSK* signals. Fig.3.(b) shows the same part of the result of differential magnitude of correlation and its Hilbert transform. We can see the peak is sharpened after difference the magnitude. Fig.3.(c) shows the result of correlation after filtered in frequency domain. We can see the periodical peaks is cleared but the main peak is flattened. Fig.3.(d) shows the result of comparison between the proposed method and other methods and the CRLB. We can see the performance of the method proposed is the best and may exceed the CRLB under certain conditions with information in advance. The CRLB is calculated by^[10]:

$$CRLB_{\sigma_{\text{dfo}}} \approx \frac{0.55}{B_s \sqrt{B T \text{SNR}}} \quad (12)$$

Where B_s is the signal bandwidth, B is the noise bandwidth at receiver input, T is the signal length and SNR is the effective input signal noise ratio. For *2FSK* signal, we can assume $B_s = B = 2f_s + \Delta f$ while $B_s = B = 2f_s + 3\Delta f$ for *4FSK* signal, where f_s is the data rate.

By using window, more information in advance is used and the search range is restricted, so the performance of the proposed method can be better than CRLB. Fig.3.(d) also shows that when SNR is above 5dB the performances have little difference. The reason is that the correlation peak is little effected by the noise when SNR is high, so the side peaks won't exceed the main peak.

4.2. Simulation for *4FSK*

The simulation conditions are as follows: the interval of frequencies is 10KHz, the baud rate is 2Kbps, the signal length is 10ms, the sample rate is 1MHz, the time delay is 20us and the simulation times is 1000.

Fig.3.(e) shows a part of the correlation result of *4FSK* signals. Fig.3.(f) shows the result of comparison between the proposed method and other methods and the CRLB. We can see that the method proposed also have better performance as a whole than others.

5. Conclusions

To the question of time delay estimation for *MFSK* signals, a hybrid method of combining window and differential magnitude of cross correlation and its Hilbert transform has been outlined based on theoretic analysis of the cause of periodical peaks of correlation. Compared with other methods, the new method has better performance as the simulation results shows. Given that periodical function has periodical

correlation function, the proposed method can be cited by the time delay estimation of these kinds of signals, such as radar signals. In the environment of multipath, although the cross correlation of received signals has multi peak without period, a window can be checked still with enough information in advance to estimate the time delay.

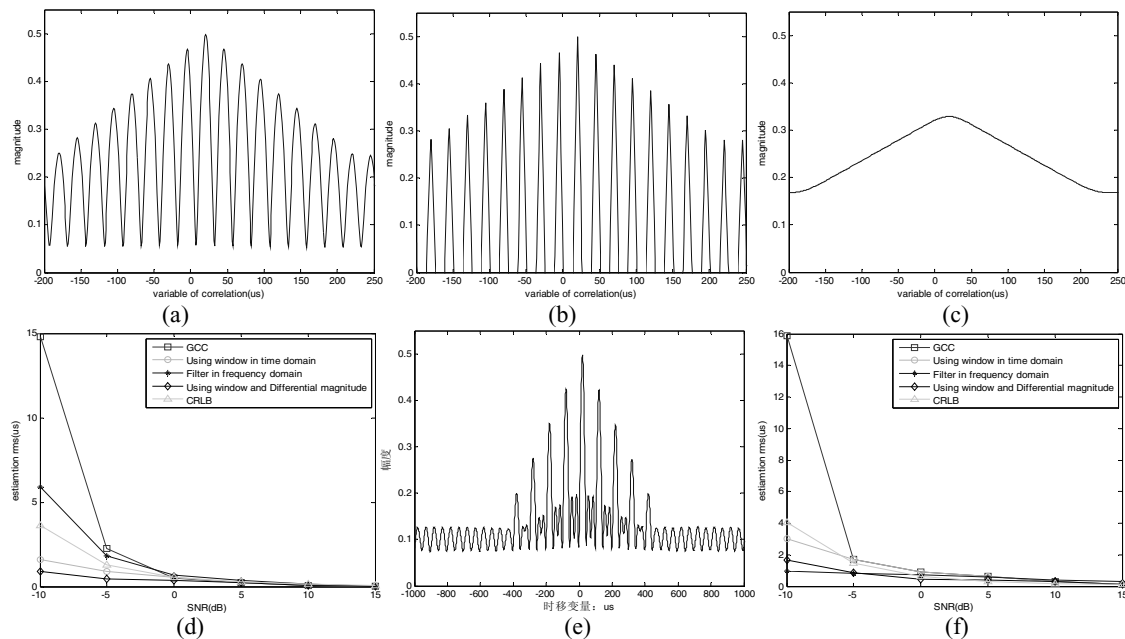


Fig. 3 (a) Cross correlation of 2FSK; (b) Result of differential magnitude; (c) Correlation Result after filtered; (d) Performance comparison of 2FSK; (e) Cross correlation of 4FSK; (f) Performance comparison of 4FSK

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