CS 230 Project Proposal: Developing a hybrid filter for bandlimited signals using neural networks

Project Category: Supervised Learning, modelling, signal processing

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1 Introduction

In the field of signal processing, filtering is among the most ubiquitous of topics. Typically, when one wants to filter a signal, one must know what type of filtering should be done (*i.e.*, low-pass filtering, high-pass filtering, *etc.*,) which also implicitly requires the prior knowledge of the bandwidth of the signal. Unfortunately, unknown signals' bandwidths are typically only found by computing their Fourier transform and observing a steep dropoff in magnitude for a given frequency. Currently, the method of computing a Fourier transform for a discrete signal can be computed in $O(n \log n)$ time [Bra78, Osg17].

The goal of this project is to use a neural network to create a multi-purpose filter for use in signal processing topics. This neural network should be able to determine the bandwidth of any signal that is input, without having to compute the Fourier transform.

2 Background

The context and background for this project is abundant, as signal processing has enjoyed a vast treatment in the past century. The Fourier series for a signal [Bra78, Osg17] is

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi int/T}. \tag{1}$$

Here, we refer to T as the *period* of the signal, c_n as the n-th *Fourier coefficient*, and $e^{2\pi int/T}$ as the n-th *harmonic*. In addition, if the coefficients $\{c_n\}_{n=-\infty}^{\infty} = 0$ for some $|n| > B/2$, then we say that the signal is *bandlimited* with *bandwidth* B.

Figure 1 below illustrates a simple example of a signal bandlimited to 2200 Hz, and its Fourier transform $F(s)$.

In the past, the typical method for determining the bandwidth of a signal has been by inspection of that signal's Fourier transform. The Fourier transform $F : \mathbf{R} \to \mathbf{R}$ of a signal $f(t)$ is defined as

$$F(s) = \int_{-\infty}^{\infty} e^{-2\pi ist} f(t) dt. \tag{2}$$

3 Challenges

In an ideal world, bandlimited signals would be clean; there would be no signal past a particular bandwidth. Unfortunately, we might sometimes encounter noise of various forms in our signal. One of the larger challenges in this project is to build the neural network to recognize when a small magnitude is either noise or a portion of the whole signal.

4 Data

An advantage of this project is that the data is readily abundant and training data can be synthetically generated efficiently. Since all signals are simply sums of sinusoids at different frequencies and amplitudes, we will generate our data by adding randomly generated

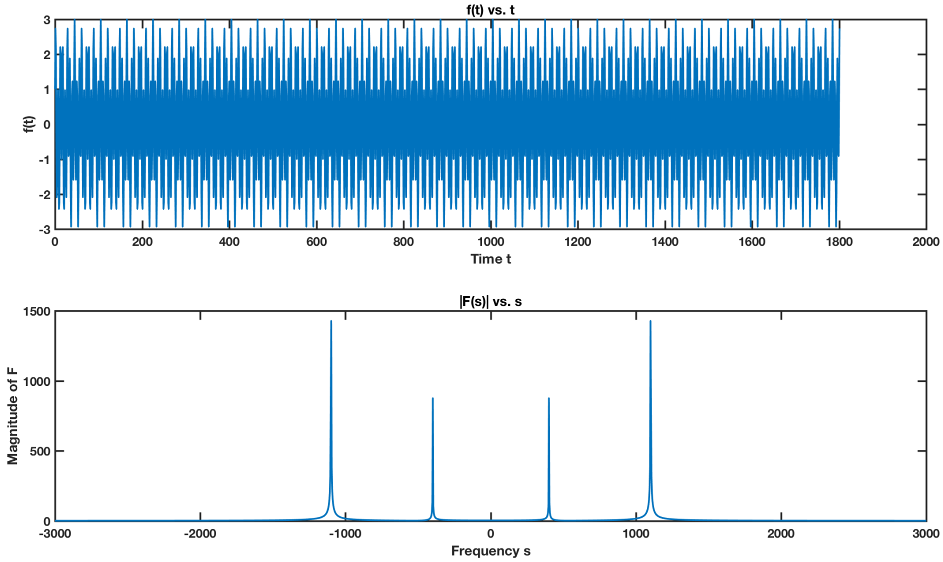

Figure 1: Bandlimited signal $f(t) = \cos(800\pi t) + 2\sin(2200\pi t)$ and the magnitude of its Fourier transform.

sinusoids together, some with various types of noise. For each example, we shall randomly choose the number of sinusoids to be added, their frequencies, and their amplitudes.

5 Method and implementation

The problem of determining the bandwidth of a signal is a supervised learning problem, and so neural networks for supervised learning problems will be used. We do not yet know the appropriate deep learning architecture for this project, but as the act of convolution is deeply related to Fourier transforms [Bra78, Osg17], we shall attempt to use architectures such as convolutional neural networks [LBBH98, RSA15].

6 Evaluation metrics

The evaluation of this project shall be based on how close the neural network estimates for a signal's bandwidth are to the actual signal's bandwidth. The metric to be evaluated is

$$E_1 = (1/m)\sum_{i=1}^{m} \mathbf{1}\{|B^{(i)} - \hat{B}^{(i)}| > \epsilon\},$$

where m is the number of samples in the set, $\mathbf{1} : \mathbf{R} \times \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ is the 0-1 indicator function, B is the actual signal bandwidth, $\hat{B}$ is the signal bandwidth estimate, and $\epsilon > 0$ is some small tolerance to be chosen.

References

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