Study on Swing-up Control of Rotary Inverted Pendulum Based on Energy Feedback

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Abstract—To study the control of the rotary inverted pendulum system, the author in this paper proposed a swing-up method based on energy control. Through analysis the energy of the pendulum rod during the whole swing-up, and introduce the angle of the pendulum as a feedback coefficient to reduce the swing-up time, the author came up with the swing-up control method. Acquire better input with the help of giving different parameters repeatedly using Matlab/Simulink simulation and finish the experimentation finally. The simulation results and the experimental data indicate that the method can drive the pendulum from its hanging-down position to the upright position well. Finally it conclude that this energy method is appropriate for the rotary inverted pendulum system.

Keywords: Rotary inverted pendulum; swing-up; energy control

I. INTRODUCTION

The classical rotary inverted pendulum system is well known in control area to demonstrate the control algorithm, which is a highly nonlinear, multi-variables, unstable, strong-coupled system. This inverted pendulum system is an under-actuated system having fewer control inputs than the degree of freedom. This makes the control more different and complicated[1-4]. Many abstract concepts such as stability, rapidity, robustness in the control field can be shown by the pendulum rod intuitively and that's why it is the ideal model for robot control system, aircraft landing system and so on[5, 6].

As a top subject in control area there are two problems about the study of inverted pendulum system: one is controlling the pendulum at up-right position reposefully and the other is to transform the pendulum rod from it's initial vertical downward position near the manageable position[7, 8]. There are lots of literatures which focus on stability control and it is relatively mature. The control method is various, such as LQR algorithm, sliding mode control approach, fuzzy control, adaptive control and so on[9]. Until now, it can achieve the simulation of four level inverted pendulum's control. Swinging inverted pendulum from initial position to the upright position is a rather new topic but it is also import to achieve the pendulum rod swinging up quickly[10, 11].

Passage[12] gives an energy based controller for rotary inverted pendulum swing-up and compare the simulation results of different stability controller. Bang-Bang-Adjust Control Algorithm is put forward in Passage[13], analyze of the open loop control parameters and apply the determination method effectively to the swing-up process of Single Circular Rail Inverted Pendulum. The optimization program calculation method of the optimal control law is established. Paper[14] divide time into intervals, limit condition of controlled quantities is formed into objective function, and an optimization question about determining controlled quantity is obtained. Based on human simulated intelligent control theory, hybrid control method is proposed in [9] for the control problem of swing-up and handstand for a rotary double inverted pendulum. Paper[15] designs a PD controller with the feedback of the angular and angular velocity of the pendulum to control the motor, and the simulation result prove this method is effective for swinging the pendulum.

In this paper, we study the swing-up and stabilization problem of the single rotary inverted pendulum, a new approach to swing-up control is proposed. The algorithm here is different from other energy controller, it analyses the energy of the pendulum in motion directly and without introducing Lyapunov function. Based on linearized the mathematical model of the pendulum system, the energy and using angle feedback compensation control methods is established, we achieve the whole swing-up control of rotary inverted pendulum. This will make the pendulum rod from it's initial downward sate swing higher and higher. When it

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is closed to the inverted position, a pole assignment controller will work and stabilize the system.

II. Modelling of the Rotary Pendulum

A. Physical model

In this part description of the model and dynamics are given. Based on the Googol Tech inverted pendulum experimental system, ignoring all kinds of friction and resistance, the single rotary inverted pendulum system can be thought consists of two parts: the driven arm rotates in the horizontal plane which is driven by the motor and an inverted pendulum attached to that arm which is free to rotate in the vertical plane. The ideal schematic diagram of our inverted pendulum system is shown in Figure 1. There we define that clockwise direction is positive direction and the aim is to control the pendulum rod from the random nature equilibrium position that is vertical and pointing down to the desired position that is vertical and pointing up. The following table 1 gives the list of the symbol and value used in this modelling system. Based on these descriptions, the Lagrangian Equation is used to establish mathematics model of this system.

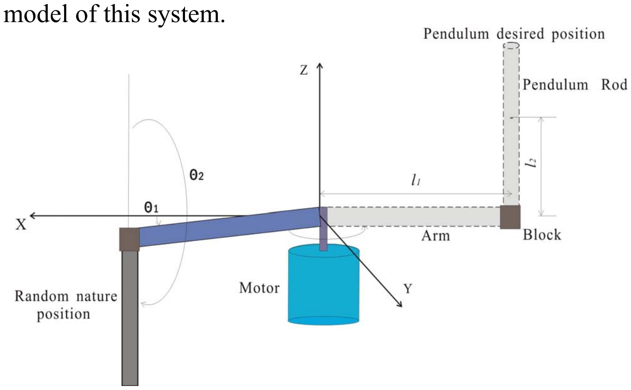

Figure.1 The ideal schematic diagram of our inverted pendulum system.

TABLE Ⅰ List of parameters of pendulum system

Symbol	*Physical meaning*	*Value*	*Unit*
θ_1	Rotation angle between horizontal arm and *X* axis		rad
θ_2	Angle between pendulum rod and vertical direction		rad
l_1	Length of the horizontal arm	0.334	m
l_2	Length to pendulum center of mass	0.273	m
m_1	Mass of horizontal arm	0.437	kg
m_2	Mass of pendulum rod	0.134	kg
m_3	Mass of the block	0.183	kg

B. Mathematical model

1) The system energy

The energy of the pendulum system consists of potential and kinetic energy which contains the energy of the driven arm, pendulum rod and the block.

The kinetic energy of the driven arm is:

$$T_{m1} = \frac{1}{2} J_1 \omega_1^2 = \frac{1}{6} m_1 l_1^2 \dot{\theta}_1^2 .$$

The kinetic energy of the pendulum rod is:

$$T_{m2} = \frac{1}{6} m_2 l_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 - 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos\theta_2) + \frac{2}{3} m_2 l_2^2 \dot{\theta}_1^2 \sin^2\theta_2 .$$

The kinetic energy of the block is:

$$T_{m3} = \frac{1}{2} m_3 l_1^2 \dot{\theta}_1^2$$

Then the kinetic energy of the system can be described as:

$$\begin{aligned} T &= T_{m1} + T_{m2} + T_{m3} \\ &= \frac{1}{6} m_1 l_1^2 \dot{\theta}_1^2 + \frac{2}{3} m_2 l_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 - 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos\theta_2) + . \\ &\quad + \frac{2}{3} l_2^2 \dot{\theta}_1^2 \sin^2\theta_2 + \frac{1}{2} m_3 l_1^2 \dot{\theta}_1^2 \end{aligned} \tag{1}$$

Let the horizontal arm plane as the zero potential energy surface, the potential energy of this system is:

$$V = m_2 g l_2 \cos\theta_2$$

2) The motion equation of the system

The Lagrangian function can be formulated as:

$$\begin{aligned} L &= T - V \\ &= \frac{1}{6} m_1 l_1^2 \dot{\theta}_1^2 + \frac{2}{3} m_2 l_2^2 \dot{\theta}_2^2 - m_2 g l_2 \cos\theta_2 + \\ &\quad + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 - 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos\theta_2) + \frac{2}{3} m_2 l_2^2 \dot{\theta}_1^2 \sin^2\theta_2 + \frac{1}{2} m_3 l_1^2 \dot{\theta}_1^2 . \end{aligned} \tag{2}$$

Taking the two generalized co-ordinates θ_1 and θ_2 , the system nonlinear motion equation can be derived through Lagrangian Equation:

$$\begin{cases} \left(\frac{1}{3} m_1 l_1^2 + m_2 l_1^2 + \frac{4}{3} m_2 l_2^2 \sin^2\theta_2 + m_3 l_1^2\right)\ddot{\theta}_1 - m_2 l_1 l_2 \cos\theta_2 \ddot{\theta}_2 + \\ \quad + \frac{4}{3} m_2 l_2^2 \dot{\theta}_1 \dot{\theta}_2 \sin\theta_2 \cos\theta_2 + m_2 l_1 l_2 \dot{\theta}_2^2 \sin\theta_2 = \tau \\ - m_2 l_1 l_2 \cos\theta_2 \ddot{\theta}_1 + \frac{4}{3} m_2 l_2^2 \ddot{\theta}_2 - m_2 g l_2 \sin\theta_2 \text{ -} \\ \quad - \frac{2}{3} m_2 l_2^2 \dot{\theta}_1^2 \sin(2\theta_2) = 0 \end{cases} \tag{3}$$

where τ is the output torque of servo motor.

3) Mathematical model

The linear equation of state space can be write as:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{cases} \tag{4}$$

Then based on $\ddot{\theta}_1$ as the input variable and linearizing (3) under the assumption of all the parameter's values near the vertical position are zero and Taylor series expansion, the system linear state equation can be described as:

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 26.923 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0.9176 \end{bmatrix} u$$

$$y = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

III. ENERGY FEEDBACK CONTROL FOR SWING-UP

The control objective is to get the pendulum rod from it's vertical downward position to near the equilibrium position and keep it at upright position. There proposed swing-up controller and stabilization controller for the control of this system. This control process contains two steps: first the swing-up controller takes the pendulum rod to the range of the stabilization control and then turn to the stabilization controller by an angle switch. Figure 2 shows the structure of this process.

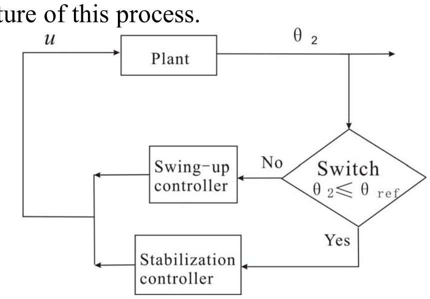

Figure.2 The structure of the control process.

A. Swing-up controller

Paper [11] proposed energy control for a pendulum after that this method get the promotion to swing-up the car inverted pendulum. Designing the swing-up controller for the pendulum system by energy method is a convenient and effective way.

Let the moment of inertia with respect to the pivot point be J ($J = 4\!/\!_3\, m_2 l_2^2$) the motion equation of the pendulum rod can be described as:

$$J\ddot{\theta}_2 - m_2 g l_2 \sin\theta + m_2 u l_1 l_2 \cos\theta_2 = 0$$

By defining the reference potential energy of the pendulum rod E_{ref} as zero at the vertical upright position and the energy of the pendulum rod can be expressed as:

$$E = \frac{1}{2} J\dot{\theta}_2^2 + m_2 g l_2 (\cos\theta_2 - 1). \tag{5}$$

$$\begin{aligned} \frac{dE}{dt} &= J\dot{\theta}_2\ddot{\theta}_2 - m_2 g l_2 \dot{\theta}_2 \sin\theta_2 \\ &= -m_2 u l_1 l_2 \dot{\theta}_2 \cos\theta_2 \end{aligned} \tag{6}$$

The energy of the pendulum rod at initial vertical downward position is: $E_0 = -2m_2 g l_2 < 0$.

There are lots of articles design the energy swing-up controller based on designing Lyapunov function:

$$V = \frac{1}{2}(E - E_{ref})^2 . \tag{7}$$

then

$$\frac{dV}{dt} = (E - E_{ref})\frac{dE}{dt} . \tag{8}$$

Based on (8), designing the controller as:

$$u = ng(E - E_{ref})\dot{\theta}_2 \cos\theta_2$$

Where n is the control parameter.

Then

$$\frac{dV}{dt} = -ngm_2 l_1 l_2 (E - E_{ref})^2 \dot{\theta}_2^2 \cos\theta_2^2 \le 0 . \tag{9}$$

In this paper based on (6) the control objective is to let the initial energy E_0<0 close to the reference energy E_{ref}=0, so only need to make sure that $\frac{dE}{dt} > 0$, then defining the control low:

$$u = -ksign(\dot{\theta}_2 \cos\theta_2)$$

Where:

$$sign(x) = \begin{cases} 1 & x \ge 0 \\ -1 & x < 0 \end{cases}$$

k is the control parameter.

Actually during the swing-up control there are more energy to overcome all kinds of the frictions and resistances. There introduce feedback coefficient of θ_2 to ensure the driven arm has enough energy to swing the pendulum rod and reducing the time that require swinging the pendulum. The coefficient function designed as:

$$f(\theta_2) = 1 + \left|\frac{\theta_2}{\pi}\right|^2$$

When the pendulum at the vertical downward position there multiply a big coefficient to quickly swing up the pendulum with a big torque, and when the pendulum near up-right position gives a small coefficient it not only provides the swing-up energy but also is advantages to switch to stabilization control. Finally we get the control low:

$$u = -kf(\theta_2)sign(\dot{\theta}_2 \cos\theta_2). \tag{9}$$

B. Stabilization controller

This paper design the pole assignment method as the stabilization controller for the above kinds of swing-up. After lots of experiments, choose the desired pole for the system linear state equation as: p_1=-3+2j; p_2=-3-2j; p_3=-5; p_4=-7, and the optimum control vector is: $K = [-16.9000\ \ 178.5343\ \ -13.5943\ \ 34.4315]^T$.

IV. Simulation Results

Based on the dynamics motion equation of this rotary inverted pendulum system, the simulation results of above swing-up control algorithm will be given by MATLAB/Simulink . Figure 3 shows the simulation model

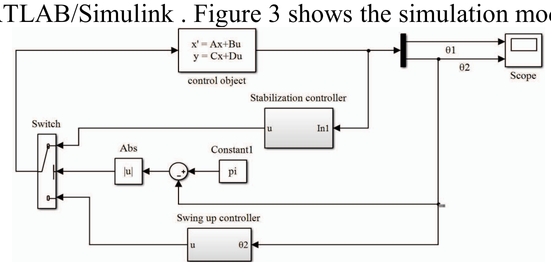

Figure.3 Simulation model.

of the swing-up control method. At first the swing-up controller will work to bring the pendulum rod near the upright position, when the pendulum angle θ_2 touches the switching angle it will trigger the stabilization controller.

As the proposed control method (10), the parameter k is unknown. In order to get this value and achieve better control effect, we need according lots of simulation results to find the best one. Select the parameter k=40 and the initial conditions are $\theta_1(0) = 0, \dot{\theta}_1(0) = 0, \theta_2(0) = \pi, \dot{\theta}_2(0) = 0$. To compare the method we proposed with other energy method Figure 4 shows the simulation results of this method and Lyapunov function method when the switching angle as 0.35 rad. Figure 5 shows the energy of the two methods which need to achieve the same control result.

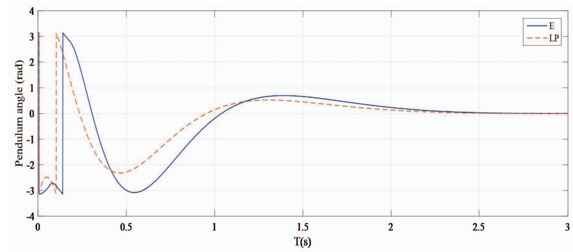

Figure.4 Simulation results of θ_2.

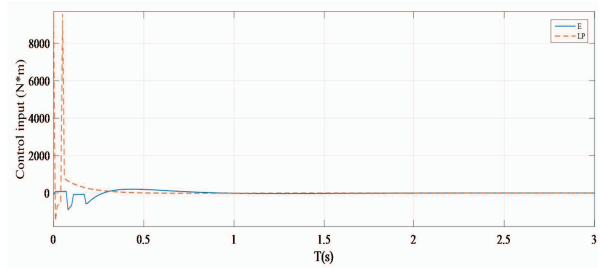

Figure.5 Simulation results of the control u.

The simulation results of θ_2 show that switching angle as 0.35 rad our swing-up controller will bring the pendulum rod from downward position near the upright position and switch to stabilization controller quickly and smoothly. Through the figure 4 and 5 we can see that to achieve the pendulum from nature position to desired position approximately at the same time the method proposed in this paper needs lower energy.

V. Experiment

As many other articles, we will work the control method in the real experiment pendulum system in order to text and verify this algorithm is effective and feasible.

In this paper we will finish our experiment by Googol Tech Furuta pendulum system. The pendulum real-time control system hardware blocks are shown in figure 6. Computer send commands that from our control program to servo driver by the movement control cards and finally guide the servo motor's movement. The servo motor drives the horizontal arm directly and control the pendulum rod swing in vertical direction. The horizontal arm's angle information is sent back to servo driver and movement

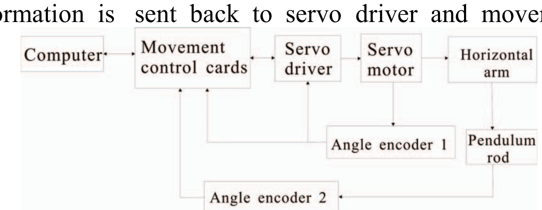

Figure.6 The control structure of real rotary inverted pendulum system.

control cards through encoder 1 that is connected to the servo motor, and the pendulum rod's angle signal is sent back to movement control cards through encoder 2. Finally the motion control cards will send these angle messages to computer's control program to achieve closed loop real-time control.

We set the initial point of pendulum rod angle 3.14 radian and the switching angle 0.35 rad as the simulation, and figure 7 shows the real time experiment results. To be safe we limit the driving torque 25 during the real time control, from figure 7 we can see that for the Lyapunov function method 25 N*m can't make the pendulum swing up as the simulation so it needs more one and half oscillation period but our method is similar with the simulation result. Figure 8 give the energy curve during the experiment. The driving torque of Lyapunov function method is the maximum during it's shaking, well our method only need 8~15 N*m to finish swing up. Figure 7 and 8 proved that our control method is effectively and more energy efficient.

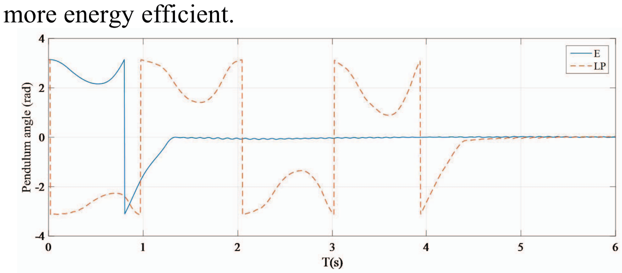

Figure.7 Experiment result of pendulum rod angle as 0.35 rad.

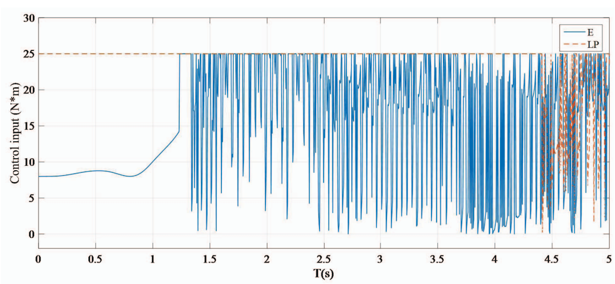

Figure.8 Experiment result of the control energy.

VI. CONCLUSION

In this paper, dynamic motion equation of rotary inverted pendulum system is established by Lagrange equation, a swing-up controller based on energy control and angle feedback is obtained. Add in balance controller and angle switching the pendulum rod can swing up from it's initial position to up-right position and stable there. Finally give the simulation and experiment results by Matlab/Simulink. It is indicates that the method we proposed is appropriate for the swing-up control of the rotary inverted pendulum and more energy efficient.

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