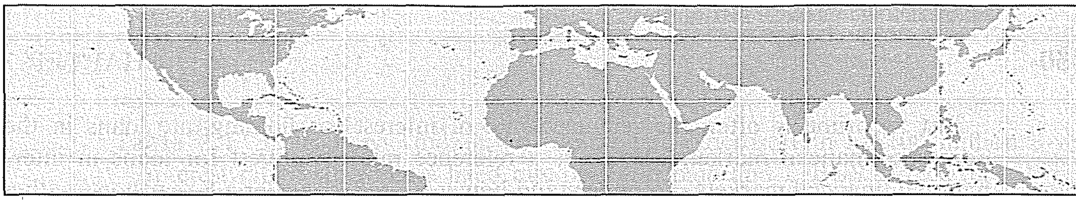


CHAPTER 7 Swaps

The first swap contracts were negotiated in the early 1980s. Since then the market has seen phenomenal growth. Swaps now occupy a position of central importance in the over-the-counter derivatives market.

A swap is an agreement between two companies to exchange cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated. Usually the calculation of the cash flows involves the future value of an interest rate, an exchange rate, or other market variable.

A forward contract can be viewed as a simple example of a swap. Suppose it is March 1, 2006, and a company enters into a forward contract to buy 100 ounces of gold for \$400 per ounce in 1 year. The company can sell the gold in 1 year as soon as it is received. The forward contract is therefore equivalent to a swap where the company agrees that on March 1, 2007, it will pay \$40,000 and receive $100S$, where S is the market price of 1 ounce of gold on that date.

Whereas a forward contract is equivalent to the exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges taking place on several future dates. In this chapter we examine how swaps are used and how they are valued. Our discussion centers on two popular swaps: plain vanilla interest rate swaps and fixed-for-fixed currency swaps. Other types of swaps are discussed in Chapter 30.

7.1 MECHANICS OF INTEREST RATE SWAPS

The most common type of swap is a "plain vanilla" interest rate swap. With this swap a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time.

LIBOR

The floating rate in most interest rate swap agreements is the London Interbank Offer Rate (LIBOR). We introduced this in Chapter 4. It is the rate of interest at which a bank is prepared to deposit money with other banks in the Eurocurrency market. Typically, 1-month, 3-month, 6-month, and 12-month LIBOR are quoted in all major currencies.

Just as prime is often the reference rate of interest for floating-rate loans in the domestic financial market, LIBOR is a reference rate of interest for loans in international financial markets. To understand how it is used, consider a 5-year bond with a rate of interest specified as 6-month LIBOR plus 0.5% per annum. The life of the bond is divided into 10 periods, each 6 months in length. For each period, the rate of interest is set at 0.5% per annum above the 6-month LIBOR rate at the beginning of the period. Interest is paid at the end of the period.

Illustration

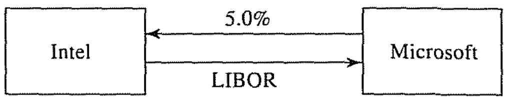
Consider a hypothetical 3-year swap initiated on March 5, 2004, between Microsoft and Intel. We suppose Microsoft agrees to pay to Intel an interest rate of 5% per annum on a notional principal of $100 million, and in return Intel agrees to pay Microsoft the 6-month LIBOR rate on the same notional principal. Microsoft is the *fixed-rate payer*; Intel is the *floating-rate payer*. We assume the agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is quoted with semiannual compounding. This swap is represented diagrammatically in Figure 7.1.

The first exchange of payments would take place on September 5, 2004, 6 months after the initiation of the agreement. Microsoft would pay Intel $2.5 million. This is the interest on the $100 million principal for 6 months at 5%. Intel would pay Microsoft interest on the $100 million principal at the 6-month LIBOR rate prevailing 6 months prior to September 5, 2004—that is, on March 5, 2004. Suppose that the 6-month LIBOR rate on March 5, 2004, is 4.2%. Intel pays Microsoft $0.5 \times 0.042 \times \$100 =$ $2.1 million.[1] Note that there is no uncertainty about this first exchange of payments because it is determined by the LIBOR rate at the time the contract is entered into.

The second exchange of payments would take place on March 5, 2005, a year after the initiation of the agreement. Microsoft would pay $2.5 million to Intel. Intel would pay interest on the $100 million principal to Microsoft at the 6-month LIBOR rate prevailing 6 months prior to March 5, 2005—that is, on September 5, 2004. Suppose that the 6-month LIBOR rate on September 5, 2004, is 4.8%. Intel pays $0.5 \times 0.048 \times \$100 =$ $2.4 million to Microsoft.

In total, there are six exchanges of payment on the swap. The fixed payments are always $2.5 million. The floating-rate payments on a payment date are calculated using the 6-month LIBOR rate prevailing 6 months before the payment date. An interest rate swap is generally structured so that one side remits the difference between the two payments to the other side. In our example, Microsoft would pay Intel $0.4 million (= $2.5 million − $2.1 million) on September 5, 2004, and $0.1 million (= $2.5 million − $2.4 million) on March 5, 2005.

Figure 7.1 Interest rate swap between Microsoft and Intel.



[1] The calculations here are simplified in that they ignore day count conventions. This point is discussed in more detail later in the chapter.

Table 7.1 Cash flows (millions of dollars) to Microsoft in a $100 million 3-year interest rate swap when a fixed rate of 5% is paid and LIBOR is received.

Date	*Six-month LIBOR rate (%)*	*Floating cash flow received*	*Fixed cash flow paid*	*Net cash flow*
Mar. 5, 2004	4.20			
Sept. 5, 2004	4.80	+2.10	−2.50	−0.40
Mar. 5, 2005	5.30	+2.40	−2.50	−0.10
Sept. 5, 2005	5.50	+2.65	−2.50	+0.15
Mar. 5, 2006	5.60	+2.75	−2.50	+0.25
Sept. 5, 2006	5.90	+2.80	−2.50	+0.30
Mar. 5, 2007		+2.95	−2.50	+0.45

Table 7.1 provides a complete example of the payments made under the swap for one particular set of 6-month LIBOR rates. The table shows the swap cash flows from the perspective of Microsoft. Note that the $100 million principal is used only for the calculation of interest payments. The principal itself is not exchanged. This is why it is termed the *notional principal.*

If the principal were exchanged at the end of the life of the swap, the nature of the deal would not be changed in any way. The principal is the same for both the fixed and floating payments. Exchanging $100 million for $100 million at the end of the life of the swap is a transaction that would have no financial value to either Microsoft or Intel. Table 7.2 shows the cash flows in Table 7.1 with a final exchange of principal added in. This provides an interesting way of viewing the swap. The cash flows in the third column of this table are the cash flows from a long position in a floating-rate bond. The cash flows in the fourth column of the table are the cash flows from a short position in a fixed-rate bond. The table shows that the swap can be regarded as the exchange of a fixed-rate bond for a floating-rate bond. Microsoft, whose position is described by Table 7.2, is long a floating-rate bond and short a fixed-rate bond. Intel is long a fixed-rate bond and short a floating-rate bond.

Table 7.2 Cash flows (millions of dollars) from Table 7.1 when there is a final exchange of principal.

Date	*Six-month LIBOR rate (%)*	*Floating cash flow received*	*Fixed cash flow paid*	*Net cash flow*
Mar. 5, 2004	4.20			
Sept. 5, 2004	4.80	+2.10	−2.50	−0.40
Mar. 5, 2005	5.30	+2.40	−2.50	−0.10
Sept. 5, 2005	5.50	+2.65	−2.50	+0.15
Mar. 5, 2006	5.60	+2.75	−2.50	+0.25
Sept. 5, 2006	5.90	+2.80	−2.50	+0.30
Mar. 5, 2007		+102.95	−102.50	+0.45

This characterization of the cash flows in the swap helps to explain why the floating rate in the swap is set 6 months before it is paid. On a floating-rate bond, interest is generally set at the beginning of the period to which it will apply and is paid at the end of the period. The calculation of the floating-rate payments in a "plain vanilla" interest rate swap such as the one in Table 7.2 reflects this.

Using the Swap to Transform a Liability

For Microsoft, the swap could be used to transform a floating-rate loan into a fixed-rate loan. Suppose that Microsoft has arranged to borrow $100 million at LIBOR plus 10 basis points. (One basis point is one-hundredth of 1%, so the rate is LIBOR plus 0.1%.) After Microsoft has entered into the swap, it has the following three sets of cash flows:

1. It pays LIBOR plus 0.1% to its outside lenders.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate payment of 5.1%. Thus, for Microsoft, the swap could have the effect of transforming borrowings at a floating rate of LIBOR plus 10 basis points into borrowings at a fixed rate of 5.1%.

For Intel, the swap could have the effect of transforming a fixed-rate loan into a floating-rate loan. Suppose that Intel has a 3-year $100 million loan outstanding on which it pays 5.2%. After it has entered into the swap, it has the following three sets of cash flows:

1. It pays 5.2% to its outside lenders.
2. It pays LIBOR under the terms of the swap.
3. It receives 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate payment of LIBOR plus 0.2% (or LIBOR plus 20 basis points). Thus, for Intel, the swap could have the effect of transforming borrowings at a fixed rate of 5.2% into borrowings at a floating rate of LIBOR plus 20 basis points. These potential uses of the swap by Intel and Microsoft are illustrated in Figure 7.2.

Using the Swap to Transform an Asset

Swaps can also be used to transform the nature of an asset. Consider Microsoft in our example. The swap could have the effect of transforming an asset earning a fixed rate of interest into an asset earning a floating rate of interest. Suppose that Microsoft owns $100 million in bonds that will provide interest at 4.7% per annum over the next 3 years.

Figure 7.2 Microsoft and Intel use the swap to transform a liability.

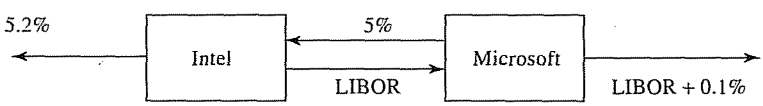
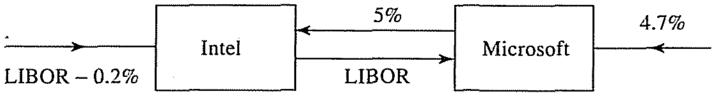


Figure 7.3 Microsoft and Intel use the swap to transform an asset.



After Microsoft has entered into the swap, it has the following three sets of cash flows:

1. It receives 4.7% on the bonds.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate inflow of LIBOR minus 30 basis points. Thus, one possible use of the swap for Microsoft is to transform an asset earning 4.7% into an asset earning LIBOR minus 30 basis points.

Next, consider Intel. The swap could have the effect of transforming an asset earning a floating rate of interest into an asset earning a fixed rate of interest. Suppose that Intel has an investment of $100 million that yields LIBOR minus 20 basis points. After it has entered into the swap, it has the following three sets of cash flows:

1. It receives LIBOR minus 20 basis points on its investment.
2. It pays LIBOR under the terms of the swap.
3. It receives 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate inflow of 4.8%. Thus, one possible use of the swap for Intel is to transform an asset earning LIBOR minus 20 basis points into an asset earning 4.8%. These potential uses of the swap by Intel and Microsoft are illustrated in Figure 7.3.

Role of Financial Intermediary

Usually two nonfinancial companies such as Intel and Microsoft do not get in touch directly to arrange a swap in the way indicated in Figures 7.2 and 7.3. They each deal with a financial intermediary such as a bank or other financial institution. "Plain vanilla" fixed-for-floating swaps on US interest rates are usually structured so that the financial institution earns about 3 or 4 basis points (0.03% or 0.04%) on a pair of offsetting transactions.

Figure 7.4 shows what the role of the financial institution might be in the situation in Figure 7.2. The financial institution enters into two offsetting swap transactions with

Figure 7.4 Interest rate swap from Figure 7.2 when financial institution is involved.

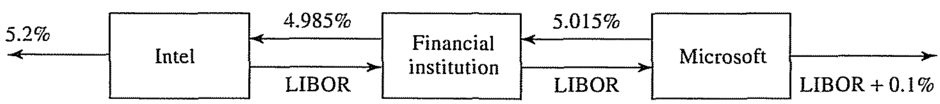
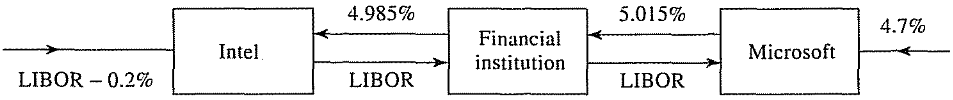


Figure 7.5 Interest rate swap from Figure 7.3 when financial institution is involved.



Intel and Microsoft. Assuming that both companies honor their obligations, the financial institution is certain to make a profit of 0.03% (3 basis points) per year multiplied by the notional principal of \$100 million. This amounts to \$30,000 per year for the 3-year period. Microsoft ends up borrowing at 5.115% (instead of 5.1%, as in Figure 7.2), and Intel ends up borrowing at LIBOR plus 21.5 basis points (instead of at LIBOR plus 20 basis points, as in Figure 7.2).

Figure 7.5 illustrates the role of the financial institution in the situation in Figure 7.3. The swap is the same as before and the financial institution is certain to make a profit of 3 basis points if neither company defaults. Microsoft ends up earning LIBOR minus 31.5 basis points (instead of LIBOR minus 30 basis points, as in Figure 7.3), and Intel ends up earning 4.785% (instead of 4.8%, as in Figure 7.3).

Note that in each case the financial institution has two separate contracts: one with Intel and the other with Microsoft. In most instances, Intel will not even know that the financial institution has entered into an offsetting swap with Microsoft, and vice versa. If one of the companies defaults, the financial institution still has to honor its agreement with the other company. The 3-basis-point spread earned by the financial institution is partly to compensate it for the risk that one of the two companies will default on the swap payments.

Market Makers

In practice, it is unlikely that two companies will contact a financial institution at the same time and want to take opposite positions in exactly the same swap. For this reason, many large financial institutions act as market makers for swaps. This means that they are prepared to enter into a swap without having an offsetting swap with another counterparty.[2] Market makers must carefully quantify and hedge the risks they are taking. Bonds, forward rate agreements, and interest rate futures are examples of the instruments that can be used for hedging by swap market makers. Table 7.3 shows quotes for plain vanilla US dollar swaps that might be posted by a market maker.[3] As mentioned earlier, the bid–offer spread is 3 to 4 basis points. The average of the bid and offer fixed rates is known as the *swap rate*. This is shown in the final column of Table 7.3.

Consider a new swap where the fixed rate equals the current swap rate. We can reasonably assume that the value of this swap is zero. (Why else would a market maker choose bid–offer quotes centered on the swap rate?) In Table 7.2 we saw that a swap can

[2] This is sometimes referred to as *warehousing* swaps.

[3] The standard swap in the United States is one where fixed payments made every 6 months are exchanged for floating LIBOR payments made every 3 months. In Table 7.1 we assumed that fixed and floating payments are exchanged every 6 months. As we shall see later, the fixed rate should in theory be the same, regardless of whether floating payments are made every 3 or every 6 months.

Table 7.3 Bid and offer fixed rates in the swap market and swap rates (percent per annum).

Maturity (years)	*Bid*	*Offer*	*Swap rate*
2	6.03	6.06	6.045
3	6.21	6.24	6.225
4	6.35	6.39	6.370
5	6.47	6.51	6.490
7	6.65	6.68	6.665
10	6.83	6.87	6.850

be characterized as the difference between a fixed-rate bond and a floating-rate bond. Define:

B_{fix}: Value of fixed-rate bond underlying the swap we are considering

B_{fl}: Value of floating-rate bond underlying the swap we are considering

Since the swap is worth zero, it follows that

$$B_{\text{fix}} = B_{\text{fl}} \tag{7.1}$$

We will use this result later in the chapter when discussing how the LIBOR/swap zero curve is determined.

7.2 DAY COUNT ISSUES

We discussed day count conventions in Section 6.1. The day count conventions affect payments on a swap, and some of the numbers calculated in the examples we have given do not exactly reflect these day count conventions. Consider, for example, the 6-month LIBOR payments in Table 7.1. Because it is a money market rate, 6-month LIBOR is quoted on an actual/360 basis. The first floating payment in Table 7.1, based on the LIBOR rate of 4.2%, is shown as \$2.10 million. Because there are 184 days between March 5, 2004, and September 5, 2004, it should be

$$100 \times 0.042 \times \frac{184}{360} = \$2.1467 \text{ million}$$

In general, a LIBOR-based floating-rate cash flow on a swap payment date is calculated as $LRn/360$, where L is the principal, R is the relevant LIBOR rate, and n is the number of days since the last payment date.

The fixed rate that is paid in a swap transaction is similarly quoted with a particular day count basis being specified. As a result, the fixed payments may not be exactly equal on each payment date. The fixed rate is usually quoted as actual/365 or 30/360. It is not therefore directly comparable with LIBOR because it applies to a full year. To make the rates comparable, either the 6-month LIBOR rate must be multiplied by 365/360 or the fixed rate must be multiplied by 360/365.

For ease of exposition, we will ignore day count issues in the calculations in the rest of this chapter.

Business Snapshot 7.1 Extract from Hypothetical Swap Confirmation

Trade date:	27-February-2004
Effective date:	5-March-2004
Business day convention (all dates):	Following business day
Holiday calendar:	US
Termination date:	5-March-2007
Fixed amounts	
Fixed-rate payer:	Microsoft
Fixed-rate notional principal:	USD 100 million
Fixed rate:	5.015% per annum
Fixed-rate day count convention:	Actual/365
Fixed-rate payment dates:	Each 5-March and 5-September, commencing 5-September-2004, up to and including 5-March-2007
Floating amounts	
Floating-rate payer:	Goldman Sachs
Floating-rate notional principal:	USD 100 million
Floating rate:	USD 6-month LIBOR
Floating-rate day count convention:	Actual/360
Floating-rate payment dates:	Each 5-March and 5-September, commencing 5-September-2004, up to and including 5-March-2007

7.3 CONFIRMATIONS

A *confirmation* is the legal agreement underlying a swap and is signed by representatives of the two parties. The drafting of confirmations has been facilitated by the work of the International Swaps and Derivatives Association (ISDA) in New York. This organization has produced a number of Master Agreements that consist of clauses defining in some detail the terminology used in swap agreements, what happens in the event of default by either side, and so on. In Business Snapshot 7.1, we show a possible extract from the confirmation for the swap shown in Figure 7.4 between Microsoft and a financial institution (assumed here to be Goldman Sachs). Almost certainly, the full confirmation would state that the provisions of an ISDA Master Agreement apply to the contract.

The confirmation specifies that the following business day convention is to be used and that the US calendar determines which days are business days and which days are holidays. This means that, if a payment date falls on a weekend or a US holiday, the payment is made on the next business day.[4] September 5, 2004, is a Sunday. The first

[4] Another business day convention that is sometimes specified is the *modified following* business day convention, which is the same as the following business day convention except that, when the next business day falls in a different month from the specified day, the payment is made on the immediately preceding business day. *Preceding* and *modified preceding* business day conventions are defined analogously.

exchange of payments in the swap between Microsoft and Goldman Sachs is therefore on Monday September 6, 2004.

7.4 THE COMPARATIVE-ADVANTAGE ARGUMENT

An explanation commonly put forward to explain the popularity of swaps concerns comparative advantages. Consider the use of an interest rate swap to transform a liability. Some companies, it is argued, have a comparative advantage when borrowing in fixed-rate markets, whereas other companies have a comparative advantage in floating-rate markets. To obtain a new loan, it makes sense for a company to go to the market where it has a comparative advantage. As a result, the company may borrow fixed when it wants floating, or borrow floating when it wants fixed. The swap is used to transform a fixed-rate loan into a floating-rate loan, and vice versa.

Illustration

Suppose that two companies, AAACorp and BBBCorp, both wish to borrow $10 million for 5 years and have been offered the rates shown in Table 7.4. AAACorp has a AAA credit rating; BBBCorp has a BBB credit rating.[5] We assume that BBBCorp wants to borrow at a fixed rate of interest, whereas AAACorp wants to borrow at a floating rate of interest linked to 6-month LIBOR. Because it has a worse credit rating than AAACorp, BBBCorp pays a higher rate of interest than AAACorp in both fixed and floating markets.

A key feature of the rates offered to AAACorp and BBBCorp is that the difference between the two fixed rates is greater than the difference between the two floating rates. BBBCorp pays 1.2% more than AAACorp in fixed-rate markets and only 0.7% more than AAACorp in floating-rate markets. BBBCorp appears to have a comparative advantage in floating-rate markets, whereas AAACorp appears to have a comparative advantage in fixed-rate markets.[6] It is this apparent anomaly that can lead to a swap being negotiated. AAACorp borrows fixed-rate funds at 4% per annum. BBBCorp borrows floating-rate funds at LIBOR plus 1% per annum. They then enter into a swap

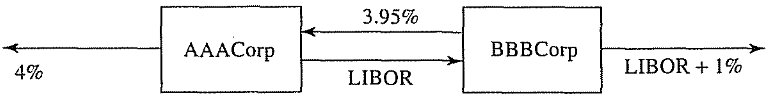
Table 7.4 Borrowing rates that provide a basis for the comparative-advantage argument.

	Fixed	*Floating*
AAACorp	4.0%	6-month LIBOR + 0.3%
BBBCorp	5.2%	6-month LIBOR + 1.0%

[5] The credit ratings assigned to companies by S&P (in order of decreasing creditworthiness) are AAA, AA, A, BBB, BB, B, and CCC. The corresponding ratings assigned by Moody's are Aaa, Aa, A, Baa, Ba, B, and Caa, respectively.

[6] Note that BBBCorp's comparative advantage in floating-rate markets does not imply that BBBCorp pays less than AAACorp in this market. It means that the extra amount that BBBCorp pays over the amount paid by AAACorp is less in this market. One of my students summarized the situation as follows: "AAACorp pays more less in fixed-rate markets; BBBCorp pays less more in floating-rate markets."

Figure 7.6 Swap agreement between AAACorp and BBBCorp when rates in Table 7.4 apply.



agreement to ensure that AAACorp ends up with floating-rate funds and BBBCorp ends up with fixed-rate funds.

To understand how this swap might work, we first assume that AAACorp and BBBCorp get in touch with each other directly. The sort of swap they might negotiate is shown in Figure 7.6. This is similar to our example in Figure 7.2. AAACorp agrees to pay BBBCorp interest at 6-month LIBOR on $10 million. In return, BBBCorp agrees to pay AAACorp interest at a fixed rate of 3.95% per annum on $10 million.

AAACorp has three sets of interest rate cash flows:

1. It pays 4% per annum to outside lenders.
2. It receives 3.95% per annum from BBBCorp.
3. It pays LIBOR to BBBCorp.

The net effect of the three cash flows is that AAACorp pays LIBOR plus 0.05% per annum. This is 0.25% per annum less than it would pay if it went directly to floating-rate markets. BBBCorp also has three sets of interest rate cash flows:

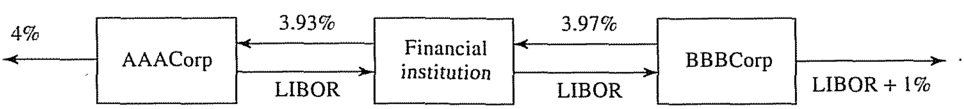
1. It pays LIBOR + 1% per annum to outside lenders.
2. It receives LIBOR from AAACorp.
3. It pays 3.95% per annum to AAACorp.

The net effect of the three cash flows is that BBBCorp pays 4.95% per annum. This is 0.25% per annum less than it would pay if it went directly to fixed-rate markets.

In this example, the swap has been structured so that the net gain to both sides is the same, 0.25%. This need not be the case. However, the total apparent gain from this type of interest rate swap arrangement is always $a - b$, where a is the difference between the interest rates facing the two companies in fixed-rate markets, and b is the difference between the interest rates facing the two companies in floating-rate markets. In this case, $a = 1.2\%$ and $b = 0.7\%$, so that the total gain is 0.5%.

If AAACorp and BBBCorp did not deal directly with each other and used a financial institution, an arrangement such as that shown in Figure 7.7 might result. (This is similar

Figure 7.7 Swap agreement between AAACorp and BBBCorp when rates in Table 7.4 apply and a financial intermediary is involved.



to the example in Figure 7.4.) In this case, AAACorp ends up borrowing at LIBOR + 0.07%, BBBCorp ends up borrowing at 4.97%, and the financial institution earns a spread of 4 basis points per year. The gain to AAACorp is 0.23%; the gain to BBBCorp is 0.23%; and the gain to the financial institution is 0.04%. The total gain to all three parties is 0.50% as before.

Criticism of the Comparative-Advantage Argument

The comparative-advantage argument we have just outlined for explaining the attractiveness of interest rate swaps is open to question. Why in Table 7.4 should the spreads between the rates offered to AAACorp and BBBCorp be different in fixed and floating markets? Now that the swap market has been in existence for some time, we might reasonably expect these types of differences to have been arbitraged away.

The reason that spread differentials appear to exist is due to the nature of the contracts available to companies in fixed and floating markets. The 4.0% and 5.2% rates available to AAACorp and BBBCorp in fixed-rate markets are 5-year rates (e.g., the rates at which the companies can issue 5-year fixed-rate bonds). The LIBOR + 0.3% and LIBOR + 1.0% rates available to AAACorp and BBBCorp in floating-rate markets are 6-month rates. In the floating-rate market, the lender usually has the opportunity to review the floating rates every 6 months. If the creditworthiness of AAACorp or BBBCorp has declined, the lender has the option of increasing the spread over LIBOR that is charged. In extreme circumstances, the lender can refuse to roll over the loan at all. The providers of fixed-rate financing do not have the option to change the terms of the loan in this way.[7]

The spreads between the rates offered to AAACorp and BBBCorp are a reflection of the extent to which BBBCorp is more likely than AAACorp to default. During the next 6 months, there is very little chance that either AAACorp or BBBCorp will default. As we look further ahead, default statistics show that on average the probability of a default by a company with a relatively low credit rating (such as BBBCorp) increases faster than the probability of a default by a company with a relatively high credit rating (such as AAACorp). This is why the spread between the 5-year rates is greater than the spread between the 6-month rates.

After negotiating a floating-rate loan at LIBOR + 1.0% and entering into the swap shown in Figure 7.7, BBBCorp appears to obtain a fixed-rate loan at 4.97%. The arguments just presented show that this is not really the case. In practice, the rate paid is 4.97% only if BBBCorp can continue to borrow floating-rate funds at a spread of 1.0% over LIBOR. If, for example, the credit rating of BBBCorp declines so that the floating-rate loan is rolled over at LIBOR + 2.0%, the rate paid by BBBCorp increases to 5.97%. The market expects that BBBCorp's spread over 6-month LIBOR will on average rise during the swap's life. BBBCorp's expected average borrowing rate when it enters into the swap is therefore greater than 4.97%.

The swap in Figure 7.7 locks in LIBOR + 0.07% for AAACorp for the whole of the next 5 years, not just for the next 6 months. This appears to be a good deal for AAACorp. The downside is that it is bearing the risk of a default by the financial institution. If it borrowed floating-rate funds in the usual way, it would not be bearing this risk.

[7] If the floating-rate loans are structured so that the spread over LIBOR is guaranteed in advance regardless of changes in credit rating, there is in practice little or no comparative advantage.

7.5 THE NATURE OF SWAP RATES

At this stage it is appropriate to examine the nature of swap rates and the relationship between swap and LIBOR markets. We explained in Section 4.1 that LIBOR is the rate of interest at which AA-rated banks borrow for periods between 1 and 12 months from other banks. As shown in Table 7.3, a swap rate is the average of (a) the fixed rate that a swap market maker is prepared to pay in exchange for receiving LIBOR (its bid rate) and (b) the fixed rate that it is prepared to receive in return for paying LIBOR (its offer rate).

Like LIBOR rates, swap rates are not risk-free lending rates. However, they are close to risk-free. A financial institution can earn the 5-year swap rate on a certain principal by doing the following:

1. Lend the principal for the first 6 months to a AA borrower and then relend it for successive 6 month periods to other AA borrowers; and
2. Enter into a swap to exchange the LIBOR income for the 5-year swap rate.

This shows that the 5-year swap rate is an interest rate with a credit risk corresponding to the situation where 10 consecutive 6-month LIBOR loans to AA companies are made. Similarly the 7-year swap rate is an interest rate with a credit risk corresponding to the situation where 14 consecutive 6-month LIBOR loans to AA companies are made. Swap rates of other maturities can be interpreted analogously.

Note that swap rates are less than AA borrowing rates. It is much more attractive to lend money for successive 6-month periods to borrowers who are always AA at the beginning of the periods than to lend it to one borrower for the whole 5 years when all we can be sure of is that the borrower is AA at the beginning of the 5 years.

7.6 DETERMINING LIBOR/SWAP ZERO RATES

We explained in Section 4.1 that derivative traders tend to use LIBOR rates as a proxies for risk-free rates when valuing derivatives. One problem with LIBOR rates is that direct observations are possible only for maturities out to 12 months. As described in Section 6.4, one way of extending the LIBOR zero curve beyond 12 months is to use Eurodollar futures. Typically Eurodollar futures are used to produce a LIBOR zero curve out to 2 years—and sometimes out to as far as 5 years. Traders then use swap rates to extend the LIBOR zero curve further. The resulting zero curve is sometimes referred to as the LIBOR zero curve and sometimes as the swap zero curve. To avoid any confusion, we will refer to it as the LIBOR/swap zero curve. We will now describe how swap rates are used in the determination of the LIBOR/swap zero curve.

The first point to note is that the value of a newly issued floating-rate bond that pays 6-month LIBOR is always equal to its principal value (or par value) when the LIBOR/swap zero curve is used for discounting.[8] The reason is that the bond provides a rate of interest of LIBOR, and LIBOR is the discount rate. The interest on the bond exactly matches the discount rate, and as a result the bond is fairly priced at par.

In equation (7.1), we showed that for a newly issued swap where the fixed rate equals the swap rate, $B_{\text{fix}} = B_{\text{fl}}$. We have just shown that B_{fl} equals the notional principal. It follows that B_{fix} also equals the swap's notional principal. Swap rates therefore define a

[8] The same is true of a newly issued bond that pays 1-month, 3-month, or 12-month LIBOR.

set of par yield bonds. For example, from the swap rates in Table 7.3, we can deduce that the 2-year LIBOR/swap par yield is 6.045%, the 3-year LIBOR/swap par yield is 6.225%, and so on.[9]

The usual method for determining the LIBOR/swap zero curve is the bootstrap method which we used to determine the Treasury zero curve in Section 4.5. LIBOR rates define the zero curve out to 1 year. Swap rates define par yield bonds that are used to determine longer-term rates.

Example 7.1

Suppose that the 6-month, 12-month, and 18-month LIBOR/swap zero rates have been determined as 4%, 4.5%, and 4.8% with continuous compounding and that the 2-year swap rate (for a swap where payments are made semiannually) is 5%. This 5% swap rate means that a bond with a principal of \$100 and a semiannual coupon of 5% per annum sells for par. It follows that, if R is the 2-year zero rate, then

$$2.5e^{-0.04\times 0.5} + 2.5e^{-0.045\times 1.0} + 2.5e^{-0.048\times 1.5} + 102.5e^{2R} = 100$$

Solving this, we obtain $R = 4.953\%$. (Note that this calculation is simplified in that it does not take the swap's day count conventions and holiday calendars into account. See Section 7.2.)

7.7 VALUATION OF INTEREST RATE SWAPS

We now move on to discuss the valuation of interest rate swaps. An interest rate swap is worth zero, or close to zero, when it is first initiated. After it has been in existence for some time, its value may become positive or negative. There are two valuation approaches. The first regards the swap as the difference between two bonds; the second regards it as a portfolio of FRAs.

Valuation in Terms of Bond Prices

Principal payments are not exchanged in an interest rate swap. However, as illustrated in Table 7.2, we can assume that principal payments are both received and paid at the end of the swap without changing its value. By doing this, we find that, from the point of view of the floating-rate payer, a swap can be regarded as a long position in a fixed-rate bond and a short position in a floating-rate bond, so that

$$V_{\text{swap}} = B_{\text{fix}} - B_{\text{fl}}$$

where V_{swap} is the value of the swap, B_{fl} is the value of the floating-rate bond (corresponding to payments that are made), and B_{fix} is the value of the fixed-rate bond (corresponding to payments that are received). Similarly, from the point of view of the fixed-rate payer, a swap is a long position in a floating-rate bond and a short

[9] Analysts frequently interpolate between swap rates before calculating the zero curve, so that they have swap rates for maturities at 6-month intervals. For example, for the data in Table 7.3 the 2.5-year swap rate would be assumed to be 6.135%; the 7.5-year swap rate would be assumed to be 6.696%; and so on.

position in a fixed-rate bond, so that the value of the swap is

$$V_{\text{swap}} = B_{\text{fl}} - B_{\text{fix}}$$

The value of the fixed rate bond, B_{fix}, can be determined as described in Section 4.4. To value the floating-rate bond, we note that the bond is worth the notional principal immediately after an interest payment. This is because at this time the bond is a "fair deal" where the borrower pays LIBOR for each subsequent accrual period.

Suppose that the notional principal is L, the next exchange of payments is at time t^*, and the floating payment that will be made at time t^* (which was determined at the last payment date) is k^*. Immediately after the payment $B_{\text{fl}} = L$ as just explained. It follows that immediately before the payment $B_{\text{fl}} = L + k^*$. The floating-rate bond can therefore be regarded as an instrument providing a single cash flow of $L + k^*$ at time t^*. Discounting this, the value of the floating-rate bond today is

$$(L + k^*)e^{-r^*t^*}$$

where r^* is the LIBOR/swap zero rate for a maturity of t^*.

Example 7.2

Suppose that a financial institution has agreed to pay 6-month LIBOR and receive 8% per annum (with semiannual compounding) on a notional principal of \$100 million. The swap has a remaining life of 1.25 years. The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month LIBOR rate at the last payment date was 10.2% (with semiannual compounding).

The calculations for valuing the swap in terms of bonds are summarized in Table 7.5. The fixed-rate bond has cash flows of 4, 4, and 104 on the three payment dates. The discount factors for these cash flows are, respectively,

$$e^{-0.1\times 0.25}, \quad e^{-0.105\times 0.75}, \quad e^{-0.11\times 1.25}$$

and are shown in the fourth column of Table 7.5. The table shows that the value of the fixed-rate bond (in millions of dollars) is 98.238.

In this example, $k^* = 0.5 \times 0.102 \times 100 = \$5.1$ million and $t^* = 0.25$, so that the floating-rate bond can be valued as though it produces a cash flow of \$105.1 million in 3 months. The table shows that the value of the floating bond (in millions of dollars) is 102.505.

Table 7.5 Valuing a swap in terms of bonds (\$ millions). Here, B_{fix} is fixed-rate bond underlying the swap, and B_{fl} is floating-rate bond underlying the swap.

Time	B_{fix} *cash flow*	B_{fl} *cash flow*	*Discount factor*	*Present value* B_{fix} *cash flow*	*Present value* B_{fl} *cash flow*
0.25	4.0	105.100	0.9753	3.901	102.505
0.75	4.0		0.9243	3.697	
1.25	104.0		0.8715	90.640	
Total:				98.238	102.505

The value of the swap is the difference between the two bond prices:

$$V_{\text{swap}} = 98.238 - 102.505 = -4.267$$

or -4.267 million dollars.

If the financial institution had been in the opposite position of paying fixed and receiving floating, the value of the swap would be +\$4.267 million. Note that our calculations do not take account of day count conventions and holiday calendars.

Valuation in Terms of FRAs

A swap can be characterized as a portfolio of forward rate agreements. Consider the swap between Microsoft and Intel in Figure 7.1. The swap is a 3-year deal entered into on March 5, 2004, with semiannual payments. The first exchange of payments is known at the time the swap is negotiated. The other five exchanges can be regarded as FRAs. The exchange on March 5, 2005, is an FRA where interest at 5% is exchanged for interest at the 6-month rate observed in the market on September 5, 2004; the exchange on September 5, 2005, is an FRA where interest at 5% is exchanged for interest at the 6-month rate observed in the market on March 5, 2005; and so on.

As shown at the end of Section 4.7, an FRA can be valued by assuming that forward interest rates are realized. Because it is nothing more than a portfolio of forward rate agreements, a plain vanilla interest rate swap can also be valued by making the assumption that forward interest rates are realized. The procedure is as follows:

1. Use the LIBOR/swap zero curve to calculate forward rates for each of the LIBOR rates that will determine swap cash flows.
2. Calculate swap cash flows on the assumption that the LIBOR rates will equal the forward rates.
3. Discount these swap cash flows (using the LIBOR/swap zero curve) to obtain the swap value.

Example 7.3

Consider again the situation in Example 7.2. Under the terms of the swap, a financial institution has agreed to pay 6-month LIBOR and receive 8% per annum (with semiannual compounding) on a notional principal of \$100 million. The swap has a remaining life of 1.25 years. The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month LIBOR rate at the last payment date was 10.2% (with semiannual compounding).

The calculations are summarized in Table 7.6. The first row of the table shows the cash flows that will be exchanged in 3 months. These have already been determined. The fixed rate of 8% will lead to a cash inflow of $100 \times 0.08 \times 0.5 = 4$ million. The floating rate of 10.2% (which was set 3 months ago) will lead to a cash outflow of $100 \times 0.102 \times 0.5 = 5.1$ million. The second row of the table shows the cash flows that will be exchanged in 9 months assuming that forward rates are realized. The cash inflow is 4.0 million as before. To calculate the cash outflow, we we must first calculate the forward rate corresponding to the period between 3 and 9 months.

Table 7.6 Valuing swap in terms of FRAs ($ millions). Floating cash flows are calculated by assuming that forward rates will be realized.

Time	*Fixed cash flow*	*Floating cash flow*	*Net cash flow*	*Discount factor*	*Present value of net cash flow*
0.25	4.0	−5.100	−1.100	0.9753	−1.073
0.75	4.0	−5.522	−1.522	0.9243	−1.407
1.25	4.0	−6.051	−2.051	0.8715	−1.787
Total:					−4.267

From equation (4.5), this is

$$\frac{0.105 \times 0.75 - 0.10 \times 0.25}{0.5} = 0.1075$$

or 10.75% with continuous compounding. From equation (4.4), the forward rate becomes 11.044% with semiannual compounding. The cash outflow is therefore $100 \times 0.11044 \times 0.5 = 5.522$ million. The third row similarly shows the cash flows that will be exchanged in 15 months assuming that forward rates are realized. The discount factors for the three payment dates are, respectively,

$$e^{-0.1\times0.25}, \quad e^{-0.105\times0.75}, \quad e^{-0.11\times1.25}$$

The present value of the exchange in three months is −1.073 million. The values of the FRAs corresponding to the exchanges in 9 months and 15 months are −1.407 and −1.787, respectively. The total value of the swap is −$4.267 million. This is in agreement with the value we calculated in Example 7.2 by decomposing the swap into bonds.

The fixed rate in an interest rate swap is chosen so that the swap is worth zero initially. This means that at the outset of a swap the sum of the values of the FRAs underlying the swap is zero. It does not mean that the value of each individual FRA is zero. In general, some FRAs will have positive values whereas others have negative values.

Consider the FRAs underlying the swap between Microsoft and Intel in Figure 7.1:

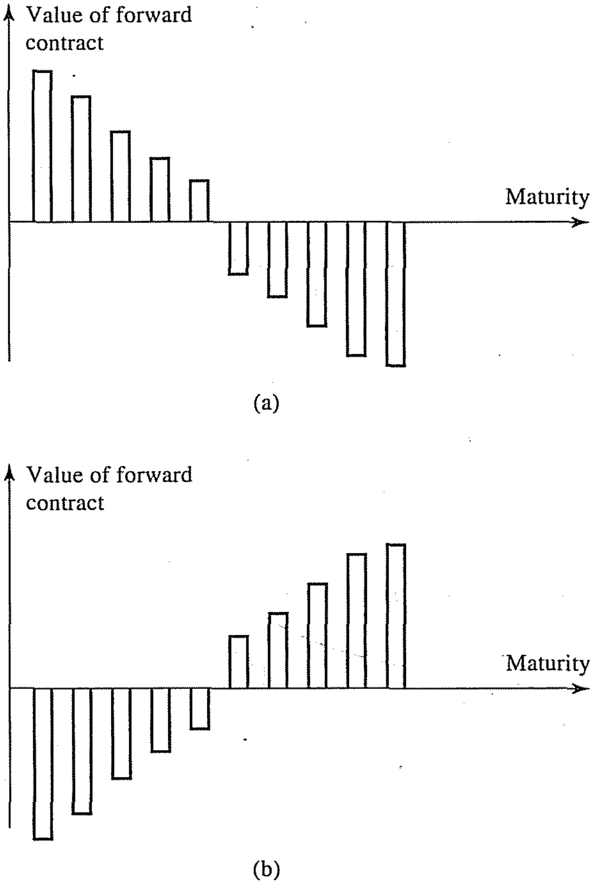
Value of FRA to Microsoft > 0 when forward interest rate > 5.0%

Value of FRA to Microsoft = 0 when forward interest rate = 5.0%

Value of FRA to Microsoft < 0 when forward interest rate < 5.0%

Suppose that the term structure of interest rates is upward-sloping at the time the swap is negotiated. This means that the forward interest rates increase as the maturity of the FRA increases. Since the sum of the values of the FRAs is zero, the forward interest rate must be less than 5.0% for the early payment dates and greater than 5.0% for the later payment dates. The value to Microsoft of the FRAs corresponding to early payment dates is therefore negative, whereas the value of the FRAs corresponding to later payment dates is positive. If the term structure of interest rates is downward-sloping at the time the swap is negotiated, the reverse is true. The impact of the shape of the term structure of interest rates on the values of the forward contracts underlying a swap is summarized in Figure 7.8.

Figure 7.8 Valuing of forward rate agreements underlying a swap as a function of maturity. In (a) the term structure of interest rates is upward-sloping and we receive fixed, or it is downward-sloping and we receive floating; in (b) the term structure of interest rates is upward-sloping and we receive floating, or it is downward-sloping and we receive fixed.

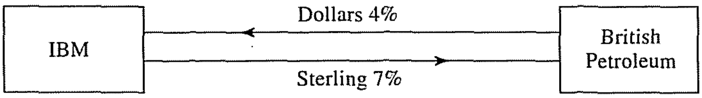


7.8 CURRENCY SWAPS

Another popular type of swap is known as a *currency swap*. In its simplest form, this involves exchanging principal and interest payments in one currency for principal and interest payments in another.

A currency swap agreement requires the principal to be specified in each of the two currencies. The principal amounts in each currency are usually exchanged at the beginning and at the end of the life of the swap. Usually the principal amounts are chosen to be approximately equivalent using the exchange rate at the swap's initiation. When they are exchanged at the end of the life of the swap, their values may be quite different.

Figure 7.9 A currency swap.



Illustration

Consider a hypothetical 5-year currency swap agreement between IBM and British Petroleum entered into on February 1, 2004. We suppose that IBM pays a fixed rate of interest of 7% in sterling and receives a fixed rate of interest of 4% in dollars from British Petroleum. Interest rate payments are made once a year and the principal amounts are $15 million and £10 million. This is termed a *fixed-for-fixed* currency swap because the interest rate in both currencies is fixed. The swap is shown in Figure 7.9. Initially, the principal amounts flow in the opposite direction to the arrows in Figure 7.9. The interest payments during the life of the swap and the final principal payment flow in the same direction as the arrows. Thus, at the outset of the swap, IBM pays $15 million and receives £10 million. Each year during the life of the swap contract, IBM receives $0.60 million (= 4% of $15 million) and pays £0.70 million (= 7% of £10 million). At the end of the life of the swap, it pays a principal of £10 million and receives a principal of $15 million. These cash flows are shown in Table 7.7.

Use of a Currency Swap to Transform Loans and Assets

A swap such as the one just considered can be used to transform borrowings in one currency to borrowings in another. Suppose that IBM can issue $15 million of US-dollar-denominated bonds at 4% interest. The swap has the effect of transforming this transaction into one where IBM has borrowed £10 million at 7% interest. The initial exchange of principal converts the proceeds of the bond issue from US dollars to sterling. The subsequent exchanges in the swap have the effect of swapping the interest and principal payments from dollars to sterling.

The swap can also be used to transform the nature of assets. Suppose that IBM can invest £10 million in the UK to yield 7% per annum for the next 5 years, but feels that

Table 7.7 Cash flows to IBM in currency swap.

Date	*Dollar cash flow (millions)*	*Sterling cash flow (millions)*
February 1, 2004	−15.00	+10.00
February 1, 2005	+0.60	−0.70
February 1, 2006	+0.60	−0.70
February 1, 2007	+0.60	−0.70
February 1, 2008	+0.60	−0.70
February 1, 2009	+15.60	−10.70

the US dollar will strengthen against sterling and prefers a US-dollar-denominated investment. The swap has the effect of transforming the UK investment into a $15 million investment in the US yielding 4%.

Comparative Advantage

Currency swaps can be motivated by comparative advantage. To illustrate this, we consider another hypothetical example. Suppose the 5-year fixed-rate borrowing costs to General Motors and Qantas Airways in US dollars (USD) and Australian dollars (AUD) are as shown in Table 7.8. The data in the table suggest that Australian rates are higher than USD interest rates, and also that General Motors is more creditworthy than Qantas Airways, because it is offered a more favorable rate of interest in both currencies. From the viewpoint of a swap trader, the interesting aspect of Table 7.8 is that the spreads between the rates paid by General Motors and Qantas Airways in the two markets are not the same. Qantas Airways pays 2% more than General Motors in the US dollar market and only 0.4% more than General Motors in the AUD market.

This situation is analogous to that in Table 7.4. General Motors has a comparative advantage in the USD market, whereas Qantas Airways has a comparative advantage in the AUD market. In Table 7.4, where a plain vanilla interest rate swap was considered, we argued that comparative advantages are largely illusory. Here we are comparing the rates offered in two different currencies, and it is more likely that the comparative advantages are genuine. One possible source of comparative advantage is tax. General Motors' position might be such that USD borrowings lead to lower taxes on its worldwide income than AUD borrowings. Qantas Airways' position might be the reverse. (Note that we assume that the interest rates in Table 7.8 have been adjusted to reflect these types of tax advantages.)

We suppose that General Motors wants to borrow 20 million AUD and Qantas Airways wants to borrow 12 million USD and that the current exchange rate (USD per AUD) is 0.6000. This creates a perfect situation for a currency swap. General Motors and Qantas Airways each borrow in the market where they have a comparative advantage; that is, General Motors borrows USD whereas Qantas Airways borrows AUD. They then use a currency swap to transform General Motors' loan into an AUD loan and Qantas Airways' loan into a USD loan.

As already mentioned, the difference between the USD interest rates is 2%, whereas the difference between the AUD interest rates is 0.4%. By analogy with the interest rate swap case, we expect the total gain to all parties to be $2.0 - 0.4 = 1.6\%$ per annum.

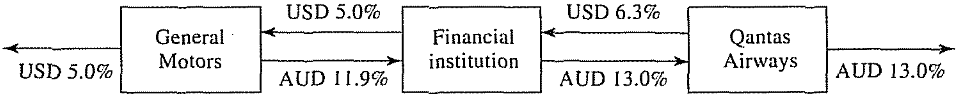
There are many ways in which the swap can be arranged. Figure 7.10 shows one way swaps might be entered into with a financial institution. General Motors borrows USD and Qantas Airways borrows AUD. The effect of the swap is to transform the USD

Table 7.8 Borrowing rates providing basis for currency swap.

	*USD**	*AUD**
General Motors	5.0%	12.6%
Qantas Airways	7.0%	13.0%

* Quoted rates have been adjusted to reflect the differential impact of taxes.

Figure 7.10 A currency swap motivated by comparative advantage.



interest rate of 5% per annum to an AUD interest rate of 11.9% per annum for General Motors. As a result, General Motors is 0.7% per annum better off than it would be if it went directly to AUD markets. Similarly, Qantas exchanges an AUD loan at 13% per annum for a USD loan at 6.3% per annum and ends up 0.7% per annum better off than it would be if it went directly to USD markets. The financial institution gains 1.3% per annum on its USD cash flows and loses 1.1% per annum on its AUD flows. If we ignore the difference between the two currencies, the financial institution makes a net gain of 0.2% per annum. As predicted, the total gain to all parties is 1.6% per annum.

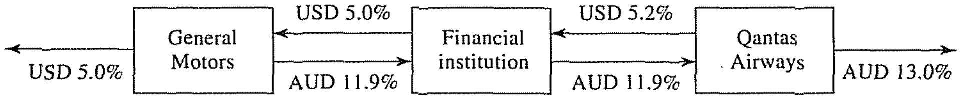
Each year the financial institution makes a gain of USD 156,000 (= 1.3% of 12 million) and incurs a loss of AUD 220,000 (= 1.1% of 20 million). The financial institution can avoid any foreign exchange risk by buying AUD 220,000 per annum in the forward market for each year of the life of the swap, thus locking in a net gain in USD.

It is possible to redesign the swap so that the financial institution makes a 0.2% spread in USD. Figures 7.11 and 7.12 present two alternatives. These alternatives are unlikely to be used in practice because they do not lead to General Motors and Qantas being free of foreign exchange risk.[10] In Figure 7.11, Qantas bears some foreign exchange risk because it pays 1.1% per annum in AUD and pays 5.2% per annum in USD. In Figure 7.12, General Motors bears some foreign exchange risk because it receives 1.1% per annum in USD and pays 13% per annum in AUD.

7.9 VALUATION OF CURRENCY SWAPS

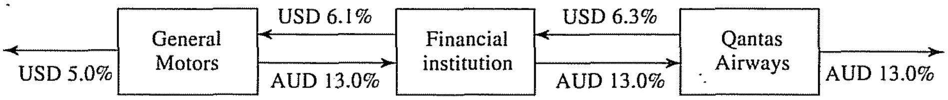
Like interest rate swaps, fixed-for-fixed currency swaps can be decomposed into either the difference between two bonds or a portfolio of forward foreign exchange contracts.

Figure 7.11 Alternative arrangement for currency swap: Qantas Airways bears some foreign exchange risk.



[10] Usually it makes sense for the financial institution to bear the foreign exchange risk, because it is in the best position to hedge the risk.

Figure 7.12 Alternative arrangement for currency swap: General Motors bears some foreign exchange risk.



Valuation in Terms of Bond Prices

If we define V_{swap} as the value in US dollars of an outstanding swap where dollars are received and a foreign currency is paid, then

$$V_{\text{swap}} = B_D - S_0 B_F$$

where B_F is the value, measured in the foreign currency, of the bond defined by the foreign cash flows on the swap and B_D is the value of the bond defined by the domestic cash flows on the swap, and S_0 is the spot exchange rate (expressed as number of dollars per unit of foreign currency). The value of a swap can therefore be determined from LIBOR rates in the two currencies, the term structure of interest rates in the domestic currency, and the spot exchange rate.

Similarly, the value of a swap where the foreign currency is received and dollars are paid is

$$V_{\text{swap}} = S_0 B_F - B_D$$

Example 7.4

Suppose that the term structure of LIBOR/swap interest rates is flat in both Japan and the United States. The Japanese rate is 4% per annum and the US rate is 9% per annum (both with continuous compounding). A financial institution has entered into a currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year. The principals in the two currencies are \$10 million and 1,200 million yen. The swap will last for another 3 years, and the current exchange rate is 110 yen = \$1.

The calculations are summarized in Table 7.9. In this case the cash flows from the dollar bond underlying the swap are as shown in the second column. The

Table 7.9 Valuation of currency swap in terms of bonds. (All amounts in millions.)

Time	*Cash flows on dollar bond (\$)*	*Present value (\$)*	*Cash flows forward on yen bond (yen)*	*Present value (yen)*
1	0.8	0.7311	60	57.65
2	0.8	0.6682	60	55.39
3	0.8	0.6107	60	53.22
3	10.0	7.6338	1,200	1,064.30
Total:		9.6439		1,230.55

present value of the cash flows using the dollar discount rate of 9% are shown in the third column. The cash flows from the yen bond underlying the swap are shown in the fourth column of the table. The present value of the cash flows using the yen discount rate of 4% are shown in the final column of the table.

The value of the dollar bond, B_D, is 9.6439 million dollars. The value of the yen bond is 1230.55 million yen. The value of the swap in dollars is therefore

$$\frac{1{,}230.55}{110} - 9.6439 = 1.5430 \text{ million}$$

Valuation as Portfolio of Forward Contracts

Each exchange of payments in a fixed-for-fixed currency swap is a forward contract. As shown in Section 5.7, forward foreign exchange contracts can be valued by assuming that forward exchange rates are realized. The forward exchange rates themselves can be calculated from equation (5.9).

Example 7.5

Consider again the situation in Example 7.4. The LIBOR/swap term structure of interest rates is flat in both Japan and the United States. The Japanese rate is 4% per annum and the US rate is 9% per annum (both with continuous compounding). A financial institution has entered into a currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year. The principals in the two currencies are \$10 million and 1,200 million yen. The swap will last for another 3 years, and the current exchange rate is 110 yen = \$1.

The calculations are summarized in Table 7.10. The financial institution pays $0.08 \times 10 = \$0.8$ million dollars and receives $1{,}200 \times 0.05 = 60$ million yen each year. In addition, the dollar principal of \$10 million is paid and the yen principal of 1,200 is received at the end of year 3. The current spot rate is 0.009091 dollar per yen. In this case $r = 4\%$ and $r_f = 9\%$, so that, from equation (5.9), the 1-year forward rate is

$$0.009091\, e^{(0.09-0.04)\times 1} = 0.009557$$

The 2- and 3-year forward rates in Table 7.10 are calculated similarly. The forward contracts underlying the swap can be valued by assuming that the forward rates are realized. If the 1-year forward rate is realized, the yen cash flow in year 1

Table 7.10 Valuation of currency swap as a portfolio of forward contracts. (All amounts in millions.)

Time	*Dollar cash flow*	*Yen cash flow*	*Forward rate*	*Dollar value of yen cash flow*	*Net cash flow (\$)*	*Present value*
1	−0.8	60	0.009557	0.5734	−0.2266	−0.2071
2	−0.8	60	0.010047	0.6028	−0.1972	−0.1647
3	−0.8	60	0.010562	0.6337	−0.1663	−0.1269
3	−10.0	1200	0.010562	12.6746	+2.6746	2.0417
Total:						1.5430

is worth $60 \times 0.009557 = 0.5734$ million dollars and the net cash flow at the end of year 1 is $0.8 - 0.5734 = -0.2266$ million dollars. This has a present value of

$$-0.2266\, e^{-0.09\times 1} = -0.2071$$

million dollars. This is the value of forward contract corresponding to the exchange of cash flows at the end of year 1. The value of the other forward contracts are calculated similarly. As shown in Table 7.10, the total value of the forward contracts is \$1.5430 million. This agrees with the value calculated for the swap in Example 7.4 by decomposing it into bonds.

The value of a currency swap is normally zero when it is first negotiated. If the two principals are worth exactly the same using the exchange rate at the start of the swap, the value of the swap is also zero immediately after the initial exchange of principal. However, as in the case of interest rate swaps, this does not mean that each of the individual forward contracts underlying the swap has zero value. It can be shown that, when interest rates in two currencies are significantly different, the payer of the currency with the high interest rate is in the position where the forward contracts corresponding to the early exchanges of cash flows have negative values, and the forward contract corresponding to final exchange of principals has a positive value. (This is the situation in our example in Table 7.10.) The payer of the currency with the low interest rate is likely to be in the opposite position; that is, the early exchanges of cash flows have positive values and the final exchange has a negative value.

For the payer of the low-interest currency, the swap will tend to have a negative value during most of its life. The forward contracts corresponding to the early exchanges of payments have positive values, and once these exchanges have taken place, there is a tendency for the remaining forward contracts to have, in total, a negative value. For the payer of the high-interest currency, the reverse is true. The value of the swap will tend to be positive during most of its life. These results are important when the credit risk in the swap is being evaluated.

7.10 CREDIT RISK

Contracts such as swaps that are private arrangements between two companies entail credit risks. Consider a financial institution that has entered into offsetting contracts with two companies (see Figure 7.4, 7.5, or 7.7). If neither party defaults, the financial institution remains fully hedged. A decline in the value of one contract will always be offset by an increase in the value of the other contract. However, there is a chance that one party will get into financial difficulties and default. The financial institution then still has to honor the contract it has with the other party.

Suppose that, some time after the initiation of the contracts in Figure 7.4, the contract with Microsoft has a positive value to the financial institution, whereas the contract with Intel has a negative value. If Microsoft defaults, the financial institution is liable to lose the whole of the positive value it has in this contract. To maintain a hedged position, it would have to find a third party willing to take Microsoft's position. To induce the third party to take the position, the financial institution would have to pay the third party an amount roughly equal to the value of its contract with Microsoft prior to the default.

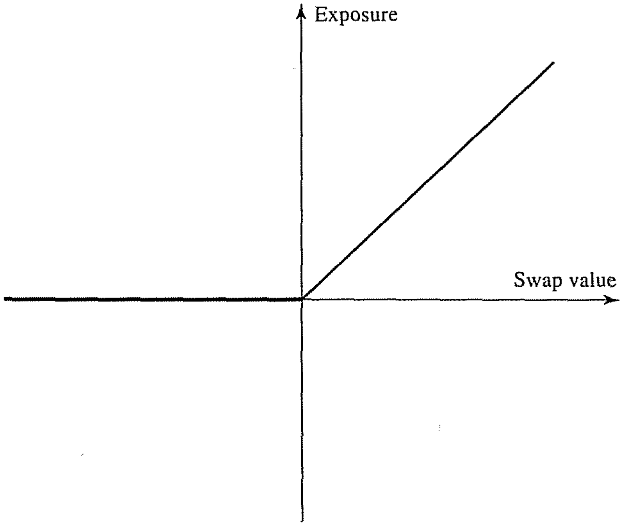
A financial institution has credit-risk exposure from a swap only when the value of the swap to the financial institution is positive. What happens when this value is negative and the counterparty gets into financial difficulties? In theory, the financial institution could realize a windfall gain, because a default would lead to it getting rid of a liability. In practice, it is likely that the counterparty would choose to sell the contract to a third party or rearrange its affairs in some way so that its positive value in the contract is not lost. The most realistic assumption for the financial institution is therefore as follows. If the counterparty goes bankrupt, there will be a loss if the value of the swap to the financial institution is positive, and there will be no effect on the financial institution's position if the value of the swap to the financial institution is negative. This situation is summarized in Figure 7.13.

Potential losses from defaults on a swap are much less than the potential losses from defaults on a loan with the same principal. This is because the value of the swap is usually only a small fraction of the value of the loan. Potential losses from defaults on a currency swap are greater than on an interest rate swap. The reason is that, because principal amounts in two different currencies are exchanged at the end of the life of a currency swap, a currency swap is liable to have a greater value at the time of a default than an interest rate swap.

It is important to distinguish between the credit risk and market risk to a financial institution in any contract. As discussed earlier, the credit risk arises from the possibility of a default by the counterparty when the value of the contract to the financial institution is positive. The market risk arises from the possibility that market variables such as interest rates and exchange rates will move in such a way that the value of a contract to the financial institution becomes negative. Market risks can be hedged by entering into offsetting contracts; credit risks are less easy to hedge.

One of the more bizarre stories in swap markets is outlined in Business Snapshot 7.2. It concerns the British Local Authority, Hammersmith and Fulham and shows that, in

Figure 7.13 The credit exposure in a swap.



Business Snapshot 7.2 The Hammersmith and Fulham Story

Between 1987 to 1989 the London Borough of Hammersmith and Fulham in Great Britain entered into about 600 interest rate swaps and related instruments with a total notional principal of about 6 billion pounds. The transactions appear to have been entered into for speculative rather than hedging purposes. The two employees of Hammersmith and Fulham that were responsible for the trades had only a sketchy understanding of the risks they were taking and how the products they were trading worked.

By 1989, because of movements in sterling interest rates, Hammersmith and Fulham had lost several hundred million pounds on the swaps. To the banks on the other side of the transactions, the swaps were worth several hundred million pounds. The banks were concerned about credit risk. They had entered into offsetting swaps to hedge their interest rate risks. If Hammersmith and Fulham defaulted, the banks would still have to honor their obligations on the offsetting swaps and would take a huge loss.

What happened was something a little different from a default. Hammersmith and Fulham's auditor asked to have the transactions declared void because Hammersmith and Fulham did not have the authority to enter into the transactions. The British courts agreed. The case was appealed and went all the way to the House of Lords, Britain's highest court. The final decision was that Hammersmith and Fulham did not have the authority to enter into the swaps, but that they ought to have the authority to do so in the future for risk-management purposes. Needless to say, banks were furious that their contracts were overturned in this way by the courts.

addition to bearing market risk and credit risk, banks trading swaps also sometimes bear legal risk.

7.11 OTHER TYPES OF SWAPS

In this chapter we have covered interest rate swaps where LIBOR is exchanged for a fixed rate of interest and currency swaps where a fixed rate of interest in one currency is exchanged for a fixed rate of interest in another currency. Many other types of swaps are traded. We will discuss many of them in detail in later chapters, such as in Chapters 21, 26, and 30. At this stage, we will provide an overview.

Variations on the Standard Interest Rate Swap

In fixed-for-floating interest rate swaps, LIBOR is the most common reference floating interest rate. In the examples in this chapter, the tenor (i.e., payment frequency) of LIBOR has been 6 months, but swaps where the tenor of LIBOR is 1 month, 3 months, and 12 months trade regularly. The tenor on the floating side does not have to match the tenor on the fixed side. (Indeed, as pointed out in footnote 3, the standard interest rate swap in the United States is one where there are quarterly LIBOR payments and semiannual fixed payments.) LIBOR is the most common floating rate, but others such as the commercial paper (CP) rate are occasionally used. Sometimes floating-for-floating interest rates swaps are negotiated. For example, the 3-month CP rate plus 10 basis points might be exchanged for 3-month LIBOR with both being applied to the same principal. (This deal would allow a company to hedge its exposure when assets and liabilities are subject to different floating rates.)

The principal in a swap agreement can be varied throughout the term of the swap to meet the needs of a counterparty. In an *amortizing swap*, the principal reduces in a predetermined way. (This might be designed to correspond to the amortization schedule on a loan.) In a *step-up swap*, the principal increases in a predetermined way. (This might be designed to correspond to drawdowns on a loan agreement.) Deferred swaps or *forward swaps*, where the parties do not begin to exchange interest payments until some future date, are also sometimes arranged. Sometimes swaps are negotiated where the principal to which the fixed payments are applied is different from the principal to which the floating payments are applied.

A *constant maturity swap* (CMS swap) is an agreement to exchange a LIBOR rate for a swap rate. An example would be an agreement to exchange 6-month LIBOR applied to a certain principal for the 10-year swap rate applied to the same principal every 6 months for the next 5 years. A *constant maturity Treasury swap* (CMT swap) is a similar agreement to exchange a LIBOR rate for a particular Treasury rate (e.g., the 10-year Treasury rate).

In a *compounding swap*, interest on one or both sides is compounded forward to the end of the life of the swap according to preagreed rules and there is only one payment date at the end of the life of the swap. In a *LIBOR-in arrears* swap, the LIBOR rate observed on a payment date is used to calculate the payment on that date. (As explained in Section 7.1, in a standard deal the LIBOR rate observed on one payment date is used to determine the payment on the next payment date.) In an *accrual swap*, the interest on one side of the swap accrues only when the floating reference rate is in a certain range.

Other Currency Swaps

In this chapter we have considered fixed-for-fixed currency swaps. Another type of swap is a fixed-for-floating currency swap, whereby a floating rate (usually LIBOR) in one currency is exchanged for a fixed rate in another currency. This is a combination of a fixed-for-floating interest rate swap and a fixed-for-fixed currency swap and is known as a *cross-currency interest rate swap*. A further type of currency swap is a *floating-for-floating currency swap*, where a floating rate in one currency is exchanged for a floating rate in another currency.

Sometimes a rate observed in one currency is applied to a principal amount in another currency. One such deal might be where 3-month LIBOR observed in the United States is exchanged for 3-month LIBOR in Britain, with both principals being applied to a principal of 10 million British pounds. This type of swap is referred to as a *diff swap or a quanto.*

Equity Swaps

An *equity swap* is an agreement to exchange the total return (dividends and capital gains) realized on an equity index for either a fixed or a floating rate of interest. For example, the total return on the S&P 500 in successive 6-month periods might be exchanged for LIBOR, with both being applied to the same principal. Equity swaps can be used by portfolio managers to convert returns from a fixed or floating investment to the returns from investing in an equity index, and vice versa.

Options

Sometimes there are options embedded in a swap agreement. For example, in an *extendable swap*, one party has the option to extend the life of the swap beyond the specified period. In a *puttable swap*, one party has the option to terminate the swap early. Options on swaps, or *swaptions*, are also available. These provide one party with the right at a future time to enter into a swap where a predetermined fixed rate is exchanged for floating.

Commodity Swaps, Volatility Swaps, and Other Exotic Instuments

Commodity swaps are in essence a series of forward contracts on a commodity with different maturity dates and the same delivery prices. In a *volatility swap* there are a series of time periods. At the end of each period, one side pays a preagreed volatility, while the other side pays the historical volatility realized during the period. Both volatilities are multiplied by the same notional principal in calculating payments.

Swaps are limited only by the imagination of financial engineers and the desire of corporate treasurers and fund mangers for exotic structures. In Chapter 30, we will describe the famous 5/30 swap entered into between Procter and Gamble and Bankers Trust, where payments depended in a complex way on the 30-day commercial paper rate, a 30-year Treasury bond price, and the yield on a 5-year Treasury bond.

SUMMARY

The two most common types of swaps are interest rate swaps and currency swaps. In an interest rate swap, one party agrees to pay the other party interest at a fixed rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time. In a currency swap, one party agrees to pay interest on a principal amount in one currency. In return, it receives interest on a principal amount in another currency.

Principal amounts are not usually exchanged in an interest rate swap. In a currency swap, principal amounts are usually exchanged at both the beginning and the end of the life of the swap. For a party paying interest in the foreign currency, the foreign principal is received, and the domestic principal is paid at the beginning of the life of the swap. At the end of the life of the swap, the foreign principal is paid and the domestic principal is received.

An interest rate swap can be used to transform a floating-rate loan into a fixed-rate loan, or vice versa. It can also be used to transform a floating-rate investment to a fixed-rate investment, or vice versa. A currency swap can be used to transform a loan in one currency into a loan in another currency. It can also be used to transform an investment denominated in one currency into an investment denominated in another currency.

There are two ways of valuing interest rate and currency swaps. In the first, the swap is decomposed into a long position in one bond and a short position in another bond. In the second it is regarded as a portfolio of forward contracts.

When a financial institution enters into a pair of offsetting swaps with different counterparties, it is exposed to credit risk. If one of the counterparties defaults when the financial institution has positive value in its swap with that counterparty, the financial institution loses money because it still has to honor its swap agreement with the other counterparty.

FURTHER READING

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Questions and Problems (Answers in Solutions Manual)

7.1. Companies A and B have been offered the following rates per annum on a \$20 million 5-year loan:

	Fixed rate	*Floating rate*
Company A:	12.0%	LIBOR + 0.1%
Company B:	13.4%	LIBOR + 0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

7.2. Company X wishes to borrow US dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes:

	Yen	*Dollars*
Company X:	5.0%	9.6%
Company Y:	6.5%	10.0%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

7.3. A $100 million interest rate swap has a remaining life of 10 months. Under the terms of the swap, 6-month LIBOR is exchanged for 12% per annum (compounded semiannually). The average of the bid–offer rate being exchanged for 6-month LIBOR in swaps of all maturities is currently 10% per annum with continuous compounding. The 6-month LIBOR rate was 9.6% per annum 2 months ago. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed?

7.4. Explain what a swap rate is. What is the relationship between swap rates and par yields?

7.5. A currency swap has a remaining life of 15 months. It involves exchanging interest at 14% on £20 million for interest at 10% on $30 million once a year. The term structure of interest rates in both the United Kingdom and the United States is currently flat, and if the swap were negotiated today the interest rates exchanged would be 8% in dollars and 11% in sterling. All interest rates are quoted with annual compounding. The current exchange rate (dollars per pound sterling) is 1.6500. What is the value of the swap to the party paying sterling? What is the value of the swap to the party paying dollars?

7.6. Explain the difference between the credit risk and the market risk in a financial contract.

7.7. A corporate treasurer tells you that he has just negotiated a 5-year loan at a competitive fixed rate of interest of 5.2%. The treasurer explains that he achieved the 5.2% rate by borrowing at 6-month LIBOR plus 150 basis points and swapping LIBOR for 3.7%. He goes on to say that this was possible because his company has a comparative advantage in the floating-rate market. What has the treasurer overlooked?

7.8. Explain why a bank is subject to credit risk when it enters into two offsetting swap contracts.

7.9. Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment:

	Fixed rate	*Floating rate*
Company X:	8.0%	LIBOR
Company Y:	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

7.10. A financial institution has entered into an interest rate swap with company X. Under the terms of the swap, it receives 10% per annum and pays 6-month LIBOR on a principal of $10 million for 5 years. Payments are made every 6 months. Suppose that company X defaults on the sixth payment date (at the end of year 3) when the interest rate (with semiannual compounding) is 8% per annum for all maturities. What is the loss to the financial institution? Assume that 6-month LIBOR was 9% per annum halfway through year 3.

7.11. A financial institution has entered into a 10-year currency swap with company Y. Under the terms of the swap, the financial institution receives interest at 3% per annum in Swiss francs and pays interest at 8% per annum in US dollars. Interest payments are exchanged once a year. The principal amounts are 7 million dollars and 10 million francs. Suppose that company Y declares bankruptcy at the end of year 6, when the exchange rate is \$0.80 per franc. What is the cost to the financial institution? Assume that, at the end of year 6, the interest rate is 3% per annum in Swiss francs and 8% per annum in US dollars for all maturities. All interest rates are quoted with annual compounding.

7.12. Companies A and B face the following interest rates (adjusted for the differential impact of taxes):

	Company A	*Company B*
US dollars (floating rate):	LIBOR + 0.5%	LIBOR + 1.0%
Canadian dollars (fixed rate):	5.0%	6.5%

Assume that A wants to borrow US dollars at a floating rate of interest and B wants to borrow Canadian dollars at a fixed rate of interest. A financial institution is planning to arrange a swap and requires a 50-basis-point spread. If the swap is equally attractive to A and B, what rates of interest will A and B end up paying?

7.13. After it hedges its foreign exchange risk using forward contracts, is the financial institution's average spread in Figure 7.10 likely to be greater than or less than 20 basis points? Explain your answer.

7.14. "Companies with high credit risks are the ones that cannot access fixed-rate markets directly. They are the companies that are most likely to be paying fixed and receiving floating in an interest rate swap." Assume that this statement is true. Do you think it increases or decreases the risk of a financial institution's swap portfolio? Assume that companies are most likely to default when interest rates are high.

7.15. Why is the expected loss from a default on a swap less than the expected loss from the default on a loan with the same principal?

7.16. A bank finds that its assets are not matched with its liabilities. It is taking floating-rate deposits and making fixed-rate loans. How can swaps be used to offset the risk?

7.17. Explain how you would value a swap that is the exchange of a floating rate in one currency for a fixed rate in another currency.

7.18. The LIBOR zero curve is flat at 5% (continuously compounded) out to 1.5 years. Swap rates for 2- and 3-year semiannual pay swaps are 5.4% and 5.6%, respectively. Estimate the LIBOR zero rates for maturities of 2.0, 2.5, and 3.0 years. (Assume that the 2.5-year swap rate is the average of the 2- and 3-year swap rates.)

Assignment Questions

7.19. The 1-year LIBOR rate is 10%. A bank trades swaps where a fixed rate of interest is exchanged for 12-month LIBOR with payments being exchanged annually. The 2- and 3-year swap rates (expressed with annual compounding) are 11% and 12% per annum. Estimate the 2- and 3-year LIBOR zero rates.

7.20. Company A, a British manufacturer, wishes to borrow US dollars at a fixed rate of interest. Company B, a US multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following rates per annum (adjusted for differential tax effects):

	Sterling	*US dollars*
Company A	11.0%	7.0%
Company B	10.6%	6.2%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 15 basis points per annum for each of the two companies.

7.21. Under the terms of an interest rate swap, a financial institution has agreed to pay 10% per annum and to receive 3-month LIBOR in return on a notional principal of $100 million with payments being exchanged every 3 months. The swap has a remaining life of 14 months. The average of the bid and offer fixed rates currently being swapped for 3-month LIBOR is 12% per annum for all maturities. The 3-month LIBOR rate 1 month ago was 11.8% per annum. All rates are compounded quarterly. What is the value of the swap?

7.22. Suppose that the term structure of interest rates is flat in the United States and Australia. The USD interest rate is 7% per annum and the AUD rate is 9% per annum. The current value of the AUD is 0.62 USD. Under the terms of a swap agreement, a financial institution pays 8% per annum in AUD and receives 4% per annum in USD. The principals in the two currencies are $12 million USD and 20 million AUD. Payments are exchanged every year, with one exchange having just taken place. The swap will last 2 more years. What is the value of the swap to the financial institution? Assume all interest rates are continuously compounded.

7.23. Company X is based in the United Kingdom and would like to borrow $50 million at a fixed rate of interest for 5 years in US funds. Because the company is not well known in the United States, this has proved to be impossible. However, the company has been quoted 12% per annum on fixed-rate 5-year sterling funds. Company Y is based in the United States and would like to borrow the equivalent of $50 million in sterling funds for 5 years at a fixed rate of interest. It has been unable to get a quote but has been offered US dollar funds at 10.5% per annum. Five-year government bonds currently yield 9.5% per annum in the United States and 10.5% in the United Kingdom. Suggest an appropriate currency swap that will net the financial intermediary 0.5% per annum.