CHAPTER 20 Credit Risk

The value-at-risk measure we covered in Chapter 18 and the Greek letters we studied in Chapter 15 are aimed at quantifying market risk. In this chapter we consider another important risk for financial institutions: credit risk. Most financial institutions devote considerable resources to the measurement and management of credit risk. Regulators have for many years required banks to keep capital to reflect the credit risks they are bearing. (This capital is in addition to the capital, described in Business Snapshot 18.1, that they are required to keep for market risk.)

Credit risk arises from the possibility that borrowers and counterparties in derivatives transactions may default. In this chapter we focus on the quantification of credit risk. We discuss a number of different approaches to estimating the probability that a company will default and explain the key difference between risk-neutral and real-world probabilities of default. We examine the nature of the credit risk in over-the-counter derivatives transactions and discuss the clauses derivatives dealers write into their contracts to reduce credit risk. Finally we cover default correlation, Gaussian copula models, and the estimation of credit value at risk.

Chapter 21 will discuss credit derivatives and show how ideas introduced in this chapter can be used to value these instruments.

20.1 CREDIT RATINGS

Rating agencies such as Moody's and S&P are in the business of providing ratings describing the creditworthiness of corporate bonds. Using the Moody's system, the best rating is Aaa. Bonds with this rating are considered to have almost no chance of defaulting. The next best rating is Aa. Following that comes A, Baa, Ba, B, and Caa. Only bonds with ratings of Baa or above are considered to be *investment grade*. The S&P ratings corresponding to Moody's Aaa, Aa, A, Baa, Ba, B, and Caa are AAA, AA, A, BBB, BB, B, and CCC, respectively. To create finer rating measures, Moody's divides the Aa rating category into Aa1, Aa2, and Aa3; it divides A into A1, A2 and A3; and so on. Similarly S&P divides its AA rating category into AA+, AA, and AA−; it divides its A rating category into A+, A, and A−; and so on. (Only the Aaa category for Moody's and the AAA category for S&P are not subdivided.)

20.2 HISTORICAL DEFAULT PROBABILITIES

Table 20.1 is typical of the data that is produced by rating agencies. It shows the default experience through time of companies that started with a certain credit rating. For example, Table 20.1 shows that a bond issue with an initial credit rating of Baa has a 0.20% chance of defaulting by the end of the first year, a 0.57% chance of defaulting by the end of the second year, and so on. The probability of a bond defaulting during a particular year can be calculated from the table. For example, the probability that a bond initially rated Baa will default during the second year of its life is $0.57 - 0.20 = 0.37\%$.

Table 20.1 shows that, for investment grade bonds, the probability of default in a year tends to be an increasing function of time (e.g., the probabilities of an A-rated bond defaulting during years 1, 2, 3, 4, and 5 are 0.02%, 0.07%, 0.14%, 0.15%, and 0.16%, respectively). This is because the bond issuer is initially considered to be creditworthy, and the more time that elapses, the greater the possibility that its financial health will decline. For bonds with a poor credit rating, the probability of default is often a decreasing function of time (e.g., the probabilities that a Caa-rated bond will default during years 1, 2, 3, 4, and 5 are 23.65%, 13.55%, 10.82%, 7.54%, and 5.27%, respectively). The reason here is that, for a bond with a poor credit rating, the next year or two may be critical. If the issuer survives this period, its financial health is likely to have improved.

Default Intensities

From Table 20.1 we can calculate the probability of a Caa bond defaulting during the third year as $48.02 - 37.20 = 10.82\%$. We will refer to this as the *unconditional default probability*. It is the probability of default during the third year as seen at time 0. The probability that the Caa-rated bond will survive until the end of year 2 is $100 - 37.20 = 62.80\%$. The probability that it will default during the third year conditional on no earlier default is therefore 0.1082/0.6280, or 17.23%. Conditional default probabilities are referred to as *default intensities* or *hazard rates*.

The 17.23% we have just calculated is for a 1-year time period. Suppose instead that we consider a short time period of length Δt. The default intensity $\lambda(t)$ at time t is then defined so that $\lambda(t)\,\Delta t$ is the probability of default between time t and $t + \Delta t$ conditional on no earlier default. If $V(t)$ is the cumulative probability of the company surviving to

Table 20.1 Average cumulative default rates (%), 1970–2003. (Source: Moody's)

Term (years)	*1*	*2*	*3*	*4*	*5*	*7*	*10*	*15*	*20*
Aaa	0.00	0.00	0.00	0.04	0.12	0.29	0.62	1.21	1.55
Aa	0.02	0.03	0.06	0.15	0.24	0.43	0.68	1.51	2.70
A	0.02	0.09	0.23	0.38	0.54	0.91	1.59	2.94	5.24
Baa	0.20	0.57	1.03	1.62	2.16	3.24	5.10	9.12	12.59
Ba	1.26	3.48	6.00	8.59	11.17	15.44	21.01	30.88	38.56
B	6.21	13.76	20.65	26.66	31.99	40.79	50.02	59.21	60.73
Caa	23.65	37.20	48.02	55.56	60.83	69.36	77.91	80.23	80.23

time t (i.e., no default by time t), then

$$V(t+\Delta t) - V(t) = -\lambda(t)V(t)\,\Delta t$$

Taking limits

$$\frac{dV(t)}{dt} = -\lambda(t)V(t)$$

from which we get

$$V(t) = e^{-\int_0^t \lambda(\tau)d\tau}$$

Define $Q(t)$ as the probability of default by time t. It follows that

$$Q(t) = 1 - e^{-\int_0^t \lambda(\tau)d\tau}$$

or

$$Q(t) = 1 - e^{\bar{\lambda}(t)t} \tag{20.1}$$

where $\bar{\lambda}(t)$ is the average default intensity between time 0 and time t.

20.3 RECOVERY RATES

When a company goes bankrupt, those that are owed money by the company file claims against the assets of the company.[1] Sometimes there is a reorganization in which these creditors agree to a partial payment of their claims. In other cases the assets are sold by the liquidator and the proceeds are used to meet the claims as far as possible. Some claims typically have priorities over other claims and are met more fully.

The recovery rate for a bond is normally defined as the bond's market value immediately after a default, as a percent of its face value. Table 20.2 provides historical data on average recovery rates for different categories of bonds in the United States. It shows that senior secured debt holders had an average recovery rate of 51.6 cents per dollar of face value while junior subordinated debt holders had an average recovery rate of only 24.5 cents per dollar of face value.

Recovery rates are significantly negatively correlated with default rates. Moody's looked at average recovery rates and average default rates each year between 1982

Table 20.2 Recovery rates on corporate bonds as a percentage of face value, 1982–2003. (Source: Moody's)

Class	*Average recovery rate (%)*
Senior secured	51.6
Senior unsecured	36.1
Senior subordinated	32.5
Subordinated	31.1
Junior subordinated	24.5

[1] In the United States, the claim made by a bondholder is the bond's face value plus accrued interest.

and 2003. It found that the following relationship provides a good fit to the data:[2]

$$\text{Average recovery rate} = 50.3 - 6.3 \times \text{Average default rate}$$

where both the average recovery rate and the average default rate are measured as percentages.

20.4 ESTIMATING DEFAULT PROBABILITIES FROM BOND PRICES

The probability of default for a company can be estimated from the prices of bonds it has issued. The usual assumption is that the only reason a corporate bond sells for less than a similar risk-free bond is the possibility of default.[3]

Consider first an approximate calculation. Suppose that a bond yields 200 basis points more than a similar risk-free bond and that the expected recovery rate in the event of a default is 40%. The holder of a corporate bond must be expecting to lose 200 basis points (or 2% per year) from defaults. Given the recovery rate of 40%, this leads to an estimate of the probability of a default per year conditional on no earlier default of $0.02/(1 - 0.4)$, or 3.33%. In general,

$$h = \frac{s}{1 - R} \tag{20.2}$$

where h is the default intensity per year, s is the spread of the corporate bond yield over the risk-free rate, and R is the expected recovery rate.

A More Exact Calculation

For a more exact calculation, suppose that the corporate bond we have been considering lasts for 5 years, provides a coupon 6% per annum (paid semiannually) and that the yield on the corporate bond is 7% per annum (with continuous compounding). The yield on a similar risk-free bond is 5% (with continuous compounding). The yields imply that the price of the corporate bond is 95.34 and the price of the risk-free bond is 104.09. The expected loss from default over the 5-year life of the bond is therefore $104.09 - 95.34$, or \$8.75. Suppose that the probability of default per year (assumed in this simple example to be the same each year) is Q. Table 20.3 calculates the expected loss from default in terms of Q on the assumption that defaults can happen at times 0.5, 1.5, 2.5, 3.5, and 4.5 years (immediately before coupon payment dates). Risk-free rates for all maturities are assumed to be 5% (with continuous compounding).

To illustrate the calculations, consider the 3.5 year row in Table 20.2. The expected value of the risk-free bond at time 3.5 years (calculated using forward interest rates) is

$$3 + 3e^{-0.05\times0.5} + 3e^{-0.05\times1.0} + 103e^{-0.05\times1.5} = 104.34$$

[2] See D. T. Hamilton, P. Varma, S. Ou, and R. Cantor, "Default and Recovery Rates of Corporate Bond Issuers," Moody's Investor's Services, January 2004. The R^2 of the regression is 0.6. The correlation is also identified and discussed in E. I. Altman, B. Brady, A. Resti, and A. Sironi, "The Link between Default and Recovery Rates: Implications for Credit Risk Models and Procyclicality," Working Paper, New York University, 2003.

[3] This assumption is not perfect. In practice the price of a corporate bond is affected by its liquidity. The lower the liquidity, the lower the price.

Table 20.3 Calculation of loss from default on a bond in terms of the default probabilities per year, Q. Notional principal = \$100.

Time (years)	*Default probability*	*Recovery amount (\$)*	*Risk-free value (\$)*	*Loss given default (\$)*	*Discount factor*	*PV of expected loss (\$)*
0.5	Q	40	106.73	66.73	0.9753	$65.08Q$
1.5	Q	40	105.97	65.97	0.9277	$61.20Q$
2.5	Q	40	105.17	65.17	0.8825	$57.52Q$
3.5	Q	40	104.34	64.34	0.8395	$54.01Q$
4.5	Q	40	103.46	63.46	0.7985	$50.67Q$
Total						$288.48Q$

Given the definition of recovery rates in the previous section, the amount recovered if there is a default is 40, so that the loss given default is 104.34 − 40, or \$64.34. The present value of this loss is 54.01. The expected loss is therefore $54.01Q$.

The total expected loss is $288.48Q$. Setting this equal to 8.75, we obtain a value for Q equal to 3.03%. The calculations we have given assume that the default probability is the same in each year and that defaults take place at just one time during the year. We can extend the calculations to assume that defaults can take place more frequently. Also, instead of assuming a constant unconditional probability of default we can assume a constant default intensity or assume a particular pattern for the variation of default probabilities with time. With several bonds we can estimate several parameters describing the term structure of default probabilities. Suppose, for example, we have bonds maturing in 3, 5, 7, and 10 years. We could use the first bond to estimate a default probability per year for the first 3 years, the second bond to estimate default probability per year for years 4 and 5, the third bond to estimate a default probability for years 6 and 7, and the fourth bond to estimate a default probability for years 8, 9, and 10 (see Problems 20.15 and 20.27). The approach is analogous to the bootstrap procedure in Section 4.5 for calculating a zero-coupon yield curve.

The Risk-Free Rate

A key issue when bond prices are used to estimate default probabilities is the meaning of the terms "risk-free rate" and "risk-free bond". In equation (20.2), the spread s is the excess of the corporate bond yield over the yield on a similar risk-free bond. In Table 20.3, the risk-free value of the bond must be calculated using the risk-free rate. The benchmark risk-free rate that is usually used in quoting corporate bond yields is the yield on similar Treasury bonds. (For example, a bond trader might quote the yield on a particular corporate bond as being a spread of 250 basis points over Treasuries.)

As discussed in Section 4.1, traders usually use LIBOR/swap rates as proxies for risk-free rates when valuing derivatives. Traders also often use LIBOR/swap rates as risk-free rates when calculating default probabilities. For example, when they determine default probabilities from bond prices, the spread s in equation (20.2) is the spread of the bond yield over the LIBOR/swap rate. Also, the risk-free discount rates used in the calculations in Table 20.3 are LIBOR/swap zero rates.

Credit default swaps (which will be discussed in the next chapter) can be used to imply the risk-free rate assumed by traders. The rate used appears to be approximately equal to the LIBOR/swap rate minus 10 basis points on average.[4] This estimate is plausible. As explained in Section 7.5, the credit risk in a swap rate is the credit risk from making a series of 6-month loans to AA-rated counterparties and 10 basis points is a reasonable default risk premium for a AA-rated 6-month instrument.

Asset Swaps

In practice, traders often use asset swap spreads as a way of extracting default probabilities from bond prices. This is because asset swap spreads provide a direct estimate of the spread of bond yields over the LIBOR/swap curve.

To explain how asset swaps work, consider the situation where an asset swap spread for a particular bond is quoted as 150 basis points. There are three possible situations:

1. The bond sells for its par value of 100. The swap then involves one side (company A) paying the coupon on the bond and the other side (company B) paying LIBOR plus 150 basis points.[5]
2. The bond sells below its par value, say, for 95. The swap is then structured so that in addition to the coupons company A pays \$5 per \$100 of notional principal at the outset.
3. The underlying bond sells above par, say, for 108. Company B would then make a payment of \$8 per \$100 of principal at the outset.

The effect of all this is that the present value of the asset swap spread is the amount by which the price of the corporate bond is exceeded by the price of a similar risk-free bond where the risk-free rate is assumed to be given by the LIBOR/swap curve (see Problem 20.24). Consider again the example in Table 20.3 where the LIBOR/swap zero curve is flat at 5%. Suppose that instead of knowing the bond's price we know that the asset swap spread is 150 basis points. This means that the amount by which the value of the risk-free bond exceeds the value of the corporate bond is the present value of 150 basis points per year for 5 years. Assuming semiannual payments, this is \$6.55 per \$100 of principal.

The total loss in Table 20.3 would in this case be set equal to \$6.55. This means that the default probability per year, Q, would be 6.55/288.48, or 2.27%.

20.5 COMPARISON OF DEFAULT PROBABILITY ESTIMATES

The default probabilities estimated from historical data are much less than those derived from bond prices. Table 20.4 illustrates this.[6] It shows, for companies that start

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[4] See J. Hull, M. Predescu, and A. White, "The Relationship between Credit Default Swap Spreads, Bond Yields, and Credit Rating Announcements," *Journal of Banking and Finance*, 28 (November 2004): 2789-2811.

[5] Note that it is the promised coupons that are exchanged. The exchanges take place regardless of whether the bond defaults.

[6] Tables 20.4 and 20.5 are taken from J. Hull, M. Predescu, and A. White, "Bond Prices, Default Probabilities, and Risk Premiums" *Journal of Credit Risk*, forthcoming.

with a particular rating, the average annual default intensity over 7 years calculated from (a) historical data and (b) bond prices.

The calculation of default intensities using historical data are based on equation (20.1) and Table 20.1. From equation (20.1), we have

$$\bar{\lambda}(7) = -\tfrac{1}{7}\ln[1 - Q(7)]$$

where $\bar{\lambda}(t)$ is the average default intensity (or hazard rate) by time t and $Q(t)$ is the cumulative probability of default by time t. The values of $Q(7)$ are taken directly from Table 20.1. Consider, for example, an A-rated company. The value of $Q(7)$ is 0.0091. The average 7-year default intensity is therefore

$$\bar{\lambda}(7) = -\tfrac{1}{7}\ln(0.9909) = 0.0013$$

or 0.13%.

The calculations using bond prices are based on equation (20.2) and bond yields published by Merrill Lynch. The results shown are averages between December 1996 and July 2004. The recovery rate is assumed to be 40% and, for the reasons discussed in the previous section, the risk-free interest rate is assumed to be the 7-year swap rate minus 10 basis points. For example, for A-rated bonds the average Merrill Lynch yield was 6.274%. The average swap rate was 5.605%, so that the average risk-free rate was 5.505%. This gives the average 7-year default probability as

$$\frac{0.06274 - 0.05505}{1 - 0.4} = 0.0128$$

or 1.28%.

Table 20.4 shows that the ratio of the default probability backed out of bond prices to the default probability calculated from historical data tends to decline as the credit quality declines with the ratio very high for investment grade companies. The difference between the two default probabilities tends to increase as credit quality declines.

Table 20.5 provides another way of looking at these results. It shows the excess return over the risk-free rate (still assumed to be the 7-year swap rate minus 10 basis points) earned by investors in bonds with different credit rating. Consider again an A-rated bond. The average spread over Treasuries is 120 basis points. Of this, 43 basis points are

Table 20.4 Seven-year average default intensities (% per annum).

Rating	*Historical default intensity*	*Default intensity from bonds*	*Ratio*	*Difference*
Aaa	0.04	0.67	16.8	0.63
Aa	0.06	0.78	13.0	0.72
A	0.13	1.28	9.8	1.15
Baa	0.47	2.38	5.1	1.91
Ba	2.47	5.07	2.1	2.67
B	7.69	9.02	1.2	1.33
Caa	16.90	21.30	1.3	4.40

Table 20.5 Expected excess return on bonds (basis points).

Rating	*Bond yield spread over Treasuries*	*Spread of risk-free rate over Treasuries*	*Spread for historical defaults*	*Expected excess return*
Aaa	83	43	2	38
Aa	90	43	4	43
A	120	43	8	69
Baa	186	43	28	115
Ba	347	43	144	160
B	585	43	449	93
Caa	1321	43	1014	264

accounted for by the average spread between 7-year Treasuries and our proxy for the risk-free rate. A spread of 8 basis points is necessary to cover expected defaults. (This equals the real-world probability of default from Table 20.4 times 1 minus the assumed recovery rate of 0.4.) This leaves an expected excess return (after expected defaults have been taken into account) of 69 basis points.

Tables 20.4 and 20.5 show that a large percentage difference between default probability estimates translates into a small (but significant) expected excess return on the bond. For Aaa-rated bonds the ratio of the two default probabilities is 16.8, but the expected excess return is only 38 basis points. The expected return tends to increase as credit quality declines.[7]

Real-World vs. Risk-Neutral Probabilities

The default probabilities implied from bond yields are risk-neutral probabilities of default. To explain why this is so, consider the calculations of default probabilities in Table 20.3. The calculations assume that expected default losses can be discounted at the risk-free rate. The risk-neutral valuation principle shows that this is a valid procedure providing the expected losses are calculated in a risk-neutral world. This means that the default probability Q in Table 20.3 must be a risk-neutral probability.

By contrast, the default probabilities implied from historical data are real-world default probabilities (sometimes also called *physical probabilities*). The expected excess return in Table 20.5 arises directly from the difference between real-world and risk-neutral default probabilities. If there were no expected excess return, then the real-world and risk-neutral default probabilities would be the same, and vice versa.

Why do we see such big differences between real-world and risk-neutral default probabilities? As we have just argued, this is the same as asking why corporate bond traders earn more than the risk-free rate on average. There are a number of potential reasons:

1. Corporate bonds are relatively illiquid and bond traders demand an extra return to compensate for this.

[7] The results for B-rated bonds in Tables 20.4 and 20.5 run counter to the overall pattern.

2. The subjective default probabilities of bond traders may be much higher than the those given in Tables 20.1. Bond traders may be allowing for depression scenarios much worse than anything seen during the period from 1970 to 2003.[8]
3. Bonds do not default independently of each other. This is the most important reason for the results in Tables 20.4 and 20.5. There are periods of time when default rates are very low and periods of time when they are very high.[9] This gives rise to systematic risk (i.e., risk that cannot be diversified away) and bond traders should require an expected excess return for bearing the risk. The variation in default rates from year to year may be due to overall economic conditions or it may be because a default by one company has a ripple effect resulting in defaults by other companies. (The latter is referred to by researchers as *credit contagion.*)
4. Bond returns are highly skewed with limited upside. As a result it is much more difficult to diversify risks in a bond portfolio than in an equity portfolio.[10] A very large number of different bonds must be held. In practice, many bond portfolios are far from fully diversified. As a result bond traders may require an extra return for bearing unsystematic risk in addition to the systematic risk mentioned above.

At this stage it is natural to ask whether we should use real-world or risk-neutral default probabilities in the analysis of credit risk. The answer depends on the purpose of the analysis. When valuing credit derivatives or estimating the impact of default risk on the pricing of instruments we should use risk-neutral default probabilities. This is because the analysis calculates the present value of expected future cash flows and almost invariably (implicitly or explicitly) involves using risk-neutral valuation. When carrying out scenario analyses to calculate potential future losses from defaults, we should use real-world default probabilities.

20.6 USING EQUITY PRICES TO ESTIMATE DEFAULT PROBABILITIES

When we use a table such as Table 20.1 to estimate a company's real-world probability of default, we are relying on the company's credit rating. Unfortunately, credit ratings are revised relatively infrequently. This has led some analysts to argue that equity prices can provide more up-to-date information for estimating default probabilities.

In 1974, Merton proposed a model where a company's equity is an option on the assets of the company.[11] Suppose, for simplicity, that a firm has one zero-coupon bond

[8] In addition to producing Table 20.1, which is based on the 1970 to 2003 period, Moody's produces a similar table based on the 1920 to 2003 period. When this table is used, historical default intensities for investment grade bonds in Table 20.4 rise somewhat. The Aaa default intensity increases from 4 to 6 basis points; the Aa increases from 6 to 22 basis points; the A increases from from 13 to 29 basis points; the Baa increases from 46 to 73 basis points.

[9] Evidence for this can be obtained by looking at the defaults rates in different years. Moody's statistics show that between 1970 and 2003 the default rate per year ranged from a low 0.09% in 1979 to a high of 3.81% in 2001.

[10] See J. D. Amato and E. M. Remolona, "The Credit Spread Puzzle," *BIS Quarterly Review*, 5, December 2003: 51–63.

[11] See R. Merton "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29 (1974): 449–70.

outstanding and that the bond matures at time T. Define:

V_0: Value of company's assets today
V_T: Value of company's assets at time T
E_0: Value of company's equity today
E_T: Value of company's equity at time T
D: Amount of debt interest and principal due to be repaid at time T
σ_V: Volatility of assets (assumed constant)
σ_E: Instantaneous volatility of equity.

If $V_T < D$, it is (at least in theory) rational for the company to default on the debt at time T. The value of the equity is then zero. If $V_T > D$, the company should make the debt repayment at time T and the value of the equity at this time is $V_T - D$. Merton's model, therefore, gives the value of the firm's equity at time T as

$$E_T = \max(V_T - D, 0)$$

This shows that the equity is a call option on the value of the assets with a strike price equal to the repayment required on the debt. The Black–Scholes formula gives the value of the equity today as

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2) \tag{20.3}$$

where

$$d_1 = \frac{\ln V_0/D + (r + \sigma_V^2/2)T}{\sigma_V\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma_V\sqrt{T}$$

The value of the debt today is $V_0 - E_0$.

The risk-neutral probability that the company will default on the debt is $N(-d_2)$. To calculate this, we require V_0 and σ_V. Neither of these are directly observable. However, if the company is publicly traded, we can observe E_0. This means that equation (20.3) provides one condition that must be satisfied by V_0 and σ_V. We can also estimate σ_E. From Itô's lemma,

$$\sigma_E E_0 = \frac{\partial E}{\partial V}\sigma_V V_0$$

or

$$\sigma_E E_0 = N(d_1)\sigma_V V_0 \tag{20.4}$$

This provides another equation that must be satisfied by V_0 and σ_V. Equations (20.3) and (20.4) provide a pair of simultaneous equations that can be solved for V_0 and σ_V.[12]

Example 20.1

The value of a company's equity is \$3 million and the volatility of the equity is 80%. The debt that will have to be paid in 1 year is \$10 million. The risk-free rate is 5% per annum. In this case $E_0 = 3$, $\sigma_E = 0.80$, $r = 0.05$, $T = 1$, and $D = 10$. Solving equations (20.3) and (20.4) yields $V_0 = 12.40$ and $\sigma_V = 0.2123$. The parameter, d_2 is 1.1408, so that the probability of default is $N(-d_2) = 0.127$, or 12.7%. The market value of the debt is $V_0 - E_0$, or 9.40. The present value of the promised payment on the debt is $10e^{-0.05\times 1} = 9.51$. The expected loss on the

[12] To solve two nonlinear equations of the form $F(x, y) = 0$ and $G(x, y) = 0$, we can use the Solver routine in Excel to find the values of x and y that minimize $[F(x, y)]^2 + [G(x, y)]^2$.

debt is therefore (9.51 − 9.40)/9.51, or about 1.2% of its no-default value. Comparing this with the probability of default gives the expected recovery in the event of a default as (12.7 − 1.2)/12.7, or about 91%.

The basic Merton model we have just presented has been extended in a number of ways. For example, one version of the model assumes that a default occurs whenever the value of the assets falls below a barrier level.

How well do the default probabilities produced by Merton's model and its extensions correspond to actual default experience? The answer is that Merton's model and its extensions produce a good ranking of default probabilities (risk-neutral or real-world). This means that monotonic transformation can be used to convert the probability of default output from Merton's model into a good estimate of either the real-world or risk-neutral default probability.[13]

20.7 CREDIT RISK IN DERIVATIVES TRANSACTIONS

The credit exposure on a derivatives transaction is more complicated than that on a loan. This is because the claim that will be made in the event of a default is more uncertain. Consider a financial institution that has one derivatives contract outstanding with a counterparty. We can distinguish three possible situations:

1. Contract is always a liability to the financial institution
2. Contract is always an asset to the financial institution
3. Contract can become either an asset or a liability to the financial institution

An example of a derivatives contract in the first category is a short option position; an example in the second category is a long option position; an example in the third category is a forward contract.

Derivatives in the first category have no credit risk to the financial institution. If the counterparty goes bankrupt, there will be no loss. The derivative is one of the counterparty's assets. It is likely to be retained, closed out, or sold to a third party. The result is no loss (or gain) to the financial institution.

Derivatives in the second category always have credit risk to the financial institution. If the counterparty goes bankrupt, a loss is likely to be experienced. The derivative is one of the counterparty's liabilities. The financial institution has to make a claim against the assets of the counterparty and may receive some percentage of the value of the derivative.

Derivatives in the third category may or may not have credit risk. If the counterparty defaults when the value of the derivative is positive to the financial institution, a claim will be made against the assets of the counterparty and a loss is likely to be experienced. If the counterparty defaults when the value is negative to the financial institution, no loss is made because the derivative will be retained, closed out, or sold to a third party.[14]

[13] Moody's KMV provides a service that transforms a default probability produced by Merton's model into a real-world default probability (which it refers to as an EDF, short for expected default frequency). CreditGrades use Merton's model to estimate credit spreads, which are closely linked to risk-neutral default probabilities.

[14] Note that a company usually defaults because of the total value of its assets and liabilities, not because of the value of any one transaction.

Adjusting Derivatives' Valuations for Counterparty Default Risk

How should a financial institution (or end-user of derivatives) adjust the value of a derivative to allow for counterparty credit risk? Consider a derivative that has a value of f_0 today assuming no defaults. Let us suppose that defaults can take place at times $t_1, t_2, \dots, t_n$ and that the value of the derivative to the financial institution (assuming no defaults) at time t_i is f_i. Define the risk-neutral probability of default at time t_i as q_i and the expected recovery rate as R.[15]

The exposure at time t_i is the financial institution's potential loss. This is $\max(f_i, 0)$. Assume that the expected recovery in the event of a default is R times the exposure. Assume also that the recovery rate and the probability of default is independent of the value of the derivative. The risk-neutral expected loss from default at time t_i is

$$q_i(1-R)\hat{E}[\max(f_i, 0)]$$

where $\hat{E}$ denotes expected value in a risk-neutral world. Taking present values leads to the cost of defaults being

$$\sum_{i=1}^{n} u_i v_i \tag{20.5}$$

where u_i equals $q_i(1-R)$ and v_i is the value today of an instrument that pays off the exposure on the derivative under consideration at time t_i.

Consider again the three categories of derivatives mentioned earlier. The first category (where the derivative is always a liability to the financial institution) is easy to deal with. The value of f_i is always negative and so the total expected loss from defaults given by equation (20.5) is always zero. The financial institution needs to make no adjustments for the cost of defaults. (Of course, the counterparty may want to take account of the possibility of the financial institution defaulting in its own pricing.)

For the second category (where the derivative is always an asset to the financial institution) f_i is always positive. The expression $\max(f_i, 0)$ is always equal to f_i. Since v_i is the present value of f_i, it always equals f_0.[16] The expected loss from default is therefore f_0 times the total probability of default during the life of the derivative times $1-R$.

Example 20.2

Consider a 2-year over-the-counter option with a value (assuming no defaults) of \$3. Suppose that the company selling the option has a risk-neutral probability of defaulting during the 2-year period of 4% and the recovery in the event of a default is 25%. The expected cost of defaults is $3 \times 0.04 \times (1-0.25)$, or \$0.09. The buyer of the option should therefore be prepared to pay only \$2.91.

For the third category of derivatives, the sign of f_i is uncertain. The variable v_i is a call option on f_i with a strike price of zero. One way of calculating v_i is to simulate the underlying market variables over the life of the derivative. Sometimes approximate analytic calculations are possible (see, e.g., Problems 20.17 and 20.18).

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[15] The probability of default could be calculated from bond prices in the way described in Section 20.4.

[16] This assumes no payoffs from the derivative prior to time t_i.

The analyses we have presented assume that the probability of default is independent of the value of the derivative. This is likely to be a reasonable approximation in circumstances when the derivative is a small part of the portfolio of the counterparty or when the counterparty is using the derivative for hedging purposes. When a counterparty wants to enter into a large derivatives transaction for speculative purposes a financial institution should be wary. When the transaction has a large negative value for the counterparty (and a large positive value for the financial institution), the chance of counterparty declaring bankruptcy may be much higher than when the situation is the other way round.

20.8 CREDIT RISK MITIGATION

In many instances the analysis we just have presented overstates the credit risk in a derivatives transaction. This is because there are a number of clauses that derivatives dealers include in their contracts to mitigate credit risk.

Netting

A clause that has become standard in over-the-counter derivatives contracts is known as *netting*. This states that if a company defaults on one contract it has with a counterparty then it must default on all outstanding contracts with the counterparty.

Netting has been successfully tested in the courts in most jurisdictions. It can substantially reduce credit risk for a financial institution. Consider, for example, a financial institution that has three contracts outstanding with a particular counterparty. The contracts are worth +\$10 million, +\$30 million, and −\$25 million to the financial institution. Suppose the counterparty runs into financial difficulties and defaults on its outstanding obligations. To the counterparty the three contracts have values of −\$10 million, −\$30 million, and +\$25 million, respectively. Without netting, the counterparty would default on the first two contracts and retain the third for a loss to the financial institution of \$40 million. With netting, it is compelled to default on all three contracts for a loss to the financial institution of \$15 million.[17]

Suppose a financial institution has a portfolio of N derivatives contracts with a particular counterparty. Suppose that the no-default value of the ith contract is V_i and the amount recovered in the event of default is the recovery rate times this no default value. Without netting, the financial institution loses

$$(1-R)\sum_{i=1}^{N}\max(V_i,\,0)$$

where R is the recovery rate. With netting, it loses

$$(1-R)\max\left(\sum_{i=1}^{N}V_i,\,0\right)$$

Without netting, its loss is the payoff from a portfolio of call options on the contract values where each option has a strike price of zero. With netting, it is the payoff from a

[17] Note that if the third contract were worth −\$45 million to the financial institution instead of −\$25 million, the counterparty would choose not to default and there would be no loss to the financial institution.

single option on the value of the portfolio of contracts. The value of an option on a portfolio is never greater than, and is often considerably less than, the value of the corresponding portfolio of options.

We can extend the analysis presented in the previous section so that equation (20.5) gives the present value of the expected loss from all contracts with a counterparty when netting agreements are in place. This is achieved by redefining v_i in the equation as the present value of a derivative that pays off the exposure at time t_i on the portfolio of all contracts with a counterparty.

A challenging task for a financial institution when considering whether it should enter into a new derivatives contract with a counterparty is to calculate the incremental effect on expected credit losses. This can be done by using equation (20.5) in the way just described to calculate expected default costs with and without the contract. It is interesting to note that, because of netting, the incremental effect of a new contract on expected default losses can be negative. This happens when the value of the new contract is negatively correlated with the value of existing contracts.

Collateralization

Another clause frequently used to mitigate credit risks is known as *collateralization*. Suppose that a company and a financial institution have entered into a number of derivatives contracts. A typical collateralization agreement specifies that the contracts be marked to market periodically using a pre-agreed formula. If the total value of the contracts to the financial institution is above a certain threshold level on a certain day, it can ask the company to post collateral. The amount of collateral posted when added to collateral already posted by the company is equal to the difference between the value of the contract to the financial institution and the threshold level. When the contract moves in favor of the company so that the difference between value of the contract to the financial institution and the threshold level is less than the total margin already posted, the company can reclaim margin. In the event of a default by the company, the financial institution can seize the collateral. If the company does not post collateral as required, the financial institution can close out the contracts.

Suppose, for example, that the threshold level for the company is \$10 million and contract is marked to market daily for the purposes of collateralization. If on a particular day the value of the contract to financial institution is \$10.5 million, it can ask for \$0.5 million of collateral. If the next day the value of the contract rises further to \$11.4 million it can ask for a further \$0.9 million of collateral. If the value of the contract falls to \$10.9 million on the following day, the company can ask for \$0.5 million of the collateral to be returned. Note that the threshold (\$10 million in this case) can be regarded as a line of credit that the financial institution is prepared to grant to the company.

The margin must be deposited by the company with the financial institution in cash or in the form of acceptable securities such as bonds. The securities are subject to a discount known as a *haircut* applied to their market value for the purposes of margin calculations. Interest is normally paid on cash.

If the collateralization agreement is a two-way agreement a threshold will also be specified for the financial institution. The company can then ask the financial institution to post collateral when the mark-to-market value of the outstanding contracts to the company exceeds the threshold.

Collateralization agreements provide a great deal of protection against the possibility of default (just as the margin accounts discussed in Chapter 2 provide protection for people who trade on an exchange). However, the threshold amount is not subject to protection. Furthermore, even when the threshold is zero, the protection is not total. When a company gets into financial difficulties, it is likely to stop responding to requests to post collateral. By the time the counterparty exercises its right to close out contracts, their value may have moved further in its favor.

Downgrade Triggers

Another credit mitigation technique used by a financial institution is known as a *downgrade trigger*. This is a clause stating that if the credit rating of the counterparty falls below a certain level, say Baa, the financial institution has the option to close out a derivatives contract at its market value. (As in the case of collateralization agreements, a formula for determining the market value must be agreed in advance.)

Downgrade triggers do not provide protection from a big jump in a company's credit rating (for example, from A to default). Also, downgrade triggers work well only if relatively little use is made of them. If a company has entered into many downgrade triggers with its counterparties, they are liable to provide relatively little protection to the counterparties (see Business Snapshot 20.1).

20.9 DEFAULT CORRELATION

The term *default correlation* is used to describe the tendency for two companies to default at about the same time. There are a number of reasons why default correlations exist. Companies in the same industry or the same geographic region tend to be affected similarly by external events and as a result may experience financial difficulties at the same time. Economic conditions generally cause average default rates to be higher in some years than in other years. A default by one company may cause a default by another—the credit contagion effect mentioned in Section 20.5. Default correlation means that credit risk cannot be completely diversified away and is the major reason why risk-neutral default probabilities are greater than real-world default probabilities (see Section 20.5).

Default correlation is important to the determination of probability distributions for default losses from a portfolio of exposures to different counterparties. Two types of default correlation models that have been suggested by researchers are referred to as *reduced form models* and *structural models*.

Reduced form models assume that the default intensities for different companies follow stochastic processes and are correlated with macroeconomic variables. When the default intensity for company A is high there is a tendency for the default intensity for company B to be high. This induces a default correlation between the two companies.

Reduced form models are mathematically attractive and reflect the tendency for economic cycles to generate default correlations. Their main disadvantage is that the range of default correlations that can be achieved is limited. Even when there is a perfect correlation between two default intensities, the corresponding correlation between defaults in any chosen period of time is usually quite low. This is liable to be a problem in some circumstances. For example, when two companies operate in the

Business Snapshot 20.1 Downgrade Triggers and Enron's Bankruptcy

In December 2001, Enron, one of the largest companies in the United States, went bankrupt. Right up to the last few days, it had an investment grade credit rating. The Moody's rating immediately prior to default was Baa3 and the S&P rating was BBB–. The default was, however, anticipated to some extent by the stock market because Enron's stock price fell sharply in the period leading up to the bankruptcy. The probability of default estimated by models such as the one described in Section 20.6 increased sharply during this period.

Enron had entered into a huge number of derivatives contracts with downgrade triggers. The downgrade triggers stated that, if its credit rating fell below investment grade (i.e., below Baa3/BBB–), its counterparties would have the option of closing out contracts. Suppose that Enron had been downgraded to below investment grade in, say, October 2001. The contracts that counterparties would choose to close out would be those with negative values to Enron (and positive values to the counterparties). So, Enron would have been required to make huge cash payments to its counterparties. It would not have been able to do this and immediate bankruptcy would result.

This example illustrates that downgrade triggers provide protection only when relatively little use is made of them. When a company enters into a huge number of contracts with downgrade triggers, they may actually cause a company to go bankrupt prematurely. In Enron's case, we could argue that it was going to go bankrupt anyway and accelerating the event by two months would not have done any harm. In fact, Enron did have a chance of survival in October 2001. Attempts were being made to work out a deal with another energy company, Dynergy, and so forcing bankruptcy in October 2001 was not in the interests of either creditors or shareholders.

The credit rating companies found themselves in a difficult position. If they downgraded Enron to recognize its deteriorating financial position, they were signing its death warrant. If they did not do so, there was a chance of Enron surviving.

same industry and the same country or when the financial health of one company is for some reason heavily dependent on the financial health of another company, a relatively high default correlation may be warranted. One approach to solving this problem is by extending the model so that the default intensity exhibits large jumps.

Structural models are based on a model similar to Merton's model (see Section 20.6). A company defaults if the value of its assets is below a certain level. Default correlation between companies A and B is introduced into the model by assuming that the stochastic process followed by the assets of company A is correlated with the stochastic process followed by the assets of company B. Structural models have the advantage over reduced form models that the correlation can be made as high as desired. Their main disadvantage is that they are liable to be computationally quite slow.

The Gaussian Copula Model for Time to Default

A reduced form default correlation model that has become a popular practical tool is the Gaussian copula model for the time to default. This quantifies the correlation between the times to default for two different companies. The model implicitly assumes that all companies will default eventually. But in any application of the model we are typically only interested in the possibility of defaults over the next 1 year, 5 years, or 10 years. We are therefore only interested in the left tail of the distribution of the time to default.

The model can be used in conjunction with either real-world or risk-neutral default probabilities. The left tail of the real-world probability distribution for the time to default of a company can be estimated from data produced by rating agencies such as that in Table 20.1. The left tail of the risk-neutral probability distribution of the time to default can be estimated from bond prices using the approach in Section 20.4.

Define t_1 as the time to default of company 1 and t_2 as the time to default of company 2. If the probability distributions of t_1 and t_2 were normal, we could assume that the joint probability distribution of t_1 and t_2 is bivariate normal. As it happens, the probability distribution of a company's time to default is not even approximately normal. This is where a Gaussian copula model comes in. We transform t_1 and t_2 into new variables x_1 and x_2 using

$$x_1 = N^{-1}[Q_1(t_1)], \qquad x_2 = N^{-1}[Q_2(t_2)]$$

where Q_1 and Q_2 are the cumulative probability distributions for t_1 and t_2, respectively, and N^{-1} is the inverse of the cumulative normal distribution ($u = N^{-1}(v)$ when $v = N(u)$). These are "percentile-to-percentile" transformations. The 5-percentile point in the probability distribution for t_1 is transformed to $x_1 = -1.645$, which is the 5-percentile point in the standard normal distribution; the 10-percentile point in the probability distribution for t_1 is transformed to $x_1 = -1.282$, which is the 10-percentile point in the standard normal distribution, and so on. The t_2-to-x_2 transformation is similar.

By construction, x_1 and x_2 have normal distributions with mean zero and unit standard deviation. We assume that the joint distribution of x_1 and x_2 is bivariate normal with correlation ρ_{12}. This assumption is referred to as using a *Gaussian copula*. The assumption is convenient because it means that the joint probability distribution of t_1 and t_2 is fully defined by the cumulative default probability distributions Q_1 and Q_2 for t_1 and t_2, together with a single correlation parameter ρ_{12}.

The attraction of the Gaussian copula model is that it can be extended to many companies. Suppose that we are considering n companies and that t_i is the time to default of the ith company. We transform each t_i into a new variable, x_i, that has a standard normal distribution. The transformation is the percentile-to-percentile transformation

$$x_i = N^{-1}[Q_i(t_i)]$$

where Q_i is the cumulative probability distribution for t_i. We then assume that the x_i are multivariate normal. The default correlation between t_i and t_j is measured as the correlation between x_i and x_j. This is referred to as the copula correlation.[18]

The Gaussian copula approach is a useful way representing the correlation structure between variables that are not normally distributed. It allows the correlation structure of the variables to be estimated separately from their marginal (unconditional) distributions. Although the variables themselves are not multivariate normal, the approach assumes that after a transformation is applied to each variable they are multivariate normal.

[18] As an approximation, the copula correlation between t_i and t_j is often assumed to be the correlation between the equity returns for companies i and j.

Example 20.3

Suppose that we wish to simulate defaults during the next 5 years in 10 companies. The copula default correlations between each pair of companies is 0.2. For each company the cumulative probability of a default during the next 1, 2, 3, 4, 5 years is 1%, 3%, 6%, 10%, 15%, respectively. When a Gaussian copula is used we sample from a multivariate normal distribution to obtain the x_i ($1 \leqslant i \leqslant 10$) with the pairwise correlation between the x_i being 0.2. We then convert the x_i to t_i, a time to default. When the sample from the normal distribution is less than $N^{-1}(0.01) = -2.33$, a default takes place within the first year; when the sample is between -2.33 and $N^{-1}(0.03) = -1.88$, a default takes place during the second year; when the sample is between -1.88 and $N^{-1}(0.06) = -1.55$, a default takes place during the third year; when the sample is between -1.55 and $N^{-1}(0.10) = -1.28$, a default takes place during the fourth year; when the sample is between -1.28 and $N^{-1}(0.15) = -1.04$, a default takes place during the fifth year. When the sample is greater than -1.04, there is no default during the 5 years.

Using Factors to Define the Correlation Structure

To avoid defining a different correlation between x_i and x_j for each pair of companies i and j in the Gaussian copula model, a one-factor model is often used. The assumption is that

$$x_i = a_i M + \sqrt{1 - a_i^2}\, Z_i \tag{20.6}$$

Here M is a common factor affecting defaults for all companies and Z_i is a factor affecting only company i. The variables M and the Z_i have independent standard normal distributions. The a_i are constant parameters between -1 and $+1$. The correlation between x_i and x_j is $a_i a_j$.[19]

Suppose that the probability that company i will default by a particular time T is $Q_i(T)$. Under the Gaussian copula model, a default happens when $N(x_i) < Q_i(T)$ or $x_i < N^{-1}[Q_i(T)]$. From equation (20.6), this condition is

$$a_i M + \sqrt{1 - a_i^2}\, Z_i < N^{-1}[Q_i(T)]$$

or

$$Z_i < \frac{N^{-1}[Q_i(T)] - a_i M}{\sqrt{1 - a_i^2}}$$

Conditional on the value of the factor M, the probability of default is therefore

$$Q_i(T|M) = N\left(\frac{N^{-1}[Q_i(T)] - a_i M}{\sqrt{1 - a_i^2}}\right) \tag{20.7}$$

A particular case of the one-factc. Gaussian model is where the probability distributions of default are the same for all i and the correlations between x_i and x_j is the same for all i and j. Suppose that $Q_i(T) = Q(T)$ for all i and that the common correlation

[19] The parameter a_i is sometimes approximated as the correlation of company i's equity returns with a well-diversified market index.

is ρ, so that $a_i = \sqrt{\rho}$ for all i. Equation (20.7) becomes

$$Q(T|M) = N\left(\frac{N^{-1}[Q(T)] - \sqrt{\rho}\,M}{\sqrt{1-\rho}}\right) \tag{20.8}$$

Binomial Correlation Measure

An alternative correlation measure used by rating agencies is the *binomial correlation measure*. For two companies A and B, this is the coefficient of correlation between:

1. A variable that equals 1 if company A defaults between times 0 and T, and 0 otherwise; and
2. A variable that equals 1 if company B defaults between times 0 and T, and 0 otherwise.

The measure is

$$\beta_{AB}(T) = \frac{P_{AB}(T) - Q_A(T)Q_B(T)}{\sqrt{[Q_A(T) - Q_A(T)^2][Q_B(T) - Q_B(T)^2]}} \tag{20.9}$$

where $P_{AB}(T)$ is the joint probability of A and B defaulting between time 0 and time T, $Q_A(T)$ is the cumulative probability that company A will default by time T, and $Q_B(T)$ is the cumulative probability that company B will default by time T. Typically $\beta_{AB}(T)$ depends on T, the length of the time period considered. Usually it increases as T increases.

From the definition of a Gaussian copula model, $P_{AB}(T) = M[x_A(T), x_B(T); \rho_{AB}]$, where $x_A(T) = N^{-1}(Q_A(T))$ and $x_B(T) = N^{-1}(Q_B(T))$ are the transformed times to default for companies A and B, and ρ_{AB} is the Gaussian copula correlation for the times to default for A and B. Here, $M(a, b; \rho)$ is the probability that, in a bivariate normal distribution where the correlation between the variables is ρ, the first variable is less than a and the second variable is less than b.[20] It follows that

$$\beta_{AB}(T) = \frac{M[x_A(T), x_B(T); \rho_{AB}] - Q_A(T)Q_B(T)}{\sqrt{[Q_A(T) - Q_A(T)^2][Q_B(T) - Q_B(T)^2]}} \tag{20.10}$$

This shows that, if $Q_A(T)$ and $Q_B(T)$ are known, $\beta_{AB}(T)$ can be calculated from ρ_{AB} and vice versa. Usually ρ_{AB} is markedly greater than $\beta_{AB}(T)$. This illustrates the important point that the magnitude of a correlation measure depends on the way it is defined.

Example 20.4

Suppose that the probability of company A defaulting in a 1-year period is 1% and the probability of company B defaulting in a 1-year period is also 1%. In this case, $x_A(1) = x_B(1) = N^{-1}(0.01) = -2.326$. If ρ_{AB} is 0.20, $M(x_A(1), x_B(1), \rho_{AB}) = 0.000337$ and equation (20.10) shows that $\beta_{AB}(T) = 0.024$ when $T = 1$.

20.10 CREDIT VaR

Credit value at risk can be defined analogously to the way we defined value at risk for market risks in Chapter 18. For example, a credit VaR with a confidence level of 99.9%

[20] See Technical Note 5 on the author's website for the calculation of $M(a, b; \rho)$.

and a 1-year time horizon is the credit loss that we are 99.9% confident will not be exceeded over 1 year.

Consider a bank with a very large portfolio of similar loans. As an approximation we assume that the probability of default is the same for each loan and the correlation between each pair of loans is the same. When the Gaussian copula model for time to default is used, the right-hand side of equation (20.8) is approximately equal to the percentage of defaults by time T as a function of M. The factor M has a standard normal distribution. We are $X\%$ certain that its value will be greater than $N^{-1}(1-X) = -N^{-1}(X)$. We are therefore $X\%$ certain that the percentage of losses over T years on a large portfolio will be less than $V(X, T)$, where

$$V(X,T) = N\left(\frac{N^{-1}[Q(T)] + \sqrt{\rho}\,N^{-1}(X)}{\sqrt{1-\rho}}\right) \tag{20.11}$$

This result was first produced by Vasicek.[21] As in equation (20.8), $Q(T)$ is the probability of default by time T and ρ is the copula correlation between any pair of loans.

A rough estimate of the credit VaR when an $X\%$ confidence level is used and the time horizon is T is therefore $L(1-R)V(X,T)$, where L is the size of the loan portfolio and R is the recovery rate. The contribution of a particular loan of size L_i to the credit VaR is $L_i(1-R)V(X,T)$. This model underlies the formulas that regulators are planning to use for credit risk capital (see Business Snapshot 20.2).

Example 20.4

Suppose that a bank has a total of $100 million of retail exposures. The 1-year probability of default averages 2% and the recovery rate averages 60%. The copula correlation parameter is estimated as 0.1. In this case,

$$V(0.999, 1) = N\left(\frac{N^{-1}(0.02) + \sqrt{0.1}\,N^{-1}(0.999)}{\sqrt{1-0.1}}\right) = 0.128$$

showing that the 99.9% worst case default rate is 12.8%. The 1-year 99.9% credit VaR is therefore $100 \times 0.128 \times (1-0.6)$ or $5.13 million.

CreditMetrics

Many bank's have developed other procedures for calculating credit VaR for internal use. One popular approach is known as CreditMetrics. This involves estimating a probability distribution of credit losses by carrying out a Monte Carlo simulation of the credit rating changes of all counterparties. Suppose we are interested in determining the probability distribution of losses over a 1-year period. On each simulation trial, we sample to determine the credit rating changes and defaults of all counterparties during the year. We then revalue our outstanding contracts to determine the total of credit losses for the year. After a large number of simulation trials, we obtain a probability distribution for credit losses. This can be used to calculate credit VaR.

This approach is liable to be computationally quite time intensive. However, it has the advantage that credit losses are defined as those arising from credit downgrades as well as defaults. Also the impact of credit mitigation clauses such as those described Section 20.8 can be approximately incorporated into the analysis.

[21] See O. Vasicek, "Probability of Loss on a Loan Portfolio," Working Paper, KMV, 1987.

Business Snapshot 20.2 Basel II

The Basel Committee on Bank Supervision is planning an overhaul of its procedures for calculating the capital banks are required to keep for the risks they are bearing. This is known as Basel II. No changes are planned to the way market risk capital is calculated (see Business Snapshot 18.1). A new capital requirement for operational risk is planned and significant changes have been proposed for the way in which capital is calculated for credit risk.

For banks eligible to use the Internal Ratings Based (IRB) approach, credit risk capital for a transaction is calculated as

$$UDR \times LGD \times EAD \times MatAd$$

Here UDR, the unexpected default rate, is the excess of the 99.9% worst case 1-year default rate over the expected 1-year default rate. It is calculated, using equation (20.11), as $V(X, T) - Q(T)$ with $X = 99.9\%$ and $T = 1$. The variable LGD is the percentage loss given default (similar to the variable we have been denoting by $1 - R$); EAD is the exposure at default; $MatAd$ is a maturity adjustment.

The rules for determining these numbers are complicated. For UDR, the 1-year probability of default, $Q(1)$, and a correlation parameter ρ are required. The 1-year probability of default is estimated by the bank and the rules for determining the correlation parameter depend on the type of exposure (retail, corporate, sovereign, etc.). For retail exposures, banks also determine LGD and EAD internally. For corporate exposures, banks using the "Advanced IRB" determine LGD and EAD internally, but for banks using the "Foundation IRB" approach there are rules prescribed for determining LGD and EAD. The maturity adjustment is an increasing function of the maturity of the instrument and equals 1.0 when the maturity of the instrument is in 1 year.

Table 20.6 is typical.of the historical data provided by rating agencies on credit rating changes and could be used as a basis for a CreditMetrics Monte Carlo simulation. It shows the percentage probability of a bond moving from one rating category to another during a 1-year period. For example, a bond that starts with an A credit rating has a 91.84% chance of still having an A rating at the end of 1 year. It has a 0.02% chance of defaulting during the year, a 0.13% chance of dropping to B, and so on.[22]

In sampling to determine credit losses, the credit rating changes for different counterparties should not be assumed to be independent. A Gaussian copula model can be used to construct a joint probability distribution of rating changes similarly to the way it is used in the previous section to describe the joint probability distribution of times to default. The copula correlation between the rating transitions for two companies is usually set equal correlation between their equity returns using a factor model similar to that in Section 20.9.

As an illustration of the CreditMetrics approach suppose that we are simulating the rating change of a Aaa and a Baa company over a 1-year period using the transition matrix in Table 20.6. Suppose that the correlation between the equities of the two

[22] Technical Note 11 on the author's website explains how a table such as Table 20.6 can be used to calculate transition matrices for periods other than 1 year.

Table 20.6 One-year ratings transition matrix (probabilities expressed as percentages). From results reported by Moody's in 2004 with adjustments for the WR category.

Initial rating	*Rating at year-end*							
	Aaa	*Aa*	*A*	*Baa*	*Ba*	*B*	*Caa*	*Default*
Aaa	92.18	7.06	0.73	0.00	0.02	0.00	0.00	0.00
Aa	1.17	90.85	7.63	0.26	0.07	0.01	0.00	0.02
A	0.05	2.39	91.84	5.07	0.50	0.13	0.01	0.02
Baa	0.05	0.24	5.20	88.48	4.88	0.80	0.16	0.18
Ba	0.01	0.05	0.50	5.45	85.13	7.05	0.55	1.27
B	0.01	0.03	0.13	0.43	6.52	83.21	3.04	6.64
Caa	0.00	0.00	0.00	0.58	1.74	4.18	67.99	25.50
Default	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

companies is 0.2. On each simulation trial, we would sample two variables x_A and x_B from normal distributions so that their correlation is 0.2. The variable x_A determines the new rating of the Aaa company and variable x_B determines the new rating of the Baa company. Since $N^{-1}(0.9218) = 1.4173$, $N^{-1}(0.9218 + 0.0706) = 2.4276$, and $N^{-1}(0.9218 + 0.0706 + 0.0073) = 3.4319$, the Aaa company stays Aaa-rated if $x_A < 1.4173$, it becomes Aa-rated if $1.4173 \leqslant x_A < 2.4276$, and it becomes A-rated if $2.4276 \leqslant x_A < 3.4319$. Similarly, since $N^{-1}(0.0005) = -3.2905$, $N^{-1}(0.0005 + 0.0024) = -2.7589$, and $N^{-1}(0.0005 + 0.0024 + 0.0520) = -1.5991$, the Baa company becomes Aaa-rated if $x_B < -3.2905$, it becomes Aa-rated if $-3.2905 \leqslant x_B < -2.7589$, and it becomes A-rated if $-2.7589 \leqslant x_B < -1.5991$. The Aaa-rated company never defaults. The BBB-rated company defaults when $x_B > N^{-1}(0.9982)$, that is, when $x_B > 2.9113$.

SUMMARY

The probability that a company will default during a particular period of time in the future can be estimated from historical data, bond prices, or equity prices. The default probabilities calculated from bond prices are risk-neutral probabilities, whereas those calculated from historical data are real-world probabilities. Real-world probabilities should be used for scenario analysis and the calculation of credit VaR. Risk-neutral probabilities should be used for valuing credit-sensitive instruments. Risk-neutral default probabilities are significantly higher than real-world probabilities.

The expected loss experienced from a counterparty default is reduced by what is known as netting. This is a clause in most contracts written by a financial institution stating that, if a counterparty defaults on one contract it has with the financial institution, it must default on all contracts it has with the financial institution. Losses are also reduced by collateralization and downgrade triggers. Collateralization requires a counterparty to post collateral and a downgrade trigger gives a company the option to close out a contract if the credit rating of a counterparty falls below a specified level.

Credit VaR can be defined similarly to the way VaR is defined for market risk. One approach to calculating it is the Gaussian copula model of time to default. This has been used by regulators in their proposals for changes to the calculation of capital for credit risk. Another popular approach for calculating credit VaR is CreditMetrics. This uses a Gaussian copula model for credit rating changes.

FURTHER READING

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Questions and Problems (Answers in the Solutions Manual)

20.1. The spread between the yield on a 3-year corporate bond and the yield on a similar risk-free bond is 50 basis points. The recovery rate is 30%. Estimate the average default intensity per year over the 3-year period.

20.2. Suppose that in Problem 20.1 the spread between the yield on a 5-year bond issued by the same company and the yield on a similar risk-free bond is 60 basis points. Assume the same recovery rate of 30%. Estimate the average default intensity per year over the 5-year period. What do your results indicate about the average default intensity in years 4 and 5?

20.3. Should researchers use real-world or risk-neutral default probabilities for (a) calculating credit value at risk and (b) adjusting the price of a derivative for defaults?

20.4. How are recovery rates usually defined?

20.5. Explain the difference between an unconditional default probability density and a default intensity.

20.6. Verify (a) that the numbers in the second column of Table 20.4 are consistent with the numbers in Table 20.1 and (b) that the numbers in the fourth column of Table 20.5 are consistent with the numbers in Table 20.4 and a recovery rate of 40%.

20.7. Describe how netting works. A bank already has one transaction with a counterparty on its books. Explain why a new transaction by a bank with a counterparty can have the effect of increasing or reducing the bank's credit exposure to the counterparty.

20.8. Suppose that the measure $\beta_{AB}(T)$ in equation (20.9) is the same in the real world and the risk-neutral world. Is the same true of the Gaussian copula measure, ρ_{AB}?

20.9. What is meant by a "haircut" in a collateralization agreement. A company offers to post its own equity as collateral. How would you respond?

20.10. Explain the difference between the Gaussian copula model for the time to default and CreditMetrics as far as the following are concerned: (a) the definition of a credit loss and (b) the way in which default correlation is modeled.

20.11. Suppose that the probability of company A defaulting during a 2-year period is 0.2 and the probability of company B defaulting during this period is 0.15. If the Gaussian copula measure of default correlation is 0.3, what is the binomial correlation measure?

20.12. Suppose that the LIBOR/swap curve is flat at 6% with continuous compounding and a 5-year bond with a coupon of 5% (paid semiannually) sells for 90.00. How would an asset swap on the bond be structured? What is the asset swap spread that would be calculated in this situation?

20.13. Show that the value of a coupon-bearing corporate bond is the sum of the values of its constituent zero-coupon bonds when the amount claimed in the event of default is the no-default value of the bond, but that this is not so when the claim amount is the face value of the bond plus accrued interest.

20.14. A 4-year corporate bond provides a coupon of 4% per year payable semiannually and has a yield of 5% expressed with continuous compounding. The risk-free yield curve is flat at 3% with continuous compounding. Assume that defaults can take place at the end of each year (immediately before a coupon or principal payment) and that the recovery rate is 30%. Estimate the risk-neutral default probability on the assumption that it is the same each year.

20.15. A company has issued 3- and 5-year bonds with a coupon of 4% per annum payable annually. The yields on the bonds (expressed with continuous compounding) are 4.5% and 4.75%, respectively. Risk-free rates are 3.5% with continuous compounding for all maturities. The recovery rate is 40%. Defaults can take place halfway through each year. The risk-neutral default rates per year are Q_1 for years 1 to 3 and Q_2 for years 4 and 5. Estimate Q_1 and Q_2.

20.16. Suppose that a financial institution has entered into a swap dependent on the sterling interest rate with counterparty X and an exactly offsetting swap with counterparty Y. Which of the following statements are true and which are false:

(a) The total present value of the cost of defaults is the sum of the present value of the cost of defaults on the contract with X plus the present value of the cost of defaults on the contract with Y.

(b) The expected exposure in 1 year on both contracts is the sum of the expected exposure on the contract with X and the expected exposure on the contract with Y.

(c) The 95% upper confidence limit for the exposure in 1 year on both contracts is the sum of the 95% upper confidence limit for the exposure in 1 year on the contract with X and the 95% upper confidence limit for the exposure in 1 year on the contract with Y.

Explain your answers.

20.17. A company enters into a 1-year forward contract to sell \$100 for AUD150. The contract is initially at the money. In other words, the forward exchange rate is 1.50. The 1-year dollar risk-free rate of interest is 5% per annum. The 1-year dollar rate of interest at which the counterparty can borrow is 6% per annum. The exchange rate volatility is 12% per annum. Estimate the present value of the cost of defaults on the contract? Assume that defaults are recognized only at the end of the life of the contract.

20.18. Suppose that in Problem 20.17, the 6-month forward rate is also 1.50 and the 6-month dollar risk-free interest rate is 5% per annum. Suppose further that the 6-month dollar rate of interest at which the counterparty can borrow is 5.5% per annum. Estimate the present value of the cost of defaults assuming that defaults can occur either at the 6-month point or at the 1-year point? (If a default occurs at the 6-month point, the company's potential loss is the market value of the contract.)

20.19. "A long forward contract subject to credit risk is a combination of a short position in a no-default put and a long position in a call subject to credit risk." Explain this statement.

20.20. Explain why the credit exposure on a matched pair of forward contracts resembles a straddle.

20.21. Explain why the impact of credit risk on a matched pair of interest rate swaps tends to be less than that on a matched pair of currency swaps.

20.22. "When a bank is negotiating currency swaps, it should try to ensure that it is receiving the lower interest rate currency from a company with a low credit risk." Explain why.

20.23. Does put–call parity hold when there is default risk? Explain your answer.

20.24. Suppose that in an asset swap B is the market price of the bond per dollar of principal, B^* is the default-free value of the bond per dollar of principal, and V is the present value of the asset swap spread per dollar of principal. Show that $V = B - B^*$.

20.25. Show that under Merton's model in Section 20.6 the credit spread on a T-year zero-coupon bond is $\ln[N(d_2) + N(-d_1)/L]/T$, where $L = De^{-rT}/V_0$.

Assignment Questions

20.26. Suppose a 3-year corporate bond provides a coupon of 7% per year payable semiannually and has a yield of 5% (expressed with semiannual compounding). The yields for all maturities on risk-free bonds is 4% per annum (expressed with semiannual compounding). Assume that defaults can take place every 6 months (immediately before a coupon payment) and the recovery rate is 45%. Estimate the default probabilities assuming (a) that the unconditional default probabilities are the same on each possible default date and (b) that the default probabilities conditional on no earlier default are the same on each possible default date.

20.27. A company has 1- and 2-year bonds outstanding, each providing a coupon of 8% per year payable annually. The yields on the bonds (expressed with continuous compounding) are 6.0% and 6.6%, respectively. Risk-free rates are 4.5% for all maturities. The recovery rate is 35%. Defaults can take place halfway through each year. Estimate the risk-neutral default rate each year.

20.28. Explain carefully the distinction between real-world and risk-neutral default probabilities. Which is higher? A bank enters into a credit derivative where it agrees to pay $100 at the end of 1 year if a certain company's credit rating falls from A to Baa or lower during the year. The 1-year risk-free rate is 5%. Using Table 20.6, estimate a value for the derivative. What assumptions are you making? Do they tend to overstate or understate the value of the derivative.

20.29. The value of a company's equity is $4 million and the volatility of its equity is 60%. The debt that will have to be repaid in 2 years is $15 million. The risk-free interest rate is 6% per annum. Use Merton's model to estimate the expected loss from default, the probability of default, and the recovery rate in the event of default. Explain why Merton's model gives a high recovery rate. (*Hint*: The Solver function in Excel can be used for this question.)

20.30. Suppose that a bank has a total of $10 million of exposures of a certain type. The 1-year probability of default averages 1% and the recovery rate averages 40%. The copula correlation parameter is 0.2. Estimate the 99.5% 1-year credit VaR.