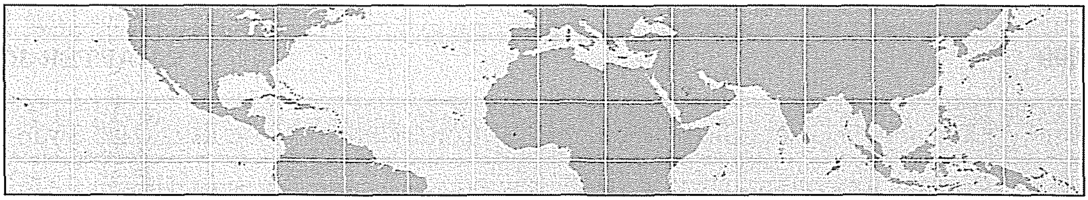

CHAPTER 30 Swaps Revisited

Swaps have been central to the success of over-the-counter derivatives markets in the 1980s and 1990s. They have proved to be very flexible instruments for managing risk. Based on the range of different contracts that now trade and the total volume of business transacted each year, swaps are arguably one the most successful innovations in financial markets ever.

In Chapter 7 we discussed how plain vanilla interest rate swaps can be valued. The standard approach can be summarized as: "Assume forward rates will be realized." The steps are as follows:

1. Calculate the swap's net cash flows on the assumption that LIBOR rates in the future equal the forward rates calculated from today's LIBOR/swap zero curve.
2. Set the value of the swap equal to the present value of the net cash flows using the LIBOR/swap zero curve for discounting.

In this chapter we describe a number of nonstandard swaps. Some can be valued using the "assume forward rates will be realized" approach; some require the application of the convexity, timing, and quanto adjustments we encountered in Chapters 27; some contain embedded options that must be valued using the techniques described in Chapters 26, 28, and 29.

30.1 VARIATIONS ON THE VANILLA DEAL

Many interest rate swaps involve relatively minor variations to the plain vanilla structure we discussed in Chapter 7. In some swaps the notional principal changes with time in a predetermined way. Swaps where the notional principal is an increasing function of time are known as *step-up swaps*. Swaps where the notional principal is a decreasing function of time are known as *amortizing swaps*. Step-up swaps could be useful for a construction company that intends to borrow increasing amounts of money at floating rates to finance a particular project and wants to swap it to fixed-rate funding. An amortizing swap could be used by a company that has fixed-rate borrowings with a certain prepayment schedule and wants to swap them to borrowings at a floating rate.

Business Snapshot 30.1 Hypothetical Confirmation for Nonstandard Swap

Trade date:	5-January-2004
Effective date:	11-January-2004
Business day convention (all dates):	Following business day
Holiday calendar:	US
Termination date:	11-January-2009
Fixed amounts	
Fixed-rate payer:	Microsoft
Fixed-rate notional principal:	USD 100 million
Fixed rate:	6% per annum
Fixed-rate day count convention:	Actual/365
Fixed-rate payment dates	Each 11-July and 11-January commencing 11-July, 2004, up to and including 11-January, 2009
Floating amounts	
Floating-rate payer	Goldman Sachs
Floating-rate notional principal	USD 120 million
Floating rate	USD 1-month LIBOR
Floating-rate day count convention	Actual/360
Floating-rate payment dates	11-July, 2004, and the 11th of each month thereafter up to and including 11-January, 2009

The principal can be different on the two sides of a swap. Also the frequency of payments can be different. Business Snapshot 30.1 illustrates this by showing a hypothetical swap between Microsoft and Goldman Sachs where the notional principal is \$120 million on the floating side and \$100 million on fixed side. Payments are made every month on the floating side and every 6 months on the fixed side. These type of variations to the basic plain vanilla structure do not affect the valuation methodology. We can still use the "assume forward rates are realized" approach.

The floating reference rate for a swap is not always LIBOR. In some swaps for instance, it is the commercial paper (CP) rate. A *basis swap* involves exchanging cash flows calculated using one floating reference rate for cash flows calculated using another floating reference rate. An example would be a swap where the 3-month CP rate plus 10 basis points is exchanged for 3-month LIBOR with both being applied to a principal of \$100 million. A basis swap could be used for risk management by a financial institution whose assets and liabilities are dependent on different floating reference rates.

Swaps where the floating reference rate is not LIBOR can be valued using the "assume forward rates are realized" approach. A zero curve other than LIBOR is necessary to calculate future cash flows on the assumption that forward rates are realized. The cash flows are discounted at LIBOR.

Business Snapshot 30.2 Hypothetical Confirmation for Compounding Swap

Trade date:	5-January-2004
Effective date:	11-January-2004
Holiday calendar:	US
Business day convention (all dates):	Following business day
Termination date:	11-January-2009
Fixed amounts	
Fixed-rate payer:	Microsoft
Fixed-rate notional principal:	USD 100 million
Fixed rate:	6% per annum
Fixed-rate day count convention:	Actual/365
Fixed-rate payment date:	11-January, 2009
Fixed-rate compounding:	Applicable at 6.3%
Fixed-rate compounding dates	Each 11-July and 11-January commencing 11-July, 2004, up to and including 11-July, 2008
Floating amounts	
Floating-rate payer:	Goldman Sachs
Floating-rate notional principal:	USD 100 million
Floating rate:	USD 6-month LIBOR plus 20 basis points
Floating-rate day count convention:	Actual/360
Floating-rate payment date:	11-January, 2009
Floating-rate compounding:	Applicable at LIBOR plus 10 basis points
Floating-rate compounding dates:	Each 11-July and 11-January commencing 11-July, 2004, up to and including 11-July, 2008

30.2 COMPOUNDING SWAPS

Another variation on the plain vanilla swap is a *compounding swap*. A hypothetical confirmation for a compounding swap is in Business Snapshot 30.2. In this example there is only one payment date for both the floating-rate payments and the fixed-rate payments. This is at the end of the life of the swap. The floating rate of interest is LIBOR plus 20 basis points. Instead of being paid, the interest is compounded forward until the end of the life of the swap at a rate of LIBOR plus 10 basis points. The fixed rate of interest is 6%. Instead of being paid this interest is compounded forward at a fixed rate of interest of 6.3% until the end of the swap.

We can use the "assume forward rates are realized" approach for valuing a compounding swap such as that in Business Snapshot 30.2. It is straightforward to deal with the fixed side of the swap because the payment that we will make at maturity is known with certainty. The "assume forward rates are realized" approach for the floating part is justifiable because we can devise a series of forward rate agreements (FRAs) where the floating-rate cash flows are exchanged for the values they would have if each floating rate equaled the corresponding forward rate.[1]

Example 30.1

A compounding swap with annual resets has a life of 3 years. A fixed rate is paid and a floating rate is received. The fixed interest rate is 4% and the floating interest rate is 12-month LIBOR. The fixed side compounds at 3.9% and the floating side compounds at 12-month LIBOR minus 20 basis points. The LIBOR zero curve is flat at 5% with annual compounding and the notional principal is $100 million.

On the fixed side, interest of $4 million is earned at the end of the first year. This compounds to $4 \times 1.039 = \$4.156$ million at the end of the second year. A second interest amount of $4 million is added at the end of the second year bringing the total compounded forward amount to $8.156 million. This compounds to $8.156 \times 1.039 = \$8.474$ million by the end of the third year when there is the third interest amount of $4 million. The cash flow at the end of the third year on the fixed side of the swap is therefore $12.474 million.

On the floating side we assume all future interest rates equal the corresponding forward LIBOR rates. Given the LIBOR zero curve, this means that we assume all future interest rates are 5% with annual compounding. The interest calculated at the end of the first year is $5 million. Compounding this forward at 4.8% (forward LIBOR minus 20 basis points) gives $5 \times 1.048 = \$5.24$ million at the end of the second year. Adding in the interest, the compounded forward amount is $10.24 million. Compounding forward to the end of the third year, we get $10.24 \times 1.048 = \$10.731$ million. Adding in the final interest gives $15.731 million.

We can value the swap by assuming that it leads to an inflow of $15.731 million and an outflow of $12.474 million at the end of year 3. The value of the swap is therefore

$$\frac{15.731 - 12.474}{1.05^3} = 2.814$$

or $2.814 million. (This analysis ignores day count issues.)

30.3 CURRENCY SWAPS

We introduced currency swaps in Chapter 7. These enable an interest rate exposure in one currency to be swapped for an interest rate exposure in another currency. Usually two principals are specified, one in each currency. The principals are exchanged at both the beginning and the end of the life of the swap as described in Section 7.8.

Suppose that the currencies involved in a currency swap are US dollars (USD) and British pounds (GBP). In a fixed-for-fixed currency swap, a fixed rate of interest is specified in each currency. The payments on one side are determined by applying the fixed rate of interest in USD to the USD principal; the payments on the other side are determined by applying the fixed rate of interest in GBP to the GBP principal. We discussed the valuation of this type of swap in Section 7.9.

Another popular type of currency swap is floating-for-floating. In this, the payments

[1] See Technical Note 18 on the author's website for the details.

on one side are determined by applying USD LIBOR (possibly with a spread added) to the USD principal; similarly the payments on the other side are determined by applying GBP LIBOR (possibly with a spread added) to the GBP principal. A third type of swap is a cross-currency interest rate swap where a floating rate in one currency is exchanged for a fixed rate in another currency.

Floating-for-floating and cross-currency interest rate swaps can be valued using the "assume forward rates are realized" rule. Future LIBOR rates in each currency are assumed to equal today's forward rates. This enables the cash flows in the currencies to be determined. The USD cash flows are discounted at the USD LIBOR zero rate. The GBP cash flows are discounted at the GBP LIBOR zero rate. The current exchange rate is then used to translate the two present values to a common currency.

An adjustment to this procedure is sometimes made to reflect the realities of the market. In theory, a new floating-for-floating swap should involve exchanging LIBOR in one currency for LIBOR in another currency (with no spreads added). In practice, macroeconomic effects give rise to spreads. Financial institutions often adjust the discount rates they use to allow for this. As an example, suppose that market conditions are such that USD LIBOR is exchanged for Japanese yen (JPY) LIBOR minus 20 basis points in new floating-for-floating swaps of all maturities. In its valuations a US financial institution would discount USD cash flows at USD LIBOR and it would discount JPY cash flows at JPY LIBOR minus 20 basis points.[2] It would do this in all swaps that involved both JPY and USD cash flows.

30.4 MORE COMPLEX SWAPS

We now move on to consider some examples of swaps where the simple rule "assume forward rates will be realized" does not work. In each case we must make adjustments to forward rates.

LIBOR-in-Arrears Swap

A plain vanilla interest rate swap is designed so that the floating rate of interest observed on one payment date is paid on the next payment date. An alternative instrument that is sometimes traded is a *LIBOR-in-arrears swap*. In this, the floating rate paid on a payment date equals the rate observed on the payment date itself.

Suppose that the reset dates in the swap are t_i for $i = 0, 1, \dots, n$, with $\tau_i = t_{i+1} - t_i$. Define R_i as the LIBOR rate for the period between t_i and t_{i+1}, F_i as the forward value of R_i, and σ_i as the volatility of this forward rate. (The value of σ_i is typically implied from caplet prices.) In a LIBOR-in-arrears swap the payment on the floating side at time t_i is based on R_i rather than R_{i-1}. As explained in Section 27.1, it is necessary to make a convexity adjustment to the forward rate when the payment is valued. The valuation should be based on the assumption that the forward rate is

$$F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1 + F_i \tau_i} \tag{30.1}$$

rather than F_i.

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[2] This adjustment is *ad hoc*, but, if it is not made, traders make an immediate profit or loss every time they trade a new JPY/USD floating-for-floating swap.

Example 30.2

In a LIBOR-in-arrears swap, the principal is \$100 million. A fixed rate of 5% is received annually and LIBOR is paid. Payments are exchanged at the ends of years 1, 2, 3, 4, and 5. The yield curve is flat at 5% per annum (measured with annual compounding). All caplet volatilities are 22% per annum.

The forward rate for each floating payment is 5%. If this were a regular swap rather than an in-arrears swap, its value would (ignoring day count conventions, etc.) be exactly zero. Because it is an in-arrears swap, we must make convexity adjustments. In equation (30.1), $F_i = 0.05$, $\sigma_i = 0.22$, and $\tau_i = 1$ for all i. The convexity adjustment changes the rate assumed at time t_i from 0.05 to

$$0.05 + \frac{0.05^2 \times 0.22^2 \times 1 \times t_i}{1 + 0.05 \times 1} = 0.05 + 0.000115t_i$$

The floating rates for the payments at the ends of years 1, 2, 3, 4, and 5 should therefore be assumed to be 5.0115%, 5.0230%, 5.0345%, 5.0460%, and 5.0575%, respectively. The net exchange on the first payment date is equivalent to a cash outflow of 0.0115% of \$100 million or \$11,500. Equivalent net cash flows for other exchanges are calculated similarly. The value of the swap is

$$-\frac{11{,}500}{1.05} - \frac{23{,}000}{1.05^2} - \frac{34{,}500}{1.05^3} - \frac{46{,}000}{1.05^4} - \frac{57{,}500}{1.05^5}$$

or −\$144,514.

CMS and CMT Swaps

A constant maturity swap (CMS) is an interest rate swap where the floating rate equals the swap rate for a swap with a certain life. For example, the floating payments on a CMS swap might be made every 6 months at a rate equal to the 5-year swap rate. Usually there is a lag so that the payment on a particular payment date is equal to the swap rate observed on the previous payment date. Suppose that rates are set at times $t_0, t_1, t_2, \dots$, payments are made at times $t_1, t_2, t_3, \dots$, and L is the notional principal. The floating payment at time t_{i+1} is

$$\tau_i L S_i$$

where $\tau_i = t_{i+1} - t_i$ and S_i is the swap rate at time t_i.

Suppose that y_i is the forward value of the swap rate S_i. To value the payment at time t_{i+1}, it turns out to be correct to make a convexity adjustment to the forward swap rate, so that the swap rate is assumed to be

$$y_i - \tfrac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + F_i \tau_i} \tag{30.2}$$

rather that y_i. In this equation, $\sigma_{y,i}$ is the volatility of the forward swap rate, F_i is the current forward interest rate between times t_i and t_{i+1}, $\sigma_{F,i}$ is the volatility of this forward rate, and ρ_i is the correlation between the forward swap rate and the forward interest rate. $G_i(x)$ is the price at time t_i of a bond as a function of its yield x. The bond pays coupons at rate y_i and has the same life and payment frequency as the swap from which the CMS rate is calculated. $G_i'(x)$ and $G_i''(x)$ are the first and second partial derivatives of G_i with respect to x. The volatilities $\sigma_{y,i}$ can be implied from swap options; the volatilities $\sigma_{F,i}$ can be implied from caplet prices; the correlation ρ_i can be estimated from historical data.

Equation (30.2) involves a convexity and a timing adjustment. The term

$$-\tfrac{1}{2}y_i^2\sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)}$$

is an adjustment similar the one we calculated in Example 27.2 of Section 27.1. It is based on the assumption that the swap rate S_i leads to only one payment at time t_i. The term

$$-\frac{y_i\tau_i F_i\rho_i\sigma_{y,i}\sigma_{F,i}\, t_i}{1+F_i\tau_i}$$

is similar to the one we calculated in Section 27.2 and allows for the fact that the payment calculated from S_i is made at time t_{i+1} rather than t_i.

Example 30.3

In a 6-year CMS swap, the 5-year swap rate is received and a fixed rate of 5% is paid on a notional principal of \$100 million. The exchange of payments is semiannual (both on the underlying 5-year swap and on the CMS swap itself). The exchange on a payment date is determined from the swap rate on the previous payment date. The term structure is flat at 5% per annum with semiannual compounding. All options on five-year swaps have a 15% implied volatility and all caplets with a 6-month tenor have a 20% implied volatility. The correlation between each cap rate and each swap rate is 0.7.

In this case, $y_i = 0.05$, $\sigma_{y,i} = 0.15$, $\tau_i = 0.5$, $F_i = 0.05$, $\sigma_{F,i} = 0.20$, and $\rho_i = 0.7$ for all i. Also,

$$G_i(x) = \sum_{i=1}^{10} \frac{2.5}{(1+x/2)^i} + \frac{100}{(1+x/2)^{10}}$$

so that $G_i'(y_i) = -437.603$ and $G_i''(y_i) = 2261.23$. Equation (30.2) gives the total convexity/timing adjustment as $0.0001197t_i$, or 1.197 basis points per year until the swap rate is observed. For example, for the purposes of valuing the CMS swap, the 5-year swap rate in 4 years' time should be assumed to be 5.0479% rather than 5% and the net cash flow received at the 4.5-year point should be assumed to be $0.5 \times 0.000479 \times 100{,}000{,}000 = \$23{,}940$. Other net cash flows are calculated similarly. Taking their present value, we find the value of the swap to be \$159,811.

A constant maturity Treasury swap (CMT swap) works similarly to a CMS swap except that the floating rate is the yield on a Treasury bond with a specified life. The analysis of a CMT swap is essentially the same as that for a CMS swap with S_i defined as the par yield on a Treasury bond with the specified life.

Differential Swaps

A *differential swap*, sometimes referred to as a *diff swap*, is an interest rate swap where the floating interest rate is observed in one currency and applied to a principal in another currency. Suppose that we observe the LIBOR rate for the period between t_i

and t_{i+1} in currency Y and apply it to a principal in currency X with the payment taking place at time t_{i+1}. Define V_i as the forward interest rate between t_i and t_{i+1} in currency Y and W_i as the forward exchange rate for a contract with maturity t_{i+1} (expressed as the number of units of currency Y that equal one unit of currency X). If the LIBOR rate in currency Y were applied to a principal in currency Y we would value the cash flow on the assumption that the LIBOR rate equaled F_i. From the analysis in Section 27.3, a quanto adjustment is necessary when it is applied to a principal in currency X. It is correct to value the cash flow on the assumption that the LIBOR rate equals

$$V_i + V_i \rho_i \sigma_{W,i} \sigma_{V,i} t_i \tag{30.3}$$

where $\sigma_{V,i}$ is the volatility of V_i, $\sigma_{W,i}$ is the volatility of W_i, and ρ_i is the correlation between V_i and W_i.

Example 30.4

Zero rates in both the US and Britain are flat at 5% per annum with annual compounding. In a 3-year diff swap agreement with annual payments, USD 12-month LIBOR is received and sterling 12-month LIBOR is paid with both being applied to a principal of 10 million pounds sterling. The volatility of all 1-year forward rates in the US is estimated to be 20%, the volatility of the forward USD/sterling exchange rate (dollars per pound) is 12% for all maturities, and the correlation between the two is 0.4.

In this case, $V_i = 0.05$, $\rho_i = 0.4$, $\sigma_{W,i} = 0.12$, $\sigma_{V,i} = 0.2$. The floating-rate cash flows dependent on the 1-year USD rate observed at time t_i should therefore be calculated on the assumption that the rate will be

$$0.05 + 0.05 \times 0.4 \times 0.12 \times 0.2 \times t_i = 0.05 + 0.00048 t_i$$

This means that the net cash flows from the swap at times 1, 2, and 3 years should be assumed to be 0, 4,800, and 9,600 pounds sterling for the purposes of valuation. The value of the swap is therefore

$$\frac{0}{1.05} + \frac{4,800}{1.05^2} + \frac{9,600}{1.05^3} = 12,647$$

or 12,647 pounds sterling.

30.5 EQUITY SWAPS

In an equity swap, one party promises to pay the return on an equity index on a notional principal, while the other promises to pay a fixed or floating return on a notional principal. Equity swaps enable a fund managers to increase or reduce their exposure to an index without buying and selling stock. An equity swap is a convenient way of packaging a series of forward contracts on an index to meet the needs of the market.

The equity index is usually a total return index where dividends are reinvested in the stocks comprising the index. An example of an equity swap is in Business Snapshot 30.3. In this, the 6-month return on the S&P 500 is exchanged for LIBOR. The principal on either side of the swap is $100 million and payments are made every 6 months.

For an equity-for-floating swap such as that in Business Snapshot 30.3 the value at the start of its life is zero. This is because a financial institution can arrange to costlessly

Business Snapshot 30.3 Hypothetical Confirmation for an Equity Swap

Trade date:	5-January-2004
Effective date:	11-January-2004
Business day convention (all dates):	Following business day
Holiday calendar:	US
Termination date:	11-January-2009
Equity amounts	
Equity payer:	Microsoft
Equity principal	USD 100 million
Equity index:	Total Return S&P 500 index
Equity payment:	$100(I_1 - I_0)/I_0$, where I_1 is the index level on the payment date and I_0 is the index level on the immediately preceding payment date. In the case of the first payment date, I_0 is the index level on 11-January, 2004
Equity payment dates:	Each 11-July and 11-January commencing 11-July, 2004, up to and including 11-January, 2009
Floating amounts	
Floating-rate payer:	Goldman Sachs
Floating-rate notional principal:	USD 100 million
Floating rate:	USD 6-month LIBOR
Floating-rate day count convention:	Actual/360
Floating-rate payment dates:	Each 11-July and 11-January commencing 11-July, 2004, up to and including 11-January, 2009

replicate the cash flows to one side by borrowing the principal on each payment date at LIBOR and investing it in the index until the next payment date with any dividends being reinvested. A similar argument shows that the swap is always worth zero immediately after a payment date.

Between payment dates we must value the equity cash flow and the LIBOR cash flow at the next payment date. The LIBOR cash flow was fixed at the last reset date and so can be valued easily. The value of the equity cash flow is LE/E_0, where L is the principal, E is the current value of the equity index, and E_0 is its value at the last reset date.[3]

30.6 SWAPS WITH EMBEDDED OPTIONS

Some swaps contain embedded options. In this section we consider some commonly encountered examples.

[3] See Technical Note 19 on the author's website for a more detailed discussion of this.

Accrual Swaps

Accrual swaps are swaps where the interest on one side accrues only when the floating reference rate is within a certain range. Sometimes the range remains fixed during the entire life of the swap; sometimes it is reset periodically.

As a simple example of an accrual swap, consider a deal where a fixed rate Q is exchanged for 3-month LIBOR every quarter. We suppose that the fixed rate accrues only on days when 3-month LIBOR is below 8% per annum. Suppose that the principal is L. In a normal swap the fixed-rate payer would pay QLn_1/n_2 on each payment date where n_1 is the number of days in the preceding quarter and n_2 is the number of days in the year. (This assumes that the day count is actual/actual.) In an accrual swap, this is changed to QLn_3/n_2, where n_3 is the number of days in the preceding quarter that the 3-month LIBOR was below 8%. The fixed-rate payer saves QL/n_2 on each day when 3-month LIBOR is above 8%.[4] The fixed-rate payer's position can therefore be considered equivalent to a regular swap plus a series of binary options, one for each day of the life of the swap. The binary options pay off QL/n_2 when the 3-month LIBOR is above 8%.

To generalize, we suppose that the LIBOR cutoff rate (8% in the case just considered) is R_K and that payments are exchanged every τ years. Consider day i during the life of the swap and suppose that t_i is the time until day i. Suppose that the τ-year LIBOR rate on day i is R_i so that interest accrues when $R_i < R_K$. Define F_i as the forward value of R_i and σ_i as the volatility of F_i. (The latter is estimated from spot caplet volatilities.) Using the usual lognormal assumption, the probability that LIBOR is greater than R_K in a world that is forward risk neutral with respect to a zero-coupon bond maturing at time $t_i + \tau$ is $N(d_2)$, where

$$d_2 = \frac{\ln(F_i/R_K) - \sigma_i^2 t_i/2}{\sigma_i\sqrt{t_i}}$$

The payoff from the binary option is realized at the swap payment date following day i. We suppose that this is at time s_i. The probability that LIBOR is greater than R_K in a world that is forward risk neutral with respect to a zero-coupon bond maturing at time s_i is given by $N(d_2^*)$, where d_2^* is calculated using the same formula as d_2, but with a small timing adjustment to F_i reflecting the difference between time $t_i + \tau$ and time s_i.

The value of the binary option corresponding to day i is

$$\frac{QL}{n_2} P(0, s_i) N(d_2^*)$$

The total value of the binary options is obtained by summing this expression for every day in the life of the swap. The timing adjustment (causing d_2 to be replaced by d_2^*) is so small that, in practice, it is frequently ignored.

Cancelable Swap

A cancelable swap is a plain vanilla interest rate swap where one side has the option to terminate on one or more payment dates. Terminating a swap is the same as entering

[4] The usual convention is that, if a day is a holiday, the applicable rate is assumed to be the rate on the immediately preceding business day.

into the offsetting (opposite) swap. Consider a swap between Microsoft and Goldman Sachs. If Microsoft has the option to cancel, it can regard the swap as a regular swap plus a long position in an option to enter into the offsetting swap. If Goldman Sachs has the cancelation option, Microsoft has a regular swap plus a short position in an option to enter into the swap.

If there is only one termination date, a cancelable swap is the same as a regular swap plus a position in a European swap option. Consider, for example, a 10-year swap where Microsoft will receive 6% and pay LIBOR. Suppose that Microsoft has the option to terminate at the end of 6 years. The swap is a regular 10-year swap to receive 6% and pay LIBOR plus long position in a 6-year European option to enter into a 4-year swap where 6% is paid and LIBOR is received. (The latter is referred to as a 6×4 European option.) The standard market model for valuing European swap options is described in Chapter 26.

When the swap can be terminated on a number of different payment dates, it is a regular swap option plus a Bermudan-style swap option. Consider, for example, the situation where Microsoft has entered into a 5-year swap with semiannual payments where 6% is received and LIBOR is paid. Suppose that the counterparty has the option to terminate on the swap on payment dates between year 2 and year 5. The swap is a regular swap plus a short position in a Bermudan-style swap option where the Bermudan style swap option is an option to enter into a swap that matures in 5 years and involves a fixed payment at 6% being received and a floating payment at LIBOR being paid. The swap option can be exercised on any payment date between year 2 and year 5. We discussed methods for valuing Bermudan swap options in Chapters 28 and 29.

Cancelable Compounding Swaps

Sometimes compounding swaps can be terminated on specified payment dates. On termination the floating-rate payer pays the compounded value of the floating amounts up to the time of termination and the fixed-rate payer pays the compounded value of the fixed payments up to the time of termination.

Some tricks can be used to value cancelable compounding swaps. Suppose first that the floating rate is LIBOR and it is compounded at LIBOR. We assume that the principal amount of the swap is paid on both the fixed and floating sides of the swap at the end of its life. This is similar to moving from Table 7.1 to Table 7.2 for a vanilla swap. It does not change the value of the swap and has the effect of ensuring that the value of the floating side is always equals the notional principal on a payment date. To make the cancelation decision, we need only look at the fixed side. We construct an interest rate tree as outlined in Chapter 28. We roll back through the tree in the usual way valuing the fixed side. At each node where the swap can be canceled, we test whether it is optimal to keep the swap or cancel it. Canceling the swap in effect sets the fixed side equal to par. If we are paying fixed and receiving floating, our objective is to minimize the value of the fixed side; if we are receiving fixed and paying floating, our objective is to maximize the value of the fixed side.

When the floating side is LIBOR plus a spread compounded at LIBOR, we can subtract cash flows corresponding to the spread rate of interest from the fixed side instead of adding them to the floating side. The option can then be valued as in the case where there is no spread.

When the compounding is at LIBOR plus a spread, an approximate approach is as follows:[5]

1. Calculate the value of the floating side of the swap at each cancelation date assuming forward rates are realized.
2. Calculate the value of the floating side of the swap at each cancelation date assuming that the floating rate is LIBOR and it is compounded at LIBOR.
3. Define the excess of step 1 over step 2 as the "value of spreads" on a cancelation date.
4. Treat the option in the way described above. In deciding whether to exercise the cancelation option, subtract the value of the spreads from the values calculated for the fixed side.

30.7 OTHER SWAPS

This chapter has discussed just a few of the different types of swaps that trade. In practice, the number of instruments that trade is limited only by the imagination of financial engineers and the appetite of corporate treasurers for innovative risk management tools.

A swap that was very popular in the United States in the mid-1990s is an *index amortizing rate swap* (sometimes also called an *indexed principal swap*). In this, the principal reduces in a way dependent on the level of interest rates. The lower the interest rate, the greater the reduction in the principal. The fixed side of an indexed amortizing swap was originally designed to mirror, at least approximately, the return obtained by an investor on a mortgage-backed security after prepayment options are taken into account. The swap therefore exchanged the return on a mortgage-backed security for a floating-rate return.

Commodity swaps are now becoming increasingly popular. A company that consumes 100,000 barrels of oil per year could agree to pay \$4 million each year for the next 10 years and to receive in return $100{,}000S$, where S is the market price of oil per barrel. The agreement would in effect lock in the company's oil cost at \$40 per barrel. An oil producer might agree to the opposite exchange, thereby locking in the price it realized for its oil at \$40 per barrel. We discussed energy derivatives in Chapter 23.

A recent innovation in swap markets is a *volatility swap*. In this, the payments depend on the volatility of a stock (or other asset). Suppose that the principal is L. On each payment date, one side pays $L\sigma$, where σ is the historical volatility calculated in the usual way by taking daily observations on the stock during the immediately preceding accrual period and the other side pays $L\sigma_K$, where σ_K is a constant prespecified volatility level. Variance swaps, correlation swaps, and covariance swaps are defined similarly.

A number of other types of swaps are discussed elsewhere in this book. For example, asset swaps are discussed in Chapter 20, and total return swaps and various types of credit default swaps are covered in Chapter 21.

[5] This approach is not perfectly accurate in that it assumes that the decision to exercise the cancelation option is not influenced by future payments being compounded at a rate different from LIBOR.

Business Snapshot 30.4 Procter and Gamble's Bizarre Deal

A particularly bizarre swap is the so-called "5/30" swap entered into between Bankers Trust (BT) and Procter and Gamble (P&G) on November 2, 1993. This was a 5-year swap with semiannual payments. The notional principal was \$200 million. BT paid P&G 5.30% per annum. P&G paid BT the average 30-day CP (commercial paper) rate minus 75 basis points plus a spread. The average commercial paper rate was calculated by taking observations on the 30-day commercial paper rate each day during the preceding accrual period and averaging them.

The spread was zero for the first payment date (May 2, 1994). For the remaining nine payment dates, it was

$$\max\left[0, \frac{98.5\left(\dfrac{\text{5-year CMT\%}}{5.78\%}\right) - (\text{30-year TSY price})}{100}\right]$$

In this, 5-year CMT is the constant maturity Treasury yield (i.e., the yield on a 5-year Treasury note, as reported by the Federal Reserve). The 30-year TSY price is the midpoint of the bid and offer cash bond prices for the 6.25% Treasury bond maturing on August 2023. Note that the spread calculated from the formula is a decimal interest rate. It is not measured in basis points. If the formula gives 0.1 and the CP rate is 6%, the rate paid by P&G is 15.25%.

P&G were hoping that the spread would be zero and the deal would enable them to exchange fixed-rate funding at 5.30% for funding at 75 basis points less than the commercial paper rate. In fact, interest rates rose sharply in early 1994, bond prices fell, and the swap proved very, very expensive (see Problem 30.10).

Bizarre Deals

Some swaps have payoffs that are calculated in quite bizarre ways. An example is a deal entered into between Procter and Gamble and Bankers Trust in 1993 (see Business Snapshot 30.4). The details of this transaction are in the public domain because it later became the subject of litigation.[6]

SUMMARY

Swaps have proved to be very versatile financial instruments. Many swaps can be valued by (a) assuming that LIBOR (or some other floating reference rate) will equal its forward value and (b) discounting the resulting cash flows at the LIBOR/swap rate. These include plain vanilla interest swaps, most types of currency swaps, swaps where the principal changes in a predetermined way, swaps where the payment dates are different on each side, and compounding swaps.

[6] See D. J. Smith, "Aggressive Corporate Finance: A Close Look at the Procter and Gamble–Bankers Trust Leveraged Swap," *Journal of Derivatives* 4, 4 (Summer 1997): 67–79.

Some swaps require adjustments to the forward rates when they are valued. These adjustments are termed convexity, timing, or quanto adjustments. Among the swaps that require adjustments are LIBOR-in-arrears swaps, CMS/CMT swaps, and differential swaps.

Equity swaps involve the return on an equity index being exchanged for a fixed or floating rate of interest. They are usually designed so that they are worth zero immediately after a payment date, but they may have nonzero values between payment dates.

Some swaps involve embedded options. An accrual swap turns out to be a regular swap plus a large portfolio of binary options (one for each day of the life of the swap). A cancelable swap turns out to be a regular swap plus a Bermudan swap option.

FURTHER READING

Chance, D., and Rich, D., "The Pricing of Equity Swap and Swaptions," *Journal of Derivatives* 5, 4 (Summer 1998): 19–31.

Demeterfi, K., Derman, E., Kamal, M., and Zou, J., "A Guide to Volatility and Variance Swaps," *Journal of Derivatives* 6, 4 (Summer 1999): 9–32.

Smith D. J., "Aggressive Corporate Finance: A Close Look at the Procter and Gamble–Bankers Trust Leveraged Swap," *Journal of Derivatives*, 4, 4 (Summer 1997): 67–79.

Questions and Problems (Answers in Solutions Manual)

30.1. Calculate all the fixed cash flows and their exact timing for the swap in Business Snapshot 30.1. Assume that the day count conventions are applied using target payment dates rather than actual payment dates.

30.2. Suppose that a swap specifies that a fixed rate is exchanged for twice the LIBOR rate. Can the swap be valued using the "assume forward rates are realized" rule?

30.3. What is the value of a 2-year fixed-for-floating compound swap where the principal is \$100 million and payments are made semiannually. Fixed interest is received and floating is paid? The fixed rate is 8% and it is compounded at 8.3% (both semiannually compounded). The floating rate is LIBOR plus 10 basis points and it is compounded at LIBOR plus 20 basis points. The LIBOR zero curve is flat at 8% with semiannual compounding.

30.4. What is the value of a 5-year swap where LIBOR is paid in the usual way and in return LIBOR compounded at LIBOR is received on the other side? The principal on both sides is \$100 million. Payment dates on the pay side and compounding dates on the receive side are every 6 months and the yield curve is flat at 5% with semiannual compounding.

30.5. Explain carefully why a bank might choose to discount cash flows on a currency swap at a rate slightly different from LIBOR.

30.6. Calculate the total convexity/timing adjustment in Example 30.3 of Section 30.4 if all cap volatilities are 18% instead of 20% and volatilities for all options on 5-year swaps are 13% instead of 15%. What should the 5-year swap rate in 3 years' time be assumed for the purpose of valuing the swap? What is the value of the swap?

30.7. Explain why a plain vanilla interest rate swap and the compounding swap in Section 30.2 can be valued using the "assume forward rates are realized" rule, but a LIBOR-in-arrears swap in Section 30.4 cannot.

30.8. In the accrual swap discussed in the text, the fixed side accrues only when the floating reference rate lies below a certain level. Discuss how the analysis can be extended to cope with a situation where the fixed side accrues only when the floating reference rate is above one level and below another.

Assignment Questions

30.9. LIBOR zero rates are flat at 5% in the United States and flat at 10% in Australia (both annually compounded). In a 4-year swap Australian LIBOR is received and 9% is paid with both being applied to a USD principal of $10 million. Payments are exchanged annually. The volatility of all 1-year forward rates in Australia is estimated to be 25%, the volatility of the forward USD/AUD exchange rate (AUD per USD) is 15% for all maturities, and the correlation between the two is 0.3. What is the value of the swap?

30.10. Estimate the interest rate paid by P&G on the 5/30 swap in Section 30.7 if (a) the CP rate is 6.5% and the Treasury yield curve is flat at 6% and (b) the CP rate is 7.5% and the Treasury yield curve is flat at 7% with semiannual compounding.

30.11. Suppose that you are trading a LIBOR-in-arrears swap with an unsophisticated counterparty who does not make convexity adjustments. To take advantage of the situation, should you be paying fixed or receiving fixed? How should you try to structure the swap as far as its life and payment frequencies?

Consider the situation where the yield curve is flat at 10% per annum with annual compounding. All cap volatilities are 18%. Estimate the difference between the way a sophisticated trader and an unsophisticated trader would value a LIBOR-in-arrears swap where payments are made annually and the life of the swap is (a) 5 years, (b) 10 years, and (c) 20 years. Assume a notional principal of $1 million.

30.12. Suppose that the LIBOR zero rate is flat at 5% with annual compounding. In a 5-year swap, company X pays a fixed rate of 6% and receives LIBOR. The volatility of the 2-year swap rate in 3 years is 20%.

(a) What is the value of the swap?

(b) Use DerivaGem to calculate the value of the swap if company X has the option to cancel after 3 years.

(c) Use DerivaGem to calculate the value of the swap if the counterparty has the option to cancel after 3 years.

(d) What is the value of the swap if either side can cancel at the end of 3 years?