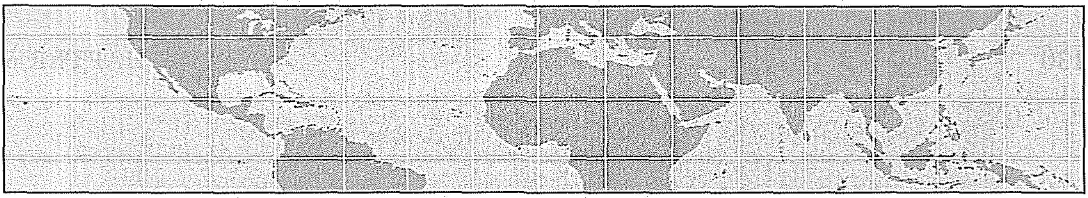

CHAPTER 6 Interest Rate Futures

So far we have covered futures contracts on commodities, stock indices, and foreign currencies. We have seen how they work, how they are used for hedging, and how futures prices are set. We now move on to consider interest rate futures.

In this chapter we explain the popular Treasury bond and Eurodollar futures contracts that trade in the United States. Many of the other interest rate futures contracts throughout the world have been modeled on these contracts. We also show how interest rate futures contracts, when used in conjunction with the duration measure introduced in Chapter 4, can be used to hedge a company's exposure to interest rate movements.

6.1 DAY COUNT CONVENTIONS

As a preliminary to the material in this chapter, we consider day count conventions. The day count defines the way in which interest accrues over time. Generally, we know the interest earned over some reference period (e.g., the time between coupon payments), and we are interested in calculating the interest earned over some other period.

The day count convention is usually expressed as X/Y. When we are calculating the interest earned between two dates, X defines the way in which the number of days between the two dates is calculated, and Y defines the way in which the total number of days in the reference period is measured. The interest earned between the two dates is

$$\frac{\text{Number of days between dates}}{\text{Number of days in reference period}} \times \text{Interest earned in reference period}$$

Three day count conventions that are commonly used in the United States are:

1. Actual/actual (in period)
2. 30/360
3. Actual/360

Business Snapshot 6.1 Day Counts Can Be Deceptive

Between February 28, 2005, and March 1, 2005, you have a choice between owning a US government bond and a US corporate bond. They pay the same coupon and have the same quoted price. Which would you prefer?

It sounds as though you should be indifferent, but in fact you should have a marked preference for the corporate bond. Under the 30/360 day count convention used for corporate bonds, there are 3 days between February 28, 2002, and March 1, 2002. Under the actual/actual (in period) day count convention used for government bonds, there is only 1 day. You would earn approximately three times as much interest by holding the corporate bond!

US Treasury Bonds

The actual/actual (in period) day count is used for Treasury bonds in the United States. This means that the interest earned between two dates is based on the ratio of the actual days elapsed to the actual number of days in the period between coupon payments. Suppose that the bond principal is \$100, coupon payment dates are March 1 and September 1, the coupon rate is 8%, and we wish to calculate the interest earned between March 1 and July 3. The reference period is from March 1 to September 1. There are 184 (actual) days in this period, and interest of \$4 is earned during the period. There are 124 (actual) days between March 1 and July 3. The interest earned between March 1 and July 3 is therefore

$$\frac{124}{184} \times 4 = 2.6957$$

US Corporate and Municipal Bonds

The 30/360 day count is used for corporate and municipal bonds in the United States. This means that we assume 30 days per month and 360 days per year when carrying out calculations. With the 30/360 day count, the total number of days between March 1 and September 1 is 180. The total number of days between March 1 and July 3 is $(4 \times 30) + 2 = 122$. In a corporate bond with the same terms as the Treasury bond just considered, the interest earned between March 1 and July 3, therefore, would be

$$\frac{122}{180} \times 4 = 2.7111$$

As shown in Business Snapshot 6.1, sometimes the 30/360 day count convention has surprising consequences.

US Money Market Instruments

The actual/360 day count is used for money market instruments in the United States. This indicates that the reference period is 360 days. The interest earned during part of a year is calculated by dividing the actual number of elapsed days by 360 and multiplying by the rate. The interest earned in 90 days is therefore exactly one-fourth of the quoted rate, and the interest earned in a whole year of 365 days is 365/360 times the quoted rate.

The prices of money market instruments are sometimes quoted using a *discount rate.* This is the interest earned as a percentage of the final face value rather than as a percentage of the initial price paid for the instrument. An example is Treasury bills in the United States. If the price of a 91-day Treasury bill is quoted as 8, this means that the annualized rate of interest earned is 8% of the face value. Suppose that the face value is \$100. Interest of \$2.0222 (= \$100 × 0.08 × 91/360) is earned over the 91-day life. This corresponds to a true rate of interest of 2.0222/(100 − 2.0222) = 2.064% for the 91-day period. In general, the relationship between the cash price and quoted price of a Treasury bill in the United States is

$$P = \frac{360}{n}(100 - Y)$$

where P is the cash price, Y is the quoted price, and n is the remaining life of the Treasury bill measured in calendar days.

6.2 QUOTATIONS FOR TREASURY BONDS

Treasury bond prices in the United States are quoted in dollars and thirty-seconds of a dollar. The quoted price is for a bond with a face value of \$100. Thus, a quote of 90-05 indicates that the quoted price for a bond with a face value of \$100,000 is \$90,156.25.

The quoted price, which traders refer to as the *clean price*, is not the same as the cash price, which traders refer to as the *dirty price*. In general,

$$\text{Cash price} = \text{Quoted price} + \text{Accrued interest since last coupon date}$$

To illustrate this formula, suppose that it is March 5, 2007, and the bond under consideration is an 11% coupon bond maturing on July 10, 2012, with a quoted price of 95-16 or \$95.50. Because coupons are paid semiannually on government bonds (and the final coupon is at maturity), the most recent coupon date is January 10, 2007, and the next coupon date is July 10, 2007. The number of days between January 10, 2007, and March 5, 2007, is 54, whereas the number of days between January 10, 2007, and July 10, 2007, is 181. On a bond with \$100 face value, the coupon payment is \$5.50 on January 10 and July 10. The accrued interest on March 5, 2007, is the share of the July 10 coupon accruing to the bondholder on March 5, 2007. Because actual/actual in period is used for Treasury bonds (see Section 6.1), this is

$$\frac{54}{181} \times \$5.5 = \$1.64$$

The cash price per \$100 face value for the bond is therefore

$$\$95.5 + \$1.64 = \$97.14$$

Thus, the cash price of a \$100,000 bond is \$97,140.

Table 6.1 Interest rate futures quotes from the *Wall Street Journal* on February 5, 2004. (Columns show month, open, high, low, settle, change, lifetime high, lifetime low, and open interest, respectively.)

Interest Rate Futures

Treasury Bonds (CBT)-$100,000; pts 32nds of 100%

Mar	111-25	111-31	109-18	111-17	-3	116-23	101-00	467,134
June	110-09	110-12	109-16	110-03	-3	116-15	104-00	31,215

Est vol 183,502; vol Tue 208,442; open int 499,090, +8,789.

Treasury Notes (CBT)-$100,000; pts 32nds of 100%

Mar	113-29	14-005	113-15	113-22	-4.5	116-10	106-29	1,130,409
June	112-17	112-17	111-29	112-03	-4.5	113-18	107-13	147,892

Est vol 489,439; vol Tue 623,701; open int 1,278,301, -9,178.

5 Yr. Treasury Notes (CBT)-$100,000; pts 32nds of 100%

Mar	12-215	112-24	12-125	112-17	-3.5	19-215	09-145	882,174

Est vol 219,841; vol Tue 268,683; open int 948,759, +6,645.

2 Yr. Treasury Notes (CBT)-$200,000; pts 32nds of 100%

Mar	07-132	07-142	07-102	07-127	-.2	07-205	106-02	164,711

Est vol 15,846; vol Tue 11,507; open int 166,044, +168.

30 Day Federal Funds (CBT)-$5,000,000; 100 - daily avg.

Feb	99.000	99.000	98.995	99.000	...	99.890	98.700	64,359
Mar	99.00	99.00	98.99	98.99	...	99.16	98.74	48,219
Apr	99.00	99.00	98.99	98.99	...	99.17	89.96	71,817
May	98.96	98.96	98.95	98.96	...	99.79	98.40	37,989
June	98.94	98.95	98.94	98.95	...	98.97	98.38	27,460
July	98.87	98.87	98.86	98.87	...	98.93	98.20	26,248
Aug	98.77	98.78	98.77	98.78	.01	98.85	98.24	4,137
Sept	98.70	98.71	98.68	98.71	-.01	98.79	98.22	5,260

Est vol 15,789; vol Tue 16,390; open int 286,642, -49,041.

10 Yr. Interest Rate Swaps (CBT)-$100,000; pts 32nds of 100%

Mar	111-15	111-19	111-03	111-10	-6	113-05	107-20	39,568

Est vol 1,060; vol Tue 968; open int 39,569, +269.

10 Yr. Muni Note Index (CBT)-$1,000 x index

Mar	103-13	103-21	103-08	103-15	1	105-04	99-21	2,249

Est vol 269; vol Tue 194; open int 2,249, +6.
Index: Close 104-15; Yield 4.44.

1 Month Libor (CME)-$3,000,000; pts of 100%

	OPEN	HIGH	LOW	SETTLE	CHG	YIELD	CHG	OPEN INT
Feb	98.90	98.90	98.89	98.89	...	1.11	...	29,195
Mar	98.89	98.89	98.89	98.89	...	1.11	...	11,060
Apr	98.86	98.86	98.86	98.86	...	1.14	...	8,279
May	98.83	98.83	98.82	98.82	...	1.18	...	2,550
Oct	98.44	98.44	98.43	98.44	...	1.56	...	51,960

Est vol 1,215; vol Tue 2,781; open int 171,119, +1,172.

Eurodollar (CME)-$1,000,000; pts of 100%

	OPEN	HIGH	LOW	SETTLE	CHG	YIELD	CHG	OPEN INT
Feb	98.86	98.86	98.86	98.86	...	1.14	...	32,246
Mar	98.84	98.84	98.83	98.84	...	1.16	...	827,925
Apr	98.80	98.80	98.79	98.80	...	1.20	...	35,531
May	98.75	98.75	98.74	98.74	...	1.26	...	14,543
June	98.69	98.69	98.66	98.68	...	1.32	...	838,794
July	98.58	98.58	98.57	98.58	...	1.42	...	2,150
Sept	98.41	98.43	98.38	98.41	...	1.59	...	794,586
Dec	98.04	98.06	98.00	98.03	...	1.97	...	600,750
Mr05	97.65	97.67	97.58	97.63	...	2.37	...	419,479
June	97.24	97.26	97.19	97.23	...	2.77	...	330,839
Sept	96.88	96.90	96.82	96.86	...	3.14	...	260,971
Dec	96.56	96.59	96.51	96.55	...	3.45	...	191,396
Mr06	96.32	96.33	96.25	96.30	-.01	3.70	.01	172,526
June	96.10	96.11	96.04	96.07	-.01	3.93	.01	128,625
Sept	95.91	95.91	95.83	95.86	-.01	4.14	.01	119,346
Dec	95.69	95.71	95.63	95.66	-.02	4.34	.02	105,045
Mr07	95.47	95.53	95.46	95.49	-.02	4.51	.02	75,659
June	95.34	95.38	95.30	95.33	-.03	4.67	.03	66,675
Sept	95.19	95.23	95.16	95.18	-.03	4.82	.03	73,288
Dec	95.05	95.09	95.02	95.04	-.03	4.96	.03	59,439
Mr08	94.92	94.97	94.90	94.92	-.03	5.08	.03	46,996
June	94.80	94.86	94.79	94.81	-.03	5.19	.03	50,074
Sept	94.71	94.75	94.68	94.71	-.03	5.29	.03	34,029
Dec	94.65	94.65	94.57	94.60	-.03	5.40	.03	26,470
Ju09	94.42	94.47	94.41	94.43	-.03	5.57	.03	9,247
Sept	94.34	94.40	94.34	94.35	-.03	5.65	.03	8,400
Dec	94.26	94.31	94.25	94.27	-.03	5.73	.03	4,633
Mr10	94.19	94.19	94.18	94.19	-.04	5.81	.04	8,192
June	94.12	94.12	94.11	94.12	-.04	5.88	.04	6,761
Sept	94.05	94.05	94.04	94.05	-.04	5.95	.04	4,683

Est vol 780,408; vol Tue 779,833; open int 5,375,781, +11,885.

Euroyen (CME)-¥100,000,000; pts of 100%

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Mar	99.91	99.91	99.91	99.91	...	99.92	99.14	11,530
June	99.91	99.91	99.91	99.91	...	99.92	99.41	9,096
Sept	99.89	99.89	99.89	99.89	...	99.90	99.35	12,320
Mr05	99.82	99.82	99.82	99.82	...	99.84	99.27	4,726

Est vol 431; vol Tue 25; open int 49,808, +775.

Short Sterling (LIFFE)-£500,000; pts of 100%

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Feb	95.82	95.82	95.82	95.82	...	95.89	95.80	1,913
Mar	95.76	95.77	95.75	95.76	...	96.80	93.01	188,159
June	95.57	95.58	95.54	95.56	...	96.71	93.04	201,882
Sept	95.37	95.40	95.34	95.36	...	96.59	93.35	153,843
Dec	95.21	95.24	95.19	95.20	...	96.48	93.25	139,045
Mr05	95.10	95.13	95.06	95.08	...	96.38	93.29	83,684
June	95.01	95.04	94.98	94.99	...	96.30	93.29	72,583
Sept	94.95	94.97	94.91	94.92	...	96.23	94.06	70,992
Dec	94.88	94.91	94.85	94.86	...	96.15	94.06	35,228
Mr06	94.82	94.84	94.80	94.82	.02	96.10	94.05	27,988
June	94.77	94.81	94.75	94.77	.02	95.97	94.04	28,423
Sept	94.74	94.78	94.72	94.74	.02	95.75	94.32	15,264
Dec	94.74	94.75	94.71	94.72	.02	95.83	94.25	6,356
Mr07	94.69	94.69	94.69	94.71	.02	95.82	94.33	527
June	94.71	94.71	94.71	94.70	.02	95.73	94.66	639

Est vol 142,996; vol Tue 184,402; open int 1,028,552, -284.

Long Gilt (LIFFE)-£100,000; pts of 100%

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Mar	107.95	108.41	107.95	108.12	.26	109.73	105.39	159,338

Est vol 50,453; vol Tue 36,817; open int 159,339, -153.

3 Month Euribor (LIFFE)-€1,000,000; pts of 100%

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Feb	97.92	97.93	97.92	97.93	.01	97.96	97.77	13,595
Mar	97.94	97.95	97.93	97.94	.01	98.29	93.83	562,698
June	97.91	97.92	97.89	97.90	.02	98.21	93.79	511,614
Sept	97.77	97.78	97.75	97.76	.03	98.08	93.73	428,741
Dec	97.55	97.57	97.53	97.55	.04	97.91	93.64	436,055
Mr05	97.32	97.34	97.30	97.31	.03	97.77	94.07	301,516
June	97.09	97.10	97.06	97.07	.03	97.60	94.29	197,768
Sept	96.88	96.89	96.84	96.86	.02	97.44	94.29	119,907
Dec	96.68	96.69	96.65	96.66	.02	97.28	94.41	95,512
Mr06	96.51	96.53	96.48	96.50	.02	97.14	94.40	41,992
June	96.35	96.36	96.32	96.33	.02	96.96	94.66	37,197
Sept	96.20	96.21	96.17	96.18	.02	96.81	94.58	22,947
Dec	96.03	96.04	96.01	96.02	.02	96.60	94.62	11,645
Mr07	95.93	95.93	95.93	95.90	.02	96.48	94.57	4,473
June	95.79	95.79	95.79	95.80	.02	96.29	94.57	2,490
Sept	95.69	95.69	95.69	95.70	.02	96.21	95.26	2,204

Est vol 547,848; vol Tue 533,760; open int 2,791,222, +50,205.

3 Month Euroswiss (LIFFE)-CHF 1,000,000; pts of 100%

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Mar	99.73	99.74	99.72	99.73	...	99.75	96.32	95,989
June	99.61	99.61	99.56	99.57	-.02	99.63	96.98	81,441
Sept	99.37	99.39	99.35	99.36	-.01	99.41	97.60	41,632
Dec	99.14	99.14	99.11	99.12	-.01	99.17	98.00	32,364
Mr05	98.87	98.87	98.86	98.87	-.01	98.93	97.90	7,335
June	98.65	98.65	98.61	98.62	-.01	98.68	97.74	9,694
Sept	98.43	98.44	98.36	98.41	-.02	98.47	97.75	4,732
Dec	98.21	98.22	98.14	98.19	-.01	98.24	97.92	2,745

Est vol 14,180; vol Tue 21,649; open int 275,932, -1,537.

Canadian Bankers Acceptance (ME)-CAD 1,000,000

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Mar	97.71	97.71	97.67	97.68	-0.02	97.78	93.77	70,087
June	97.78	97.78	97.72	97.75	-0.03	97.88	95.34	97,819
Sept	97.71	97.71	97.65	97.68	-0.03	97.81	94.22	35,605
Dec	97.51	97.51	97.45	97.47	-0.04	97.62	94.10	17,196
Mr05	97.21	97.21	97.16	97.18	-0.04	97.33	94.45	9,163
Sept	96.53	96.53	96.53	96.51	-0.04	96.64	95.21	1,200

Est vol 24,925; vol Tue 21,162; open int 238,828, +585.

10 Yr. Canadian Govt. Bonds (ME)-CAD 100,000

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Mar	110.58	110.64	110.07	110.39	-0.22	111.61	106.90	90,003

Est vol 6,222; vol Tue 12,898; open int 90,003, +6,036.

3 Yr. Commonwealth T-Bonds (SFE)-AUD 100,000								
Mar	94.38	94.48	94.37	94.47	0.09	94.56	93.96	609,295
Est vol 130,882; vol Tue 72,788; open int 609,295, +83,322.								
Euroyen (SGX)-¥100,000,000; pts of 100%								
Mar	99.91	99.91	99.91	99.91	...	99.92	98.19	60,509
June	99.91	99.91	99.91	99.92	0.01	99.92	99.45	71,194
Sept	99.89	99.90	99.89	99.90	0.01	99.90	99.34	43,155
Dec	99.87	99.87	99.87	99.87	...	99.87	99.22	45,234
Mr05	99.81	99.82	99.81	99.82	0.01	99.85	99.18	23,103
June	99.78	99.78	99.78	99.78	0.01	99.85	99.10	20,948
Sept	99.70	99.71	99.70	99.71	0.02	99.74	98.95	14,023
Dec	99.61	99.62	99.61	99.61	0.01	99.77	98.80	3,635
Mr06	99.50	99.50	99.50	99.50	0.01	99.76	98.84	3,405
June	99.42	99.42	99.42	99.43	0.02	99.75	98.55	1,380
Dec	99.23	99.23	99.23	99.23	0.02	99.71	98.35	1,851
Est vol 3,160; vol Tue 5,292; open int 295,306, −1,880.								
5 Yr. Euro-BOBL (EUREX)-€100,000; pts of 100%								
Mar	111.59	111.66	111.47	111.56	...	112.06	108.71	743,330
June	110.79	110.80	110.71	110.75	...	111.16	109.50	7,545
vol Wed 582,579; open int 750,875, +21,654.								
10 Yr. Euro-BUND (EUREX)-€100,000; pts of 100%								
Mar	114.30	114.45	114.15	114.26	−0.02	117.76	110.73	945,187
June	113.31	113.43	113.26	113.28	−0.01	114.11	110.62	27,345
vol Wed 841,211; open int 972,534, −23,916.								
2 Yr. Euro-SCHATZ (EUREX)-€100,000; pts of 100%								
Mar	106.18	106.20	106.13	106.17	...	106.35	104.95	683,537
June	105.80	105.84	105.79	105.80	...	105.88	105.21	28,066
vol Wed 437,442; open int 711,603, +22,620.								

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6.3 TREASURY BOND FUTURES

Table 6.1 shows interest rate futures quotes as they appeared in the *Wall Street Journal* on February 5, 2004. One of the most popular long-term interest rate futures contracts is the Treasury bond futures contract traded on the Chicago Board of Trade (CBOT). In this contract, any government bond that has more than 15 years to maturity on the first day of the delivery month and is not callable within 15 years from that day can be delivered. As will be explained later in this section, the CBOT has developed a procedure for adjusting the price received by the party with the short position according to the particular bond delivered.

The Treasury note and 5-year Treasury note futures contract in the United States are also very popular. With Treasury note futures, any government bond (or note) with a maturity between $6\frac{1}{2}$ and 10 years can be delivered. In the 5-year Treasury note futures contract, the bond delivered has a life that is about 4 or 5 years.

The remaining discussion in this section focuses on CBOT Treasury bond futures. The Treasury note futures traded in the United States and many other futures contracts in the rest of the world are designed in a similar way to CBOT Treasury bond futures, so that many of the points we will make are applicable to these contracts as well.

Quotes

Treasury bond futures prices are quoted in the same way as the Treasury bond prices themselves (see Section 6.2). Table 6.1 shows that the settlement price on February 4, 2004, for the June 2004 contract was 110-03, or $110\frac{3}{32}$. One contract involves the delivery of \$100,000 face value of the bond. Thus, a \$1 change in the quoted futures price would lead to a \$1,000 change in the value of the futures contract. Delivery can take place at any time during the delivery month.

Conversion Factors

As mentioned, the Treasury bond futures contract allows the party with the short position to choose to deliver any bond that has a maturity of more than 15 years and that is not callable within 15 years. When a particular bond is delivered, a parameter known as its *conversion factor* defines the price received by the party with the short position. The quoted price applicable to the delivery is the product of the conversion factor and the most recent settlement price. Taking accrued interest into account, as described in Section 6.2, the cash received for each \$100 face value of bond delivered is

$$(\text{Settlement price} \times \text{Conversion factor}) + \text{Accrued interest}$$

Each contract is for the delivery of \$100,000 face value of bonds. Suppose the settlement price is 90-00, the conversion factor for the bond delivered is 1.3800, and the accrued interest on this bond at the time of delivery is \$3 per \$100 face value. The cash received by the party with the short position (and paid by the party with the long position) is then

$$(1.3800 \times 90.00) + 3.00 = \$127.20$$

per \$100 face value. A party with the short position in one contract would deliver bonds with a face value of \$100,000 and receive \$127,200.

The conversion factor for a bond is equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per annum (with semiannual compounding). The bond maturity and the times to the coupon payment dates are rounded down to the nearest 3 months for the purposes of the calculation. The practice enables the CBOT to produce comprehensive tables. If, after rounding, the bond lasts for an exact number of 6-month periods, the first coupon is assumed to be paid in 6 months. If, after rounding, the bond does not last for an exact number of 6-month periods (i.e., there are an extra 3 months), the first coupon is assumed to be paid after 3 months and accrued interest is subtracted.

As a first example of these rules, consider a 10% coupon bond with 20 years and 2 months to maturity. For the purposes of calculating the conversion factor, the bond is assumed to have exactly 20 years to maturity. The first coupon payment is assumed to be made after 6 months. Coupon payments are then assumed to be made at 6-month intervals until the end of the 20 years when the principal payment is made. Assume that the face value is \$100. When the discount rate is 6% per annum with semiannual compounding (or 3% per 6 months), the value of the bond is

$$\sum_{i=1}^{40} \frac{5}{1.03^i} + \frac{100}{1.03^{40}} = \$146.23$$

Dividing by the face value gives a conversion factor of 1.4623.

As a second example of the rules, consider an 8% coupon bond with 18 years and 4 months to maturity. For the purposes of calculating the conversion factor, the bond is assumed to have exactly 18 years and 3 months to maturity. Discounting all the payments back to a point in time 3 months from today at 6% per annum (compounded semiannually) gives a value of

$$4 + \sum_{i=1}^{36} \frac{4}{1.03^i} + \frac{100}{1.03^{36}} = \$125.83$$

The interest rate for a 3-month period is $\sqrt{1.03} - 1$, or 1.4889%. Hence, discounting back to the present gives the bond's value as $125.83/1.014889 = \$123.99$. Subtracting the accrued interest of 2.0, this becomes \$121.99. The conversion factor is therefore 1.2199.

Cheapest-to-Deliver Bond

At any given time during the delivery month, there are many bonds that can be delivered in the CBOT Treasury bond futures contract. These vary widely as far as coupon and maturity are concerned. The party with the short position can choose which of the available bonds is "cheapest" to deliver. Because the party with the short position receives

$$(\text{Settlement price} \times \text{Conversion factor}) + \text{Accrued interest}$$

and the cost of purchasing a bond is

$$\text{Quoted bond price} + \text{Accrued interest}$$

the cheapest-to-deliver bond is the one for which

$$\text{Quoted bond price} - (\text{Settlement price} \times \text{Conversion factor})$$

is least. Once the party with the short position has decided to deliver, it can determine the cheapest-to-deliver bond by examining each of the bonds in turn.

Example 6.1

The party with the short position has decided to deliver and is trying to choose between the three bonds in Table 6.2. Assume the most recent settlement price is 93-08, or 93.25.

Table 6.2 Deliverable bonds in the Example 6.1.

Bond	*Quoted bond price ($)*	*Conversion factor*
1	99.50	1.0382
2	143.50	1.5188
3	119.75	1.2615

The cost of delivering each of the bonds is as follows:

Bond 1: $99.50 - (93.25 \times 1.0382) = \$2.69$

Bond 2: $143.50 - (93.25 \times 1.5188) = \$1.87$

Bond 3: $119.75 - (93.25 \times 1.2615) = \$2.12$

The cheapest-to-deliver bond is Bond 2.

A number of factors determine the cheapest-to-deliver bond. When bond yields are in excess of 6%, the conversion factor system tends to favor the delivery of low-coupon long-maturity bonds. When yields are less than 6%, the system tends to favor the delivery of high-coupon short-maturity bonds. Also, when the yield curve is upward-sloping, there is a tendency for bonds with a long time to maturity to be favored, whereas when it is downward-sloping, there is a tendency for bonds with a short time to maturity to be delivered.

In addition to the cheapest-to-deliver bond option, the party with a short position has an option known as the wild card play. This is described in Business Snapshot 6.2.

Business Snapshot 6.2 The Wild Card Play

Trading in the CBOT Treasury bond futures contract ceases at 2:00 p.m. Chicago time. However, Treasury bonds themselves continue trading in the spot market until 4:00 p.m. Furthermore, a trader with a short futures position has until 8:00 p.m. to issue to the clearinghouse a notice of intention to deliver. If the notice is issued, the invoice price is calculated on the basis of the settlement price that day. This is the price at which trading was conducted just before the closing bell at 2:00 p.m.

This practice gives rise to an option known as the *wild card play*. If bond prices decline after 2:00 p.m. on the first day of the delivery month, the party with the short position can issue a notice of intention to deliver at, say, 3:45 p.m. and proceed to buy cheapest-to-deliver bonds for delivery at the 2:00 p.m. futures price. If the bond price does not decline, the party with the short position keeps the position open and waits until the next day when the same strategy can be used.

As with the other options open to the party with the short position, the wild card play is not free. Its value is reflected in the futures price, which is lower than it would be without the option.

Determining the Futures Price

An exact theoretical futures price for the Treasury bond contract is difficult to determine because the short party's options concerned with the timing of delivery and choice of the bond that is delivered cannot easily be valued. However, if we assume that both the cheapest-to-deliver bond and the delivery date are known, the Treasury bond futures contract is a futures contract on a security providing the holder with known income.[1] Equation (5.2) then shows that the futures price, F_0, is related to the spot price, S_0, by

$$F_0 = (S_0 - I)e^{rT} \tag{6.1}$$

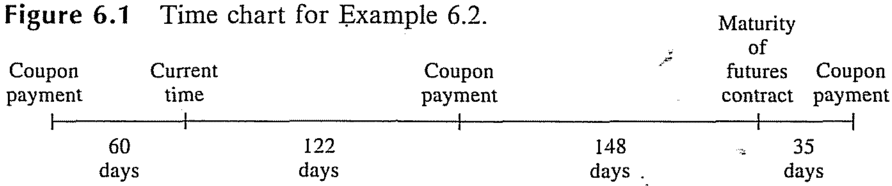
where I is the present value of the coupons during the life of the futures contract, T is the time until the futures contract matures, and r is the risk-free interest rate applicable to a time period of length T.

Example 6.2

Suppose that, in a Treasury bond futures contract, it is known that the cheapest-to-deliver bond will be a 12% coupon bond with a conversion factor of 1.4000. Suppose also that it is known that delivery will take place in 270 days. Coupons are payable semiannually on the bond. As illustrated in Figure 6.1, the last coupon date was 60 days ago, the next coupon date is in 122 days, and the coupon date thereafter is in 305 days. The term structure is flat, and the rate of interest (with continuous compounding) is 10% per annum. Assume that the current quoted bond price is \$120. The cash price of the bond is obtained by adding to this quoted price the proportion of the next coupon payment that accrues to the

[1] In practice, for the purposes of determining the cheapest-to-deliver in this calculation, analysts usually assume that zero rates at the maturity of the futures contract will equal today's forward rates.

Figure 6.1 Time chart for Example 6.2.



holder. The cash price is therefore

$$120 + \frac{60}{60 + 122} \times 6 = 121.978$$

A coupon of \$6 will be received after 122 days (= 0.3342 years). The present value of this is

$$6e^{-0.1 \times 0.3342} = 5.803$$

The futures contract lasts for 270 days (= 0.7397 years). The cash futures price, if the contract were written on the 12% bond, would therefore be

$$(121.978 - 5.803)e^{0.1 \times 0.7397} = 125.094$$

At delivery, there are 148 days of accrued interest. The quoted futures price, if the contract were written on the 12% bond, is calculated by subtracting the accrued interest

$$125.094 - 6 \times \frac{148}{148 + 35} = 120.242$$

From the definition of the conversion factor, 1.4000 standard bonds are considered equivalent to each 12% bond. The quoted futures price should therefore be

$$\frac{120.242}{1.4000} = 85.887$$

6.4 EURODOLLAR FUTURES

The most popular interest rate futures contract in the United States is the 3-month Eurodollar futures contract traded on the Chicago Mercantile Exchange (CME). A Eurodollar is a dollar deposited in a US or foreign bank outside the United States. The Eurodollar interest rate is the rate of interest earned on Eurodollars deposited by one bank with another bank. It is essentially the same as the London Interbank Offer Rate (LIBOR) introduced in Chapter 4.

Three-month Eurodollar futures contracts are futures contracts on the 3-month (90-day) Eurodollar interest rate. They allow an investor to lock in an interest rate on \$1 million for a future 3-month period. The 3-month period to which the interest rate applies starts on the third Wednesday of the delivery month. The contracts have delivery months of March, June, September, and December for up to 10 years into the future. This means that in 2004 an investor can use Eurodollar futures to lock in an interest rate for 3-month periods that are as far into the future as 2014. (Table 6.1 shows quotes out to 2010.) Short-maturity contracts trade for months other than March, June, September, and December. For example, from Table 6.1 we see that Eurodollar futures with maturities in February, April, May, and July 2004 trade on February 4, 2004. However, these have relatively low open interest.

To understand how Eurodollar futures contracts work, consider the March 2005 contract in Table 6.1. This has a settlement price of 97.63. The contract ends on the third Wednesday of the delivery month. In the case of this contract, the third Wednesday of the delivery month is March 16, 2005. The contract is marked to market in the usual way until that date. However, on March 16, 2005, the settlement price is set equal to $100 - R$, where R is the actual 3-month Eurodollar interest rate on that day, expressed with quarterly compounding and an actual/360 day count convention. (Thus, if the 3-month Eurodollar interest rate on March 16, 2005, turned out to be 2%, the final settlement price would be 98.) There is a final marking to market reflecting this settlement price and all contracts are declared closed.

The contract is designed so that a 1 basis point (= 0.01) move in the futures quote corresponds to a gain or loss of \$25 per contract. When a Eurodollar futures quote increases by 1 basis point, a trader who is long one contract gains \$25 and a trader who is short one contract loses \$25. Similarly, when the quote decreases by 1 basis point a trader who is long one contract loses \$25 and a trader who is short one contract gains \$25. This is consistent with the point made earlier: that the contract locks in an interest rate on \$1 million dollars for 3 months. When an interest rate per year changes by 1 basis point, the interest earned on 1 million dollars for 3 months changes by

$$1{,}000{,}000 \times 0.0001 \times 0.25 = 25$$

or \$25. Because the futures quote is 100 minus the futures interest rate, an investor who is long gains when interest rates fall and an investor who is short gains when interest rates rise.

Example 6.3

On February 4, 2004, an investor wants to lock in the interest rate that will be earned on \$5 million for 3 months starting on March 16, 2005. The investor goes long five March05 Eurodollar futures contracts at 97.63. On March 16, 2005, the 3-month LIBOR interest rate is 2%, so that the final settlement price proves to be 98.00. The investor gains $5 \times 25 \times (9{,}800 - 9{,}763) = \$4{,}625$ on the long futures position. The interest earned on the \$5 million for 3 months at 2% is

$$5{,}000{,}000 \times 0.25 \times 0.02 = 25{,}000$$

or \$25,000. The gain on the futures contract brings this up to \$29,625. This is the interest that would have been earned if the interest rate had been 2.37% ($5{,}000{,}000 \times 0.25 \times 0.0237 = 29{,}625$). This illustration shows that the futures trade has the effect of locking in an interest rate equal to 2.37%, or $(100 - 97.63)\%$.

The exchange defines the contract price as

$$10{,}000[100 - 0.25(100 - Q)] \qquad \textbf{(6.2)}$$

where Q is the quote. Thus, the settlement price of 97.63 for the March 2005 contract in

Table 6.1 corresponds to a contract price of

$$10{,}000[100 - 0.25(100 - 97.63)] = \$994{,}075$$

In Example 6.3, the final contract price is

$$10{,}000[100 - 0.25(100 - 98)] = \$995{,}000$$

and the difference between the initial and final contract price is $925, so that an investor with a long position in five contracts gains 5×925 dollars, or $4,625, as in Example 6.3. This is consistent with the "$25 per 1 basis point move" rule.

We can see that the interest rate term structure in the United States was upward-sloping on February 4, 2004. The futures rate for a 3-month period beginning in March 17, 2004, was 1.16%; for a 3-month period beginning March 16, 2005, it was 2.37%; for a 3-month period beginning March 21, 2007, it was 4.51%; and for a 3-month period beginning March 17, 2010, it was 5.81%.

Other contracts similar to the CME Eurodollar futures contract trade on interest rates in other countries. As shown in Table 6.1, the CME trades Euroyen contracts. The London International Financial Futures and Options Exchange trades 3-month Euribor contracts (i.e., contracts on the 3-month LIBOR rate for the euro) and 3-month Euroswiss futures.

Forward vs. Futures Interest Rates

The Eurodollar futures contract is similar to a forward rate agreement (FRA: see Section 4.7) in that it locks in an interest rate for a future period. For short maturities (up to a year or so), the two contracts can be assumed to be the same and the Eurodollar futures interest rate can be assumed to be the same as the corresponding forward interest rate. For longer-dated contracts, differences between the contracts become important. Compare a Eurodollar futures contract on an interest rate for the period between times T_1 and T_2 with an FRA for the same period. The Eurodollar futures contract is settled daily. The final settlement is at time T_1 and reflects the realized interest rate for the period between times T_1 and T_2. By contrast the FRA is not settled daily and the final settlement reflecting the realized interest rate between times T_1 and T_2 is made at time T_2.[2]

There are therefore two components to the difference between a Eurodollar futures contract and an FRA. These are:

1. The difference between a Eurodollar futures contract and a similar contract where there is no daily settlement. The latter is a forward contract where a payoff equal to the difference between the forward interest rate and the realized interest rate is paid at time T_1.
2. The difference between a forward contract where there is settlement at time T_1 and a forward contract where there is settlement at time T_2.

These two components to the difference between the contracts cause some confusion in practice. Both decrease the forward rate relative to the futures rate, but for long-dated contracts the reduction caused by the second difference is much smaller than that

[2] As mentioned in Section 4.7, settlement may occur at time T_1, but it is then equal to the present value of the normal forward contract payoff at time T_2.

caused by the first. The reason why the first difference (daily settlement) decreases the forward rate follows from the arguments in Section 5.8. Suppose you have a contract where the payoff is $R_M - R_F$ at time T_1, where R_F is a predetermined rate for the period between T_1 and T_2, and R_M is the realized rate for this period, and you have the option to switch to daily settlement. In this case daily settlement leads to cash inflows when rates are high and cash outflows when rates are low. You would therefore find switching to daily settlement to be attractive because you tend to have more money in your margin account when rates are high. As a result the market would therefore set R_F higher for the daily settlement alternative (reducing your cumulative expected payoff). To put this the other way round, switching from daily settlement to settlement at time T_1 reduces R_F.

To understand the reason why the second difference reduces the forward rate, suppose that the payoff of $R_M - R_F$ is at time T_2 instead of T_1 (as it is for a regular FRA). If R_M is high, the payoff is positive. Because rates are high, the cost to you of having the payoff that you receive at time T_2 rather than time T_1 is relatively high. If R_M is low, the payoff is negative. Because rates are low, the benefit to you of having the payoff you make at time T_2 rather than time T_1 is relatively low. Overall you would rather have the payoff at time T_1. If it is at time T_2 rather than T_1, you must be compensated by a reduction in R_F.[3]

Analysts make what is known as a *convexity adjustment* to account for the total differences between the two rates. One popular adjustment is[4]

$$\text{Forward rate} = \text{Futures rate} - \tfrac{1}{2}\sigma^2 T_1 T_2 \qquad \textbf{(6.3)}$$

where, as above, T_1 is the time to maturity of the futures contract and T_2 is the time to the maturity of the rate underlying the futures contract. The variable σ is the standard deviation of the change in the short-term interest rate in 1 year. Both rates are expressed with continuous compounding.[5] A typical value for σ is 1.2% or 0.012.

Example 6.4

Consider the situation where $\sigma = 0.012$ and we wish to calculate the forward rate when the 8-year Eurodollar futures price quote is 94. In this case $t_1 = 8$, $t_2 = 8.25$, and the convexity adjustment is

$$\tfrac{1}{2} \times 0.012^2 \times 8 \times 8.25 = 0.00475$$

or 0.475% (47.5 basis points). The futures rate is 6% per annum on an actual/360 basis with quarterly compounding. This corresponds to 1.5% per 90 days or an annual rate of $(365/90)\ln 1.015 = 6.038\%$ with continuous compounding and an actual/365 day count. The estimate of the forward rate given by equation (6.3), therefore, is $6.038 - 0.475 = 5.563\%$ per annum with continuous compounding. Table 6.3 shows how the size of the adjustment increases with the time to maturity.

[3] Quantifying the effect of this type of timing difference on the value of a derivative is discussed further in Chapter 27.

[4] See Technical Note 1 on the author's website for a proof of this.

[5] This formula is based on the Ho–Lee interest rate model, which will be discussed in Chapter 28. See T. S. Y. Ho and S.-B. Lee, "Term structure movements and pricing interest rate contingent claims," *Journal of Finance*, 41 (December 1986), 1011–29.

Table 6.3 Convexity adjustment for the futures rate in Example 6.4.

Maturity of futures (years)	*Convexity adjustments (basis points)*
2	3.2
4	12.2
6	27.0
8	47.5
10	73.8

We can see from Table 6.3 that the size of the adjustment is roughly proportional to the square of the time to maturity of the futures contract. Thus the convexity adjustment for the 8-year contract is approximately 16 times that for a 2-year contract.

Using Eurodollar Futures to Extend the LIBOR Zero Curve

The LIBOR zero curve out to 1 year is determined by the 1-month, 3-month, 6-month, and 12-month LIBOR rates. Once the convexity adjustment just described has been made, Eurodollar futures are often used to extend the zero curve. Suppose that the ith Eurodollar futures contract matures at time T_i ($i = 1, 2, \dots$). It is usually assumed that the forward interest rate calculated from the ith futures contract applies exactly to the period T_i to T_{i+1}. (In practice this is close to true.) This enables a bootstrap procedure to be used to determine zero rates. Suppose that F_i is the forward rate calculated from the ith Eurodollar futures contract and R_i is the zero rate for a maturity T_i. From equation (4.5), we have

$$F_i = \frac{R_{i+1}T_{i+1} - R_iT_i}{T_{i+1} - T_i}$$

so that

$$R_{i+1} = \frac{F_i(T_{i+1} - T_i) + R_iT_i}{T_{i+1}} \tag{6.4}$$

Other Euro rates such as Euroswiss, Euroyen, and Euribor are used in a similar way.

Example 6.5

The 400-day LIBOR zero rate has been calculated as 4.80% with continuous compounding and, from Eurodollar futures quotes, it has been calculated that (a) the forward rate for a 90-day period beginning in 400 days is 5.30% with continuous compounding, (b) the forward rate for a 90-day period beginning in 491 days is 5.50% with continuous compounding, and (c) the forward rate for a 90-day period beginning in 589 days is 5.60% with continuous compounding. We can use equation (6.4) to obtain the 491-day rate as

$$\frac{0.053 \times 91 + 0.048 \times 400}{491} = 0.04893$$

or 4.893%. Similarly we can use the second forward rate to obtain the 589-day rate as

$$\frac{0.055 \times 98 + 0.04893 \times 491}{589} = 0.04994$$

or 4.994%. The next forward rate of 5.60% would be used to determine the zero curve out to the maturity of the next Eurodollar futures contract. (Note that, even though the rate underlying the Eurodollar futures contract is a 90-day rate, it is assumed to apply to the 91 or 98 days elapsing between Eurodollar contract maturities.)

6.5 DURATION-BASED HEDGING STRATEGIES

We discussed duration in Section 4.8. Consider the situation where a position in an asset that is interest rate dependent, such as a bond portfolio or a money market security, is being hedged using an interest rate futures contract. Define:

F_C: Contract price for the interest rate futures contract

D_F: Duration of the asset underlying the futures contract at the maturity of the futures contract

P: Forward value of the portfolio being hedged at the maturity of the hedge (in practice, this is usually assumed to be the same as the value of the portfolio today)

D_P: Duration of the portfolio at the maturity of the hedge

If we assume that the change in the yield, Δy, is the same for all maturities, which means that only parallel shifts in the yield curve can occur, it is approximately true that

$$\Delta P = -PD_P \, \Delta y$$

To a reasonable approximation, it is also true that

$$\Delta F_C = -F_C D_F \, \Delta y$$

The number of contracts required to hedge against an uncertain Δy, therefore, is

$$N^* = \frac{PD_P}{F_C D_F} \tag{6.5}$$

This is the *duration-based hedge ratio*. It is sometimes also called the *price sensitivity hedge ratio*.[6] Using it has the effect of making the duration of the entire position zero.

When the hedging instrument is a Treasury bond futures contract, the hedger must base D_F on an assumption that one particular bond will be delivered. This means that the hedger must estimate which of the available bonds is likely to be cheapest to deliver at the time the hedge is put in place. If, subsequently, the interest rate environment changes so that it looks as though a different bond will be cheapest to deliver, then the hedge has to be adjusted and its performance may be worse than anticipated.

When hedges are constructed using interest rate futures, it is important to bear in

[6] For a more detailed discussion of equation (6.5), see R.J. Rendleman, "Duration-Based Hedging with Treasury Bond Futures," *Journal of Fixed Income* 9, 1 (June 1999): 84–91.

mind that interest rates and futures prices move in opposite directions. When interest rates go up, an interest rate futures price goes down. When interest rates go down, the reverse happens, and the interest rate futures price goes up. Thus, a company in a position to lose money if interest rates drop should hedge by taking a long futures position. Similarly, a company in a position to lose money if interest rates rise should hedge by taking a short futures position.

The hedger tries to choose the futures contract so that the duration of the underlying asset is as close as possible to the duration of the asset being hedged. Eurodollar futures tend to be used for exposures to short-term interest rates, whereas Treasury bond and Treasury note futures contracts are used for exposures to longer-term rates.

Example 6.6

It is August 2 and a fund manager with $10 million invested in government bonds is concerned that interest rates are expected to be highly volatile over the next 3 months. The fund manager decides to use the December T-bond futures contract to hedge the value of the portfolio. The current futures price is 93-02, or 93.0625. Because each contract is for the delivery of $100,000 face value of bonds, the futures contract price is $93,062.50.

We suppose that the duration of the bond portfolio in 3 months will be 6.80 years. The cheapest-to-deliver bond in the T-bond contract is expected to be a 20-year 12% per annum coupon bond. The yield on this bond is currently 8.80% per annum, and the duration will be 9.20 years at maturity of the futures contract.

The fund manager requires a short position in T-bond futures to hedge the bond portfolio. If interest rates go up, a gain will be made on the short futures position, but a loss will be made on the bond portfolio. If interest rates decrease, a loss will be made on the short position, but there will be a gain on the bond portfolio. The number of bond futures contracts that should be shorted can be calculated from equation (6.5) as

$$\frac{10{,}000{,}000}{93{,}062.50} \times \frac{6.80}{9.20} = 79.42$$

Rounding to the nearest whole number, the portfolio manager should short 79 contracts.

6.6 HEDGING PORTFOLIOS OF ASSETS AND LIABILITIES

Financial institutions frequently attempt to hedge themselves against interest rate risk by ensuring that the average duration of their assets equals the average duration of their liabilities. (The liabilities can be regarded as short positions in bonds.) This strategy is known as *duration matching* or *portfolio immunization*. When implemented, it ensures that a small parallel shift in interest rates will have little effect on the value of the portfolio of assets and liabilities. The gain (loss) on the assets should offset the loss (gain) on the liabilities.

Duration matching does not immunize a portfolio against nonparallel shifts in the zero curve. This is a weakness of the approach. In practice, short-term rates are usually more volatile than, and are not perfectly correlated with, long-term rates. Sometimes it

Business Snapshot 6.3 Asset–Liability Management by Banks

In the 1960s interest rates were low and not very volatile. Many banks got into the habit of accepting short-term deposits and making long-term loans. In the 1970s interest rates rose and some of these banks found that they were funding the low-interest long-term loans made in the 1960s with relatively expensive short-term deposits. As a result there were some spectacular bank failures.

The asset–liability management (ALM) committees of banks now monitor their exposure to interest rates very carefully. Matching the durations of assets and liabilities is a first step, but this does not protect a bank against nonparallel shifts in the yield curve. A popular approach is known as *GAP management*. This involves dividing the zero-coupon yield curve into segments, known as *buckets*. The first bucket might be 0 to 1 month, the second 1 to 3 months, and so on. The ALM committee then investigates the effect on the values of both assets and liabilities of the zero rates corresponding to one bucket changing while those corresponding to all other buckets staying the same.

If there is a mismatch, corrective action is usually taken. Luckily banks today have many more tools to manage their exposures to interest rates than they had in the 1960s. These tools include swaps, FRAs, bond futures, Eurodollar futures, and other interest rate derivatives.

even happens that short- and long-term rates move in opposite directions to each other. Duration matching is therefore only a first step and financial institutions have developed other tools to help them manage their interest rate exposure. See Business Snapshot 6.3.

SUMMARY

Two very popular interest rate contracts are the Treasury bond and Eurodollar futures contracts that trade in the United States. In the Treasury bond futures contracts, the party with the short position has a number of interesting delivery options:

1. Delivery can be made on any day during the delivery month.
2. There are a number of alternative bonds that can be delivered.
3. On any day during the delivery month, the notice of intention to deliver at the 2:00 p.m. settlement price can be made any time up to 8:00 p.m.

These options all tend to reduce the futures price.

The Eurodollar futures contract is a contract on the 3-month rate on the third Wednesday of the delivery month. Eurodollar futures are frequently used to estimate LIBOR forward rates for the purpose of constructing a LIBOR zero curve. When long-dated contracts are used in this way, it is important to make what is termed a convexity adjustment to allow for the marking to market in the futures contract.

The concept of duration is important in hedging interest rate risk. It enables a hedger to assess the sensitivity of a bond portfolio to small parallel shifts in the yield curve. It also enables the hedger to assess the sensitivity of an interest rate futures price to small changes in the yield curve. The number of futures contracts necessary to protect the bond portfolio against small parallel shifts in the yield curve can therefore be calculated.

The key assumption underlying the duration-based hedging scheme is that all interest rates change by the same amount. This means that only parallel shifts in the term structure are allowed for. In practice, short-term interest rates are generally more volatile than are long-term interest rates, and hedge performance is liable to be poor if the duration of the bond underlying the futures contract differs markedly from the duration of the asset being hedged.

FURTHER READING

Burghardt, G., and W. Hoskins. "The Convexity Bias in Eurodollar Futures," *Risk*, 8, 3 (1995): 63–70.

Duffie, D. "Debt Management and Interest Rate Risk," in W. Beaver and G. Parker (eds.), *Risk Management: Challenges and Solutions*. New York: McGraw-Hill, 1994.

Grinblatt, M., and N. Jegadeesh. "The Relative Price of Eurodollar Futures and Forward Contracts," *Journal of Finance*, 51, 4 (September 1996): 1499–1522.

Questions and Problems (Answers in Solutions Manual)

6.1. A US Treasury bond pays a 7% coupon on January 7 and July 7. How much interest accrues per $100 of principal to the bondholder between July 7, 2004, and August 9, 2004? How would your answer be different if it were a corporate bond?

6.2. It is January 9, 2005. The price of a Treasury bond with a 12% coupon that matures on October 12, 2009, is quoted as 102-07. What is the cash price?

6.3. How is the conversion factor of a bond calculated by the Chicago Board of Trade? How is it used?

6.4. A Eurodollar futures price changes from 96.76 to 96.82. What is the gain or loss to an investor who is long two contracts?

6.5. What is the purpose of the convexity adjustment made to Eurodollar futures rates? Why is the convexity adjustment necessary?

6.6. The 350-day LIBOR rate is 3% with continuous compounding and the forward rate calculated from a Eurodollar futures contract that matures in 350 days is 3.2% with continuous compounding. Estimate the 440-day zero rate.

6.7. It is January 30. You are managing a bond portfolio worth $6 million. The duration of the portfolio in 6 months will be 8.2 years. The September Treasury bond futures price is currently 108-15, and the cheapest-to-deliver bond will have a duration of 7.6 years in September. How should you hedge against changes in interest rates over the next 6 months?

6.8. The price of a 90-day Treasury bill is quoted as 10.00. What continuously compounded return (on an actual/365 basis) does an investor earn on the Treasury bill for the 90-day period?

6.9. It is May 5, 2005. The quoted price of a government bond with a 12% coupon that matures on July 27, 2011, is 110-17. What is the cash price?

6.10. Suppose that the Treasury bond futures price is 101-12. Which of the following four bonds is cheapest to deliver?

Bond	*Price*	*Conversion factor*
1	125-05	1.2131
2	142-15	1.3792
3	115-31	1.1149
4	144-02	1.4026

6.11. It is July 30, 2005. The cheapest-to-deliver bond in a September 2005 Treasury bond futures contract is a 13% coupon bond, and delivery is expected to be made on September 30, 2005. Coupon payments on the bond are made on February 4 and August 4 each year. The term structure is flat, and the rate of interest with semiannual compounding is 12% per annum. The conversion factor for the bond is 1.5. The current quoted bond price is $110. Calculate the quoted futures price for the contract.

6.12. An investor is looking for arbitrage opportunities in the Treasury bond futures market. What complications are created by the fact that the party with a short position can choose to deliver any bond with a maturity of over 15 years?

6.13. Suppose that the 9-month LIBOR interest rate is 8% per annum and the 6-month LIBOR interest rate is 7.5% per annum (both with actual/365 and continuous compounding). Estimate the 3-month Eurodollar futures price quote for a contract maturing in 6 months.

6.14. Suppose that the 300-day LIBOR zero rate is 4% and Eurodollar quotes for contracts maturing in 300, 398, and 489 days are 95.83, 95.62, and 95.48. Calculate 398-day and 489-day LIBOR zero rates. Assume no difference between forward and futures rates for the purposes of your calculations.

6.15. Suppose that a bond portfolio with a duration of 12 years is hedged using a futures contract in which the underlying asset has a duration of 4 years. What is likely to be the impact on the hedge of the fact that the 12-year rate is less volatile than the 4-year rate?

6.16. Suppose that it is February 20 and a treasurer realizes that on July 17 the company will have to issue $5 million of commercial paper with a maturity of 180 days. If the paper were issued today, the company would realize $4,820,000. (In other words, the company would receive $4,820,000 for its paper and have to redeem it at $5,000,000 in 180 days' time.) The September Eurodollar futures price is quoted as 92.00. How should the treasurer hedge the company's exposure?

6.17. On August 1, a portfolio manager has a bond portfolio worth $10 million. The duration of the portfolio in October will be 7.1 years. The December Treasury bond futures price is currently 91-12 and the cheapest-to-deliver bond will have a duration of 8.8 years at maturity. How should the portfolio manager immunize the portfolio against changes in interest rates over the next 2 months?

6.18. How can the portfolio manager change the duration of the portfolio to 3.0 years in Problem 6.17?

6.19. Between October 30, 2006, and November 1, 2006, you have a choice between owning a US government bond paying a 12% coupon and a US corporate bond paying a 12% coupon. Consider carefully the day count conventions discussed in this chapter and decide which of the two bonds you would prefer to own. Ignore the risk of default.

6.20. Suppose that a Eurodollar futures quote is 88 for a contract maturing in 60 days. What is the LIBOR forward rate for the 60- to 150-day period? Ignore the difference between futures and forwards for the purposes of this question.

6.21. The 3-month Eurodollar futures price for a contract maturing in 6 years is quoted as 95.20. The standard deviation of the change in the short-term interest rate in 1 year is 1.1%. Estimate the forward LIBOR interest rate for the period between 6.00 and 6.25 years in the future.

6.22. Explain why the forward interest rate is less than the corresponding futures interest rate calculated from a Eurodollar futures contract.

Assignment Questions

6.23. Assume that a bank can borrow or lend money at the same interest rate in the LIBOR market. The 90-day rate is 10% per annum, and the 180-day rate is 10.2% per annum, both expressed with continuous compounding and actual/actual day count. The Eurodollar futures price for a contract maturing in 91 days is quoted as 89.5. What arbitrage opportunities are open to the bank?

6.24. A Canadian company wishes to create a Canadian LIBOR futures contract from a US Eurodollar futures contract and forward contracts on foreign exchange. Using an example, explain how the company should proceed. For the purposes of this problem, assume that a futures contract is the same as a forward contract.

6.25. The futures price for the June 2005 CBOT bond futures contract is 118-23.
  (a) Calculate the conversion factor for a bond maturing on January 1, 2021, paying a coupon of 10%.
  (b) Calculate the conversion factor for a bond maturing on October 1, 2026, paying a coupon of 7%.
  (c) Suppose that the quoted prices of the bonds in (a) and (b) are 169.00 and 136.00, respectively. Which bond is cheaper to deliver?
  (d) Assuming that the cheapest-to-deliver bond is actually delivered, what is the cash price received for the bond?

6.26. A portfolio manager plans to use a Treasury bond futures contract to hedge a bond portfolio over the next 3 months. The portfolio is worth $100 million and will have a duration of 4.0 years in 3 months. The futures price is 122, and each futures contract is on $100,000 of bonds. The bond that is expected to be cheapest to deliver will have a duration of 9.0 years at the maturity of the futures contract. What position in futures contracts is required?
  (a) What adjustments to the hedge are necessary if after 1 month the bond that is expected to be cheapest to deliver changes to one with a duration of 7 years?
  (b) Suppose that all rates increase over the next 3 months, but long-term rates increase less than short-term and medium-term rates. What is the effect of this on the performance of the hedge?