CHAPTER 14

Options on Stock Indices, Currencies, and Futures

In this chapter we tackle the problem of valuing options on stock indices, currencies, and futures contracts. As a first step, we produce results for options on a stock paying a known dividend yield. We then argue that stock indices, currencies, and futures prices are analogous to stocks paying known dividend yield. This enables the results for options on a stock paying a dividend yield to be applied to value options on these other assets.

14.1 RESULTS FOR A STOCK PAYING A KNOWN DIVIDEND YIELD

This section provides a simple rule that enables results produced for European options on a non-dividend-paying stock to be extended so that they apply to European options on a stock paying a known dividend yield.

Dividends cause stock prices to reduce on the ex-dividend date by the amount of the dividend payment. The payment of a dividend yield at rate q therefore causes the growth rate in the stock price to be less than it would otherwise be by an amount q. If, with a dividend yield of q, the stock price grows from S_0 today to S_T at time T, then in the absence of dividends it would grow from S_0 today to $S_T e^{qT}$ at time T. Alternatively, in the absence of dividends, it would grow from $S_0 e^{-qT}$ today to S_T at time T.

This argument shows that we get the same probability distribution for the stock price at time T in each of the following two cases:

1. The stock starts at price S_0 and provides a dividend yield at rate q.
2. The stock starts at price $S_0 e^{-qT}$ and pays no dividends.

This leads to a simple rule. When valuing a European option lasting for time T on a stock paying a known dividend yield at rate q, we reduce the current stock price from S_0 to $S_0 e^{-qT}$ and then value the option as though the stock pays no dividends.

Lower Bounds for Option Prices

As a first application of this rule, consider the problem of determining bounds for the price of a European option on a stock providing a dividend yield equal to q. Substituting S_0e^{-qT} for S_0 in equation (9.1), we see that the lower bound for the European call option price c is

$$c \geqslant \max(S_0e^{-qT} - Ke^{-rT}, 0) \tag{14.1}$$

To obtain a lower bound for a European put option, we can similarly replace S_0 by S_0e^{-qT} in equation (9.2), to get

$$p \geqslant \max(Ke^{-rT} - S_0e^{-qT}, 0) \tag{14.2}$$

These results can also be proved using no-arbitrage arguments (see Problem 14.36).

Put–Call Parity

Replacing S_0 by S_0e^{-qT} in equation (9.3), we obtain put–call parity for a stock providing a dividend yield equal to q:

$$c + Ke^{-rT} = p + S_0e^{-qT} \tag{14.3}$$

This result can also be proved using no-arbitrage arguments (see Problem 14.36).

14.2 OPTION PRICING FORMULAS

By replacing S_0 by S_0e^{-qT} in the Black–Scholes formulas, equations (13.20) and (13.21), we obtain the price c of a European call and the price p of a European put on a stock providing a dividend yield at rate q as

$$c = S_0e^{-qT}N(d_1) - Ke^{-rT}N(d_2) \tag{14.4}$$

$$p = Ke^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1) \tag{14.5}$$

Since

$$\ln\left(\frac{S_0e^{-qT}}{K}\right) = \ln\frac{S_0}{K} - qT$$

the parameters d_1 and d_2 are given by

$$d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

These results were first derived by Merton.[1] As discussed in Section 13.12, the word "dividend" should be defined as the reduction of the stock price on the ex-dividend date arising from any dividends declared. If the dividend yield is not constant during the life of the option, equations (14.4) and (14.5) are still true, with q equal to the average

[1] See R. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4 (Spring 1973): 141–83.

annualized dividend yield during the life of the option. The dividend yield should be expressed with continuous compounding (see Section 5.6).

Differential Equation and Risk-Neutral Valuation

To prove the results in equations (14.4) and (14.5) more formally, we can either solve the differential equation that the option price must satisfy or use risk-neutral valuation.

When we include a dividend yield of q in the analysis in Section 13.6, the differential equation (13.16) becomes[2]

$$\frac{\partial f}{\partial t} + (r-q)S\frac{\partial f}{\partial S} + \tfrac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \tag{14.6}$$

Like equation (13.16), this does not involve any variable affected by risk preferences. Therefore, the risk-neutral valuation procedure, described in Section 13.7, can be used.

In a risk-neutral world, the total return from the stock must be r. The dividends provide a return of q. The expected growth rate in the stock price must therefore be $r-q$. So the risk-neutral process for the stock price is given by

$$dS = (r-q)S\,dt + \sigma S\,dz \tag{14.7}$$

To value a derivative dependent on a stock that provides a dividend yield equal to q, we set the expected growth rate of the stock equal to $r-q$ and discount the expected payoff at rate r. When the expected growth rate in the stock price is $r-q$, the expected stock price at time T is $S_0e^{(r-q)T}$. A similar analysis to that in the appendix of Chapter 13 gives the expected payoff in a risk-neutral world as

$$e^{(r-q)T}S_0N(d_1) - KN(d_2)$$

where d_1 and d_2 are defined as above. Discounting at rate r for time T leads to equation (14.4).

Binomial Trees

Binomial trees can be used to value an option on a stock paying a known dividend yield in the way described in Chapter 11. To match the stock price volatility, we set

$$u = e^{\sigma\sqrt{\Delta t}} \quad \text{and} \quad d = e^{-\sigma\sqrt{\Delta t}}$$

where Δt is the length of the time step. The risk-neutral probability p of an up movement is chosen so that the expected return is $r-q$. This means that

$$pSu + (1-p)Sd = e^{(r-q)\Delta t}$$

or

$$p = \frac{a-d}{u-d}$$

where

$$a = e^{(r-q)\Delta t}$$

This was the result we used in Section 11.9.

---

[2] See Technical Note 6 on the author's website for a proof of this.

14.3 OPTIONS ON STOCK INDICES

As discussed in Chapter 8, several exchanges trade options on stock indices. Some of the indices track the movement of the market as a whole. Others are based on the performance of a particular sector (e.g., computer technology, oil and gas, transportation, or telecommunications).

Quotes

Table 14.1 shows quotes for options on the Dow Jones Industrial Average (DJX) and S&P 500 (SPX) as they appeared in the Money and Investing section of the *Wall Street*

Table 14.1 Quotes for stock index options from the *Wall Street Journal*, February 5, 2004.

Wednesday, Feb. 4, 2004
Volume, last, net change and open interest for all contracts. Volume figures are unofficial. Open interest reflects previous trading day. p-Put c-Call. The totals for call and put volume are midday figures

CHICAGO

STRIKE		VOL	LAST	NET CHG	OPEN INT
DJ INDUS AVG(DJX)					
Mar	90p	5	0.15	−0.05	5,844
Mar	92p	105	0.20	0.05	14,161
Apr	92p	1	0.55	0.05	40
Mar	96p	310	0.40	...	11,814
Feb	98p	40	0.10	−0.05	7,602
Mar	98p	775	0.60	0.05	4,211
Feb	99c	10	5.90	0.10	328
Feb	99p	200	0.15	...	2,190
Apr	99p	3	1.35	0.05	606
Feb	100p	179	0.20	...	6,935
Mar	100p	3	0.90	0.05	21,574
Apr	100p	3	1.35	−0.05	2,248
Mar	101c	10	4.80	−0.70	3,075
Mar	101p	3	1	−0.10	4,772
Feb	102p	151	0.40	−0.10	2,925
Apr	102p	2,133	2	0.15	2,206
Feb	104c	40	1.75	−0.05	5,265
Feb	104p	422	1.05	0.15	7,282
Mar	104c	378	2.50	−0.30	11,255
Mar	104p	458	2.10	0.20	12,458
Apr	104p	5	2.85	0.10	1,799
Feb	105c	2,068	1.05	−0.15	13,467
Feb	105p	2,335	1.50	0.20	15,555
Mar	105c	646	1.90	−0.15	39,444
Mar	105p	122	2.35	0.05	21,489
Apr	105c	200	2.75	0.05	1,914
Apr	105p	102	3	...	895
Feb	106c	1,071	0.65	−0.10	4,647
Feb	106p	65	2.10	0.15	3,485
Mar	106p	30	3	0.25	5,426
Apr	106p	5	3.80	0.20	1,547
Feb	107c	118	0.35	−0.10	4,414
Mar	107p	2	3.50	...	125
Apr	107c	10	1.50	−0.75	617
Apr	107p	5	4.30	0.20	742
Feb	108c	6	0.20	−0.10	3,305
Feb	108p	2	3.90	0.40	2,585
Mar	108c	182	0.85	...	11,472
Mar	108p	41	4	...	614
Apr	108p	40	5	0.20	88
Feb	112p	23	7.30	0.10	435
Call Vol.		8,251	Open Int.		313,904
Put Vol.		14,484	Open Int.		370,073
S & P 500(SPX)					
Feb	850p	10	0.05	...	1,434
Mar	850p	430	0.40	0.10	29,388
Apr	850p	10	1.05	...	311
Feb	875p	5	0.05	...	613
Apr	875p	5	1.65	0.20	16
Mar	900p	5	0.80	...	37,089
Apr	900p	85	1.90	−0.15	2
Feb	925c	140	199.50	2.50	690
Feb	925p	4	0.10	−0.05	3,579
Mar	925p	96	1.05	0.05	14,592
Feb	950p	200	0.40	0.30	17,129
Feb	975p	2,090	0.25	0.10	18,301
Mar	975c	10	155	6.50	9,718
Mar	975p	360	2.05	0.05	40,001
Apr	975p	26	5.20	1.20	2,027
Feb	995p	23	0.30	...	13,445
Mar	995p	2,004	2.70	0.10	27,317
Apr	995p	4	5.70	...	2,658
Feb	1005p	256	0.35	0.05	36,093
Mar	1005c	11	125	−12.50	2,370
Mar	1005p	1,173	2.90	−0.10	25,947
Feb	1025c	10	100.50	−9.50	5,757
Feb	1025p	6,227	0.60	0.05	45,995
Mar	1025p	515	4.60	0.50	55,930
Apr	1025p	225	9	1.00	5,171
Feb	1035p	306	0.70	...	2,864
Feb	1040p	10	1	0.25	4,270
Feb	1050c	1,789	76.30	−15.20	9,986
Feb	1050p	1,929	1.10	...	42,107
Mar	1050c	10	84	−5.80	19,676
Mar	1050p	36	6.90	0.90	48,190
Feb	1055p	130	1.40	0.10	3,134
Mar	1060c	1	73	−6.50	3,391
Mar	1060p	2,305	8.10	1.30	7,272
Mar	1070p	600	9	0.90	7,919
Feb	1075c	27	57.80	−3.70	11,711
Feb	1075p	11,023	2.70	0.60	28,638
Mar	1075c	16	64.50	−3.50	33,222
Mar	1075p	519	10.40	1.80	38,840
Apr	1075p	185	16.40	1.30	1,138
Feb	1085c	4	48.80	−0.20	204
Feb	1085p	583	3.70	1.00	6,492
Mar	1085p	305	12	2.50	5,608
Feb	1090c	30	43.60	−5.40	319
Feb	1090p	85	4	0.70	5,371
Feb	1100c	447	31.30	−8.10	21,191
Feb	1100p	2,617	6.40	2.10	32,392
Mar	1100c	33	40.50	−6.50	40,878
Mar	1100p	4,203	15.80	1.80	44,776
Apr	1100c	32	50	−4.50	462
Apr	1100p	8,895	24	4.00	9,380
Feb	1105p	124	7.20	1.70	1,734
Feb	1110c	6	26.50	−5.50	13
Feb	1110p	3,828	8.50	2.20	6,048
Mar	1110c	11	34.40	−9.80	20,786
Mar	1110p	688	18.10	2.10	18,829
Feb	1115c	4	20.60	−5.20	973
Feb	1115p	115	10.30	3.10	9,530
Feb	1120c	93	18	−6.00	152
Feb	1120p	255	12.10	3.50	6,774
Feb	1125c	1,803	14	−7.00	19,486
Feb	1125p	1,570	14.50	4.50	32,185
Mar	1125c	4,980	24.70	−4.90	80,288
Mar	1125p	4,764	25	4.00	78,162
Apr	1125c	36	32.90	−4.80	2,641
Apr	1125p	327	32	3.20	2,931
Feb	1130c	1,156	11.30	−5.20	4,741
Feb	1130p	2,693	16.60	4.20	11,001
Mar	1130c	2,829	21.50	−5.00	12,667
Mar	1130p	2,864	27	3.80	13,475
Feb	1135c	322	9	−4.50	1,262
Feb	1135p	396	19.90	5.60	2,600
Mar	1135c	413	19.90	−4.10	9,978
Mar	1135p	851	30	5.00	9,651
Feb	1140c	1,779	7	−5.00	6,401
Feb	1140p	948	22	5.50	8,040
Mar	1140c	1,401	18	−3.00	3,698
Mar	1140p	1	30	2.00	2,151
Feb	1145c	52	6.60	−2.90	944
Feb	1145p	47	26	5.00	1,584
Feb	1150c	3,479	4.20	−3.30	26,943
Feb	1150p	943	28.70	6.20	6,483
Mar	1150c	520	13	−3.70	35,491
Mar	1150p	52	38	5.00	23,226
Apr	1150c	23	20.60	−3.40	2,122
Feb	1155c	179	3.80	−2.00	1,557
Feb	1160c	1,351	2.55	−1.85	6,062
Feb	1160p	126	37	7.80	1,159
Mar	1160c	402	10	−2.90	2,098
Mat	1160p	1	45	1.00	14
Feb	1170c	3,054	1.50	−0.90	6,733
Feb	1170p	13	42.80	3.80	261
Feb	1175c	1,617	1.15	−0.85	28,065
Feb	1175p	55	49	6.00	2,196
Mar	1175c	614	6	−2.00	26,761
Mar	1175p	3	57.10	8.10	2,304
Apr	1175c	558	12.20	−1.50	2,095
Feb	1180c	420	0.80	−0.55	1,543
Feb	1185c	7	0.85	−0.25	731
Feb	1190c	86	0.50	−0.35	2,597
Mar	1190c	104	4.70	...	...
Feb	1200c	1,259	0.35	−0.20	22,677
Feb	1200p	16	70.70	5.70	315
Mar	1200c	1,965	3	−0.50	23,307
Mar	1200p	1	73.40	4.90	463
Apr	1200c	25	7	−0.60	3,481
Feb	1210c	10	0.25	−0.10	1,424
Feb	1215c	13	0.25	−0.15	963
Feb	1225c	72	0.10	−0.10	5,124
Mar	1225c	1	1.20	−0.30	3,018
Mar	1225p	2	96.90	−4.60	11
Apr	1225c	20	3.20	−0.50	2,845
Feb	1250c	55	0.05	−0.10	8,403
Mar	1250c	14	0.55	−0.05	11,441
Mar	1250p	30	120	0.50	515
Apr	1250c	3	1.50	−0.40	410
Call Vol.		37,739	Open Int.		1,200,003
Put Vol.		85,508	Open Int.		1,976,864
LEAPS-LONG TERM					
DJ INDUS AVG - CB					
Dec 05	76p	10	2	...	...
Dec 05	104c	1	9.20	0.40	11,701
Dec 05	108c	500	6.90	0.40	82
Dec 05	108p	500	10	1.00	20
Call Vol.		501	Open Int.		13,617
Put Vol.		510	Open Int.		12,357
S & P 500 - CB					
Dec 04	80c	60	33.10	...	7,895
Dec 05	80p	10	1.75	0.05	12,238
Dec 04	90p	132	1.60	0.20	38,870
Dec 05	90c	61	26	...	24,696
Dec 05	90p	3	3.50	0.50	18,418
Dec 04	95p	10	2.20	0.05	5,595
Dec 04	100p	87	3.10	0.15	25,728
Dec 05	100p	10	5	0.30	28,972
Dec 04	105p	8	4.30	0.10	2,324
Dec 04	110p	12	5.60	0.60	25,390
Dec 06	110c	11	14.70	−1.00	4,328
Dec 06	110p	10	9.10	0.70	33,811
Dec 04	120c	4	3.40	−0.20	9,911
Call Vol.		136	Open Int.		480,710
Put Vol.		282	Open Int.		435,054

Source: Reprinted by permission of Dow Jones, Inc., via Copyright Clearance Center, Inc.

Journal on Thursday February 5, 2004. The *Wall Street Journal* also shows quotes for options on a number of other indices including the Nasdaq 100 (NDX), Russell 2000 (RUT), and S&P 100 (OEX). All the options trade on the Chicago Board Options Exchange and all are European, except the contract on the S&P 100, which is American. The quotes refer to the price at which the last trade was made on Wednesday, February 4, 2004. The closing prices of the DJX and SPX on February 4, 2004, were 104.71 and 1,126.52, respectively.

One index option contract is on 100 times the index. (Note that the Dow Jones index used for index options is 0.01 times the usually quoted Dow Jones index.) Index options are settled in cash. This means that, on exercise of the option, the holder of a call option contract receives $(S - K) \times 100$ in cash and the writer of the contract pays this amount in cash, where S is the value of the index at the close of trading on the day of the exercise and K is the strike price. Similarly, the holder of a put option contract receives $(K - S) \times 100$ in cash and the writer of the contract pays this amount in cash.

Table 14.1 shows that, in addition to relatively short-dated options, the exchanges trade longer-maturity contracts known as LEAPS. The acronym LEAPS stands for "long-term equity anticipation securities" and was originated by the CBOE. LEAPS are exchange-traded options that last up to 3 years. (Note when interpreting Table 14.1 that the S&P 500 index is divided by 10 for the purpose of defining LEAPS contracts.) The usual expiration month for LEAPS on indices is December. As mentioned in Chapter 8, the CBOE and several other exchanges also trade LEAPS on many individual stocks. These have expirations in January.

The CBOE also trades *flex options* on indices. As mentioned in Chapter 8, these are options where the trader can choose the expiration date, the strike price, and whether the option is American or European.

Valuation

In valuing index futures in Chapter 5, we assumed that the index could be treated as a security paying a known dividend yield. In valuing index options, we make similar assumptions. This means that equations (14.1) and (14.2) provide a lower bound for European index options; equation (14.3) is the put–call parity result for European index options; and equations (14.4) and (14.5) can be used to value European options on an index. In all cases, S_0 is equal to the value of the index, σ is equal to the volatility of the index, and q is equal to the average annualized dividend yield (continuously compounded) on the index during the life of the option. The calculation of q should include only dividends whose ex-dividend date occurs during the life of the option.

In the United States ex-dividend dates tend to occur during the first week of February, May, August, and November. At any given time, the correct value of q is therefore likely to depend on the life of the option. This is even more true for some foreign indices. In Japan, for example, all companies tend to use the same ex-dividend dates.

Example 14.1

Consider a European call option on the S&P 500 that is 2 months from maturity. The current value of the index is 930, the exercise price is 900, the risk-free interest rate is 8% per annum, and the volatility of the index is 20% per annum. Dividend yields of 0.2% and 0.3% are expected in the first month and the second month, respectively. In this case, $S_0 = 930$, $K = 900$, $r = 0.08$, $\sigma = 0.2$, and $T = 2/12$. The total dividend yield during the option's life is $0.2 + 0.3 = 0.5\%$. This is 3% per annum. Hence, $q = 0.03$, and

$$d_1 = \frac{\ln(930/900) + (0.08 - 0.03 + 0.2^2/2) \times 2/12}{0.2\sqrt{2/12}} = 0.5444$$

$$d_2 = \frac{\ln(930/900) + (0.08 - 0.03 - 0.2^2/2) \times 2/12}{0.2\sqrt{2/12}} = 0.4628$$

$$N(d_1) = 0.7069, \qquad N(d_2) = 0.6782$$

so that the call price c is given by equation (14.4) as

$$c = 930 \times 0.7069e^{-0.03\times 2/12} - 900 \times 0.6782e^{-0.08\times 2/12} = 51.83$$

One contract would cost \$5,183.

If the absolute amount of the dividend that will be paid on the stocks underlying the index (rather than the dividend yield) is assumed to be known, the basic Black–Scholes formula can be used with the initial stock price being reduced by the present value of the dividends. This is the approach recommended in Chapter 13 for a stock paying known dividends. However, it may be difficult to implement for a broadly based stock index because it requires a knowledge of the dividends expected on every stock underlying the index.

Binomial Trees

In some circumstances it is optimal to exercise American put and call options on an index prior to the expiration date. Binomial trees can be used to value American-style index options as discussed in Section 11.9. An example of the use of binomial trees for index options is in Example 11.1 and Figure 11.11.

Portfolio Insurance

Portfolio managers can use index options to limit their downside risk. Suppose that the value of an index today is S_0. Consider a manager in charge of a well-diversified portfolio whose beta is 1.0. A beta of 1.0 implies that the returns from the portfolio mirror those from the index. Assuming the dividend yield from the portfolio is the same as the dividend yield from the index, the percentage changes in the value of the portfolio can be expected to be approximately the same as the percentage changes in the value of the index. Each contract on the S&P 500 is on 100 times the index. It follows that the value of the portfolio is protected against the possibility of the index falling below K if, for each $100S_0$ dollars in the portfolio, the manager buys one put option contract with strike price K. Suppose that the manager's portfolio is worth \$500,000 and the value of the index is 1,000. The portfolio is worth 500 times the index. The manager can obtain insurance against the value of the portfolio dropping below \$450,000 in the next 3 months by buying five put option contracts with a strike price

Business Snapshot 14.1 Can We Guarantee that Stocks Will Beat Bonds in the Long Run?

It is often said that if you are a long-term investor you should buy stocks rather than bonds. Consider a US fund manager who is trying to persuade investors to buy as a long-term investment an equity fund that is expected to mirror the S&P 500. The manager might be tempted to offer purchasers of the fund a guarantee that their return will be at least as good as the return on risk-free bonds over the next 10 years. Historically stocks have outperformed bonds in the United States over almost any 10-year period. It appears that the fund manager would not be giving much away.

In fact, this type of guarantee is surprisingly expensive. Suppose that an equity index is 1,000 today, the dividend yield on the index is 1% per annum, the volatility of the index is 15% per annum, and the 10-year risk-free rate is 5% per annum. To outperform bonds, the stocks underlying the index must earn more than 5% per annum. The dividend yield will provide 1% per annum. The capital gains on the stocks must therefore provide 4% per annum. This means that we require the index level to be at least $1{,}000e^{0.04\times10} = 1{,}492$ in 10 years.

A guarantee that the return on \$1,000 invested in the index will be greater than the return on \$1,000 invested in bonds over the next 10 years is therefore equivalent to the right to sell the index for 1,492 in 10 years. This is a European put option on the index and can be valued from equation (14.5) with $S_0 = 1{,}000$, $K = 1{,}492$, $r = 5\%$, $\sigma = 15\%$, $T = 10$, and $q = 1\%$. The value of the put option is 169.7. This shows that the guarantee contemplated by the fund manager is worth about 17% of the fund—hardly something that should be given away!

of 900. Suppose that the risk-free rate is 12%, the dividend yield on the index is 4%, and the volatility of the index is 22%. The parameters of the option are:

$$S_0 = 1000, \quad K = 900, \quad r = 0.12, \quad \sigma = 0.22, \quad T = 0.25, \quad q = 0.04$$

From equation (14.5), the value of the option is \$6.48. The cost of the insurance is therefore $5 \times 100 \times 6.48 = \$3{,}240$.

To illustrate how the insurance works, consider the situation where the index drops to 880 in 3 months. The portfolio will be worth about \$440,000. The payoff from the options will be $5 \times (900 - 880) \times 100 = \$10,000$, bringing the total value of the portfolio up to the insured value of \$450,000 (or \$446,760 when the cost of the options are taken into account).

It is sometimes argued that the return from stocks is certain to beat the return from bonds in the long run. If this were true, long-dated portfolio insurance where the strike price equaled the future value of a bond portfolio would not cost very much. In fact, as indicated in Business Snapshot 14.1, it is quite expensive.

When the Portfolio's Beta Is Not 1.0

If the portfolio's returns are not expected to equal those of an index, the capital asset pricing model can be used. This model asserts that the expected excess return of a portfolio over the risk-free interest rate equals beta times the excess return of a market index over the risk-free interest rate. Suppose that the \$500,000 portfolio just considered

Table 14.2 Relationship between value of index and value of portfolio for beta = 2.0.

Value of index in 3 months	*Value of portfolio in 3 months ($)*
1,080	570,000
1,040	530,000
1,000	490,000
960	450,000
920	410,000
880	370,000

has a beta of 2.0 instead of 1.0. As before, we assume that the S&P 500 index is currently 1,000, the risk-free rate is 12% and the dividend yield on the index is 4%. Table 14.2 shows the expected relationship between the level of the index and the value of the portfolio in 3 months. To illustrate the sequence of calculations necessary to derive Table 14.2, Table 14.3 shows the calculations for the case when the value of the index in 3 months proves to be 1,040.

Suppose that S_0 is the value of the index. It can be shown that, for each $100S_0$ dollars in the portfolio, a total of beta put contracts should be purchased. The strike price should be the value that the index is expected to have when the value of the portfolio reaches the insured value. Assume that the required insured value is $450,000, as in the beta = 1.0 case. Table 14.2 shows that the appropriate strike price for the put options purchased is 960. The option parameters are:

$$S = 1000, \quad K = 960, \quad r = 0.12, \quad \sigma = 0.22, \quad T = 0.25, \quad q = 0.04$$

and equation (14.5) gives the value of the option as $19.21. In this case, $100S_0 = \$100{,}000$ and beta = 2.0, so that two put contracts are required for each $100,000 in the portfolio.

Table 14.3 Calculations for Table 14.2 when the value of the index is 1,040 in 3 months.

Value of index in 3 months:	1,040
Return from change in index:	40/1,000, or 4% per 3 months
Dividends from index:	$0.25 \times 4 = 1\%$ per 3 months
Total return from index:	$4 + 1 = 5\%$ per 3 months
Risk-free interest rate:	$0.25 \times 12 = 3\%$ per 3 months
Excess return from index over risk-free interest rate:	$5 - 3 = 2\%$ per 3 months
Excess return from portfolio over risk-free interest rate:	$2 \times 2 = 4\%$ per 3 months
Return from portfolio:	$3 + 4 = 7\%$ per 3 months
Dividends from portfolio:	$0.25 \times 4 = 1\%$ per 3 months
Increase in value of portfolio:	$7 - 1 = 6\%$ per 3 months
Value of portfolio:	$\$500,000 \times 1.06 = \$530,000$

Since the portfolio is worth \$500,000, a total of 10 contracts should be purchased. The total cost of the insurance is therefore $10 \times 100 \times 19.21 = \$19{,}210$.

To illustrate that the required result is obtained, consider what happens if the value of the index falls to 880. As shown in Table 14.2, the value of the portfolio is then about \$370,000. The put options pay off $(960 - 880) \times 10 \times 100 = \$80{,}000$, and this is exactly what is necessary to move the total value of the portfolio manager's position up from \$370,000 to the required level of \$450,000. (After the cost of the options are taken into account the value of the portfolio is \$430,790.)

There are two reasons why the cost of hedging increases as the beta of a portfolio increases: more put options are required, and they have a higher strike price.

14.4 CURRENCY OPTIONS

Currency options are primarily traded in the over-the-counter market. The advantage of this market is that large trades are possible with strike prices, expiration dates, and other features tailored to meet the needs of corporate treasurers. European and American options do trade on the Philadelphia Stock Exchange in the United States, but the exchange-traded market is much smaller than the over-the-counter market.

For a corporation wishing to hedge a foreign exchange exposure, foreign currency options are an interesting alternative to forward contracts. A company due to receive sterling at a known time in the future can hedge its risk by buying put options on sterling that mature at that time. The strategy guarantees that the value of the sterling will not be less than the strike price, while allowing the company to benefit from any favorable exchange-rate movements. Similarly, a company due to pay sterling at a known time in the future can hedge by buying calls on sterling that mature at that time. The approach guarantees that the cost of the sterling will not be greater than a certain amount while allowing the company to benefit from favorable exchange-rate movements. Whereas a forward contract locks in the exchange rate for a future transaction, an option provides a type of insurance. This insurance is not free. It costs nothing to enter into a forward transaction, whereas options require a premium to be paid up front.

Valuation

To value currency options, we define S_0 as the spot exchange rate. To be precise, S_0 is the value of one unit of the foreign currency in US dollars. As explained in Section 5.10, a foreign currency is analogous to a stock paying a known dividend yield. The owner of foreign currency receives a yield equal to the risk-free interest rate, r_f, in the foreign currency. Equations (14.1) and (14.2), with q replaced by r_f, provide bounds for the European call price, c, and the European put price, p:

$$c \geqslant S_0 e^{-r_f T} - K e^{-rT}$$

$$p \geqslant K e^{-rT} - S_0 e^{-r_f T}$$

Equation (14.3), with q replaced by r_f, provides the put–call parity result for currency options:

$$c + K e^{-rT} = p + S_0 e^{-r_f T}$$

Finally, equations (14.4) and (14.5) provide the pricing formulas for currency options when q is replaced by r_f:

$$c = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2) \tag{14.7}$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-r_f T} N(-d_1) \tag{14.8}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r - r_f + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - r_f - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Both the domestic interest rate, r, and the foreign interest rate, r_f, are the rates for a maturity T. Put and call options on a currency are symmetrical in that a put option to sell currency A for currency B at an exercise price K is the same as a call option to buy B with A at $1/K$.

Example 14.2

Consider a 4-month European call option on the British pound. Suppose that the current exchange rate is 1.6000, the exercise price is 1.6000, the risk-free interest rate in the United States is 8% per annum, the risk-free interest rate in Britain is 11% per annum, and the option price is 4.3 cents. In this case, $S_0 = 1.6$, $K = 1.6$, $r = 0.08$, $r_f = 0.11$, $T = 0.3333$, and $c = 0.043$. The implied volatility can be calculated by trial and error. A volatility of 20% gives an option price of 0.0639, a volatility of 10% gives an option price of 0.0285, and so on. The implied volatility is 14.1%.

From equation (5.9), the forward rate F_0 for a maturity T is given by

$$F_0 = S_0 e^{(r - r_f)T}$$

Thus, equations (14.7) and (14.8) can be simplified to

$$c = e^{-rT}[F_0 N(d_1) - K N(d_2)] \tag{14.9}$$

$$p = e^{-rT}[K N(-d_2) - F_0 N(-d_1)] \tag{14.10}$$

where

$$d_1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Note that, for equations (14.9) and (14.10), to be the correct equations for valuing a European option on the spot foreign exchange rate, the maturities of the forward contract and the option must be the same.

Binomial Trees

In some circumstances it is optimal to exercise American currency options prior to maturity. Thus, American currency options are worth more than their European counterparts. In general, call options on high-interest currencies and put options on low-interest currencies are the most likely to be exercised prior to maturity. The reason is that a high-interest currency is expected to depreciate and a low-interest currency is expected to appreciate. Binomial trees can be used to value American-style currency options as described in Section 11.9. An example of the valuation of a currency option is given in Example 11.2 and Figure 11.12.

14.5 FUTURES OPTIONS

Options on futures contracts, or futures options, are now traded on many different exchanges. They are American-style options and require the delivery of an underlying futures contract when exercised. If a call futures option is exercised, the holder acquires a long position in the underlying futures contract plus a cash amount equal to the most recent settlement futures price minus the strike price. If a put futures option is exercised, the holder acquires a short position in the underlying futures contract plus a cash amount equal to the strike price minus the most recent settlement futures price. As the following examples show, the effective payoff from a call futures option is the futures price at the time of exercise less the strike price; the effective payoff from a put futures option is the strike price less the futures price at the time of exercise.

Example 14.3

Suppose it is August 15 and an investor has one September futures call option contract on copper with a strike price of 70 cents per pound. One futures contract is on 25,000 pounds of copper. Suppose that the futures price of copper for delivery in September is currently 81 cents, and at the close of trading on August 14 (the last settlement) it was 80 cents. If the option is exercised, the investor receives a cash amount of

$$25{,}000 \times (80 - 70)\,\text{cents} = \$2{,}500$$

plus a long position in a futures contract to buy 25,000 pounds of copper in September. If desired, the position in the futures contract can be closed out immediately. This would leave the investor with the $2,500 cash payoff plus an amount

$$25{,}000 \times (81 - 80)\,\text{cents} = \$250$$

reflecting the change in the futures price since the last settlement. The total payoff from exercising the option on August 15 is $2,750, which equals $25{,}000(F - K)$, where F is the futures price at the time of exercise and K is the strike price.

Example 14.4

An investor has one December futures put option on corn with a strike price of 200 cents per bushel. One futures contract is on 5,000 bushels of corn. Suppose that the current futures price of corn for delivery in December is 180, and the most recent settlement price is 179 cents. If the option is exercised, the investor receives a cash amount of

$$5{,}000 \times (200 - 179)\,\text{cents} = \$1{,}050$$

plus a short position in a futures contract to sell 5,000 bushels of corn in December.

If desired, the position in the futures contract can be closed out. This would leave the investor with the $1,050 cash payoff minus an amount

$$5{,}000 \times (180 - 179)\,\text{cents} = \$50$$

reflecting the change in the futures price since the last settlement. The net payoff from exercise is $1,000, which equals $5{,}000(K - F)$, where F is the futures price at the time of exercise and K is the strike price.

Quotes

Futures options are referred to by the month in which the underlying futures contract matures—not by the expiration month of the option. As mentioned earlier, futures options are American. The expiration date of a futures option contract is usually on, or a few days before, the earliest delivery date of the underlying futures contract. (For example, the CBOT Treasury bond futures option expires on the Friday preceding by at least two business days the end of the month before the futures contract expiration month.) An exception is the CME mid-curve Eurodollar contract, where the futures contract expires either one or two years after the options contract.

Table 14.4 shows quotes for futures options as they appeared in the *Wall Street Journal* on February 5, 2004. The most popular contracts (as measured by open interest) are those on corn, soybeans, cotton, sugar-world, crude oil, natural gas, gold, Treasury bonds, Treasury notes, 5-year Treasury notes, 30-day federal funds, Eurodollars, 1-year and 2-year mid-curve Eurodollars, Euribor, Eurobunds, and the S&P 500.

Options on Interest Rate Futures

The most actively traded interest rate options offered by exchanges in the United States are those on Treasury bond futures, Treasury note futures, and Eurodollar futures. Table 14.4 shows the closing prices for these instruments on February 4, 2004.

A Treasury bond futures option is an option to enter a Treasury bond futures contract. As mentioned in Chapter 6, one Treasury bond futures contract is for the delivery of $100,000 of Treasury bonds. The price of a Treasury bond futures option is quoted as a percentage of the face value of the underlying Treasury bonds to the nearest sixty-fourth of 1%. Table 14.4 gives the price of the March call futures option on a Treasury bond on February 4, 2004, as 2-06, or $2\frac{6}{64}\%$ of the bond principal, when the strike price is 110. This means that one contract costs $2,093.75. The quotes for options on Treasury notes are similar.

An option on Eurodollar futures is an option to enter into a Eurodollar futures contract. As explained in Chapter 6, when the Eurodollar futures quote changes by 1 basis point, or 0.01%, there is a gain or loss on a Eurodollar futures contract of $25. Similarly, in the pricing of options on Eurodollar futures, 1 basis point represents $25. The *Wall Street Journal* quote for the CME Eurodollar futures contract in Table 14.4 should be multiplied by 10 to get the CME quote in basis points. For example, the 5.90 quote for the CME March call futures option when the strike price is 98.25 in Table 14.4 indicates that the CME quote is 59.0 basis points and one contract costs $59.0 \times \$25 = \$1{,}475.00$.

Table 14.4 Closing prices of futures options on February 4, 2004.

Wednesday, February 4, 2004

Final or settlement prices of selected contracts. Volume and open interest are totals in all contract months.

STRIKE CALLS-SETTLE PUTS-SETTLE

Grain and Oilseed

Corn (CBT)

5,000 bu.; cents per bu.

Price	Mar	May	Jly	Mar	May	Jly
260	11.875	20.250	26.750	1.625	5.250	8.500
270	5.500	14.750	21.250	5.250	9.500	13.500
280	2.250	10.500	17.250	12.000	15.500	19.000
290	.750	7.375	14.000	20.500	22.000	25.625
300	.250	5.125	11.375	30.000	29.625	32.625
310	.125	3.500	9.250	...	...	...

Est vol 14,610 Tu 8,885 calls 6,364 puts
Op int Tues 323,990 calls 227,010 puts

Soybeans (CBT)

5,000 bu.; cents per bu.

Price	Mar	May	Jly	Mar	May	Jly
760	47.500	58.500	60.000	1.875	13.000	28.500
780	31.250	46.500	50.250	5.500	20.750	38.500
800	18.875	36.250	42.000	13.125	30.750	50.000
820	10.250	28.500	35.000	24.500	42.250	62.750
840	5.125	22.000	29.500	39.375	56.000	77.000
860	2.500	17.000	24.750	56.625	70.750	92.000

Est vol 17,482 Tu 16,204 calls 6,863 puts
Op int Tues 153,237 calls 125,007 puts

Soybean Meal (CBT)

100 tons; $ per ton

Price	Mar	May	Jly	Mar	May	Jly
235	...	...	...	...	...	...
240	9.00	13.50	14.50	2.00	7.25	11.75
245	...	...	...	...	...	...
250	3.75	9.30	10.90	6.75	12.60	18.25
255	...	...	...	...	...	...
260	1.35	6.50	8.50	14.40	19.75	25.70

Est vol 2,445 Tu 2,767 calls 2,418 puts
Op int Tues 39,831 calls 36,748 puts

Soybean Oil (CBT)

60,000 lbs.; cents per lb.

Price	Mar	May	Jly	Mar	May	Jly
290	1.080	1.770	2.070	.250	1.000	1.620
295	.750	1.545	1.870	.400	1.280	...
300	.550	1.325	1.700	.700	1.570	2.240
305	...	...	...	...	...	...
310	.250	1.000	1.410	...	...	...
315	...	...	...	...	...	...

Est vol 6,036 Tu 2,484 calls 2,045 puts
Op int Tues 55,851 calls 44,819 puts

Wheat (CBT)

5,000 bu.; cents per bu.

Price	Mar	May	Jly	Mar	May	Jly
360	19.250	32.375	34.500	3.250	10.000	17.250
370	12.750	26.500	29.750	6.750	14.000	22.500
380	8.000	21.500	25.250	12.000	19.000	28.000
390	4.500	17.500	21.500	18.500	25.000	34.250
400	2.500	14.125	18.250	26.375	31.500	41.000
410	1.375	11.250	15.500	35.250	38.625	48.000

Est vol 4,768 Tu 2,369 calls 1,615 puts
Op int Tues 76,609 calls 56,869 puts

Wheat (KC)

5,000 bu.; cents per bu.

Price	Mar	May	Jly	Mar	May	Jly
360	22.500	30.625	36.375	2.000	10.250	16.000
370	15.000	24.875	31.250	4.500	14.500	20.750
380	9.125	20.000	26.625	8.625	19.500	26.125
390	5.250	16.125	22.750	14.750	25.625	32.125
400	2.875	14.000	19.375	22.375	32.500	38.750
410	2.000	10.500	16.500	31.000	...	...

Est vol 2,045 Tu 437 calls 315 puts
Op int Tues 21,347 calls 19,365 puts

STRIKE CALLS-SETTLE PUTS-SETTLE

Food and Fiber

Cotton (NYCE)

50,000 lbs.; cents per lb.

Price	Mar	May	Jly	Mar	May	Jly
67	2.44	5.85	7.13	.19	1.60	1.87
68	1.64	5.21	6.47	.39	1.95	2.20
69	.90	4.60	5.86	.65	2.34	2.58
70	.46	4.04	5.28	1.21	2.78	3.00
71	.28	3.54	4.75	2.03	3.27	3.46
72	.15	3.07	4.25	2.90	3.80	3.95

Est vol 9,021 Tu 8,443 calls 5,904 puts
Op int Tues 217,446 calls 113,615 puts

Orange Juice (NYCE)

15,000 lbs.; cents per lb.

Price	Mar	May	Jly	Mar	May	Jly
50	11.65	14.45	17.10	.05	.15	.25
55	6.75	9.75	12.60	.10	.40	.75
60	2.40	5.75	8.25	.75	1.35	1.40
65	.45	3.05	5.05	3.50	3.50	3.00
70	.15	1.55	2.95	8.35	7.05	5.90
75	.10	.80	1.70	13.35	11.40	9.55

Est vol 412 Tu 1,547 calls 843 puts
Op int Tues 42,351 calls 14,369 puts

Coffee (CSCE)

37,500 lbs.; cents per lb.

Price	Mar	Apr	May	Mar	Apr	May
67.5	5.40	8.17	9.06	0.30	0.98	1.94
70	3.35	6.38	7.53	0.75	1.85	2.90
72.5	1.85	4.94	6.24	1.75	2.79	4.10
75	1.00	3.82	5.18	3.30	4.17	5.52
77.5	0.49	2.98	4.30	5.39	5.82	7.14
80	0.23	2.34	3.58	7.63	7.68	8.91

Est vol 9,420 Tu 2,864 calls 2,718 puts
Op int Tues 78,119 calls 38,500 puts

Sugar-World (CSCE)

112,000 lbs.; cents per lb.

Price	Mar	Apr	May	Mar	Apr	May
450	1.19	1.39	1.40	0.01	0.01	0.02
500	0.69	0.89	0.93	0.01	0.02	0.06
550	0.25	0.47	0.55	0.07	0.09	0.17
600	0.02	0.18	0.27	0.34	0.30	0.39
650	0.01	0.05	0.12	0.83	0.67	0.74
700	0.01	0.01	0.06	1.33	1.13	1.17

Est vol 2,533 Tu 1,814 calls 1,889 puts
Op int Tues 154,632 calls 112,414 puts

Cocoa (CSCE)

10 metric tons; $ per ton

Price	Mar	Apr	May	Mar	Apr	May
1500	84	108	133	3	39	64
1550	42	78	105	11	59	86
1600	14	54	81	33	85	112
1650	4	36	61	73	117	142
1700	1	24	45	120	155	176
1750	1	15	34	170	196	214

Est vol 1,663 Tu 439 calls 341 puts
Op int Tues 18,472 calls 15,125 puts

Petroleum

Crude Oil (NYM)

1,000 bbls.; $ per bbl.

Price	Mar	Apr	May	Mar	Apr	May
3200	1.53	1.36	1.39	0.43	1.37	2.07
3250	1.20	1.12	1.18	0.60	1.63	2.36
3300	0.91	0.93	1.00	0.81	1.94	2.67
3350	0.66	0.75	0.84	1.06	2.26	3.01
3400	0.49	0.61	0.70	1.39	2.62	3.37
3450	0.33	0.50	0.00	1.73	3.00	...

Est vol 43,517 Tu 13,264 calls 17,244 puts
Op int Tues 341,383 calls 486,295 puts

Heating Oil No.2 (NYM)

42,000 gal.; $ per gal.

STRIKE CALLS-SETTLE PUTS-SETTLE

Price	Mar	Apr	May	Mar	Apr	May
87	.0437	.0426	.0335	.0240	.0540	...
88	.0381	.0386	.0303	.0284	.0600	.0850
89	.0330	.0348	.0275	.0333	.0661	...
90	.0290	.0314	.0248	.0393	.0727	...
91	.0245	.0282	.0224	.0448	.0795	...
92	.0210	.0254	...	.0513	.0866	...

Est vol 815 Tu 800 calls 300 puts
Op int Tues 27,374 calls 19,492 puts

Gasoline-Unlead (NYM)

42,000 gal.; $ per gal.

Price	Mar	Apr	May	Mar	Apr	May
97	.0462	...	.0932	.0305	.0301	.0484
98	.0409	.0842	.0877	.0352	.0338	.0528
99	.0361	...	.0823	.0404	.0378	.0574
100	.0318	.0725	.0773	.0461	.0421	.0623
101	.0279	.0671	.0724	.0522	.0466	.0674
102	.0243	.0619	.0680	.0586	.0514	...

Est vol 2,854 Tu 1,831 calls 1,008 puts
Op int Tues 21,736 calls 17,368 puts

Natural Gas (NYM)

10,000 MMBtu.; $ per MMBtu.

Price	Mar	Apr	May	Mar	Apr	May
555	.382	.276	...	.278	.486	...
560	.358	.260	.238	.304	.519	...
565	.334	.244	...	.330	.553	...
570	.313	.230	.210	.359	.589	...
575	.293	.216	.197	.389	.625	...
580	.275	.203	.185	.421	.662	...

Est vol 37,627 Tu 17,111 calls 19,795 puts
Op int Tues 316,788 calls 386,608 puts

Brent Crude (IPE)

1,000 net bbls.; $ per bbl.

Price	Mar	Apr	May	Mar	Apr	May
	Data not available from source.				...	...
...	...	...				...
	...	...	...	...	...	...
	...	...	...	...	...	...
	...	...	...	...	...	...
	...	...	...	...	...	...
	...	...	...	...	...	...

Est vol Tu calls puts
Op int Tues calls puts

Livestock

Cattle-Feeder (CME)

50,000 lbs.; cents per lb.

Price	Mar	Apr	May	Mar	Apr	May
8000	4.00	5.50	6.28	3.00	2.80	3.10
8100	...	...	...	...	...	...
8200	2.50	4.00	...	3.50	3.30	3.90
8300	2.10	...	...	4.10	...	...
8400	1.60	3.10	4.00	4.60	4.40	4.80
8500	1.20	...	...	5.20	...	...

Est vol 534 Tu 183 calls 261 puts
Op int Tues 3,298 calls 5,427 puts

Cattle-Live (CME)

40,000 lbs.; cents per lb.

Price	Feb	Mar	Apr	Feb	Mar	Apr
73	1.50	...	2.00	0.50	...	4.05
74	0.80	...	1.70	0.80	...	4.75
75	0.35	...	1.50	1.35	...	5.55
76	0.18	...	1.25	2.18	...	6.28
77	0.08	...	1.05	3.08	...	7.08
78	0.03	...	0.85	4.03	...	7.88

Est vol 1,903 Tu 690 calls 855 puts
Op int Tues 40,381 calls 42,076 puts

Hogs-Lean (CME)

40,000 lbs.; cents per lb.

Price	Feb	Apr	May	Feb	Apr	May
57	2.63	3.80	...	0.20	1.93	...
58	1.78	3.18	4.80	0.35	2.30	2.18
59	1.08	2.65	...	0.65	2.78	...
60	0.53	2.15	3.63	1.10	3.28	2.98
61	0.28	1.73	...	...	...	...
62	0.15	1.35	2.68	2.73	...	...

Est vol 243 Tu 207 calls 358 puts
Op int Tues 5,619 calls 6,176 puts

Continued on next page

Table 14.4—Continued

STRIKE CALLS-SETTLE PUTS-SETTLE

Metals

Copper (CMX)

25,000 lbs.; cents per lb.

Price	Mar	Apr	May	Mar	Apr	May
114	5.00	6.00	7.15	1.55	2.90	4.65
116	3.70	4.90	6.15	2.25	3.80	5.60
118	2.55	3.95	5.20	3.10	4.85	6.65
120	1.75	3.10	4.40	4.30	6.00	7.85
122	1.15	1.80	3.70	5.70	9.65	9.10
124	0.70	1.00	3.05	7.25	13.90	10.50

Est vol 1,650 Tu 247 calls 23 puts
Op int Tues 12,848 calls 3,638 puts

Gold (CMX)

100 troy ounces; $ per troy ounce

Price	Mar	Apr	Jun	Mar	Apr	Jun
390	13.50	16.80	21.80	1.90	5.10	9.20
395	10.00	13.70	19.00	3.30	7.00	11.40
400	7.00	11.00	17.50	5.30	9.30	14.90
405	4.80	8.80	14.30	8.10	12.10	16.60
410	3.20	6.60	12.50	11.50	14.90	19.70
415	2.10	5.50	10.80	15.40	18.80	23.00

Est vol 18,000 Tu 4,487 calls 5,463 puts
Op int Tues 306,159 calls 227,854 puts

Silver (CMX)

5,000 troy ounces; cts per troy ounce

Price	Mar	Apr	May	Mar	Apr	May
610	20.30	30.50	38.40	15.50	24.40	32.20
620	15.90	26.30	34.30	21.10	30.10	38.10
625	14.00	24.40	32.40	24.20	33.20	41.20
630	12.40	22.70	30.70	27.60	36.40	44.40
640	9.70	19.50	27.50	34.90	43.20	51.20
650	7.60	16.80	24.70	42.70	50.50	58.40

Est vol 1,800 Tu 1,474 calls 1,954 puts
Op int Tues 66,669 calls 26,556 puts

Interest Rate

T-Bonds (CBT)

$100,000; points and 64ths of 100%

Price	Mar	Apr	May	Mar	Apr	May
110	2-06	2-03	2-35	0-36	1-61	2-29
111	1-28	1-36	...	0-58	2-30	...
112	0-58	1-11	1-42	1-24	3-04	...
113	0-34	0-54	...	2-00	3-48	...
114	0-19	0-39	...	2-49	4-32	...
115	0-10	0-27	0-49	3-40	5-20	...

Est vol 23,701;
Tu vol 14,191 calls 17,000 puts
Op int Tues 412,644 calls 444,891 puts

T-Notes (CBT)

$100,000; points and 64ths of 100%

Price	Mar	Apr	May	Mar	Apr	May
112	2-00	1-30	1-52	0-20	1-25	1-46
113	1-17	1-00	...	0-37	1-58	...
114	0-44	0-41	0-59	1-00	...	...
115	0-20	0-24	0-40	1-40	...	...
116	0-08	0-14	0-26	2-28	...	...
117	0-03	0-08	0-16	...	...	...

Est vol 150,806 Tu 61,052 calls 65,301 puts
Op int Tues 1,045,055 calls 1,083,950 puts

5 Yr Treas Notes (CBT)

$100,000; points and 64ths of 100%

Price	Mar	Apr	May	Mar	Apr	May
11150	1-16	0-49	0-62	0-15	1-08	1-21
11200	0-56	0-36	...	0-22	1-27	...
11250	0-36	0-25	...	0-34	...	...
11300	0-22	0-17	...	0-52	...	...
11350	0-12	0-11	...	1-10	...	...
11400	0-06	...	...	1-36	...	...

Est vol 17,994 Tu 4,736 calls 25,086 puts
Op int Tues 125,023 calls 426,615 puts

30 Day Federal Funds (CBT)

$5,000,000; 100 minus daily average

Price	Feb	Mar	Apr	Feb	Mar	Apr
988750	.127	.117	.120	.002	.002	.005
989375	.065	.062	.060	.002	.007	.007
990000	.007	.007	.007	.007	.017	.017
990625	...	.002	.002	...	...	...
991250	.002	.002	...	...	...	...
991875	...	...	...	...	...	...

Est vol 330 Tu 1,199 calls 1,303 puts
Op int Tues 128,420 calls 162,873 puts

Eurodollar (CME)

$ million; pts. of 100%

Price	Feb	Mar	Apr	Feb	Mar	Apr
9825	...	5.90	...	0.00	0.00	0.12
9850	...	3.42	2.22	0.00	0.02	0.37
9875	0.95	1.02	0.45	0.05	0.12	1.10
9900	...	0.05	0.02	...	1.65	...
9925	...	0.00	...	...	4.10	...
9950	...	0.00	...	...	6.60	...

Est vol 288,753;
Tu vol 83,303 calls 142,595 puts
Op int Tues 4,268,863 calls 4,408,535 puts

1 Yr. Mid-Curve Eurodlr (CME)

$1,000,000 contract units; pts. of 100%

Price	Feb	Mar	Apr	Feb	Mar	Apr
9725	4.02	4.65	2.75	0.17	0.80	2.95
9750	2.05	2.87	1.60	0.70	1.52	...
9775	0.70	1.55	0.82	1.85	2.70	...
9800	0.15	0.65	0.35	3.80	4.30	...
9825	0.02	0.20	0.15	...	6.35	...
9850	0.00	0.05	...	...	...	...

Est vol 210,600 Tu 61,545 calls 129,840 puts
Op int Tues 934,544 calls 932,093 puts

2 Yr. Mid-Curve Eurodlr (CME)

$1,000,000 contract units; pts. of 100%

Price	Mar	Jun	Sep	Mar	Jun	Sep
9575	6.00	5.70	...	0.50	2.45	...
9600	4.00	4.10	4.05	1.00	3.35	5.40
9625	2.45	2.82	2.90	1.95	4.57	...
9650	1.27	1.85	...	3.27	6.07	...
9675	0.60	1.12	...	5.10	...	...
9700	0.17	0.50	...	7.17	...	...

Est vol 800 Tu 8,400 calls 0 puts
Op int Tues 158,035 calls 33,178 puts

Euribor (LIFFE)

Euro 1,000,000

Price	Feb	Mar	Apr	Feb	Mar	Apr
97750	0.18	0.19	0.17	...	0.00	0.02
97875	0.06	0.07	0.08	0.00	0.01	0.06
98000	0.01	0.03	0.03	0.07	0.09	0.13
98125	...	0.01	0.01	0.19	0.20	0.24
98250	...	0.00	0.00	0.31	0.32	0.35
98375	...	...	0.00	0.44	0.44	0.48

Vol Wd 327,805 calls 29,183 puts
Op int Tues 5,655,304 calls 1,807,541 puts

Euro-BUND (EUREX)

100,000; pts. in 100%

Price	Mar	Apr	May	Mar	Apr	May
11350	1.01	0.78	1.02	0.25	1.00	1.24
11400	0.68	0.56	0.78	0.42	1.28	1.50
11450	0.42	0.39	0.61	0.66	1.61	1.83
11500	0.22	0.26	0.46	0.96	1.98	2.18
11550	0.11	0.17	0.34	1.35	2.39	2.56
11600	0.06	0.10	...	1.80	2.82	...

Vol Wd 35,857 calls 42,186 puts
Op int Tues 366,384 calls 479,188 puts

Currency

Japanese Yen (CME)

12,500,000 yen; cents per 100 yen

Price	Feb	Mar	Apr	Feb	Mar	Apr
9400	1.03	1.72	2.30	0.06	0.75	1.04
9450	0.60	1.44	2.03	0.13	0.97	1.27
9500	0.30	1.19	1.78	0.33	1.22	1.52
9550	0.13	0.98	1.56	0.66	...	...
9600	0.06	0.81	1.36	1.09	1.84	...
9650	0.04	0.67	...	...	...	...

Est vol 1,352 Tu 1,271 calls 531 puts
Op int Tues 23,459 calls 20,676 puts

Canadian Dollar (CME)

100,000 Can.$, cents per Can.$

Price	Feb	Mar	Apr	Feb	Mar	Apr
7400	...	1.35	...	0.07	0.50	...
7450	0.51	1.04	...	0.16	0.69	...
7500	0.23	0.77	1.03	0.38	0.92	1.38
7550	0.10	0.57	...	0.75	1.22	...
7600	0.05	0.42	...	1.19	1.57	...
7650	0.02	0.31	...	1.67	1.96	...

Est vol 419 Tu 219 calls 163 puts
Op int Tues 12,761 calls 9,409 puts

British Pound (CME)

62,500 pounds; cents per pound

Price	Feb	Mar	Apr	Feb	Mar	Apr
1810	2.01	3.13	...	0.24	1.36	...
1820	1.13	2.53	...	0.36	1.76	...
1830	0.68	2.04	...	0.91	2.27	...
1840	0.34	1.60	...	1.57	2.83	...
1850	0.16	1.24	1.66	...	...	...
1860	0.08	0.96	1.42	3.31	...	...

Est vol 755 Tu 242 calls 625 puts
Op int Tues 6,257 calls 5,097 puts

Swiss Franc (CME)

125,000 francs; cents per franc

Price	Feb	Mar	Apr	Feb	Mar	Apr
7900	1.10	1.70	...	0.08	0.68	...
7950	0.69	1.39	...	0.17	0.87	...
8000	0.37	1.11	...	0.35	1.09	...
8050	0.18	0.89	...	0.66	1.37	...
8100	0.10	0.71	...	1.08	1.69	...
8150	0.05	0.55	...	1.53	2.03	...

Est vol 189 Tu 44 calls 384 puts
Op int Tues 1,690 calls 2,356 puts

Euro Fx (CME)

125,000 euros; cents per euro

Price	Feb	Mar	Apr	Feb	Mar	Apr
12400	1.35	2.36	2.82	0.15	1.16	1.91
12450	0.98	2.07	2.55	0.28	1.37	2.14
12500	0.66	1.81	2.31	0.46	1.61	2.40
12550	0.42	1.56	2.08	0.72	1.86	2.67
12600	0.26	1.34	1.87	1.06	2.14	2.96
12650	0.15	1.14	1.68	1.45	2.44	3.27

Est vol 3,767 Tu 3,252 calls 2,088 puts
Op int Tues 39,137 calls 43,286 puts

Index

DJ Industrial Avg (CBOT)

$100 times premium

Price	Feb	Mar	Apr	Feb	Mar	Apr
102	28.50	37.00	42.40	4.50	13.25	20.50
103	21.00	30.00	35.50	7.00	16.25	...
104	14.50	24.00	29.75	10.50	20.00	...
105	9.00	18.50	24.25	15.00	24.50	...
106	5.50	14.00	19.50	21.50	30.00	...
107	3.00	10.00	...	29.00	...	...

Est vol 124 Tu 111 calls 72 puts
Op int Tues 5,861 calls 5,480 puts

S&P 500 Stock Index (CME)

$250 times premium

Price	Feb	Mar	Apr	Feb	Mar	Apr
1115	19.70	29.90	37.80	10.80	21.00	29.90
1120	16.60	26.90	34.90	12.70	23.00	32.00
1125	13.80	24.00	32.00	14.90	25.10	34.10
1130	11.30	21.40	29.30	17.40	27.50	36.40
1135	9.10	19.00	26.80	20.20	30.10	...
1140	7.20	16.70	24.30	23.30	32.80	41.30

Est vol 14,455 Tu 4,759 calls 10,464 puts
Op int Tues 88,723 calls 228,763 puts

Other Options

Nasdaq 100 (CME)

$100 times NASDAQ 100 Index

Price	Feb	Mar	Apr	Feb	Mar	Apr
1460	...	...	...	...	...	...

Est vol 41 Tu 3 calls 2 puts
Op int Tues 2,185 calls 958 puts

NYSE Composite (NYFE)

$50 times premium

Price	Feb	Mar	Apr	Feb	Mar	Apr
6500	7450	12100	16400	6500	11150	16450

Est vol 0 Tu 3 calls 20 puts
Op int Tues 1 calls 9,514 puts

Interest rate futures option contracts work in the same way as the other futures options contracts discussed in this chapter. For example, the payoff from a call is $\max(F-K, 0)$, where F is the futures price at the time of exercise and K is the strike price. In addition to the cash payoff, the option holder obtains a long position in the futures contract when the option is exercised and the option writer obtains a corresponding short position.

Interest rate futures prices increase when bond prices increase (i.e., when interest rates fall). They decrease when bond prices decrease (i.e., when interest rates rise). An investor who thinks that short-term interest rates will rise can speculate by buying put options on Eurodollar futures, whereas an investor who thinks the rates will fall can speculate by buying call options on Eurodollar futures. An investor who thinks that long-term interest rates will rise can speculate by buying put options on Treasury note futures or Treasury bond futures, whereas an investor who thinks the rates will fall can speculate by buying call options on these instruments.

Example 14.5

It is February and the futures price for the June Eurodollar contract is 93.82 (corresponding to a 3-month Eurodollar interest rate of 6.18% per annum). The price of a call option on the contract with a strike price of 94.00 is quoted at the CME as 0.1, or 10 basis points (corresponding to a *Wall Street Journal* quote of 1.00). This option could be attractive to an investor who feels that interest rates are likely to come down. Suppose that short-term interest rates do drop by about 100 basis points and the investor exercises the call when the Eurodollar futures price is 94.78 (corresponding to a 3-month Eurodollar interest rate of 5.22% per annum). The payoff is $25 \times (94.78 - 94.00) = \$1{,}950$. The cost of the contract is $10 \times 25 = \$250$. The investor's profit is therefore \$1,700.

Example 14.6

It is August and the futures price for the December Treasury bond contract traded on the CBOT is 96-09 (or $96\frac{9}{32} = 96.28125$). The yield on long-term government bonds is about 6.4% per annum. An investor who feels that this yield will fall by December might choose to buy December calls with a strike price of 98. Assume that the price of these calls is 1-04 (or $1\frac{4}{64} = 1.0625\%$ of the principal). If long-term rates fall to 6% per annum and the Treasury bond futures price rises to 100-00, the investor will make a net profit per \$100 of bond futures of

$$100.00 - 98.00 - 1.0625 = 0.9375$$

Since one option contract is for the purchase or sale of instruments with a face value of \$100,000, the investor would make a profit of \$937.50 per option contract bought.

Reasons for the Popularity of Futures Options

It is natural to ask why people choose to trade options on futures rather than options on the underlying asset. The main reason appears to be that a futures contract is, in many circumstances, more liquid and easier to trade than the underlying asset. Furthermore, a futures price is known immediately from trading on the futures exchange, whereas the spot price of the underlying asset may not be so readily available.

Consider Treasury bonds. The market for Treasury bond futures is much more active than the market for any particular Treasury bond. Moreover, a Treasury bond futures price is known immediately from trading on the CBOT. By contrast, the current market price of a bond can be obtained only by contacting one or more dealers. It is not surprising that investors would rather take delivery of a Treasury bond futures contract than Treasury bonds.

Futures on commodities are also often easier to trade than the commodities themselves. For example, it is much easier and more convenient to make or take delivery of a live-hogs futures contract than it is to make or take delivery of the hogs themselves.

An important point about a futures option is that exercising it does not usually lead to delivery of the underlying asset. This is because, in most circumstances, the underlying futures contract is closed out prior to delivery. Futures options are therefore normally eventually settled in cash. This is appealing to many investors, particularly those with limited capital who may find it difficult to come up with the funds to buy the underlying asset when an option is exercised.

Another advantage sometimes cited for futures options is that futures and futures options are traded in pits side by side in the same exchange. This facilitates hedging, arbitrage, and speculation. It also tends to make the markets more efficient.

A final point is that futures options tend to entail lower transactions costs than spot options in many situations.

Put–Call Parity

In Chapter 9, we derived a put–call parity relationship for European stock options. We now present a similar argument to derive a put–call parity relationship for European futures options on the assumption that there is no difference between the payoffs from futures and forward contracts.

Consider European call and put futures options, both with strike price K and time to expiration T. We can form two portfolios:

Portfolio A: a European call futures option plus an amount of cash equal to Ke^{-rT}

Portfolio B: a European put futures option plus a long futures contract plus an amount of cash equal to F_0e^{-rT}

In portfolio A, the cash can be invested at the risk-free rate r and will grow to K at time T. Let F_T be the futures price at maturity of the option. If $F_T > K$, the call option in portfolio A is exercised and portfolio A is worth F_T. If $F_T \leqslant K$, the call is not exercised and portfolio A is worth K. The value of portfolio A at time T is therefore given by

$$\max(F_T, K)$$

In portfolio B, the cash can be invested at the risk-free rate to grow to F_0 at time T. The put option provides a payoff of $\max(K - F_T, 0)$. The futures contract provides a payoff of $F_T - F_0$. The value of portfolio B at time T is therefore given by

$$F_0 + (F_T - F_0) + \max(K - F_T, 0) = \max(F_T, K)$$

Since the two portfolios have the same value at time T and there are no early exercise opportunities, it follows that they are worth the same today. The value of portfolio A today is

$$c + Ke^{-rT}$$

where c is the price of the call futures option. The marking-to-market process ensures that the futures contract in portfolio B is worth zero today. Therefore, portfolio B is worth

$$p + F_0 e^{-rT}$$

where p is the price of the put futures option. Hence,

$$c + Ke^{-rT} = p + F_0 e^{-rT} \tag{14.11}$$

This is the same as put–call parity for options on a non-dividend-paying stock in equation (9.3) except that the stock price is replaced by the futures price times e^{-rT}

For American options, the put–call parity relationship is (see Problem 14.38)

$$F_0 e^{-rT} - K \leqslant C - P \leqslant F_0 - Ke^{-rT}$$

Example 14.7

Suppose that the price of a European call option on silver futures for delivery in 6 months is \$0.56 per ounce when the exercise price is \$8.50. Assume that the silver futures price for delivery in 6 months is currently \$8.00 and the risk-free interest rate for an investment that matures in 6 months is 10% per annum. From a rearrangement of equation (14.11), the price of a European put option on silver futures with the same maturity and exercise price as the call option is

$$0.56 + 8.50e^{-0.1\times 0.5} - 8.00e^{-0.1\times 0.5} = 1.04$$

14.6 VALUATION OF FUTURES OPTIONS USING BINOMIAL TREES

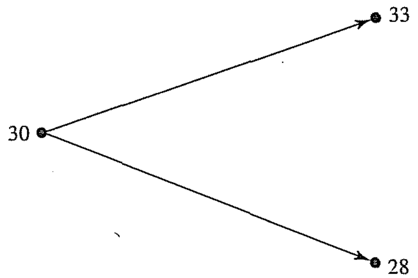
This section examines, more formally than in Chapter 11, how binomial trees can be used to price futures options. The key difference between futures options and stock options is that there are no up-front costs when a futures contract is entered into.

Suppose that the current futures price is 30 and it is expected to move either up to 33 or down to 28 over the next month. We consider a 1-month call option on the futures with a strike price of 29 and ignore daily settlement. The situation is shown in Figure 14.1. If the futures price proves to be 33, then the payoff from the option is 4 and the value of the futures contract is 3. If the futures price proves to be 28, then the payoff from the option is zero and the value of the futures contract is –2.[3]

To set up a riskless hedge, we consider a portfolio consisting of a short position in one option contract and a long position in Δ futures contracts. If the futures price moves up to 33, the value of the portfolio is $3\Delta - 4$; if it moves down to 28, the value of the portfolio is -2Δ. The portfolio is riskless when these are the same—that is,

[3] There is an approximation here in that the gain or loss on the futures contract is not realized at time T. It is realized day by day between time 0 and time T. However, as the length of the time step in a binomial tree becomes shorter, the approximation becomes better, and in the limit, as the time step tends to zero, an accurate answer is obtained.

Figure 14.1 Futures price movements in numerical example.



when

$$3\Delta - 4 = -2\Delta$$

or $\Delta = 0.8$.

For this value of Δ, we know the portfolio will be worth $3 \times 0.8 - 4 = -1.6$ in 1 month. Assume a risk-free interest rate of 6%. The value of the portfolio today must be

$$-1.6e^{-0.06 \times 0.08333} = -1.592$$

The portfolio consists of one short option and Δ futures contracts. Since the value of the futures contract today is zero, the value of the option today must be 1.592.

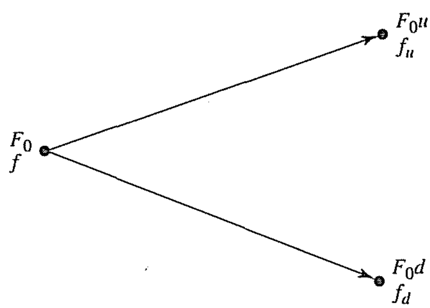
A Generalization

We can generalize this analysis by considering a futures price that starts at F_0 and is anticipated to rise to F_0u or move down to F_0d over the time period T. We consider a derivative maturing at the end of the time period, and we suppose that its payoff is f_u if the futures price moves up and f_d if it moves down. The situation is summarized in Figure 14.2.

The riskless portfolio in this case consists of a short position in one option combined with a long position in Δ futures contracts, where

$$\Delta = \frac{f_u - f_d}{F_0u - F_0d}$$

Figure 14.2 Futures price and option price in general situation.



The value of the portfolio at the end of the time period, then, is always

$$(F_0u - F_0)\Delta - f_u$$

Denoting the risk-free interest rate by r, we obtain the value of the portfolio today as

$$[(F_0u - F_0)\Delta - f_u]e^{-rT}$$

Another expression for the present value of the portfolio is $-f$, where f is the value of the option today. It follows that

$$-f = [(F_0u - F_0)\Delta - f_u]e^{-rT}$$

Substituting for Δ and simplifying reduces this equation to

$$f = e^{-rT}[pf_u + (1-p)f_d] \tag{14.12}$$

where

$$p = \frac{1-d}{u-d} \tag{14.13}$$

In the numerical example in Figure 14.1, $u = 1.1$, $d = 0.9333$, $r = 0.06$, $T = 0.08333$, $f_u = 4$, and $f_d = 0$. From equation (14.13), we have

$$p = \frac{1 - 0.9333}{1.1 - 0.9333} = 0.4$$

and, from equation (14.12),

$$f = e^{-0.06\times 0.08333}(0.4 \times 4 + 0.6 \times 0) = 1.592$$

This result agrees with the answer obtained for this example earlier.

Multistep Trees

In practice, trees are used to value American-style futures options in the same way as they are used to value options on stocks. This is explained in Section 11.9. An example is in Example 11.3 and Figure 11.13.

14.7 THE DRIFT OF FUTURES PRICES IN A RISK-NEUTRAL WORLD

There is a general result that allows us to use the analysis in Section 14.1 for futures options. This result is that in a risk-neutral world a futures price behaves in the same way as a stock paying a dividend yield at the domestic risk-free interest rate r.

One clue that this might be so is given by noting that the equation for p in a binomial tree for a futures price is the same as that for a stock paying a dividend yield equal to q when $q = r$. Another clue is that the put–call parity relationship for futures options prices is the same as that for options on a stock paying a dividend yield at rate q when the stock price is replaced by the futures price and $q = r$.

To prove the result formally, we calculate the drift of a futures price in a risk-neutral world. We define F_t as the futures price at time t. If we enter into a long futures contract today, its value is zero. At time Δt (the first time it is marked to market) it provides a payoff of $F_{\Delta t} - F_0$. If r is the very-short-term (Δt-period) interest rate at time 0, risk-neutral valuation gives the value of the contract at time 0 as

$$e^{-r\Delta t}\hat{E}[F_{\Delta t} - F_0]$$

where $\hat{E}$ denotes expectations in a risk-neutral world. We must therefore have

$$e^{-r\Delta t}\hat{E}(F_{\Delta t} - F_0) = 0$$

showing that

$$\hat{E}(F_{\Delta t}) = F_0$$

Similarly, $\hat{E}(F_{2\Delta t}) = F_{\Delta t}$, $\hat{E}(F_{3\Delta t}) = F_{2\Delta t}$, and so on. Putting many results like this together, we see that

$$\hat{E}(F_T) = F_0$$

for any time T

The drift of the futures price in a risk-neutral world is therefore zero. From equation (14.7), then, the futures price behaves like a stock providing a dividend yield q equal to r. This result is a very general one. It is true for all futures prices and does not depend on any assumptions about interest rates, volatilities, etc.[4]

The usual assumption made for the process followed by a futures price F in the risk-neutral world is

$$dF = \sigma F\,dz \tag{14.14}$$

where σ is a constant.

Differential Equation

For another way of seeing that a futures price behaves like a stock paying a dividend yield at rate q, we can derive the differential equation satisfied by a derivative dependent on a futures price in the same way as we derived the differential equation for a derivative dependent on a non-dividend-paying stock in Section 13.6. This is[5]

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf \tag{14.15}$$

It has the same form as equation (14.6) with q set equal to r. This confirms that, for the purpose of valuing derivatives, a futures price can be treated in the same way as a stock providing a dividend yield at rate r.

14.8 BLACK'S MODEL FOR VALUING FUTURES OPTIONS

European futures options can be valued by extending the results we have produced. Fischer Black was the first to show this in a paper published in 1976.[6] The underlying

[4] As we will discover in Chapter 25, a more precise statement of the result is: "A futures price has zero drift in the traditional risk-neutral world where the numeraire is the money market account." A zero-drift stochastic process is known as a martingale. A forward price is a martingale in a different risk-neutral world. This is one where the numeraire is a zero-coupon bond maturing at time T.

[5] See Technical Note 7 on the author's website for a proof of this.

[6] See F. Black, "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3 (March 1976): 167–79.

assumption is that futures prices have the same lognormal property that we assumed for stock prices in Chapter 13. The European call price c and the European put price p for a futures option are given by equations (14.4) and (14.5) with S_0 replaced by F_0 and $q = r$:

$$c = e^{-rT}[F_0 N(d_1) - KN(d_2)] \tag{14.16}$$

$$p = e^{-rT}[KN(-d_2) - F_0 N(-d_1)] \tag{14.17}$$

where

$$d_1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

and σ is the volatility of the futures price. When the cost of carry and the convenience yield are functions only of time, it can be shown that the volatility of the futures price is the same as the volatility of the underlying asset. Note that Black's model does not require the option contract and the futures contract to mature at the same time.

Example 14.8

Consider a European put futures option on crude oil. The time to the option's maturity is 4 months, the current futures price is \$20, the exercise price is \$20, the risk-free interest rate is 9% per annum, and the volatility of the futures price is 25% per annum. In this case, $F_0 = 20$, $K = 20$, $r = 0.09$, $T = 4/12$, $\sigma = 0.25$, and $\ln(F_0/K) = 0$, so that

$$d_1 = \frac{\sigma\sqrt{T}}{2} = 0.07216$$

$$d_2 = -\frac{\sigma\sqrt{T}}{2} = -0.07216$$

$$N(-d_1) = 0.4712, \qquad N(-d_2) = 0.5288$$

and the put price p is given by

$$p = e^{-0.09\times 4/12}(20 \times 0.5288 - 20 \times 0.4712) = 1.12$$

or \$1.12.

14.9 FUTURES OPTIONS vs. SPOT OPTIONS

In this section we compare options on futures and options on spot when they have the same strike price and time to maturity. An *option on spot* or *spot option* is a regular option to buy or sell the underlying asset in the spot market.

The payoff from a European spot call option with strike price K is

$$\max(S_T - K,\ 0)$$

where S_T is the spot price at the option's maturity. The payoff from a European futures call option with the same strike price is

$$\max(F_T - K,\ 0)$$

where F_T is the futures price at the option's maturity. If the European futures option matures at the same time as the futures contract, $F_T = S_T$ and the two options are in theory equivalent. If the European call futures option matures before the futures contract, it is worth more than the corresponding spot option in a normal market (where futures prices are higher than spot prices) and less than the corresponding spot option in an inverted market (where futures prices are lower than spot prices).

Similarly, a European futures put option is worth the same as its spot option counterpart when the futures option matures at the same time as the futures contract. If the European put futures option matures before the futures contract, it is worth less than the corresponding spot option in a normal market and more than the corresponding spot option in an inverted market.

Results for American Options

Traded futures options are, in practice, usually American. Assuming that the risk-free rate of interest, r, is positive, there is always some chance that it will be optimal to exercise an American futures option early. American futures options are, therefore, worth more than their European counterparts.

It is not generally true that an American futures option is worth the same as the corresponding American spot option when the futures and options contracts have the same maturity. Suppose, for example, that there is a normal market with futures prices consistently higher than spot prices prior to maturity. This is the case with most stock indices, gold, silver, low-interest currencies, and some commodities. An American call futures option must be worth more than the corresponding American spot call option. The reason is that in some situations the futures option will be exercised early, in which case it will provide a greater profit to the holder. Similarly, an American put futures option must be worth less than the corresponding American spot put option. If there is an inverted market with futures prices consistently lower than spot prices, as is the case with high-interest currencies and some commodities, the reverse must be true. American call futures options are worth less than the corresponding American spot call option, whereas American put futures options are worth more than the corresponding American spot put option.

The differences just described between American futures options and American spot options hold true when the futures contract expires later than the options contract as well as when the two expire at the same time. In fact, the differences tend to be greater the later the futures contract expires.

SUMMARY

The Black–Scholes formula for valuing European options on a non-dividend-paying stock can be extended to cover European options on a stock providing a known dividend yield. This is a useful result because a number of other assets on which options are written can be considered to be analogous to a stock providing a dividend yield. In particular:

1. An index is analogous to a stock providing a dividend yield. The dividend yield is the average dividend yield on the stocks composing the index.

2. A foreign currency is analogous to a stock providing a dividend yield where the dividend yield is the foreign risk-free interest rate.
3. A futures price is analogous to a stock providing a dividend yield where the dividend yield is equal to the domestic risk-free interest rate.

The extension to Black–Scholes can, therefore, be used to value European options on indices, foreign currencies, and futures contracts.

Index options are settled in cash. Upon exercise of an index call option, the holder receives the amount by which the index exceeds the strike price at close of trading. Similarly, upon exercise of an index put option, the holder receives the amount by which the strike price exceeds the index at close of trading. Index options can be used for portfolio insurance. If the portfolio has a β of 1.0, it is appropriate to buy one put option for each $100S_0$ dollars in the portfolio, where S_0 is the value of the index; otherwise, β put options should be purchased for each $100S_0$ dollars in the portfolio, where β is the beta of the portfolio calculated using the capital asset pricing model. The strike price of the put options purchased should reflect the level of insurance required.

Currency options are traded both on organized exchanges and over the counter. They can be used by corporate treasurers to hedge foreign exchange exposure. For example, a US corporate treasurer who knows that sterling will be received at a certain time in the future can hedge by buying put options that mature at that time. Similarly, a US corporate treasurer who knows that the company will be paying sterling at a certain time in the future can hedge by buying call options that mature at that time.

Futures options require the delivery of the underlying futures contract upon exercise. When a call is exercised, the holder acquires a long futures position plus a cash amount equal to the excess of the futures price over the strike price. Similarly, when a put is exercised, the holder acquires a short position plus a cash amount equal to the excess of the strike price over the futures price. The futures contract that is delivered typically expires slightly later than the option. If we assume that the two expiration dates are the same, a European futures option is worth exactly the same as the corresponding European spot option. However, this is not true of American options. If the futures market is normal, an American call futures option is worth more than the corresponding American spot call option, while an American put futures is worth less than the corresponding American spot put option. If the futures market is inverted, the reverse is true.

FURTHER READING

General

Merton, R. C. "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4 (Spring 1973): 141–83.

Bodie, Z. "On the Risk of Stocks in the Long Run," *Financial Analysts Journal*, 51, 3 (1995): 18–22.

On Options on Currencies

Amin, K., and R.A. Jarrow. "Pricing Foreign Currency Options under Stochastic Interest Rates," *Journal of International Money and Finance*, 10 (1991): 310–29.

Biger, N., and J.C. Hull. "The Valuation of Currency Options," *Financial Management*, 12 (Spring 1983): 24–28.

Garman, M. B., and S. W. Kohlhagen. "Foreign Currency Option Values," *Journal of International Money and Finance*, 2 (December 1983): 231–37.

Giddy, I. H. and G. Dufey. "Uses and Abuses of Currency Options," *Journal of Applied Corporate Finance*, 8, 3 (1995): 49–57.

Grabbe, J. O. "The Pricing of Call and Put Options on Foreign Exchange," *Journal of International Money and Finance*, 2 (December 1983): 239–53.

Jorion, P. "Predicting Volatility in the Foreign Exchange Market," *Journal of Finance* 50, 2 (1995): 507–28.

On Options on Futures

Black, F. "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3 (March 1976): 167–79.

Hilliard, J. E., and J. Reis. "Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot," *Journal of Financial and Quantitative Analysis*, 33, 1 (March 1998): 61–86.

Miltersen, K. R., and E. S. Schwartz. "Pricing of Options on Commodity Futures with Stochastic Term Structures of Convenience Yields and Interest Rates," *Journal of Financial and Quantitative Analysis*, 33, 1 (March 1998), 33–59.

Questions and Problems (Answers in Solutions Manual)

14.1. A portfolio is currently worth \$10 million and has a beta of 1.0. The S&P 100 is currently standing at 500. Explain how a put option on the S&P 100 with a strike of 480 can be used to provide portfolio insurance.

14.2. "Once we know how to value options on a stock paying a dividend yield, we know how to value options on stock indices, currencies, and futures." Explain this statement.

14.3. A stock index is currently 300, the dividend yield on the index is 3% per annum, and the risk-free interest rate is 8% per annum. What is a lower bound for the price of a 6-month European call option on the index when the strike price is 290?

14.4. A currency is currently worth \$0.80. Over each of the next 2 months it is expected to increase or decrease in value by 2%. The domestic and foreign risk-free interest rates are 6% and 8%, respectively. What is the value of a 2-month European call option with a strike price of \$0.80?

14.5. Explain the difference between a call option on yen and a call option on yen futures.

14.6. Explain how currency options can be used for hedging.

14.7. Calculate the value of a 3-month at-the-money European call option on a stock index when the index is at 250, the risk-free interest rate is 10% per annum, the volatility of the index is 18% per annum, and the dividend yield on the index is 3% per annum.

14.8. Consider an American call futures option where the futures contract and the option contract expire at the same time. Under what circumstances is the futures option worth more than the corresponding American option on the underlying asset?

14.9. Calculate the value of an 8-month European put option on a currency with a strike price of 0.50. The current exchange rate is 0.52, the volatility of the exchange rate is 12%, the domestic risk-free interest rate is 4% per annum, and the foreign risk-free interest rate is 8% per annum.

14.10. Why are options on bond futures more actively traded than options on bonds?

14.11. "A futures price is like a stock paying a dividend yield." What is the dividend yield?

14.12. A futures price is currently 50. At the end of 6 months it will be either 56 or 46. The risk-free interest rate is 6% per annum. What is the value of a 6-month European call option with a strike price of 50?

14.13. Calculate the value of a 5-month European put futures option when the futures price is \$19, the strike price is \$20, the risk-free interest rate is 12% per annum, and the volatility of the futures price is 20% per annum.

14.14. A total return index tracks the return, including dividends, on a certain portfolio. Explain how you would value (a) forward contracts and (b) European options on the index.

14.15. The S&P 100 index currently stands at 696 and has a volatility of 30% per annum. The risk-free rate of interest is 7% per annum and the index provides a dividend yield of 4% per annum. Calculate the value of a 3-month European put with strike price 700.

14.16. What is the put–call parity relationship for European currency options?

14.17. A foreign currency is currently worth \$1.50. The domestic and foreign risk-free interest rates are 5% and 9%, respectively. Calculate a lower bound for the value of a 6-month call option on the currency with a strike price of \$1.40 if it is (a) European and (b) American.

14.18. Consider a stock index currently standing at 250. The dividend yield on the index is 4% per annum and the risk-free rate is 6% per annum. A 3-month European call option on the index with a strike price of 245 is currently worth \$10. What is the value of a 3-month European put option on the index with a strike price of 245?

14.19. Would you expect the volatility of a stock index to be greater or less than the volatility of a typical stock? Explain your answer.

14.20. Does the cost of portfolio insurance increase or decrease as the beta of the portfolio increases? Explain your answer.

14.21. Suppose that a portfolio is worth \$60 million and the S&P 500 is at 1200. If the value of the portfolio mirrors the value of the index, what options should be purchased to provide protection against the value of the portfolio falling below \$54 million in 1 year's time?

14.22. Consider again the situation in Problem 14.21. Suppose that the portfolio has a beta of 2.0, that the risk-free interest rate is 5% per annum, and that the dividend yield on both the portfolio and the index is 3% per annum. What options should be purchased to provide protection against the value of the portfolio falling below \$54 million in 1 year's time?

14.23. Suppose you buy a put option contract on October gold futures with a strike price of \$400 per ounce. Each contract is for the delivery of 100 ounces. What happens if you exercise when the October futures price is \$377 and the most recent settlement price is \$380?

14.24. Suppose you sell a call option contract on April live-cattle futures with a strike price of 70 cents per pound. Each contract is for the delivery of 40,000 pounds. What happens if the contract is exercised when the futures price is 76 cents and the most recent settlement price is 75 cents?

14.25. Consider a 2-month call futures option with a strike price of 40 when the risk-free interest rate is 10% per annum. The current futures price is 47. What is a lower bound for the value of the futures option if it is (a) European and (b) American?

14.26. Consider a 4-month put futures option with a strike price of 50 when the risk-free interest rate is 10% per annum. The current futures price is 47. What is a lower bound for the value of the futures option if it is (a) European and (b) American?

14.27. A futures price is currently 60. It is known that over each of the next two 3-month periods it will either rise by 10% or fall by 10%. The risk-free interest rate is 8% per annum. What is the value of a 6-month European call option on the futures with a strike price of 60? If the call were American, would it ever be worth exercising it early?

14.28. In Problem 14.27, what is the value of a 6-month European put option on futures with a strike price of 60? If the put were American, would it ever be worth exercising it early? Verify that the call prices calculated in Problem 14.27 and the put prices calculated here satisfy put–call parity relationships.

14.29. A futures price is currently 25, its volatility is 30% per annum, and the risk-free interest rate is 10% per annum. What is the value of a 9-month European call on the futures with a strike price of 26?

14.30. A futures price is currently 70, its volatility is 20% per annum, and the risk-free interest rate is 6% per annum. What is the value of a 5-month European put on the futures with a strike price of 65?

14.31. Suppose that a futures price is currently 35. A European call option and a European put option on the futures with a strike price of 34 are both priced at 2 in the market. The risk-free interest rate is 10% per annum. Identify an arbitrage opportunity. Both options have 1 year to maturity.

14.32. "The price of an at-the-money European call futures option always equals the price of a similar at-the-money European put futures option." Explain why this statement is true.

14.33. Suppose that a futures price is currently 30. The risk-free interest rate is 5% per annum. A 3-month American call futures option with a strike price of 28 is worth 4. Calculate bounds for the price of a 3-month American put futures option with a strike price of 28.

14.34. Can an option on the yen/euro exchange rate be created from two options, one on the dollar/euro exchange rate, and the other on the dollar–yen exchange rate? Explain your answer.

14.35. A corporation knows that in 3 months it will have \$5 million to invest for 90 days at LIBOR minus 50 basis points and wishes to ensure that the rate obtained will be at least 6.5%. What position in exchange-traded interest rate options should it take?

14.36. Prove the results in equations (14.1), (14.2), and (14.3) using the following portfolios:

Portfolio A: one European call option plus an amount of cash equal to Ke^{-rT}
Portfolio B: e^{-qT} shares, with dividends being reinvested in additional shares
Portfolio C: one European put option plus e^{-qT} shares, with dividends on the shares being reinvested in additional shares
Portfolio D: an amount of cash equal to Ke^{-rT}

14.37. Show that, if C is the price of an American call with strike price K and maturity T on a stock providing a dividend yield of q, and P is the price of an American put on the same stock with the same strike price and exercise date, then

$$S_0e^{-qT} - K \leqslant C - P \leqslant S_0 - Ke^{-rT}$$

where S_0 is the stock price, r is the risk-free interest rate, and $r > 0$. (*Hint*: To obtain the first half of the inequality, consider possible values of:

Portfolio A: a European call option plus an amount K invested at the risk-free rate
Portfolio B: an American put option plus e^{-qT} of stock with dividends being reinvested in the stock

To obtain the second half of the inequality, consider possible values of:

Portfolio C: an American call option plus an amount Ke^{-rT} invested at the risk-free rate
Portfolio D: a European put option plus one stock, with dividends being reinvested in the stock.)

14.38. Show that, if C is the price of an American call option on a futures contract when the strike price is K and the maturity is T, and P is the price of an American put on the same futures contract with the same strike price and exercise date, then

$$F_0e^{-rT} - K \leqslant C - P \leqslant F_0 - Ke^{-rT}$$

where F_0 is the futures price and r is the risk-free rate. Assume that $r > 0$ and that there is no difference between forward and futures contracts. (*Hint*: Use an analogous approach to that indicated for Problem 14.37.)

14.39. If the price of currency A expressed in terms of the price of currency B follows the process

$$dS = (r_B - r_A)S\,dt + \sigma S\,dz$$

where r_A is the risk-free interest rate in currency A and r_B is the risk-free interest rate in currency B. What is the process followed by the price of currency B expressed in terms of currency A?

Assignment Questions

14.40. Use the DerivaGem software to calculate implied volatilities for the March 104 call and the March 104 put on the Dow Jones Industrial Average (DJX) in Table 14.1. The value of the DJX on February 4, 2004, was 104.71. Assume that the risk-free rate was 1.2% and that the dividend yield was 3.5%. The options expire on March 20, 2004. Are the quotes for the two options consistent with put–call parity?

14.41. A stock index currently stands at 300. It is expected to increase or decrease by 10% over each of the next two time periods of 3 months. The risk-free interest rate is 8% and the dividend yield on the index is 3%. What is the value of a 6-month put option on the index with a strike price of 300 if it is (a) European and (b) American?

14.42. Suppose that the spot price of the Canadian dollar is US $0.75 and that the Canadian dollar/US dollar exchange rate has a volatility of 4% per annum. The risk-free rates of interest in Canada and the United States are 9% and 7% per annum, respectively. Calculate the value of a European call option to buy one Canadian dollar for US $0.75 in 9 months. Use put–call parity to calculate the price of a European put option to sell one Canadian dollar for US $0.75 in 9 months. What is the price of a call option to buy US $0.75 with one Canadian dollar in 9 months?

14.43. A mutual fund announces that the salaries of its fund managers will depend on the performance of the fund. If the fund loses money, the salaries will be zero. If the fund makes a profit, the salaries will be proportional to the profit. Describe the salary of a fund manager as an option. How is a fund manager motivated to behave with this type of remuneration package?

14.44. A futures price is currently 40. It is known that at the end of 3 months the price will be either 35 or 45. What is the value of a 3-month European call option on the futures with a strike price of 42 if the risk-free interest rate is 7% per annum?

14.45. Calculate the implied volatility of soybean futures prices from the following information concerning a European put on soybean futures:

Current futures price	525
Exercise price	525
Risk-free rate	6% per annum
Time to maturity	5 months
Put price	20

14.46. Use the DerivaGem software to calculate implied volatilities for the July options on corn futures in Table 14.4. Assume the futures prices in Table 2.2 apply and that the risk-free rate is 1.1% per annum. Treat the options as American and use 100 time steps. The options mature on June 19, 2004. Can you draw any conclusions from the pattern of implied volatilities you obtain?