CHAPTER 21

Credit Derivatives

The most exciting developments in derivatives markets since the late 1990s have been in the credit derivatives area. In 2000 the total notional principal for outstanding credit derivatives contracts was about \$800 billion. By 2003 this had grown to over \$3 trillion. Credit derivatives are contracts where the payoff depends on the creditworthiness of one or more companies or countries. In this chapter we explain how credit derivatives work and discuss some valuation issues.

Credit derivatives allow companies to trade credit risks in much the same way that they trade market risks. Banks and other financial institutions used to be in the position where they could do little once they had assumed a credit risk except wait (and hope for the best). Now they can actively manage their portfolios of credit risks, keeping some and entering into credit derivatives contracts to protect themselves from others. As indicated in Business Snapshot 21.1, banks have been the biggest buyers of credit protection and insurance companies have been the biggest sellers.

21.1 CREDIT DEFAULT SWAPS

The most popular credit derivative is a *credit default swap* (CDS). This is a contract that provides insurance against the risk of a default by particular company. The company is known as the *reference entity* and a default by the company is known as a *credit event*. The buyer of the insurance obtains the right to sell bonds issued by the company for their face value when a credit event occurs and the seller of the insurance agrees to buy the bonds for their face value when a credit event occurs.[1] The total face value of the bonds that can be sold is known as the credit default swap's *notional principal*.

The buyer of the CDS makes periodic payments to the seller until the end of the life of the CDS or until a credit event occurs. These payments are typically made in arrears every quarter, every half year, or every year. The settlement in the event of a default involves either physical delivery of the bonds or a cash payment.

An example will help to illustrate how a typical deal is structured. Suppose that two

[1] The face value (or par value) of a bond is the principal amount that the issuer will repay at maturity if it does not default.

Business Snapshot 21.1 Who Bears the Credit Risk?

Traditionally banks have been in the business of making loans and then bearing the credit risk that the borrower will default. But this is changing. Banks have for some time been reluctant to keep loans on their balance sheets. This is because, after the capital required by regulators has been accounted for, the average return earned on loans is often less attractive than that on other assets. During the 1990s, banks created asset-backed securities (similar to the mortgage-backed securities discussed in Chapter 29) to pass loans (and their credit risk) on to investors. In the late 1990s and early 2000s, banks have made extensive use of credit derivatives to shift the credit risk in their loans to other parts of the financial system.

If banks have been net buyers of credit protection, who have been net sellers? The answer is insurance companies. Insurance companies are not regulated in the same way as banks and as a result are sometimes more willing to bear credit risks than banks.

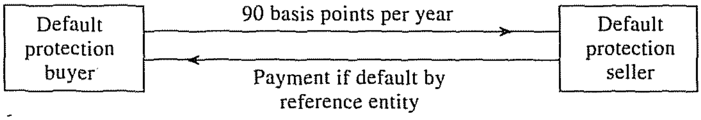
The result of all this is that the financial institution bearing the credit risk of a loan is often different from the financial institution that did the original credit checks. Whether this proves to be good for the overall health of the financial system remains to be seen.

parties enter into a 5-year credit default swap on March 1, 2004. Assume that the notional principal is \$100 million and the buyer agrees to pay 90 basis points annually for protection against default by the reference entity.

The CDS is shown in Figure 21.1. If the reference entity does not default (i.e., there is no credit event), the buyer receives no payoff and pays \$900,000 on March 1 of each of the years 2005, 2006, 2007, 2008, and 2009. If there is a credit event, a substantial payoff is likely. Suppose that the buyer notifies the seller of a credit event on June 1, 2007 (a quarter of the way into the fourth year). If the contract specifies physical settlement, the buyer has the right to sell bonds issued by the reference entity with a face value of \$100 million for \$100 million. If the contract requires cash settlement, an independent calculation agent will poll dealers to determine the mid-market value of the cheapest deliverable bond a predesignated number of days after the credit event. Suppose this bond is worth \$35 per \$100 of face value. The cash payoff would be \$65 million.

The regular quarterly, semiannual, or annual payments from the buyer of protection to the seller of protection cease when there is a credit event. However, because these payments are made in arrears, a final accrual payment by the buyer is usually required. In our example, the buyer would be required to pay to the seller the amount of the annual payment accrued between March 1, 2007, and June 1, 2007 (approximately \$225,000), but no further payments would be required.

Figure 21.1 Credit default swap.



The total amount paid per year, as a percent of the notional principal, to buy protection is known as the *CDS spread.* Several large banks are market makers in the credit default swap market. When quoting on a new 5-year credit default swap on Ford Motor Credit, a market maker might bid 250 basis points and offer 260 basis points. This means that the market maker is prepared to buy protection on Ford by paying 250 basis points per year (i.e., 2.5% of the principal per year) and to sell protection on Ford for 260 basis points per year (i.e., 2.6% of the principal per year).

Credit Default Swaps and Bond Yields

A CDS can be used to hedge a position in a corporate bond. Suppose that an investor buys a 5-year corporate bond yielding 7% per year for its face value and at the same time enters into a 5-year CDS to buy protection against the issuer of the bond defaulting. Suppose that the CDS spread is 2% per annum. The effect of the CDS is to convert the corporate bond to a risk-free bond (at least approximately). If the bond issuer does not default the investor earns 5% per year (when the CDS spread is netted against the corporate bond yield). If the bond does default the investor earns 5% up to the time of the default. Under the terms of the CDS, the investor is then able to exchange the bond for its face value. This face value can be invested at the risk-free rate for the remainder of the 5 years.

The n-year CDS spread should be approximately equal to the excess of the par yield on an n-year corporate bond over the par yield on an n-year risk-free bond. If it is markedly less than this, an investor can earn more than the risk-free rate by buying the corporate bond and buying protection. If it is markedly greater than this, an investor can borrow at less than the risk-free rate by shorting the corporate bond and selling CDS protection. These are not perfect arbitrages, but they do give a good guide to the relationship between CDS spreads and bond yields. CDS spreads can be used to imply the risk-free rates used by market participants. As discussed in Section 20.4, the average implied risk-free rate appears to be approximately equal to the LIBOR/swap rate minus 10 basis points.

The Cheapest-to-Deliver Bond

As explained in Section 20.3 the recovery rate on a bond is defined as the value of the bond immediately after default as a percent of face value. This means that the payoff from a CDS is $L(1 - R)$, where L is the notional principal and R is the recovery rate.

Usually a CDS specifies that a number of different bonds can be delivered in the event of a default. The bonds typically have the same seniority, but they may not sell for the same percentage of face value immediately after a default.[2] This gives the holder of a CDS a cheapest-to-deliver bond option. When a default happens the buyer of protection (or the calculation agent in the event of cash settlement) will review alternative deliverable bonds and choose for delivery the one that can be purchased most cheaply.

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[2] There are a number of reasons for this. The claim that is made in the event of a default is typically equal to the bond's face value plus accrued interest. Bonds with high accrued interest at the time of default therefore tend to have higher prices immediately after default. Also the market may judge that in the event of a reorganization of the company some bond holders will fare better than others.

21.2 CREDIT INDICES

Participants in credit derivatives markets have developed indices to track credit default swap spreads. In 2004, there were agreements between different producers of indices which led to some consolidation. Among the indices now used are:

1. The 5- and 10-year CDX NA IG indices tracking the credit spread for 125 investment grade North American companies; and
2. The 5- and 10-year iTraxx Europe indices tracking the credit spread for 125 investment grade European companies

In addition to monitoring credit spreads, indices provide a way market participants can easily buy or sell a portfolio of credit default swaps. For example, an investment bank, acting as market maker might quote the CDX NA IG 5-year index as bid 65 basis points and offer 66 basis points. An investor could then buy $800,000 of 5-year CDS protection on each the 125 underlying companies for $660,000 per year. The investor can sell $800,000 million of 5-year CDS protection on each of the 125 underlying names for $650,000 per year. When a company defaults, the annual payment is reduced by $660,000/125 = $5,280.[3]

21.3 VALUATION OF CREDIT DEFAULT SWAPS

Mid-market CDS spreads on individual reference entities (i.e., the average of the bid and offer CDS spreads quoted by brokers) can be calculated from default probability estimates. We will illustrate how this is done with a simple example.

Suppose that the probability of a reference entity defaulting during a year conditional on no earlier default is 2%.[4] Table 21.1 shows survival probabilities and unconditional default probabilities (i.e., default probabilities as seen at time zero) for each of the 5 years. The probability of a default during the first year is 0.02 and the probability the

Table 21.1 Unconditional default probabilities and survival probabilities.

Time (years)	*Default probability*	*Survival probability*
1	0.0200	0.9800
2	0.0196	0.9604
3	0.0192	0.9412
4	0.0188	0.9224
5	0.0184	0.9039

[3] The index is slightly lower than the average of the credit default swap spreads for the companies in the portfolio. To understand the reason for this, consider two companies, one with a spread of 1000 basis points and the other with a spread of 10 basis points. To buy protection on both companies would cost slightly less than 505 basis points per company. This is because the 1000 basis points is not expected to be paid for as long as the 10 basis points and should therefore carry less weight.

[4] As mentioned in Section 20.2, conditional default probabilities are known as default intensities.

Table 21.2 Calculation of the present value of expected payments. Payment = s per annum.

Time (years)	*Probability of survival*	*Expected payment*	*Discount factor*	*PV of expected payment*
1	0.9800	$0.9800s$	0.9512	$0.9322s$
2	0.9604	$0.9604s$	0.9048	$0.8690s$
3	0.9412	$0.9412s$	0.8607	$0.8101s$
4	0.9224	$0.9224s$	0.8187	$0.7552s$
5	0.9039	$0.9039s$	0.7788	$0.7040s$
Total				$4.0704s$

reference entity will survive until the end of the first year is 0.98. The probability of a default during the second year is $0.02 \times 0.98 = 0.0196$ and the probability of survival until the end of the second year is $0.98 \times 0.98 = 0.9604$. The probability of default during the third year is $0.02 \times 0.9604 = 0.0192$, and so on.

We will assume that defaults always happen halfway through a year and that payments on the credit default swap are made once a year, at the end of each year. We also assume that the risk-free (LIBOR) interest rate is 5% per annum with continuous compounding and the recovery rate is 40%. There are three parts to the calculation. These are shown in Tables 21.2, 21.3, and 21.4.

Table 21.2 shows the calculation of the expected present value of the payments made on the CDS assuming that payments are made at the rate of s per year and the notional principal is \$1. For example, there is a 0.9412 probability that the third payment of s is made. The expected payment is therefore $0.9412s$ and its present value is $0.9412se^{-0.05\times 3} = 0.8101s$. The total present value of the expected payments is $4.0704s$.

Table 21.3 shows the calculation of the expected present value of the payoff assuming a notional principal of \$1. As mentioned earlier, we are assuming that defaults always happen halfway through a year. For example, there is a 0.0192 probability of a payoff halfway through the third year. Given that the recovery rate is 40% the expected payoff at this time is $0.0192 \times 0.6 \times 1 = 0.0115$. The present value of the expected payoff is $0.0115e^{-0.05\times 2.5} = 0.0102$. The total present value of the expected payoffs is \$0.0511.

Table 21.3 Calculation of the present value of expected payoff. Notional principal = \$1.

Time (years)	*Probability of default*	*Recovery rate*	*Expected payoff (\$)*	*Discount factor*	*PV of expected payoff (\$)*
0.5	0.0200	0.4	0.0120	0.9753	0.0117
1.5	0.0196	0.4	0.0118	0.9277	0.0109
2.5	0.0192	0.4	0.0115	0.8825	0.0102
3.5	0.0188	0.4	0.0113	0.8395	0.0095
4.5	0.0184	0.4	0.0111	0.7985	0.0088
Total					0.0511

Table 21.4 Calculation of the present value of accrual payment.

Time (years)	*Probability of default*	*Expected accrual payment*	*Discount factor*	*PV of expected accrual payment*
0.5	0.0200	$0.0100s$	0.9753	$0.0097s$
1.5	0.0196	$0.0098s$	0.9277	$0.0091s$
2.5	0.0192	$0.0096s$	0.8825	$0.0085s$
3.5	0.0188	$0.0094s$	0.8395	$0.0079s$
4.5	0.0184	$0.0092s$	0.7985	$0.0074s$
Total				$0.0426s$

As a final step we evaluate in Table 21.4 the accrual payment made in the event of a default. For example, there is a 0.0192 probability that there will be a final accrual payment halfway through the third year. The accrual payment is $0.5s$. The expected accrual payment at this time is therefore $0.0192 \times 0.5s = 0.0096s$. Its present value is $0.0096se^{-0.05\times2.5} = 0.0085s$. The total present value of the expected accrual payments is $0.0426s$.

From Tables 21.1 and 21.3, the present value of the expected payments is

$$4.0704s + 0.0426s = 4.1130s$$

From Table 21.2, the present value of the expected payoff is 0.0511. Equating the two, we obtain the CDS spread for a new CDS as

$$4.1130s = 0.0511$$

or $s = 0.0124$. The mid-market spread should be 0.0124 times the principal or 124 basis points per year. This example is designed to illustrate the calculation methodology. In practice, we are likely to find that calculations are more extensive than those in Tables 21.2 to 21.4 because (a) payments are often made more frequently than once a year and (b) we might want to assume that defaults can happen more frequently than once a year.

Marking to Market a CDS

At the time it is negotiated, a CDS, like most other swaps, is worth close to zero. Later it may have a positive or negative value. Suppose, for example the credit default swap in our example had been negotiated some time ago for a spread of 150 basis points, the present value of the payments by the buyer would be $4.1130 \times 0.0150 = 0.0617$ and the present value of the payoff would be 0.0511 as above. The value of swap to the seller would therefore be $0.0617 - 0.0511$, or 0.0106 times the principal. Similarly the mark-to-market value of the swap to the buyer of protection would be -0.0106 times the principal.

Estimating Default Probabilities

The default probabilities used to value a CDS should be risk-neutral default probabilities, not real-world default probabilities (see Section 20.5 for a discussion of the difference between the two). Risk-neutral default probabilities can be estimated from bond prices or asset swaps as explained in Chapter 20. An alternative is to imply them

Table 21.5 Calculation of the present value of expected payoff from a binary credit default swap. Principal = \$1.

Time (years)	*Probability of default*	*Expected payoff (\$)*	*Discount factor*	*PV of expected payoff (\$)*
0.5	0.0200	0.0200	0.9753	0.0195
1.5	0.0196	0.0196	0.9277	0.0182
2.5	0.0192	0.0192	0.8825	0.0170
3.5	0.0188	0.0188	0.8395	0.0158
4.5	0.0184	0.0184	0.7985	0.0147
Total				0.0852

from CDS quotes. The latter approach is similar to the practice in options markets of implying volatilities from the prices of actively traded options.

Suppose we change the example in Tables 21.2, 21.3 and 21.4 so that we do not know the default probabilities. Instead we know that the mid-market CDS spread for a newly issued 5-year CDS is 100 basis points per year. We can reverse engineer our calculations to conclude that the implied default probability per year (conditional on no earlier default) is 1.61% per year.[5]

Binary Credit Default Swaps

A binary credit default swap is structured similarly to a regular credit default swap except that the payoff is a fixed dollar amount. Suppose that in the example we have considered in Tables 21.1 to 21.4 the payoff is \$1, instead of $(1 - R)$ dollars, and the swap spread is s. Tables 21.1, 21.2 and 21.4 are the same, but Table 21.3 is replaced by Table 21.5. The CDS spread for a new binary CDS is given by

$$4.1130s = 0.0852$$

so that the CDS spread, s, is 0.0207 or 207 basis points.

How Important is the Recovery Rate?

Whether we use CDS spreads or bond prices to estimate default probabilities we need an estimate of the recovery rate. However, provided that we use the same recovery rate for (a) estimating risk-neutral default probabilities and (b) valuing a CDS, the value of the CDS (or the estimate of the CDS spread) is not very sensitive to the recovery rate. This is because the implied probabilities of default are approximately proportional to $1/(1 - R)$ and the payoffs from a CDS are proportional to $1 - R$.

This argument does not apply to the valuation of binary CDS. Implied probabilities of default are still proportional to $1/(1 - R)$. However, for a binary CDS, the payoffs from the CDS are independent of R. If we have CDS spread for both a plain vanilla CDS and a binary CDS, we can estimate both the recovery rate and the default probability (see Problem 21.25).

[5] Ideally we would like to estimate a different default probability for each year instead of a single default intensity. We could do this if we had spreads for 1-, 2-, 3-, 4-, and 5-year CDS swaps or bond prices.

Business Snapshot 21.2 Is the CDS Market a Fair Game?

There is one important difference between credit default swaps and the other over-the-counter derivatives that we have considered in this book. The other over-the-counter derivatives depend on interest rates, exchange rates, equity indices, commodity prices, and so on. There is no reason to assume that any one market participant has better information than any other market participant about these variables.

Credit default swaps spreads depend on the probability that a particular company will default during a particular period of time. Arguably some market participants have more information to estimate this probability than others. A financial institution that works closely with a particular company by providing advice, making loans, and handling new issues of securities is likely to have more information about the creditworthiness of the company than another financial institution that has no dealings with the company. Economists refer to this as an *asymmetric information* problem.

Whether asymmetric information will curtail the expansion of the credit default swap market remains to be seen. Financial institutions emphasize that the decision to buy protection against the risk of default by a company is normally made by a risk manager and is not based on any special information that many exist elsewhere in the financial institution about the company.

The Future of the CDS Market

The market for credit default swaps has grown rapidly in the late 1990s and early 2000s. Credit default swaps account for about 70% of all credit derivatives. They have become important tools for managing credit risk. A financial institution can reduce its credit exposure to particular companies by buying protection. It can also use CDSs to diversify credit risk. For example, if a financial institution has too much credit exposure to a particular business sector, it can buy protection against defaults by companies in the sector and at the same time sell protection against default by companies in other unrelated sectors.

Some market participants think the growth of the CDS market will continue and that it will be as big as the interest rate swap market by 2010. Others are less optimistic. There is a potential asymmetric information problem in the CDS market that is not present in other over-the-counter derivatives markets (see Business Snapshot 21.2).

21.4 CDS FORWARDS AND OPTIONS

Once the CDS market was well established, it was natural for derivatives dealers to trade forwards and options on credit default swap spreads.[6]

A forward credit default swap is the obligation to buy or sell a particular credit default swap on a particular reference entity at a particular future time T. If the reference entity defaults before time T, the forward contract ceases to exist. Thus a bank could enter into a forward contract to sell 5-year protection on Ford Motor

[6] The valuation of these instruments is discussed in J. C. Hull and A. White, "The Valuation of Credit Default Swap Options," *Journal of Derivatives*, 10, 5 (Spring 2003): 40–50.

Credit for 280 basis points starting in 1 year. If Ford defaults during the next year, the bank's obligation under the forward contract ceases to exist.

A credit default swap option is an option to buy or sell a particular credit default swap on a particular reference entity at a particular future time T. For example, an investor could negotiate the right to buy 5-year protection on Ford Motor Credit starting in 1 year for 280 basis points. This is a call option. If the 5-year CDS spread for Ford in 1 year turns out to be more than 280 basis points, the option will be exercised; otherwise it will not be exercised. The cost of the option would be paid up front. Similarly an investor might negotiate the right to sell 5-year protection on Ford Motor Credit for 280 basis points starting in 1 year. This is a put option. If the 5-year CDS spread for Ford in 1 year turns out to be less than 280 basis points the option will be exercised; otherwise it will not be exercised. Again the cost of the option would be paid up front. Like CDS forwards, CDS options are usually structured so that they will cease to exist if the reference entity defaults before option maturity.

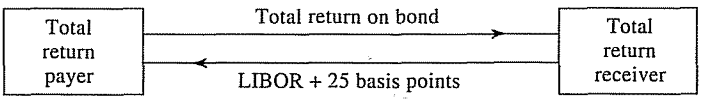
An option contract that has become popular in the credit derivatives market is a call option on a basket of reference entities. If there are m reference entities in the basket that have not defaulted by the option maturity, the option gives the holder the right to buy a portfolio of CDSs on the names for mK basis points, where K is the strike price. In addition, the holder gets the usual CDS payoff on any reference entities that do default during the life of the contract.

21.5 TOTAL RETURN SWAPS

A *total return swap* is a type of credit derivative. It is an agreement to exchange the total return on a bond (or any portfolio of assets) for LIBOR plus a spread. The total return includes coupons, interest, and the gain or loss on the asset over the life of the swap.

An example of a total return swap is a 5-year agreement with a notional principal of \$100 million to exchange the total return on a corporate bond for LIBOR plus 25 basis points. This is illustrated in Figure 21.2. On coupon payment dates the payer pays the coupons earned on an investment of \$100 million in the bond. The receiver pays interest at a rate of LIBOR plus 25 basis points on a principal of \$100 million. (LIBOR is set on one coupon date and paid on the next as in a plain vanilla interest rate swap.) At the end of the life of the swap there is a payment reflecting the change in value of the bond. For example, if the bond increases in value by 10% over the life of the swap, the payer is required to pay \$10 million (= 10% of \$100 million) at the end of the 5 years. Similarly, if the bond decreases in value by 15%, the receiver is required to pay \$15 million at the end of the 5 years. If there is a default on the bond, the swap is usually terminated and the receiver makes a final payment equal to the excess of \$100 million over the market value of the bond.

Figure 21.2 Total return swap.



If we add the notional principal to both sides at the end of the life of the swap, we can characterize the total return swap as follows. The payer pays the cash flows on an investment of $100 million in the corporate bond. The receiver pays the cash flows on a $100 million bond paying LIBOR plus 25 basis points. If the payer owns the corporate bond, the total return swap allows it to pass the credit risk on the bond to the receiver. If it does not own the bond, the total return swap allows it to take a short position in the bond.

Total return swaps are often used as a financing tool. One scenario that could lead to the swap in Figure 21.2 is as follows. The receiver wants financing to invest $100 million in the reference bond. It approaches the payer (which is likely to be a financial institution) and agrees to the swap. The payer then invests $100 million in the bond. This leaves the receiver in the same position as it would have been if it had borrowed money at LIBOR plus 25 basis points to buy the bond. The payer retains ownership of the bond for the life of the swap and faces less credit risk than it would have done if it had lent money to the receiver to finance the purchase of the bond, with the bond being used as collateral for the loan. If the receiver defaults the payer does not have the legal problem of trying to realize on the collateral. Total return swaps are similar to repos (see Section 4.1) in that they are structured to minimize credit risk when securities are being financed.

The spread over LIBOR received by the payer is compensation for bearing the risk that the receiver will default. The payer will lose money if the receiver defaults at a time when the reference bond's price has declined. The spread therefore depends on the credit quality of the receiver, the credit quality of the bond issuer, and the default correlation between the two.

There are a number of variations on the standard deal we have described. Sometimes, instead of there being a cash payment for the change in value of the bond, there is physical settlement where the payer exchanges the underlying asset for the notional principal at the end of the life of the swap. Sometimes the change-in-value payments are made periodically rather than all at the end.

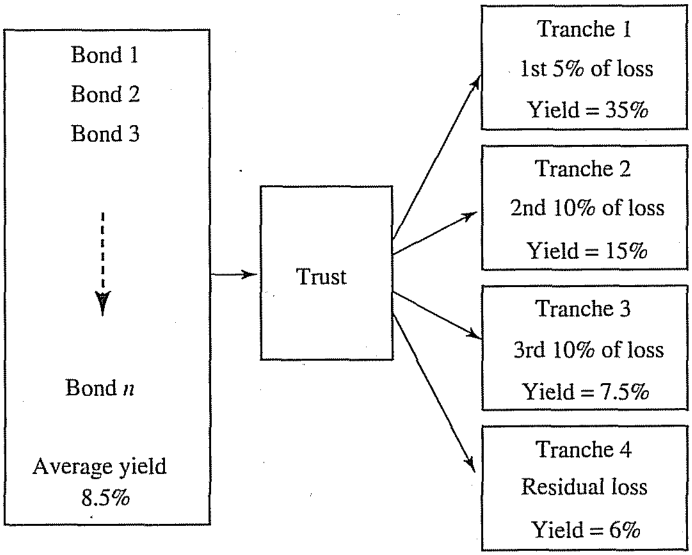
21.6 BASKET CREDIT DEFAULT SWAPS

In what is referred to as a *basket credit default swap* there are a number of reference entities. An *add-up basket* CDS provides a payoff when any of the reference entities default. A *first-to-default* CDS provides a payoff only when the first default occurs. A *second-to-default* CDS provides a payoff only when the second default occurs. More generally an *nth-to-default* CDS provides a payoff only when the nth default occurs. Payoffs are calculated in the same way as for a regular CDS. After the relevant default has occurred, there is a settlement. The swap then terminates and there are no further payments by either party.

21.7 COLLATERALIZED DEBT OBLIGATIONS

A collateralized debt obligation (CDO) is a way of creating securities with widely different risk characteristics from a portfolio of debt instruments. An example is shown in Figure 21.3. In this four types of securities (or tranches) are created from a portfolio

Figure 21.3 Collateralized debt obligation.



of bonds. The first tranche has 5% of the total bond principal and absorbs all credit losses from the portfolio during the life of the CDO until they have reached 5% of the total bond principal. The second tranche has 10% of the principal and absorbs all losses during the life of the CDO in excess of 5% of the principal up to a maximum of 15% of the principal. The third tranche has 10% of the principal and absorbs all losses in excess of 15% of the principal up to a maximum of 25% of the principal. The fourth tranche has 75% of the principal absorbs all losses in excess of 25% of the principal. The yields in Figure 21.3 are the rates of interest paid to tranche holders. These rates are paid on the balance of the principal remaining in the tranche after losses have been paid. Consider the first tranche. Initially the return of 35% is paid on the whole amount invested by the tranche 1 holders. But after losses equal to 1% of the total bond principal have been experienced, tranche 1 holders have lost 20% of their investment and the return is paid on only 80% of the original amount invested. Tranche 1 is referred to the *equity tranche.* A default loss of 2.5% on the bond portfolio translates into a loss of 50% of the tranche's principal. Tranche 4 by contrast is usually given an Aaa rating. Defaults on the bond portfolio must exceed 25% before the holders of this tranche are responsible for any credit losses.

The creator of the CDO normally retains the equity tranche and sells the remaining tranches in the market. A CDO provides a way of creating high quality debt from average quality (or even low quality) debt.

Synthetic CDOs

The CDO in Figure 21.3 is referred to as a *cash CDO*. An alternative structure which has become popular is a *synthetic CDO* where the creator of the CDO sells a portfolio of credit default swaps to third parties. It then passes on the default risk to the synthetic CDO's tranche holders. Analogously to Figure 21.3, the first tranche might be responsible for the payoffs on the credit default swaps until they have reached 5% of the total notional principal; the second tranche might be responsible for the payoffs between 5% and 15% of the total notional principal; and so on. The income from the credit default swaps is distributed to the tranches in a way that reflects the risk they are bearing. For example, the first tranche might get 3,000 basis points; the second tranche 1,000 basis points, and so on. As in a cash CDO this would be paid on a principal that declined as defaults for which the tranche is responsible occur.

Single Tranche Trading

In Section 21.2 we discussed the portfolios of 125 companies that are used to generate CDX and iTraxx indices. The market uses these portfolios to define standard CDO tranches. The trading of these standard tranches is known as *single tranche trading*. A single tranche trade is an agreement where one side agrees to sell protection against losses on a tranche and the other side agrees to buy the protection. The tranche is not part of a synthetic CDO but cash flows are calculated in the same way as if it were part of a synthetic CDO. The tranche is referred to as "unfunded" because it has not been created by selling credit default swaps or buying bonds.

In the case of the CDX NA IG index, the equity tranche covers losses between 0% and 3% of the principal. The second tranche, which is referred to as the *mezzanine tranche*, covers losses between 3% and 7%. The remaining tranches cover losses from 7% and 10%, 10% to 15%, and 15% to 30%. In the case of the iTraxx Europe index, the equity tranche covers losses between 0% and 3%. The mezzanine tranche covers losses between 3% and 6%. The remaining tranches cover losses from 6% to 9%, 9% to 12%, and 12% to 22%.

Table 21.6 shows the mid-market quotes for 5-year CDX and iTraxx tranches on August 4, 2004. On that date the CDX index level was 63.25 basis points and the iTraxx index was 42 basis points. For example, the mid-market price of mezzanine protection for the CDX IG NA was 347 basis points per year while that for iTraxx Europe was 168 basis points per year. Note that the equity tranche is quoted differently from the other tranches. The market quote of 41.75% for CDX means that the protection seller receives an initial payment of 41.75% of the principal plus a spread of 500 basis points per year. Similarly the market quote of 27.6% for iTraxx means that the protection seller receives an initial payment of 27.6% of the principal plus a spread of 500 basis points per year.

Table 21.6 Five-year CDX IG NA and iTraxx Europe tranches on August 4, 2004. Quotes are in basis points except for 0%–3% tranche. (*Source*: GFI)

CDX IG NA					
Tranche	0%–3%	3%–7%	7%–10%	10%–15%	15%–30%
Quote	41.8%	347	135.5	47.5	14.5
iTraxx Europe					
Tranche	0%–3%	3%–6%	6%–9%	9%–12%	12%–22%
Quote	27.6%	168	70	43	20

21.8 VALUATION OF A BASKET CDS AND CDO

The spread for an nth-to-default CDS or the tranche of a CDO is critically dependent on default correlation. Suppose that a basket of 100 reference entities is used to define a 5-year nth-to-default CDS and that each reference entity has a risk-neutral probability of 2% of defaulting during the 5 years. When the default correlation between the reference entities is zero the binomial distribution shows that the probability of one or more defaults during the 5 years is 86.74% and the probability of 10 or more defaults is 0.0034%. A first-to-default CDS is therefore quite valuable whereas a tenth-to-default CDS is worth almost nothing.

As the default correlation increases the probability of one or more defaults declines and the probability of 10 or more defaults increases. In the limit where the default correlation between the reference entities is perfect the probability of one or more defaults equals the probability of ten or more defaults and is 2%. This is because in this extreme situation the reference entities are essentially the same. Either they all default (with probability 2%) or none of them default (with probability 98%).

The valuation of a tranche of a CDO is similarly dependent on default correlation. If the correlation is low the junior equity tranche is very risky and the senior tranches are very safe. As the default correlation increases the junior tranches become less risky and the senior tranches become more risky. In the limit where the default correlation is perfect the tranches are equally risky.

Using the Gaussian Copula Model of Time to Default

The one-factor Gaussian copula model of time to default presented above has become the standard market model for valuing an nth-to-default CDS or a tranche of a CDO.

Consider a portfolio of N reference entities. From equation (20.7)

$$Q_i(T \mid M) = N\left(\frac{N^{-1}[Q_i(T)] - a_i M}{\sqrt{1 - a_i^2}}\right) \tag{21.1}$$

where $Q_i(T \mid M)$ is the probability of the ith entity defaulting by time T conditional on the value of the factor, M. Denote the probability of more than k defaults by time T as $P(k, T)$ and the corresponding probability conditional on the value of M as $P(k, T \mid M)$. When we fix the value of M, the default probabilities are independent. This facilitates the calculation of $P(k, T \mid M)$.

In the standard market model it is assumed that the time-to-default distribution is the same for all reference entities in the portfolio and that the copula correlation is the same for all pairs of reference entities.[7] This means that $Q_i(T \mid M) = Q(T \mid M)$ is the same for all i and equation (20.8) can be used instead of equation (20.7), so that

$$Q(T \mid M) = N\left(\frac{N^{-1}[Q(T)] - \sqrt{\rho}\, M}{\sqrt{1 - \rho}}\right) \tag{21.2}$$

[7] For a discussion of the more general model where these assumptions are not made, see J. C. Hull and A. White, "Vauation of a CDO and nth-to-Default Swap without Monte Carlo Simulation," *Journal of Derivatives*, 12, 2 (Winter 2004), 823.

Business Snapshot 21.3 Correlation Smiles

Credit derivatives dealers imply default correlations from the spreads on tranches. If the implied correlations were the same for all tranches, we could deduce that market prices are consistent with the one-factor Gaussian copula model for time to default. In practice, we find that the implied correlations for the most junior (equity) and most senior tranches are higher than those for the intermediate tranches. For example, in Table 21.6, the implied correlations for the five CDX IG NA tranches (starting with the equity tranche) are 21.0%, 4.2%, 17.7%, 19.0%, and 27.4%. Similarly the implied correlations for the corresponding iTraxx Europe tranches are 20.4%, 5.5%, 16.1%, 23.3%, and 31.1%.

The existence of a volatility smile in options markets indicates that the Black–Scholes model (although widely used) does not reflect the beliefs of market participants (see Chapter 16). In the same way, the existence of a correlation smile in CDO markets indicates that the one-factor Gaussian copula model (although widely used) does not reflect the beliefs of market participants.

From the properties of the binomial distribution,

$$P(k, T \mid M) = \frac{N!}{(N-k)!\,k!} Q(T \mid M)^k [1 - Q(T \mid M)]^{N-k}$$

The probability that the nth default will happen between times T_1 and T_2 conditional on M is $P(n, T_2 \mid M) - P(n, T_1 \mid M)$. This gives the probability distribution for the time of the nth default conditional on M. The factor M has a standard normal probability distribution. By integrating over the distribution for M we obtain the unconditional probability distribution for the time of the nth default.[8] With this distribution in hand, we can value an nth-to-default CDS in exactly the way as a regular CDS. To value the tranche of a CDO, we calculate expected payoffs and payments on the tranche conditional on M and then integrate over M.

Derivatives dealers calculate the implied copula correlation, ρ, in equation (21.2) from the spreads quoted in the market for nth-to-default CDSs and tranches of CDOs and tend to quote these rather than the spreads themselves. This is similar to the practice in options markets of quoting Black–Scholes implied volatilities rather than dollar prices. As discussed in Business Snapshot 21.3, there is a correlation smile phenomenon in CDO markets similar to the volatility smile phenomenon in options markets.

21.9 CONVERTIBLE BONDS

Convertible bonds are bonds issued by a company where the holder has the option to exchange the bonds for the company's stock at certain times in the future. The *conversion ratio* is the number of shares of stock obtained for one bond (this can be a function of time). The bonds are almost always callable (i.e., the issuer has the right to buy them back at certain times at a predetermined prices). The holder always has the right to convert the

[8] Integration over M can be accomplished in a fast and efficient way using a procedure known as Gaussian quadrature.

bond once it has been called. The call feature is therefore usually a way of forcing conversion earlier than the holder would otherwise choose. Sometimes the holder's call option is conditional on the price of the company's stock being above a certain level.

Credit risk plays an important role in the valuation of convertibles. If we ignore credit risk, we will get poor prices because the coupons and principal payments on the bond will be overvalued.

Ingersoll provides a way of valuing convertibles using a model similar to Merton's (1974) model discussed in Section 20.6.[9] He assumes geometric Brownian motion for the issuer's total assets and models the company's equity, its convertible debt, and its other debt as claims contingent on the value of the assets. Credit risk is taken into account because the debt holders get repaid in full only if the value of the assets exceeds the amount owing to them.

A simpler model that is widely used in practice involves modeling the issuer's stock price. It is assumed that the stock follows geometric Brownian motion except that there is a probability $\lambda\,\Delta t$ that there will be a default in each short period of time Δt. In the event of a default the stock price falls to zero and there is a recovery on the bond. The variable λ is the risk-neutral default intensity introduced in Section 20.2.

We can represent the stock price process by varying the usual binomial tree so that at each node there is:

1. A probability p_u of a percentage up movement of size u over the next time period of length Δt
2. A probability p_d of a percentage down movement of size d over the next time period of length Δt
3. A probability $\lambda\,\Delta t$, or more accurately $1 - e^{-\lambda\Delta t}$, that there will be a default with the stock price moving to zero over the next time period of length Δt

Parameter values, chosen to match the first two moments of the stock price distribution, are:

$$p_u = \frac{a - de^{-\lambda\Delta t}}{u-d}, \qquad p_d = \frac{ue^{-\lambda\Delta t} - a}{u-d}, \qquad u = e^{\sqrt{(\sigma^2-\lambda)\Delta t}}, \qquad d = \frac{1}{u}$$

where $a = e^{(r-q)\Delta t}$, r is the risk-free rate, and q is the dividend yield on the stock.

The life of the tree is set equal to the life of the convertible bond. The value of the convertible at the final nodes of the tree is calculated based on any conversion options that the holder has at that time. We then roll back through the tree. At nodes where the terms of the instrument allow conversion we test whether conversion is optimal. We also test whether the position of the issuer can be improved by calling the bonds. If so, we assume that the bonds are called and retest whether conversion is optimal. This is equivalent to setting the value at a node equal to

$$\max[\min(Q_1, Q_2), Q_3]$$

where Q_1 is the value given by the rollback (assuming that the bond is neither converted nor called at the node), Q_2 is the call price, and Q_3 is the value if conversion takes place.

[9] See J.E. Ingersoll, "A Contingent Claims Valuation of Convertible Securities," *Journal of Financial Economics*, 4, (May 1977), 289–322.

Example 21.1

Consider a 9-month zero-coupon bond issued by company XYZ with a face value of \$100. Suppose that it can be exchanged for two shares of company XYZ's stock at any time during the 9 months. Assume also that it is callable for \$113 at any time. The initial stock price is \$50, its volatility is 30% per annum, and there are no dividends. The default intensity λ is 1% per year, and all risk-free rates for all maturities are 5%. We assume that in the event of a default the bond is worth \$40 (i.e., the recovery rate as it is usually defined is 40%).

Figure 21.4 shows the stock price tree that can be used to value the convertible when there are three time steps ($\Delta t = 0.25$). The upper number at each node is the stock price; the lower number is the price of the convertible bond. The tree parameters are:

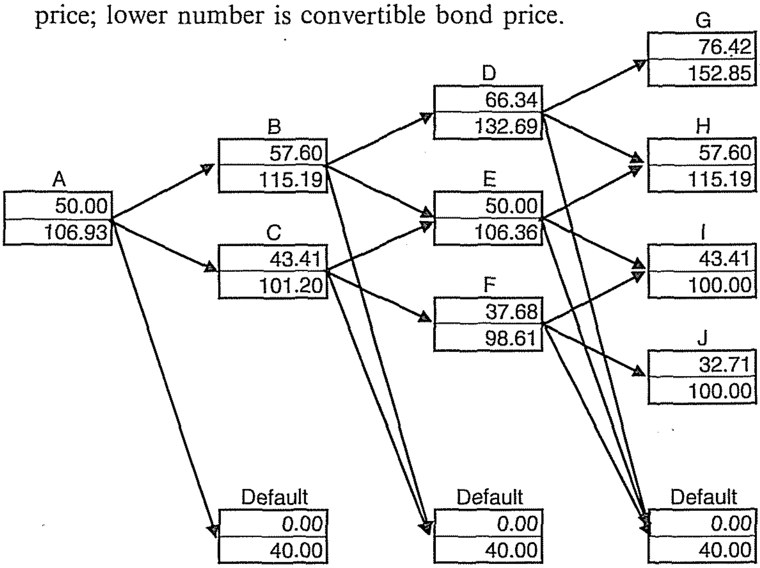
$$u = e^{\sqrt{(0.09-0.01)\times 0.25}} = 1.1519, \quad d = 1/u = 0.8681$$
$$a = e^{0.05\times 0.25} = 1.0126, \quad p_u = 0.5167, \quad p_d = 0.4808$$

The probability of a default (i.e., of moving to the lowest nodes on the tree is $1 - e^{-0.01\times 0.25} = 0.002497$. At the three default nodes the stock price is zero and the bond price is 40.

We first consider the final nodes. At nodes G and H the bond should be converted and is worth twice the stock price. At nodes I and J the bond should not be converted and is worth 100.

We then move back through the tree calculating the value at earlier nodes. Consider for example node E. The value if the bond is converted is $2 \times 50 = \$100$. If it is not converted, then there is (a) a probability 0.5167 that it will move to node H, where the bond is worth 115.19, (b) a 0.4808 probability that it will move down to node I, where the bond is worth 100, and (c) a 0.002497

Figure 21.4 Tree for valuing convertible. Upper number at each node is stock price; lower number is convertible bond price.



probability that it will default and be worth 40. The value of the bond if it is not converted is therefore

$$(0.5167 \times 115.19 + 0.4808 \times 100 + 0.002497 \times 40) \times e^{-0.05\times0.25} = 106.36$$

This is more than the value of 100 that it would have if converted. We deduce that it is not worth converting the bond at node F. Finally, we note that the bond issuer would not call the bond at node F because this would be offering 113 for a bond worth 106.36.

As another example consider node B. The value of the bond if it is converted is $2 \times 57.596 = 115.19$. If it is not converted a similar calculation to that just given for node E gives its value as 118.31. The convertible bond holder will therefore choose not to convert. However, at this stage the bond issuer will call the bond for 113 and the bond holder will then decide that converting is better than being called. The value of the bond at node B is therefore 115.19. A similar argument is used to arrive at the value at node D. With no conversion the value is 132.79. However, the bond is called, forcing conversion and reducing the value at the node to 132.69.

The value of the convertible is its value at the initial node A, or 106.93.

When interest is paid on the debt, it must be taken into account. At each node when valuing the bond on the assumption that it is not converted, we include the present value of any interest payable on the bond in the next time step. The risk-neutral default intensity λ can be estimated from either bond prices or credit default swap spreads. In a more general implementation, λ, σ, and r are functions of time. This can be handled using a trinomial rather than a binomial tree (see Section 17.4).

One disadvantage of the model we have presented is that the probability of default is independent of the stock price. This has led some researchers to suggest an implicit finite difference method implementation of the model where the default intensity λ is a function of the stock price as well as time.[10]

SUMMARY

Credit derivatives enable banks and other financial institutions to actively manage their credit risks. They can be used to transfer credit risk from one company to another and to diversify credit risk by swapping one type of exposure for another.

The most common credit derivative is a credit default swap. This is a contract where one company buys insurance against another company defaulting on its obligations. The payoff is usually the difference between the face value of a bond issued by the second company and its value immediately after a default. Credit default swaps can be analyzed by calculating the present value of the expected payments and the present value of the expected payoff.

A forward credit default swap is an obligation to enter into a particular credit default swap on a particular date. A credit default swap option is the right to enter into a

[10] See, e.g., L. Andersen and D. Buffum, "Calibration and Implementation of Convertible Bond Models," *Journal of Computational Finance*, 7, 1 (Winter 2003/04), 1–34. These authors suggest assuming that the default intensity is inversely proportional to S^α, where S is the stock price and α is a positive constant.

particular credit default swap on a particular date. Both instruments cease to exist if the reference entity defaults before the date.

A total return swap is an instrument where the total return on a portfolio of credit-sensitive assets is exchanged for LIBOR plus a spread. Total return swaps are often used as financing vehicles. A company wanting to purchase a portfolio of bonds will approach a financial institution, who will buy the bonds on its behalf. The financial institution will then enter into a total return swap where it pays the return on the bonds to the company and receives LIBOR. The advantage of this type of arrangement is that the financial institution reduces its exposure to defaults by the company.

An nth-to-default CDS is defined as a CDS that pays off when the nth default occurs in a portfolio of companies. In a collateralized debt obligation a number of different securities are created from a portfolio of corporate bonds or commercial loans. There are rules for determining how credit losses are allocated to the securities. The result of the rules is that securities with both very high and very low credit ratings are created from the portfolio. A synthetic collateralized debt obligation creates a similar set of securities from credit default swaps. The standard market model for pricing both an nth-to-default CDS and tranches of a CDO is the one-factor Gaussian copula model for time to default.

Convertible bonds are bonds that can be converted to the issuer's equity according to prespecified terms. Credit risk has to be considered in the valuation of a convertible bond. This is because, if the bond is not converted, the promised payments on the bond are subject to credit risk. One popular procedure for valuing convertible bonds is to model equity prices on the assumption that there is a certain default intensity. In the event of a default the equity price drops to zero and the debt drops to a price reflecting its recovery rate.

FURTHER READING

Andersen, L., J. Sidenius, and S. Basu, "All Your Hedges in One Basket," *Risk*, November 2003.

Das S. *Credit Derivatives: Trading & Management of Credit & Default Risk*. Singapore: Wiley, 1998.

Hull, J. C., and A. White, "Valuation of a CDO and nth-to-Default Swap without Monte Carlo Simulation," *Journal of Derivatives*, 12, 2 (Winter 2004), 8–23.

Tavakoli, J. M., *Credit Derivatives: A Guide to Instruments and Applications*. New York: Wiley, 1998.

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Questions and Problems (Answers in Solutions Manual)

21.1. Explain the difference between a regular credit default swap and a binary credit default swap.

21.2. A credit default swap requires a semiannual payment at the rate of 60 basis points per year. The principal is \$300 million and the credit default swap is settled in cash. A default occurs after 4 years and 2 months, and the calculation agent estimates that the price of the cheapest deliverable bond is 40% of its face value shortly after the default. List the cash flows and their timing for the seller of the credit default swap.

21.3. Explain the two ways a credit default swap can be settled.

21.4. Explain how a CDO and a synthetic CDO are created.

21.5. Explain what a first-to-default credit default swap is. Does its value increase or decrease as the default correlation between the companies in the basket increases? Explain.

21.6. Explain the difference between risk-neutral and real-world probabilities.

21.7. Explain why a total return swap can be useful as a financing tool.

21.8. Suppose that the risk-free zero curve is flat at 7% per annum with continuous compounding and that defaults can occur halfway through each year in a new 5-year credit default swap. Suppose that the recovery rate is 30% and the default probabilities each year conditional on no earlier default is 3%. Estimate the credit default swap spread. Assume payments are made annually.

21.9. What is the value of the swap in Problem 21.8 per dollar of notional principal to the protection buyer if the credit default swap spread is 150 basis points?

21.10. What is the credit default swap spread in Problem 21.8 if it is a binary CDS?

21.11. How does a 5-year nth-to-default credit default swap work? Consider a basket of 100 reference entities where each reference entity has a probability of defaulting in each year of 1%. As the default correlation between the reference entities increases what would you expect to happen to the value of the swap when (a) $n = 1$ and (b) $n = 25$. Explain your answer.

21.12. How is the recovery rate of a bond usually defined?

21.13. Show that the spread for a new plain vanilla CDS should be $(1 - R)$ times the spread for a similar new binary CDS, where R is the recovery rate.

21.14. Verify that if the CDS spread for the example in Tables 21.1 to 21.4 is 100 basis points and the probability of default in a year (conditional on no earlier default) must be 1.61%. How does the probability of default change when the recovery rate is 20% instead of 40%? Verify that your answer is consistent with the implied probability of default being approximately proportional to $1/(1 - R)$, where R is the recovery rate.

21.15. A company enters into a total return swap where it receives the return on a corporate bond paying a coupon of 5% and pays LIBOR. Explain the difference between this and a regular swap where 5% is exchanged for LIBOR.

21.16. Explain how forward contracts and options on credit default swaps are structured.

21.17. "The position of a buyer of a credit default swap is similar to the position of someone who is long a risk-free bond and short a corporate bond." Explain this statement.

21.18. Why is there a potential asymmetric information problem in credit default swaps?

21.19. Does valuing a CDS using real-world default probabilities rather than risk-neutral default probabilities overstate or understate its value? Explain your answer.

21.20. What is the difference between a total return swap and an asset swap?

21.21. Consider an 18-month zero-coupon bond with a face value of \$100 that can be converted into five shares of the company's stock at any time during its life. Suppose that the current share price is \$20, no dividends are paid on the stock, the risk-free rate for all maturities is 6% per annum with continuous compounding, and the share price volatility is 25% per annum. Assume that the default intensity is 3% per year and the recovery rate is 35%. The bond is callable at \$110. Use a three-time-step tree to calculate the value of the bond. What is the value of the conversion option (net of the issuer's call option)?

21.22. Suppose that in a one-factor Gaussian copula model the 5-year probability of default for each of 125 names is 3% and the pairwise copula correlation is 0.2. Calculate, for factor values of -2, -1, 0, 1, and 2:
(a) The default probability conditional on the factor value
(b) The probability of more than 10 defaults conditional on the factor value when the factor value

21.23. What is a CDO squared? How about a CDO cubed?

Assignment Questions

21.24. Suppose that the risk-free zero curve is flat at 6% per annum with continuous compounding and that defaults can occur at times 0.25 years, 0.75 years, 1.25 years, and 1.75 years in a 2-year plain vanilla credit default swap with semiannual payments. Suppose that the recovery rate is 20% and the unconditional probabilities of default (as seen at time zero) are 1% at times 0.25 years and 0.75 years, and 1.5% at times 1.25 years and 1.75 years. What is the credit default swap spread? What would the credit default spread be if the instrument were a binary credit default swap?

21.25. Assume that the default probability for a company in a year, conditional on no earlier defaults is λ and the recovery rate is R. The risk-free interest rate is 5% per annum. Default always occurs halfway through a year. The spread for a 5-year plain vanilla CDS where payments are made annually is 120 basis points and the spread for a 5-year binary CDS where payments are made annually is 160 basis points. Estimate R and λ.

21.26. Explain how you would expect the yields offered on the various tranches in a CDO to change when the correlation between the bonds in the portfolio increases.

21.27. Suppose that:
(a) The yield on a 5-year risk-free bond is 7%.
(b) The yield on a 5-year corporate bond issued by company X is 9.5%.
(c) A 5-year credit default swap providing insurance against company X defaulting costs 150 basis points per year.

What arbitrage opportunity is there in this situation? What arbitrage opportunity would there be if the credit default spread were 300 basis points instead of 150 basis points? Give two reasons why arbitrage opportunities such as those you identify are less than perfect.

21.28. A 3-year convertible bond with a face value of \$100 has been issued by company ABC. It pays a coupon of \$5 at the end of each year. It can be converted into ABC's equity at the end of the first year or at the end of the second year. At the end of the first year, it can be exchanged for 3.6 shares immediately after the coupon date. At the end of the second year, it can be exchanged for 3.5 shares immediately after the coupon date. The current stock price is \$25 and the stock price volatility is 25%. No dividends are paid on the stock. The risk-free interest rate is 5% with continuous compounding. The yield on bonds issued by ABC is 7% with continuous compounding and the recovery rate is 30%.
(a) Use a three-step tree to calculate the value of the bond.
(b) How much is the conversion option worth?

(c) What difference does it make to the value of the bond and the value of the conversion option if the bond is callable any time within the first 2 years for \$115?

(d) Explain how your analysis would change if there were a dividend payment of \$1 on the equity at the 6-month, 18-month, and 30-month points. Detailed calculations are not required.

(*Hint*: Use equation (20.2) to estimate the default intensity.)