A Two-Level Optimization Framework for Vector-Borne Disease Transmission with Beliefs and Information Quality

1 Introduction

Dudley et al. (2016) model malaria transmission using a deterministic SIR model from the perspective of a central planner via integer linear programming; individuals are passive - i.e. there is no behavioral component. Fenichel (2011) incorporates endogenous behavioral responses into epidemiological models by making the transmission rate dependent on individuals' choices, but assumes perfect information about prevalence across all agents and does not consider vector-borne diseases.

I try to combine the strengths of both frameworks while introducing a new *information channel*. In a simplified and tractable formulation, my model features:

- a central planner choosing intervention intensity x and information quality Q;
- a representative individual choosing myopic protection effort $e(t)$;
- deterministic belief dynamics $b(t)$ driven by information quality;
- a behavior-modified SIR model of vector-borne disease transmission;
- a closed feedback loop linking information, beliefs, behavior, and transmission.

This simplified system preserves the conceptual contributions of the full model, while remaining numerically tractable for simulation.

2 Central Planner Decisions

The planner chooses:

$$(Q, x) \in [0, 1]^2,$$

where:

- Q is information quality (e.g., campaigns, communication, monitoring);
- x is intervention coverage (e.g., LLINs, IRS).

The planner does *not* solve an optimization problem in this simplified version; instead, (Q, x) are treated as policy scenarios and the equilibrium behavior–disease outcomes are simulated.

3 Individual Behavior

A representative individual chooses protection effort $e(t) \in [0,1]$ myopically by solving:

$$e(t) = \arg\max_{e \in [0,1]} \left\{ -c_e e - \frac{1}{2} e^2 - \rho\kappa\, p_{\text{inf}}(t) \right\},$$

where:

- c_e is the marginal cost of protection effort,
- κ is the utility loss from infection,
- ρ is a risk preference parameter,
- $p_{\text{inf}}(t)$ is the perceived infection probability.

The perceived infection probability is:

$$p_{\text{inf}}(t) = 1 - \exp\left(-\lambda(t)\, b(t)\right),$$

where $b(t)$ is a subjective belief about risk.

The first-order condition yields:

$$c_e + e(t) + \rho\kappa \frac{\partial p_{\text{inf}}}{\partial e} = 0, \tag{1}$$

which (given p_{inf}) can be solved numerically for $e(t)$ at each time t.

4 Belief Dynamics

Beliefs evolve according to a simple deterministic learning rule:

$$\dot{b}(t) = \alpha(Q) \left(\frac{I(t)}{N} - b(t) \right), \tag{2}$$

where:

- $b(t)$ is perceived infection prevalence,
- $\alpha(Q)$ is a learning rate increasing in information quality ($\alpha'(Q) > 0$),
- $I(t)/N$ is true prevalence.

This captures the idea that high-quality information speeds convergence to accurate beliefs.

5 Disease Dynamics

We use a behavior-modified SIR model with population size $N = S + I + R$:

$$\dot{S}(t) = -\lambda(t) S(t), \tag{3}$$
$$\dot{I}(t) = \lambda(t) S(t) - \gamma I(t), \tag{4}$$
$$\dot{R}(t) = \gamma I(t). \tag{5}$$

The force of infection incorporates both public interventions and private behavior:

$$\lambda(t) = \beta_0 \left(1 - \theta_p e(t) - \theta_x x\right) \frac{I(t)}{N}, \tag{6}$$

where:

- β_0 is the baseline transmission rate,
- θ_p is private protection effectiveness,
- θ_x is public intervention effectiveness,
- $e(t)$ is endogenously chosen behavior.

6 Model Summary

The full simplified equilibrium system consists of:

$$(Q, x) \to b(t) \to e(t) \to \lambda(t) \to I(t) \to b(t).$$

Explicitly, the system is:

$$e(t) = \arg\max_{e\in[0,1]} \left\{-c_e e - \tfrac{1}{2}e^2 - \rho\kappa \left[1 - \exp\left(-\lambda(t)b(t)\right)\right]\right\}, \tag{7}$$

$$\dot{b}(t) = \alpha(Q)\left(\frac{I(t)}{N} - b(t)\right), \tag{8}$$

$$\lambda(t) = \beta_0 \left(1 - \theta_p e(t) - \theta_x x\right) \frac{I(t)}{N}, \tag{9}$$

$$\dot{S}(t) = -\lambda(t)S(t), \tag{10}$$

$$\dot{I}(t) = \lambda(t)S(t) - \gamma I(t), \tag{11}$$

$$\dot{R}(t) = \gamma I(t). \tag{12}$$

7 Variable Definitions

Variable	Description
Q	Information quality chosen by planner
x	Intervention coverage chosen by planner
$b(t)$	Individual belief about prevalence
$e(t)$	Protection effort chosen by individual
$\alpha(Q)$	Learning rate increasing in Q
$S(t), I(t), R(t)$	Susceptible, infected, recovered populations
N	Total population size
$\lambda(t)$	Force of infection
β_0	Baseline transmission rate
θ_p	Effectiveness of private protection
θ_x	Effectiveness of public interventions
c_e	Cost of protection effort
ρ, κ	Risk aversion and infection cost
γ	Recovery rate