A Stable Voronoi-based Algorithm for Medial Axis Extraction through Labeling Sample Points

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Medial Axis (MA)

The medial axis (also called skeleton) is the closure of the set of points in an object $\mathcal{O}$ that have at least two closest points on the object's boundary $\partial\mathcal{O}$

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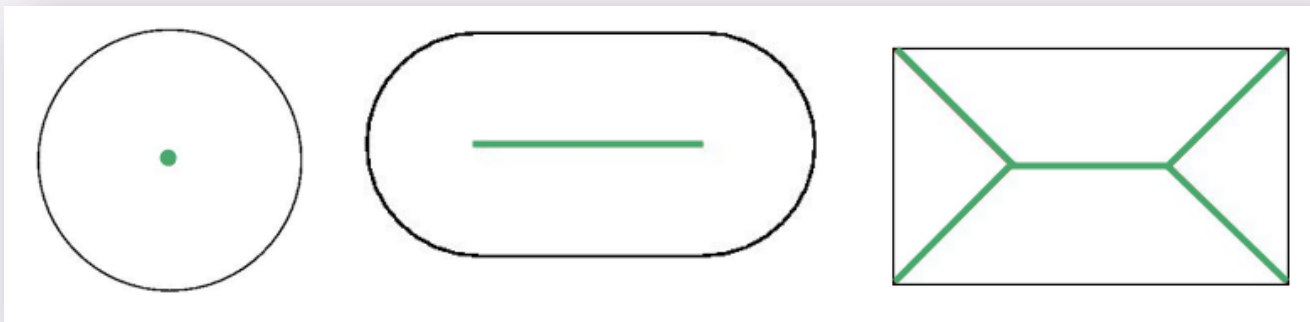
Medial Axis (MA)

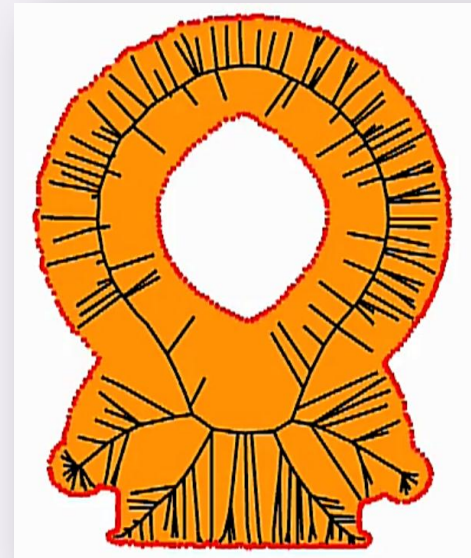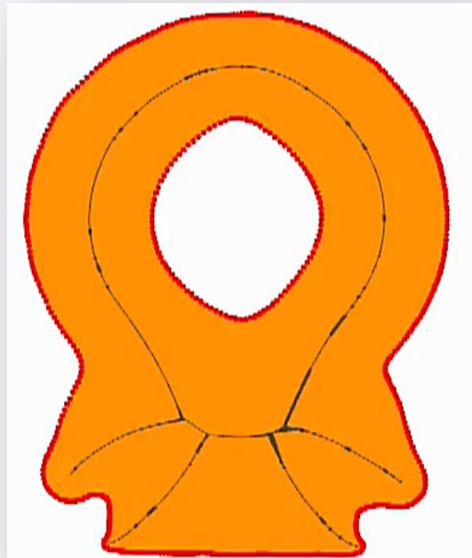**Properties:**

– There is a unique MA for a given shape.

– The MA is topologically equivalent to its shape.

– There is a one-to-one relation between a shape and its MA: a shape can be reconstructed from its MA
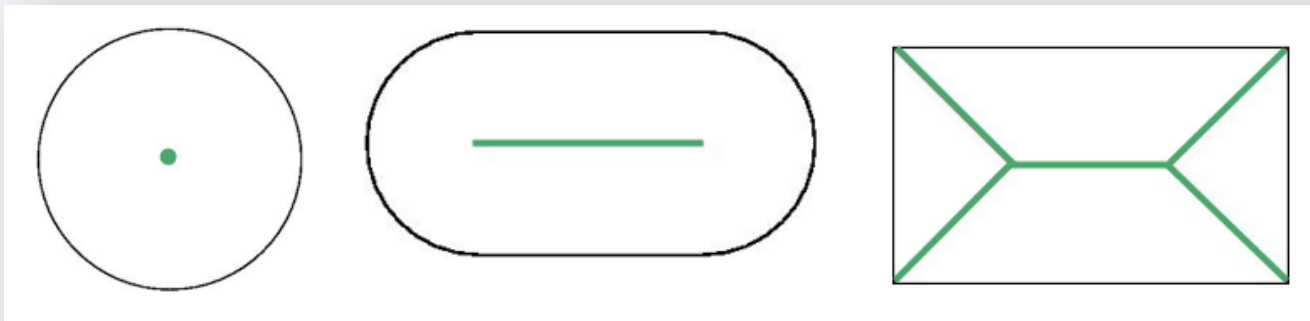
Instability of Medial Axis

- The MA is very sensitive to small changes of the boundary, which produces many irrelevant branches in the MA.

- Two very similar shapes can have significantly different MAs:

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Instability of Medial Axis

- **Solution:** Filtering extraneous branches
  - **Simplification:** Smooth the boundary before computing the MA (pre-processing).
  - **Pruning:** Remove the irrelevant branches of the extracted MA (post-processing).

- **Disadvantage:** May alter the topological or geometrical structure of the MA

Instability of Medial Axis

Our approach:

A modification to a Voronoi-based MA extraction algorithm (i.e., one-step crust and skeleton extraction) through labeling the sample points

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One-Step Crust and Skeleton Extraction

- Proposed by Gold and Snoeyink (1999)

- Extracts both the boundary (crust) and the MA (skeleton), simultaneously

- Every Voronoi/Delaunay edge is either part of the crust (Delaunay) or the skeleton (Voronoi), which is determined by *inCircle* test
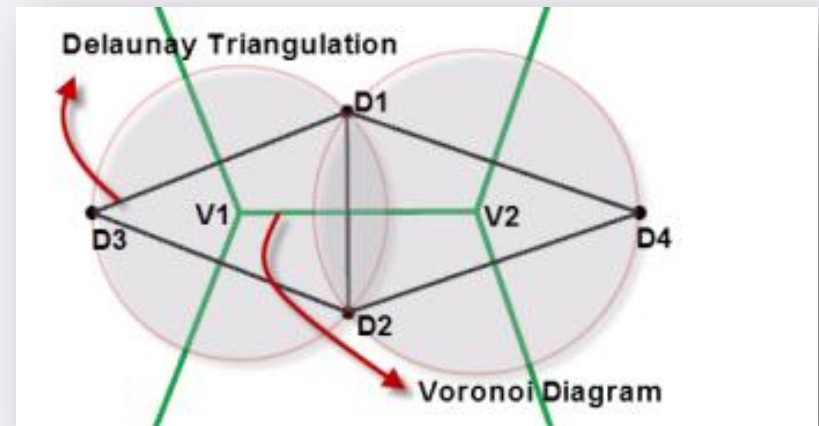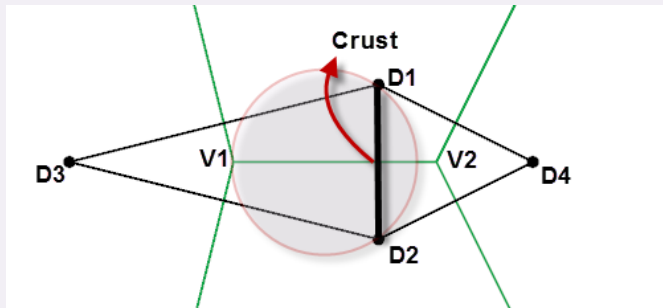

$$InCircle\ (D_1, D_2, V_1, V_2) = \begin{bmatrix} x_{D1} & y_{D1} & x_{D1}^2 + y_{D1}^2 & 1 \\ x_{D2} & y_{D2} & x_{D2}^2 + y_{D2}^2 & 1 \\ x_{V1} & y_{V1} & x_{V1}^2 + y_{V1}^2 & 1 \\ x_{V2} & y_{V2} & x_{V2}^2 + y_{V2}^2 & 1 \end{bmatrix}$$
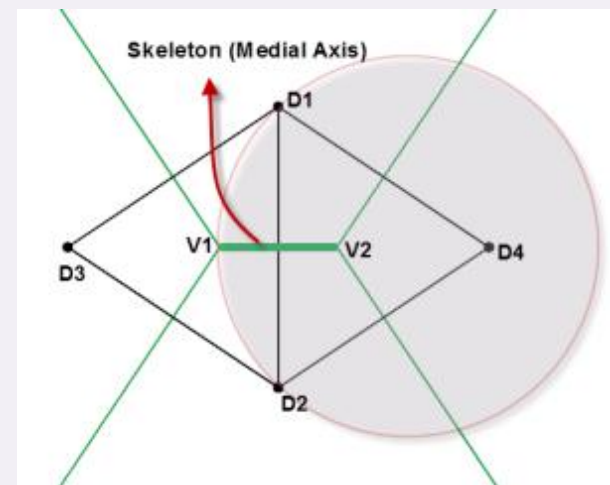
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One-Step Crust and Skeleton Extraction

- The *InCircle*($D1$, $D2$, $V1$, $V2$) determines the position of $V2$ respect to the circle passes through $D1$, $D2$ and $V1$:

  a) $V2$ is outside the circle → $D1D2$ belongs to the crust

  b) $V2$ is inside the circle → $V1V2$ belongs to the skeleton

(a)

(b)

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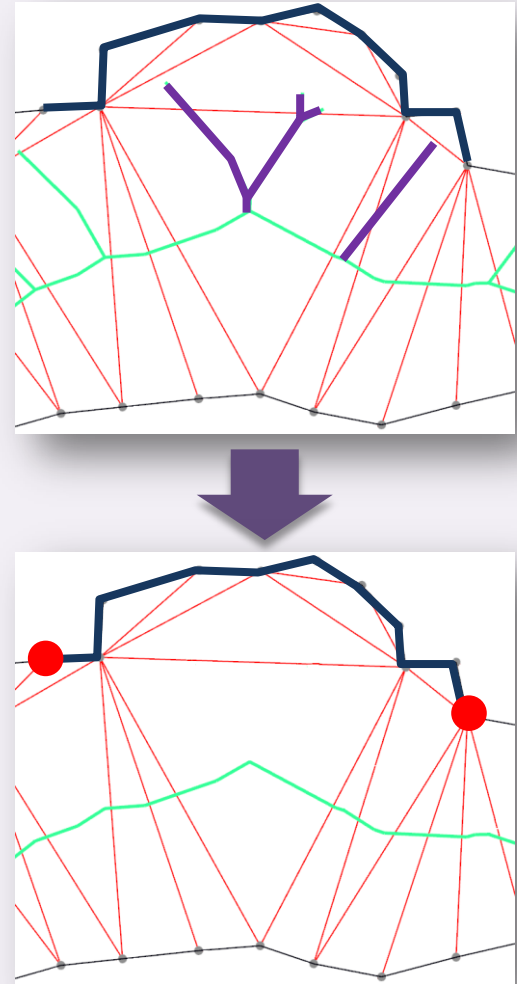*Department of Geomatics*
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Our Approach

- **Observation:** extraneous edges are the Voronoi edges created between the sample points that lie on the same segment of the curve

- **Solution:** Labeling the sample points to automatically avoid appearing such edges in the medial axis
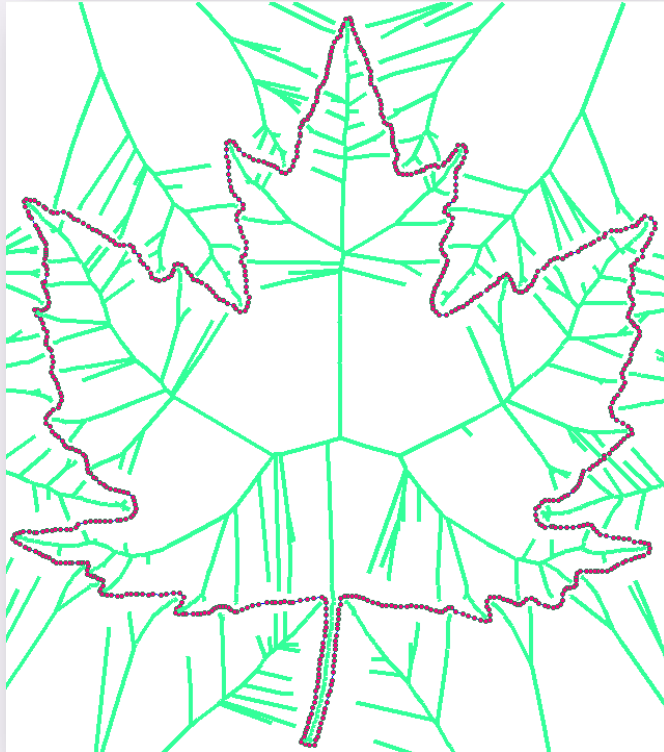
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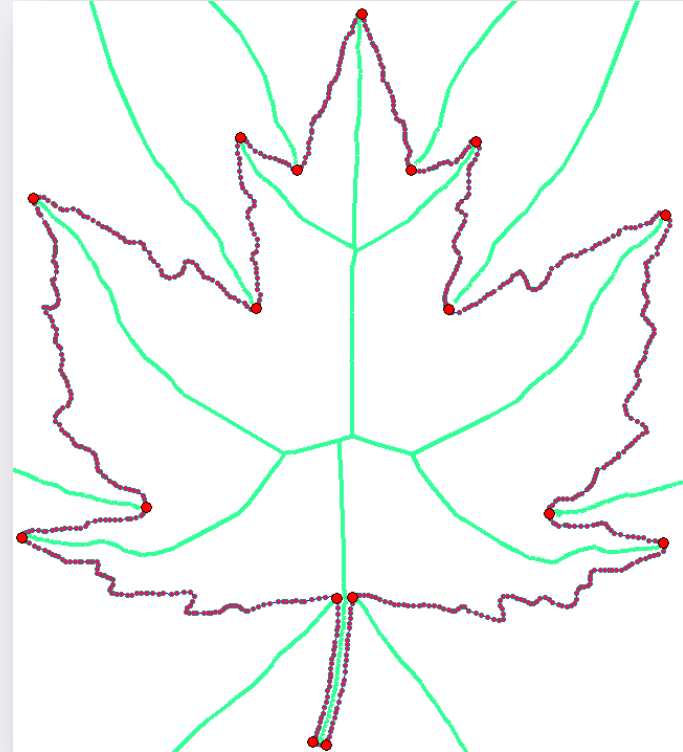*University of Tehran*

Our Approach

- Modifications applied to one-step algorithm:
  - **Labeling the sample points:** Each segment of the shape is assigned a unique label; and all of its sample points are assigned the same label.
  - **Extracting the crust and MA:** each Delaunay edge passes the *InCircle* test
    - If *InCircle<0* **and the corresponding Delaunay vertices have the same labels**, that Delaunay edge is added to the crust.
    - If *InCircle>0* **and the corresponding Delaunay vertices have different labels**, its dual is added to the MA.
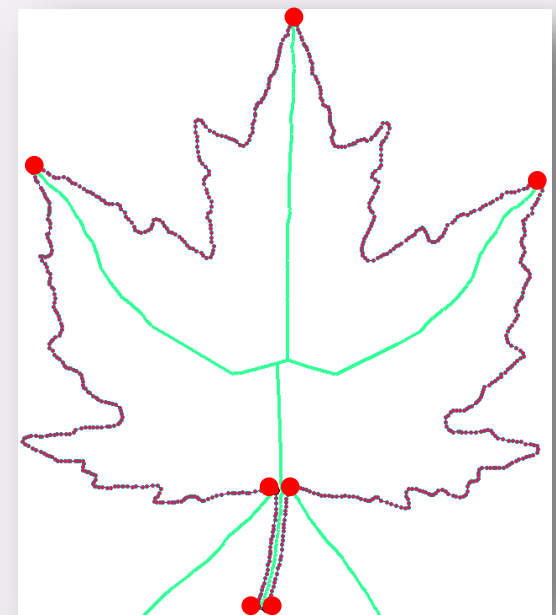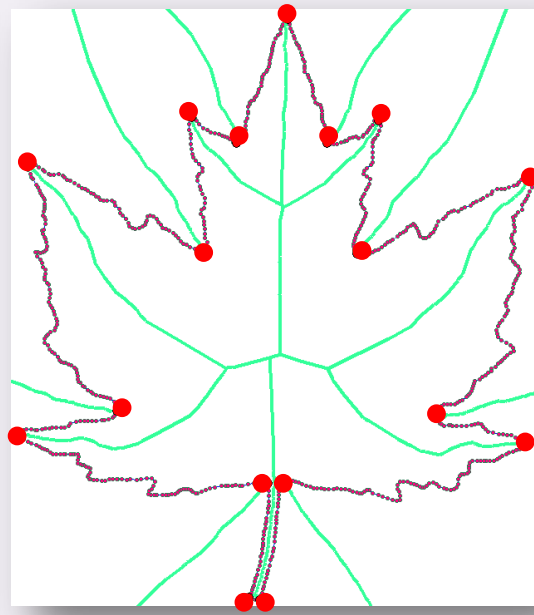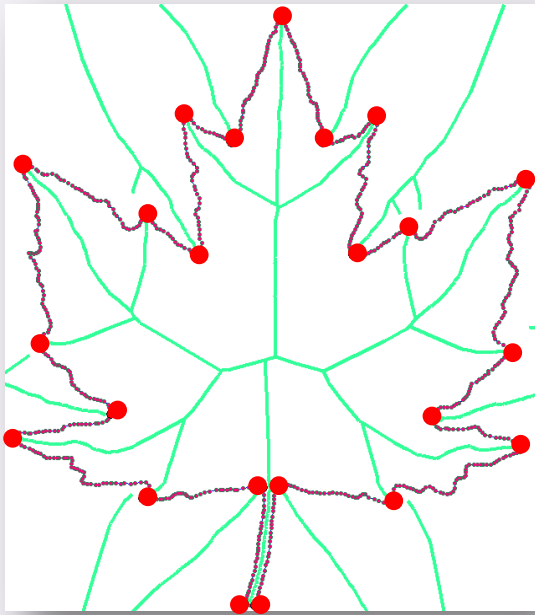
Results

One-step crust and skeleton algorithm
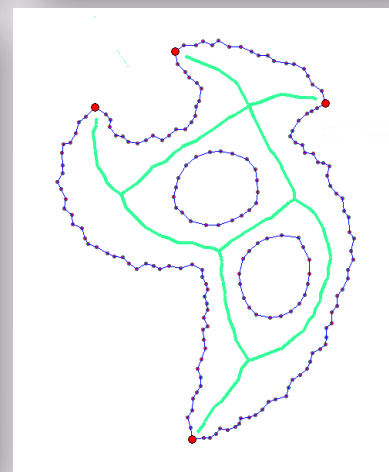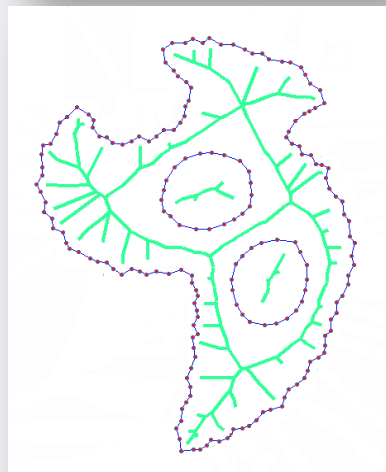

Our algorithm
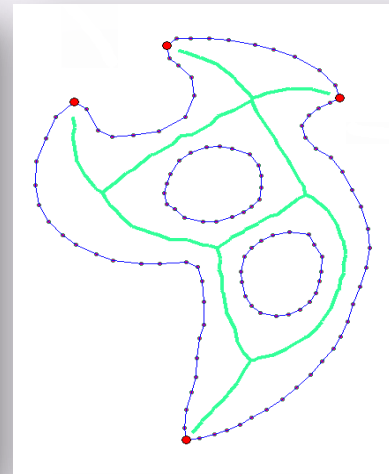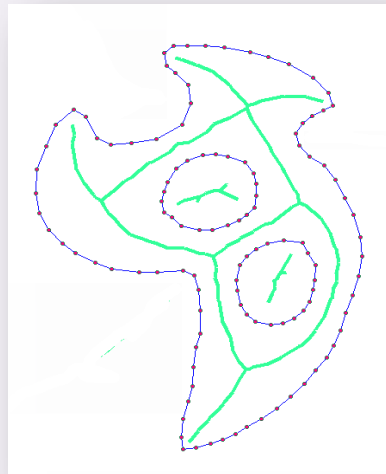
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Results

- Different segmentations lead to different MAs:

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Discussion

- **Stability**

Noise

One-step algorithm   Our algorithm

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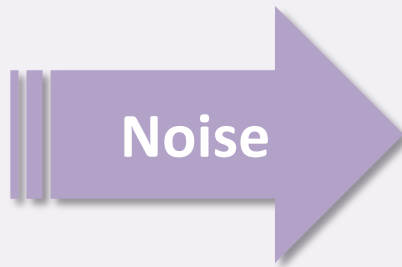
Discussion

- **Flexibility**

One-step algorithm

Our algorithm

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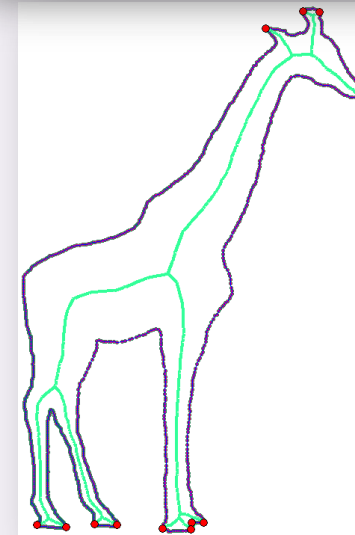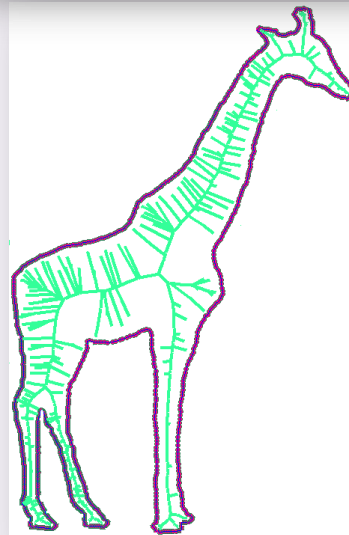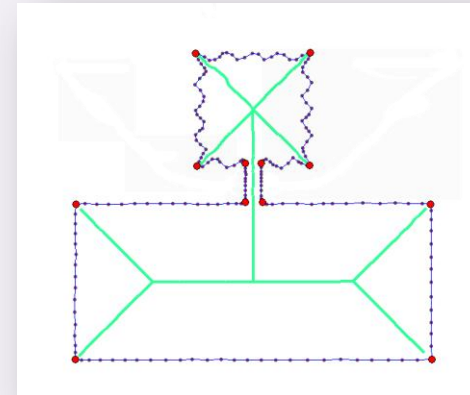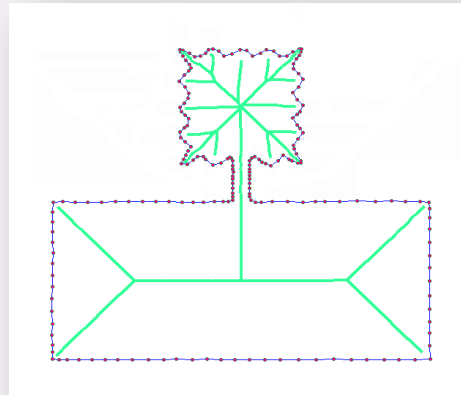
Discussion

- **Open curves**

One-step algorithm

Our algorithm

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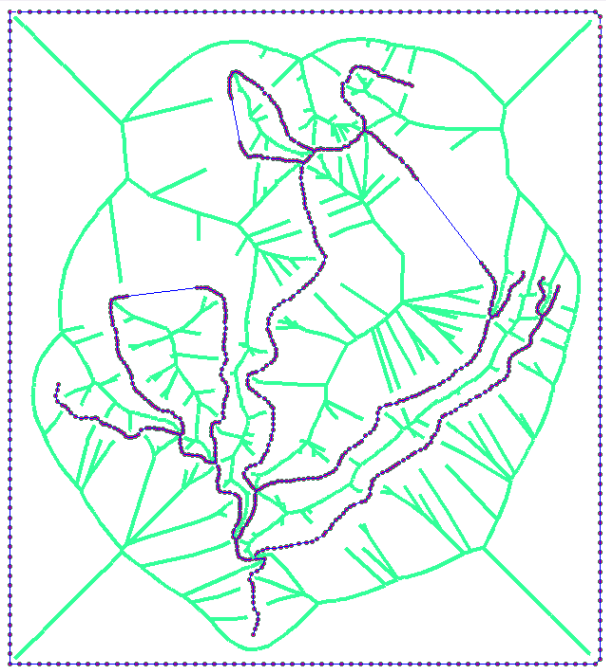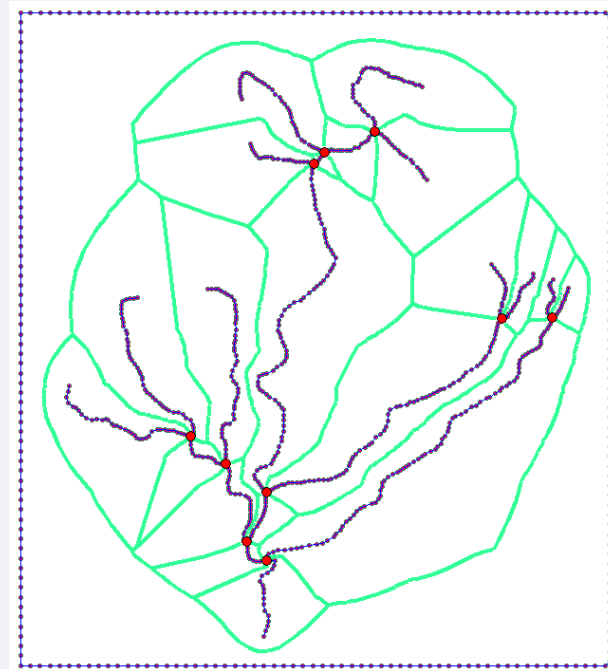
Conclusion and Future Work

- Improving the one-step crust and skeleton algorithm through labeling the sample points

- The solution is
  - Simple
  - Easy to implement
  - Robust to boundary perturbations
  - Able to handle sharp corners and open curves
  - Stable, even in the presence of significant noise and perturbations

- Extending the approach for surface reconstruction and 3D MA extraction

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Thank you for your attention

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