Cross-Scale Analysis of Sub-Pixel Variations in Digital Elevation Models

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Abstract: Terrain is generally modeled on a grid of pixels, assuming that elevation values are constant within any single pixel of a Digital Elevation Model (DEM) ('rigid pixel' paradigm). This paradigm can generate imprecise measurements, because it does not account for the slope and curvature of the terrain within each pixel, leading to precious information being lost. In order to improve the precision of interpolated points, this paper relaxes the rigid pixel assumption, allowing possible sub-pixel variations ('surface-adjusted' paradigm) to be used to interpolate elevation of arbitrary points given a regular grid. Tests based on interpolating elevation values for 5,000 georeferenced random points from a DEM using the rigid pixel paradigm and different interpolation methods (e.g., weighted average, linear, bilinear, bi-quadratic, bi-cubic, and best fitting polynomials) and different contiguity configurations (i.e., incorporating first and second order neighbors) are presented. Based on these tests, this paper examines the sensitivity of surface adjustment to a progression of spatial resolutions (i.e., 10, 30, 100, and 1000m DEMs), using sub-pixel variations that can be directly measured from 3m resolution LiDAR data.

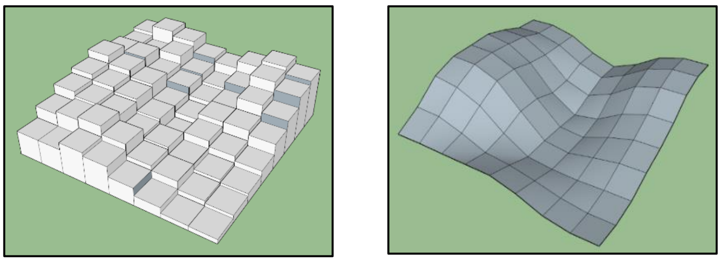
Keywords: Digital Elevation Models, Rigid Pixel Paradigm, Sub-pixel Variations, Surface-Adjusted Analysis

1. Introduction

Terrain models guide scientists and planners in multiple ways and have fundamental impacts on society, safety, and resource management. For example, digital terrain data is used by natural hazards scientists to model flooding inundation and debris flows (e.g., Griffin et al. 2015) and by hydrologists to model water flow direction and accumulation (e.g., Stanislawski et al. 2015). Ecologists use terrain models to delineate habitats and nesting territory or to predict the presence of certain species (e.g., Czarnecka and Chabudziński 2014). Avalanche risk is measured using solar insolation metrics based on slope and aspect (Jaboyedoff et al. 2012). Civil engineers plan construction of road and railroad networks on the basis of elevation and slope in conjunction with other factors (Zhao et al. 2005). This research contests a prevailing paradigm for terrain-based analysis, which forms the foundation for nearly all environmental models.

Digital terrain is currently modeled in a grid of pixels, assuming that elevation values are constant within any single pixel of a DEM (*'rigid pixel' paradigm*) (Figure 1-a). This paradigm generates imprecise measurements, because it does not account for the slope and curvature of the terrain within each pixel and precious information is lost. Projection of terrain features into a planar surface leads to some distortion in the geometry of terrain elements. In addition, because a horizontal plane is considered as the height value of each sample point, an abrupt height change (i.e., discontinuity) occurs between horizontal planes. Discrepancies (error magnitudes) are expected to vary with DEM resolution, terrain roughness and landscape conditions (precipitation regime) (Buttenfield et al. 2016). With advances in computer processing speed and increased data availability, the scientific community must address limitations of existing modeling methods to provide decision-makers the best possible information about critical environmental issues including biophysical and anthropogenic factors and interactions. This paper relaxes the assumption of the rigid pixel paradigm using sub-pixel variations (surface-adjusted paradigm) to interpolate elevation of an arbitrary point given a regular grid (or DEM) in order to improve the precision of interpolated points (Figure 1-b).

This *surface-adjusted* paradigm incorporates elevation, slope, aspect, and curvature of DEM pixels into distance computations. This paper in particular uses interpolation methods as one possible strategy for the incorporation of sub-pixel variations into these computations. That is, interpolation methods can reconstruct the 3d surface of each pixel using the information from adjacent pixels. For example, a bilinear polynomial is used to estimate a *surface* at each data point using the four closets pixel centroids. Thus, a series of contiguous bilinear surfaces generate the whole terrain surface. Higher-order polynomials also can be employed to construct the sub-surfaces, but due to the complexity and unpredictable undulations, they are not normally used (though these higher-order polynomials are also evaluated in this paper). Initial results indicate that for any *individual* pixel, the improvement in measurements can be relatively small, however, the *additive* effect across the study area can become quite significant.



(a) (b)

Fig 1. (a) The rigid pixel paradigm, and (b) the surface-adjusted paradigm

In the last decade, Light Detection and Ranging (LiDAR) has provided new opportunities for studying Earth surface processes, providing more precise measurements (Tarolli 2014). Although it is a fair assumption that elevation values are constant within any single pixel in High Resolution Topography, this data provides some other challenges; filtering needs to be done on LiDAR data as a preprocessing step to remove artefacts and extract bare earth points. The filtering process reduces the accuracy of the original data and might eliminate or distort significant features (Passalacqua et al. 2015). LiDAR data has become more affordable, but efficient processing of this data is still a challenge due to high data storage costs and long processing times. Furthermore, in most of GIS projects, various data layers need to be integrated. High Resolution ancillary data (e.g., vegetation, precipitation, soil, etc.) is not generally available, making it difficult to harmonize LiDAR data with other data sources. Finally and most importantly, the current availability of LiDAR data is far from comprehensive in developed nations, and non-existent in many rural and undeveloped regions. Given the emerging efforts of regional and global modeling of social and natural processes that use terrain, and recognizing the need for methods that benefit society in developed and less developed regions, automated data collection and processing technologies have advanced to the point where it becomes compelling to reconsider the state of the art in terrain analysis.

In the following sections, tests based on interpolating the elevation of 5,000 georeferenced random points from DEM using the rigid pixel paradigm and different interpolation methods (weighted average, linear, bi-linear, bi-quadratic, bi-cubic, and best fitting polynomials) and different contiguity configurations (incorporating first and second order neighbors) are presented. This research then examines the sensitivity of surface adjustments to a progression of spatial resolutions (10, 30, 100, and 1000 meter DEMs), using sub-pixel variations that can be directly measured from 3 meters resolution LiDAR data as validation. Additional tests compute Root Mean Square Error (RMSE) between DEM and LiDAR outputs to assess differences in the various methods and resolutions.

2. Dataset and Study Area

The study area was chosen based on the availability of 3 meter resolution DEMs to be used for validation. While other resolutions are available for the continental US, coverage at such a fine resolution is much more limited. The study area is centered on the coordinates of 35.798 degrees N and 81.473 degrees W, Its location at the southeast end of the Appalachian mountain range, provides a variety of elevations, ranging from 209 to 1602 meters. Its location where the Blue Ridge Mountains run out towards the coastal plains is a humid, hilly landscape, averaging 51 inches (129.5 cm) annual precipitation. The study area provides a mix of uninhabited land with smaller rural settlements as well.

DEM data will be tested at 3, 10, 30, 100, and 1000m resolutions. The 3m, 10m, and 30m resolutions were from the USGS National Elevation Dataset (NED) and were downloaded from the Geospatial Data Gateway[1]. The accuracy of NED varies spatially due to the diversity of the source DEMs. The overall absolute vertical accuracy of this dataset has been reported as the root mean square error (RMSE) of 2.44 meters (Gesch et al. 2014). The source for 100m and 1000m resolutions was SRTM[2]. The absolute vertical accuracy of this dataset has been reported as the root mean square error (RMSE) of 4.01 meters (Gesch et al. 2012). The DEMs are in NAD_1983_UTM_Zone_17N coordinate system.

3. Methods

Tests will interpolate the elevation of 5,000 georeferenced random points using different methods and contiguity configurations, across a progression of DEM resolutions. The tested methods all incorporate elevation, but differ in contextual information about surrounding pixels to gain a progression of surface adjustment methods. Several methods are compared for DEMs spanning a range of resolutions for the study area and validated against a 3 meter (m) LiDAR data benchmark (Figure 2). In this section, different combinations of first and second order neighbors used in interpolation techniques are discussed. Then different interpolating techniques used in this research are presented, compared, and the results are evaluated. Figure 3 shows the research methodology, which is discussed in more detail below.

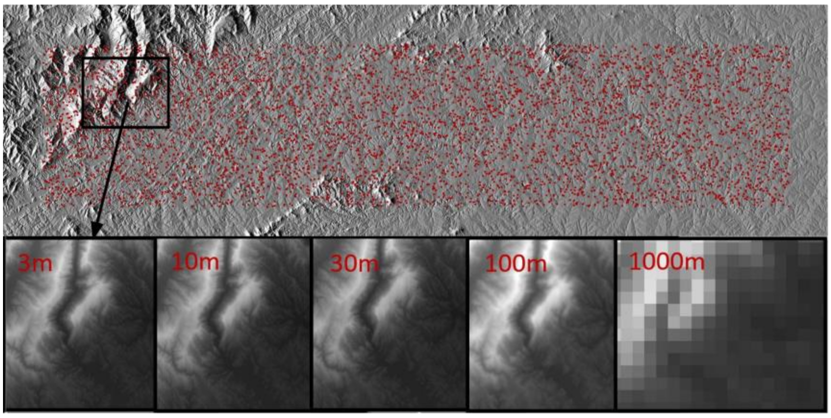


Fig 2. 3m LiDAR DEM with four other resolutions. Red points are 5000 random points generated within the study area.

[1] (https://gdg.sc.egov.usda.gov/)
[2] (http://dds.cr.usgs.gov/srtm/version2_1/)

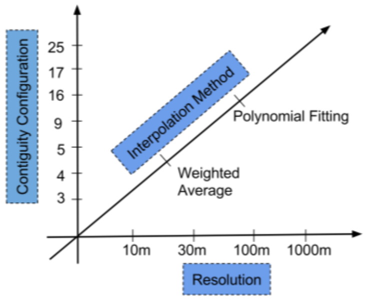


Fig 3. A three-factor research methodology is used in this paper. A range of contiguity configurations increase the number of pixels incorporated into elevation estimation. DEMs spanning four common spatial resolutions are tested. Weighted average and a selection of polynomial fitting methods are compared, based on various contiguity configurations and spatial resolutions.

3.1 Contiguity Configuration

To calculate sub-pixel variations, different contiguity configurations can be investigated. In this research first and second order neighboring pixels are combined in different ways to configure the contiguity structure of sub-pixel variations (Figure 4). Contiguity configurations are based upon the number of neighbors (here 3, 4, 5, 9, 16, 17, 25). A 1-pixel contiguity configuration will be used as a control to compare the various surface-adjusted methods with a planar "within-pixel" method, discussed later in this paper. In 3, 4, and 16 contiguity configurations, 3, 4, and 16 pixel centroids that are closest to the sample points are selected, respectively. Some interpolation methods are an exact interpolation technique and can only be tested on one contiguity configuration. For example, the bi-linear interpolation method is based upon four neighbors. On the other hand, the weighted average can be tested on different contiguity configurations.

3.2 Interpolation Methods

To interpolate the value of an arbitrary point given a regular grid within the defined neighborhood, different interpolation techniques will generate differing elevation estimates. The methods to be compared in this paper include weighted average (an exact interpolator), exact polynomial surfaces (linear, bi-linear, bi-quadratic, and bi-cubic), and least-squares polynomial fitting, which is an inexact interpolator.

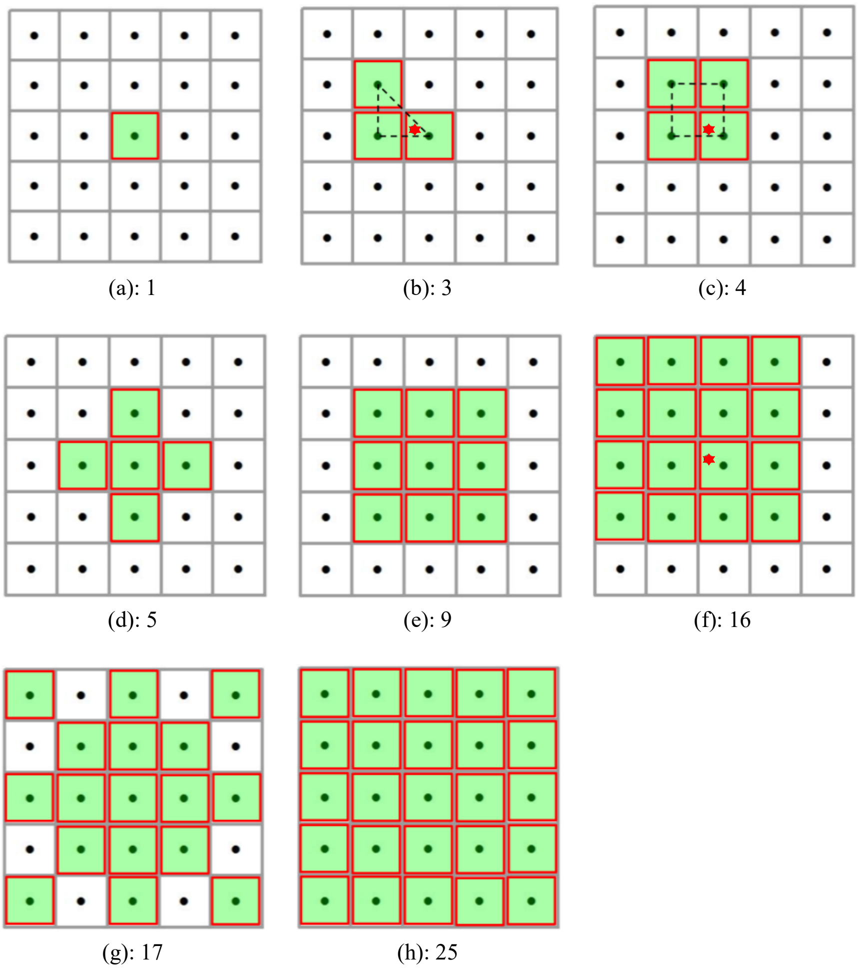


Fig 4. Contiguity configurations; in b, c, and f, the sample point is represented as a star red point and 3, 4, and 16 closest pixels are highlighted respectively.

• ***Within-Pixel (rigid pixel paradigm):*** This method is based upon the rigid pixel paradigm that each point represents a small area around itself (i.e., the region of influence) by a horizontal planar surface. It is utilized in this paper as a "straw horse", a control metric by which to compare the current convention for estimating elevation with the various methods for surface adjustment. Although this approach is quite simple and easy to implement, it does not model the actual terrain surface due to the planar surfaces and discontinuities in the model. So, the elevation at the pixel centroid is assigned to any sample point that falls within that pixel.

• ***Weighted Average:*** is a deterministic interpolation technique that is based on a weighted average of the elevation of neighbor pixels. Distance to neighboring pixel centroids is commonly used as a weighting function, with shorter distances carrying larger weight. Weighted distance assumes that points that are closer together are more similar than those that are farther apart. Different distance functions can be used conceptualized this concept. The simplest and most common way is the "Inverse Distance Weighting" (IDW) method. This method is commonly used for continuous data such as terrain:

$$z(x,y) = \frac{\sum_{i=1}^{n} w_i \, z_i}{\sum_{i=1}^{n} w_i} \;; \quad w_i = \frac{1}{d_i^{p}}$$

Where w_i and z_i are the weight and elevation of the ith source points, respectively. Distance (d) in the weight has an exponent (p); a larger exponent leads to smaller weights to the observations in large distances.

• ***Exact Polynomial Surfaces:*** This is a deterministic method whose estimates are based on a polynomial equation with varying amounts of terms and powers of the X and Y coordinates that goes exactly through a set of given point. The polynomial equations with more terms create a surface with more freedom to undulate. (Li et al. 2004). Four different exact polynomial interpolators are used in this research:

Linear: A triangle is based on the first three terms of a polynomial. Linear interpolation needs the three closest points to solve the equation (Figure 4-b). The mathematical function is a plane as follows:

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y}$$

Bi-linear: The first order polynomial, bi-linear, takes into account four terms determined by four known points (Figure 4-c). It fits a first order polynomial to the four surrounding cell.

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y}$$

Bi-quadratic: The bi-quadratic function is a second order polynomial that uses nine terms as opposed the bi-linear's four, allowing for more freedom of undulation. Nine closest neighbors are selected to solve the bi-quadratic equations (Figure 4-e)

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y} + a_4\,x^2 + a_5\,y^2 + a_6\,x^2y^2 + a_7\,x^2\,y \; + a_8\,xy^2$$

Bi-cubic: the bi-cubic function is a third order polynomial that increases the surrounding pixels to sixteen closest neighbors (Figure 4-f). This method calculates a surface with more local maxima and minima.

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y} + a_4\,x^2 + a_5\,y^2 + a_6\,x^2y^2 + a_7\,x^2\,y + a_8\,xy^2 + a_9\,x^3 + a_{10}\,y^3 + a_{11}\,x^3y^3 + a_{12}\,x^3y^2 + a_{13}x^2y^3 + a_{14}\,x^3\mathrm{y} + a_{15}\,xy^3$$

• ***Best Fitting Polynomials:*** Terrain is a complex surface and using high-order polynomials in an exact fitting approach can lead to fluctuations in the resulting elevation estimate. Least-Squares Polynomial Best Fitting is a common way to resolve fluctuations by minimizing the sum of squared errors. This is an inexact interpolator, but is not necessarily a better approximation of the terrain surface. A best-fitting surface can be linear or a smoothed curved surface. Polynomials of order 1, 2, 3, and 4 are used as follows:

$$Order\ 1\text{: } z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y}$$

$$Order\ 2\text{: } z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y} + a_4\,x^2 + a_5\,y^2$$

$$Oder\ 3\text{: } z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y} + a_4\,x^2 + a_5\,y^2 + a_6\,x^2\,y + a_7\,xy^2 + a_8\,x^3 + a_9\,y^3$$

$$Order\ 4\text{: } z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y} + a_4\,x^2 + a_5\,y^2 + a_6\,x^2\,y + a_7\,xy^2 + a_8\,x^3 + a_9\,y^3 + a_{10}\,x^4 + a_{11}\,y^4 + a_{12}x^2y^2 + a_{13}\,x^3\mathrm{y} + a_{14}\,xy^3$$

3.3 Accuracy Assessment

Accuracy assessment is conducted on the interpolated elevation of sample point by comparing its observed elevation from 3m resolution LiDAR data. The residuals (e) are calculated as the interpolated value minus the observed value.

There are several ways to examine the accuracy of an interpolated sample points. Here we examine standard deviation (STD), root mean square error (RMSE), mean absolute error (MAE), and RMSE based on 95% confidence interval. For 95% RMSE, the high and low residuals are eliminated based on MBE +/- 1.96 (STD).

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}}\ ;\ MAE = \frac{\sum_{i=1}^{n} |e_i|}{n}$$

4. Results and Discussion

4.1 Analysis of Residuals

Although it is usually assumed that the data do not contain systematic errors and the errors are normally distributed in the digital elevation data, it is useful to note that DEM vertical errors are usually not normally distributed due to the small biases or major and minor outliers (Fisher and Tate 2006; Zandbergen 2011).

Table 1 represents basic descriptive parameters of the residuals (mean, median, minimum, maximum, standard deviation, and 95th percentile–lower and upper bound) in different resolutions. The mean value slightly positive (10, 100, and 1000m) or negative (30m). Therefore, the elevations in 10, 100, and 1000m resolution are overestimated and in 30m resolution is underestimated by a small bias. Furthermore, the high values of minima and maxima are due to the occurrence of both positive and negative outliers.

Two standard normality tests also are run on the residuals ─ the Kolmogorov–Smirnov test (Lilliefors 1967) and the Anderson–Darling test (Anderson and Darling 1952). The results of these tests verify that residuals are not normally distributed. Based on the American Society for Photogrammetry and Remote Sensing (ASPRS) (Flood 2004) and the National Digital Elevation Program (NDEP) (NDEP 2004) guidelines, the 95th percentile can be used when the error distribution is not normal.

Table 1. Summary statistics of residuals for different resolutions

Resolution	Mean	Median	Min	Max	Standard Deviation	95th percentile-lower bound	95th percentile-higher bound
10	0.01	0.0	-10.91	10.63	1.07	-1.53	1.60
30	-0.02	-0.12	-35.53	41.26	3.94	-5.60	6.28
100	5.11	3.71	-40.90	66.37	7.95	-4.75	19.52
1000	4.06	4.09	-399.81	381.55	41.89	-35.52	47.39

Outliers are an indication to inspect the spatial pattern of residuals more closely. Local Moran's I (Anselin 1995) was calculated for the residuals in order to visualize clusters (whether residuals with similar values (either high values or low values) ae clustered) and outliers. Residuals display spatial heterogeneity due to the varied character of terrain. The circles in Figure 5 show the most extreme residuals because the points in that part of the study area are located in rough terrain. These imply that terrain type is an important factor needing further testing. As a result, studying the results of terrain type on sub-pixel estimation would be an interesting future investigation.

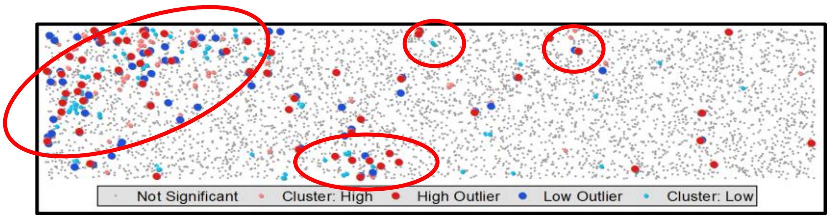


Fig 5. Clusters and outliers of residuals in 10 meter DEM using the bi-linear interpolation

Figure 6 represents the correlation between the residuals and elevation, slope, aspect, curvature, local relief (the range of elevation in a 10 by 10 focal window), and roughness (the standard deviation of elevation in a 5 by 5 window) based on 30 m DEM. All of the correlation coefficients are significant at 95 percent confidence level, though the relationship with aspect is very weak. Slope has the highest correlation with the residual. Therefore, the accuracy of DEM in terrain with high slopes is lower than with flatter slopes. In addition, there is a strong relationship between residuals and terrain roughness and local relief and this also reflects the slope relationship.

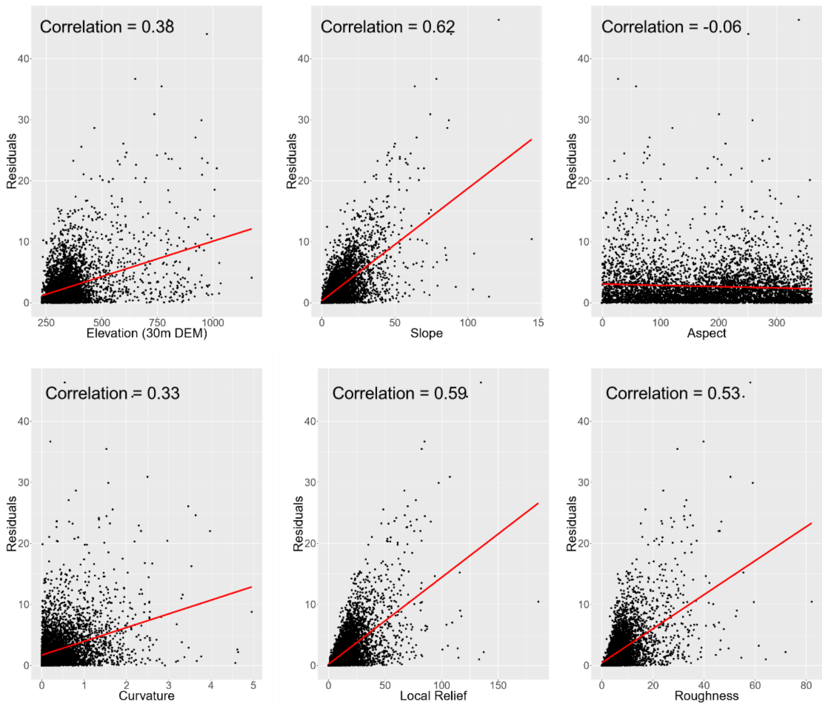


Fig 6. Residuals are plotted against elevation, slope, aspect, curvature, local relief, and roughness (30 m DEM is used here).

4.2 Weighted Average

Table 2 shows the RMSEs resulting from the Weighted Average method for all contiguity configurations (3, 4, 5, 9, 16, 17, and 25 neighbors), and across all resolutions (10, 30, 100, and 1000 meters). There is not one optimum contiguity configuration in all of the resolutions. However, the range of RMSEs in each resolution is quite small, and 9 contiguity configurations, which is the most widely used contiguity configurations for WA, was chosen for the further analysis in Section 4.4.

Table 2. RMSEs for the weighted average method using different number of neighbors in different resolutions; Red indicates the lowest RMSE in each resolution

	10m	**30m**	**100m**	**1000m**
WA3	1.09	4.01	9.46	42.57
WA4	1.08	3.97	9.46	42.24
WA5	1.10	4.02	9.51	42.53
WA9	1.08	3.95	9.50	41.56
WA16	1.08	3.88	9.53	40.93
WA17	1.08	3.91	9.54	41.15
WA25	1.08	3.89	9.61	40.80
Range of RMSEs	*0.02m*	*0.14m*	*0.15m*	*1.64m*

4.3 Best Fitting Polynomials

Table 3 shows the RMSEs for the best fitting polynomial method. Different orders and number of neighbors are tested.

Table 3. RMSEs for the best fitting method using various polynomial orders and different number of neighbors in different resolutions. Red indicates the lowest RMSE in each resolution.

Polynomial order	**Contiguity Configuration**	**9m**	**30m**	**100m**	**1000m**
1	4	1.07	3.94	9.45	42.09
	5	1.07	3.87	9.47	40.85
	9	1.08	3.82	9.65	39.24
	16	1.13	3.90	10.12	39.45
	25	1.22	4.14	10.76	40.75
2	9	1.08	3.82	9.65	39.26
	16	1.13	3.90	10.12	39.48
	17	1.18	4.02	10.44	40.17
	25	1.22	4.14	10.76	40.77
3	9	1.08	3.82	9.67	39.51
	16	1.13	3.90	10.12	39.50
	17	1.18	4.01	10.44	40.20
	25	1.22	4.14	10.76	40.80
4	16	1.14	3.91	10.18	39.58
	17	1.19	4.02	10.51	40.71
	25	1.22	4.14	10.76	40.82

In the Table, polynomial order 1 shows the lowest RMSE. Reading down the columns for a given order, there is a general trend where RMSE increases with increasing numbers of neighboring pixels. In 1000m resolution the opposite occurs; polynomial order 1 with contiguity configuration 4 has the highest RMSE. Again, there is not an optimum method in all of the resolutions. Polynomial order 1 with 5 neighbors was selected for further analysis, having the lowest RMSE.

4.4 Comparing Surface-Adjusted Elevations with the Rigid Pixel Paradigm

The comparison of interpolated elevations with the rigid pixel paradigm is accomplished by comparing results discussed so far with the Within-Pixel (WP) method. Recall that the WP method uses only a single pixel for estimating elevation. Table 4 shows the WP method comparison.

Table 4. Accuracy assessment parameters for different interpolation methods in different resolutions; Columns show interpolation methods (left to right): within a pixel, weighted average using 9 neighbors, linear using 3 neighbors, bi-linear using 4 neighbors, bi-quadratic using 9 neighbors, bi-cubic using 16 neighbors, best fitting polynomial of order 1 using 5 neighbors. Red indicates the lowest RMSE in each resolution.

		WP	**WA9**	**Li3**	**BiLi4**	**BiQ9**	**BiC16**	**Bf1_5**
10m	*RMSE*	1.19	1.08	1.08	1.07	1.08	1.13	1.07
	RMSE (95%)	0.75	0.67	0.66	0.65	0.68	0.73	0.67
	MAE	0.72	0.66	0.65	0.65	0.66	0.71	0.65
	STD	1.19	1.09	1.08	1.07	1.08	1.13	1.07
30m	*RMSE*	4.34	3.95	3.96	3.94	3.82	3.90	3.87
	RMSE (95%)	2.75	2.60	2.57	2.58	2.55	2.64	2.53
	MAE	2.70	2.53	2.53	2.53	2.50	2.61	2.50
	STD	4.34	3.95	3.96	3.95	3.82	3.90	3.87
100m	*RMSE*	9.85	9.50	9.47	9.45	9.65	10.12	9.47
	RMSE (95%)	8.25	8.01	7.97	7.97	8.06	8.37	7.99
	MAE	6.87	6.66	6.62	6.61	6.75	7.08	6.63
	STD	8.42	8.01	7.97	7.95	8.19	8.72	7.99
1000 m	*RMSE*	45.08	41.56	42.04	42.09	39.26	39.50	40.85
	RMSE (95%)	23.77	22.07	22.22	22.33	21.51	21.84	22.04
	MAE	24.69	23.20	23.47	23.50	22.36	22.42	22.97
	STD	44.92	41.36	41.85	41.90	39.05	39.28	40.65

Error magnitudes vary with DEM resolution and with interpolation method. Accuracy assessment parameters show a general trend of increase in the residuals at coarser resolutions. The Within-pixel method shows highest magnitude of errors for all resolutions, except the 100m in which the bi-cubic method has the highest amount of error. Bilinear has the least RMSE in the 10 and 100m resolution. Also the best fitting method has the least RMSE in the 30m resolution. Bi-quadratic and bi-cubic methods work better in 1000m resolutions. This means that polynomial functions of higher orders can better model the slope and curvature of pixels in courser resolutions, but the improvement from bicubic equations seems barely worth the added complexity when the bilinear performs fine.

5. Summary

The current paradigm assumes that the terrain surface is uniform, ignoring slope and curvature of the pixels in DEM. This research employed realistic surface geometries of terrain using different interpolation methods and the information from adjacent pixels for a more accurate and precise interpolation of points. Sub-pixel variations in DEMs through different resolutions was investigated in order to develop the foundation of surface-adjusted computations on DEMs, resulting in more realistic analysis in terrain models. This work also advances understanding of how spatial error propagates through resolutions, a cross-scale terrain analytics component that is at present not well understood. In the next stage of this project, we are going to investigate how the results of this research do vary with geographic conditions and how this affects different modeling applications. Finally, the assumption of non-rigid pixels carries an additional computational load, as it must incorporates not only elevation and pixel size, but also slope and curvature, in effect, adjusting for the changing terrain surface at sub-pixel resolutions. The next step of this research addresses the balance between the increased computations needed to measure surface-adjusted elevation against the improvement in precision.

Acknowledgements

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