Indian Journal of Science and Technology Vol. 5 No. 7 (July 2012) ISSN: 0974- 6846

Crust and skeleton approximation from samples

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Abstract

Samples of shape boundary carry the information of the shape for a sufficiently dense sampling. There is not any connectivity information between sample points and the geometry or topology of the original shape is not known. These samples are used for crust and skeleton approximation. In this article, we focus on the Voronoi-based methods which the shape is defined by a set of sample points on the shape boundary and then the shape and its skeleton is approximated with the Voronoi diagram of these points. These methods are extremely sensitive to noise and boundary perturbation. To overcome the problems of crust and skeleton extraction, we propose a new method by labeling the sample points. The conceptual structure and the results illustrate that our method are stable, easy to implement, robust and able to handle sharp corners and open curves, even in the presence of significant noise and perturbations.

Keywords: Sample points, Crust, Skeleton, Voronoi diagram, Delaunay triangulation.

Introduction

Samples of shape boundary carry the information of the shape for a sufficiently dense sampling. There is not any connectivity information between sample points and the geometry or topology of the original shape is not known. These sample points play an important role in many applications for crust and skeleton approximation (Blum & Nagel, 1978; Bookstein, 1979; C. Gold, 1999; CM Gold & Dakowicz, 2005; C. Gold & Snoeyink, 2001).

Many methods have been proposed for curve reconstruction. In this research, we focus on the Voronoi-based methods and well-known algorithms that use the crust structure for curve reconstruction is introduced. Curve reconstruction is not possible from any set of samples; Amenta *et al*. (Amenta *et al.*, 1998) proposed some theories for sampling criteria. These criteria illustrates that the required sampling density varies locally and less detailed sections of the curve can be reconstructed from fewer samples and do not have to be sampled as densely.

The skeleton (medial axis) was first introduced by Blum (Blum *et al.*, 1967) to describe biological shape and as a tool in image analysis. Grassfire model is the most popular definition of the skeleton with an intuitive concept that defined by Blum; consider starting a fire on the boundary of a shape in the plane. The fire starts at the same moment, everywhere on the boundary and it propagates with homogeneous velocity in every direction. The skeleton is the set of points where the front of the fire collides with itself, or other fire front. Alternatively, in mathematical language: it is the set of points that are equidistant from at least two points on the boundary of shape.

Sometimes, crust cannot reconstruct curves at sharp edges and corners. Also, algorithms that use Delaunay triangulation and Voronoi diagram of sample points for crust and skeleton approximation can be used just for closed curves. On the other hands, a major disadvantage of the skeleton is its inherent instability under small perturbations, because skeleton is very sensitive to the small changes of the boundary. These perturbations produce many irrelevant branches in the skeleton corresponding to non-significant parts of the boundary.

To overcome the problems of crust and skeleton approximation, we propose a new method by labeling the sample points. Each main part of the shape considers as a curve segment and its sample points get same and unique label. The conceptual structure and the results illustrate that our method are stable, easy to implement, robust and able to handle sharp corners and open curves, even in the presence of significant noise and perturbations.

The problem can be extended to 3D (Hoppe *et al.,* 1992; Attali, 1998; Amenta & Bern, 1999) where a set of sample points in R^3 are used to reconstruct a surface and extract its skeleton. The focus of this paper is on 2D space, but the results can properly be extended to 3D.

This article is organized as follows: Section 2 represents some geometric definitions, including Delaunay triangulation, Voronoi diagram and skeleton. In section 3, the crust and the one-step crust and skeleton algorithms are described. We propose an algorithm by labeling the sample points in section 4. Furthermore, we compare our results with the one-step crust and skeleton algorithms in this section. Finally, section 6 concludes some concluding remarks and discusses possible future work directions.

Geometric definitions

This section represents some geometric preliminaries, including Delaunay triangulation, Voronoi diagram and skeleton. In this section, $\mathcal{O}$ is a 2D object, $\partial\mathcal{O}$ is its boundary and $S \subset \partial\mathcal{O}$ is a dense sampling of $\partial\mathcal{O}$.

Delaunay Triangulation

Definition 1. Given a point set S in the plane, the *Delaunay triangulation* (DT) is a unique triangulation (if the points are in general position) of the points in S that satisfies the circum-circleproperty: the circum-circle of each triangle does not contain any other point $s \in S$ (Ledoux, 2006). Fig. 1.a illustrates a 2D example.

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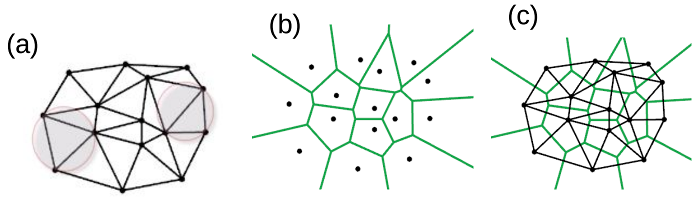
Fig. 1. (a) Delaunay triangulation and (b) Voronoi diagram of a set of points in the plane; and (c) their duality



Voronoi Diagram

Definition 2. Let S be a set of points in R^2. The Voronoi cell of a point $p \in S$, denoted as $V_p(S)$, is the set of points $x \in R^2$ that are closer to p than to any other point in S:

$$V_p(S) = \{ x \in R^2 \mid \|x - p\| \le \|x - q\|, q \in S, q \neq p\} \quad (1)$$

The union of the Voronoi cells of all points $s \in S$ forms the *Voronoi diagram* of S, denoted as $VD(S)$:

$$VD(S) = \cup V_p(S), p \in S \quad (2)$$

Fig. 1.b shows the Voronoi diagrams of a set of 2D points. Delaunay triangulation and Voronoi diagram are dual structures: the centers of circum-circles of Delaunay triangulation are the Voronoi vertices; and joining the adjacent generator points in a Voronoi diagram yields their Delaunay triangulation (Fig. 1.c) (Karimipour *et al.*, 2010).

For Voronoi diagram of sample points S, the Voronoi vertices are classified into *inner* and *outer vertices*, which lie inside and outside, respectively. Then, the Voronoi edges are classified into three groups: edges between two inner vertices (*inner Voronoi edges*), edges between two outer vertices (*outer Voronoi edges*), and edges between an inner and an outer vertices (*mixed Voronoi edges*).

A *Voronoi ball* is centered at a Voronoi vertex and its radius is its distance to the closest sample point. Again, Voronoi balls are classified into *inner* and *outer balls* depending on type of their center points (Giesen *et al.*, 2007).

Skeleton

Definition 3. The *skeleton* is (the closure of) the set of points in $\mathcal{O}$ that have at least two closest points on the object's boundary $\partial\mathcal{O}$ (Amenta et al., 1998). Another description defines the *skeleton* as the centers of the set of maximal disks contained in $\mathcal{O}$ (a maximal disk is a disk contained in the shape not exactly covered by another disk contained in the shape) (Fig. 2).

Fig.2. Skeleton of a 2D curve (rectangle)

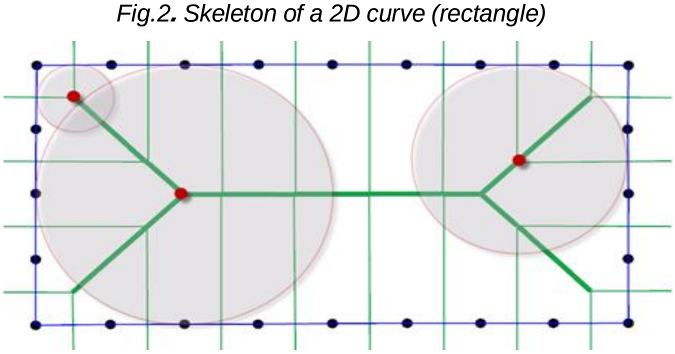
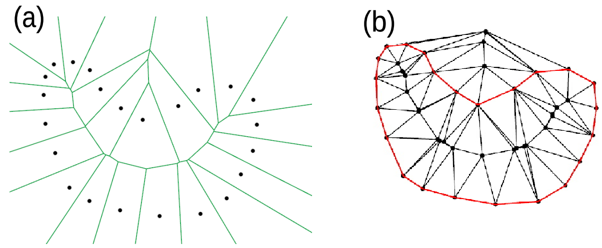


Fig. 3. Curve reconstruction using the crust algorithm: (a) Voronoi diagram of the sample points; (b) Delaunay triangulation of the sample points and Voronoi vertices; and selecting the Delaunay edges whose endpoints belong to S (red lines), which approximate the curve.



Algorithms

Many methods have been proposed in order to extract the crust and skeleton (Arcelli & Frucci, 1992; Lam *et al., 1992*; Borgefors, 1993; Ogniewicz & Kubler, 1995; Ramanathan & Gurumoorthi, 2003). In this research, we focus on the Voronoi-based methods and some well-known algorithms and their implementations are described.

Crust Algorithm

Amenta *et al.* (Amenta *et al.*, 1998) proposed a Voronoi-based algorithm (called crust algorithm) to reconstruct the boundary from a set of sample points forming the boundary of a shape. In this algorithm, the crust is a subset of the edges of the Delaunay triangulation of the sample points.

To compute the crust, let S be the sample points and V be the vertices of the Voronoi diagram of the sample points. Then:

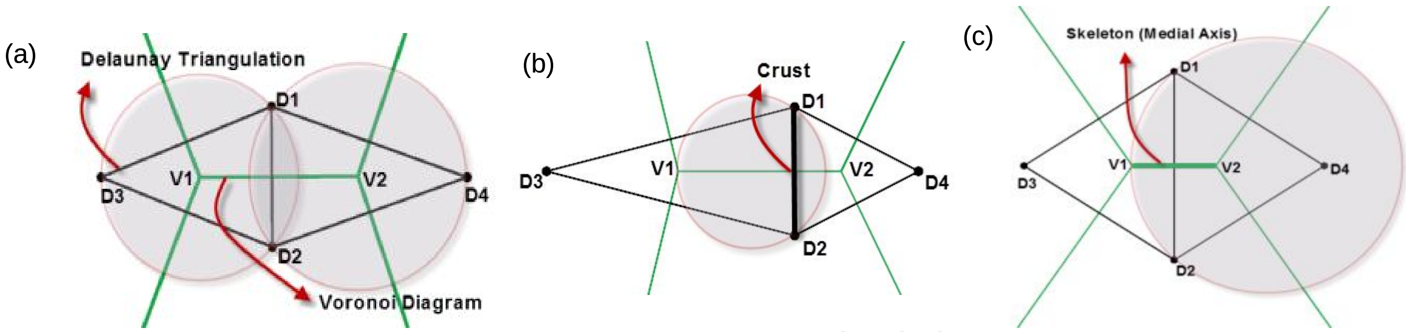
1. Compute the Voronoi diagram of the sample points S (Fig. 3.a).
2. Compute the Delaunay triangulation of $S \cup V$ (Fig. 3.b).
3. The edges of the above Delaunay triangulation whose endpoints belong to S form the crust, which is an approximation of the shape (Fig. 3.b).

The crust algorithm is based on the fact that an edge e of the DT belongs to the crust if e has a circum-circle that contains neither sample points nor Voronoi vertices of S. It means that a global test is needed to check the position of every sample points and Voronoi vertices respect to this circle.

One-Step Crust and Skeleton Algorithm

The above crust algorithm was improved by Gold and Snoeyink (C. Gold & Snoeyink, 2001). They coined the name "one-step crust and skeleton" for this algorithm, because it extracts both crust and skeleton at the same time. This algorithm is fast and easy to implement. Here, every Voronoi / Delaunay edge is either part of the crust (Delaunay) or the skeleton (Voronoi), which can be determined by a simple *inCircle* test. Each Delaunay edge (D_1D_2 in Fig. 4.a) belongs to two triangles ($D_1D_2D_3$

Fig.4. One-step crust and skeleton extraction algorithm:
(a) Delaunay triangulation and Voronoi diagram of four sample points D_1 to D_4;
(b) V_2 is outside the circle passes through D_1, D_2 and V_1, so D_1D_2 belongs to the crust;
(c) V_2 is inside the circle passes through D_1, D_2 and V_1, so V_1V_2 belongs to the skeleton.



and $D_1D_2D_4$ in Fig. 4.a). For each Delaunay edge, there is a dual Voronoi edge (V_1V_2 in Fig. 4.a).

In the crust algorithm (section 3.2), a Delaunay edge belongs to the crust if there is a circle that contains the edge, but does not contain any Voronoi vertices. However, in the one-step crust algorithm, this global test is replaced with a local test that uses only the two endpoints of the dual Voronoi edge.

Suppose two triangles $D_1D_2D_3$ and $D_1D_2D_4$ have a common edge D_1D_2 whose dual Voronoi edge is V_1V_2. The *In Circle* (D_1, D_2, V_1, V_2) determines the position of V_2 respect to the circle passes through D_1, D_2 and V_1. If V_2 is outside the circle, D_1D_2 belongs to the crust (Fig. 4.b). If V_2 is inside, however, V_1V_2 belongs to the skeleton (Fig. 4.c).

The value of *InCircle* (D_1, D_2, V_1, V_2) test is calculated using the following determinant:

$$InCircle\,(D_1, D_2, V_1, V_2) = \begin{bmatrix} x_{D1} & y_{D1} & {x_{D1}}^2 + {y_{D1}}^2 & 1 \\ x_{D2} & y_{D2} & {x_{D2}}^2 + {y_{D2}}^2 & 1 \\ x_{V1} & y_{V1} & {x_{V1}}^2 + {y_{V1}}^2 & 1 \\ x_{V2} & y_{V2} & {x_{V2}}^2 + {y_{V2}}^2 & 1 \end{bmatrix} \quad (3)$$

D_1D_2 belongs to the crust if this determinant is negative, otherwise V_1V_2 belongs to the skeleton.

The pseudo-code for this algorithm is as follows:

One-step crust and skeleton extraction

Input : Sample point S

Output: Crust and skeleton of the shape approximated by S

1. DT $\leftarrow$ Delaunay Triangulation of *S*
2. *E* $\leftarrow$ Edges of DT
3. For every *e* $\in$ E do
4. S_1, S_2 $\leftarrow$ triangles that contain *e*
5. D_1, D_2 $\leftarrow$ end points of *e*
6. V_1, V_2 $\leftarrow$ centers of the circum-circles of S_1 and S_2
7. *H* $\leftarrow$ *InCircle*(D_1, D_2, V_1, V_2)
8. If $H < 0$ then $D_1D_2 \in$ Crust
9. else $V_1V_2 \in$ Skeleton

Proposed method

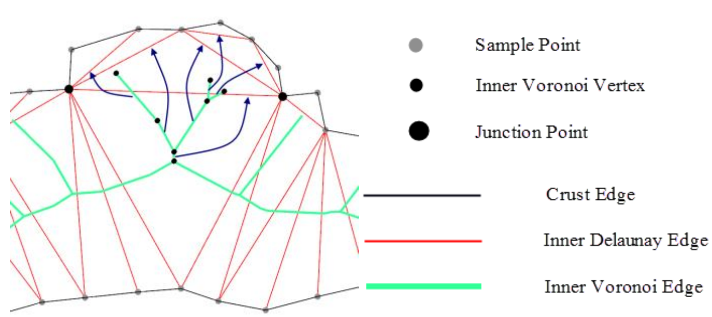
In this section we propose an improvement to the one-step crust and skeleton algorithm using labeling the sample points as a pre-processing; and show how our proposed approach improves the results.

We classified Voronoi vertices and Voronoi edges in geometric definitions. Similar definitions can be used for Delaunay triangulation. Delaunay edges of sample points *S* are classified into three classes. *Mixed Delaunay edges* that connect two consecutive points and belongs to the crust and *inner (outer) Delaunay edges* that connect two non-consecutive points and are completely inside (outside) $\mathcal{O}$ (it should be noted that all Delaunay vertices lie on the boundary $\partial\mathcal{O}$). it is provable that inner and outer Voronoi edges do not intersect $\partial\mathcal{O}$ and mixed Voronoi edges intersect $\partial\mathcal{O}$ in one single point (Giesen *et al.*, 2007).We consider the shape boundary as separate curve segments (each curve segments should be a main part of the shape):

$$\partial O = \bigcup_{i=1}^{n} \partial O_i \quad (4)$$

Afterwards, we use the fact that inner/outer/mixed Voronoi edges are dual to the inner/outer/mixed Delaunay edges. We observed that if the end points of one inner Delaunay edge lie on a curve segment, it's dual in inner Voronoi edges will be an irrelevant edge, while the dual of major edges in skeleton structure are inner

Fig. 5. Extraneous edges in skeleton are inner Voronoi edges that its corresponding inner Delaunay edges created between the sample points which lie on the same curve segment.



Delaunay edges whose end points lie on two different curve segment (Fig. 5). Thus, the main idea is to remove all inner Voronoi edges whose corresponding Delaunay vertices lie on the same boundary curve. Consequently, if sample points of each curve segment have same label, we can eliminate irrelevant edge from skeleton structure simply.

In proposed method, before skeleton extraction, it is necessary to label sample points. Each main part of the shape considers as a curve segment and its sample points get same and unique label. Points which are common between two curve segments called junction. Junction points should have unique and negative labels too.

To apply our proposed approach in the one-step crust algorithm, the lines 8 and 9 of the one step algorithm pseudo-code are modified as follows:

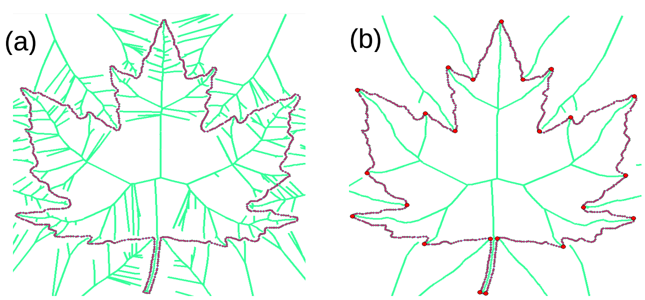
8. If $H < 0$ and label(D_1)=label(D_2) or label(D_1)*label(D_2)<0 then $D_1D_2 \in$ Crust
9. else if label(D_1) ~= label(D_2) then $V_1V_2 \in$ Skeleton

Here, to extract the crust and skeleton, each Delaunay edge passes the In Circle test: If the determinant is negative and the corresponding Delaunay vertices have the same labels or one of them is a junction, that Delaunay edge is added to the crust. Otherwise, if the determinant is positive and the corresponding Delaunay vertices have different labels, its dual is added to the skeleton.

The results of the one-step algorithm and our proposed approach are illustrated in Fig. 6.

Conclusion and future works

Fig. 6. Crust and skeleton approximation: (a) One step algorithm and (b) our proposed method.



In this paper, we reviewed the crust and skeleton approximation methods that use Voronoi diagram in their approach and improved the one-step crust and skeleton algorithm by labeling. It leads to a solution that is simple and easy to implement, robust to boundary perturbations, able to handle sharp corners and open curves.

The present study was important from some different aspects. First, we showed that how labeling sample points can improve the result of crust and skeleton approximation. Another important advantage of this study over previous studies was that it can be used for any open curves. Furthermore, the proposed method never produces extraneous branches, which are common when using the known methods. Finally we can say, the proposed method have solved the weakness of most of the commonly used methods.

In the future, we will extend the approach for surface reconstruction and 3D skeleton extraction. We will also study in more details the relationship between crust and skeleton extraction and its application in some new fields.

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