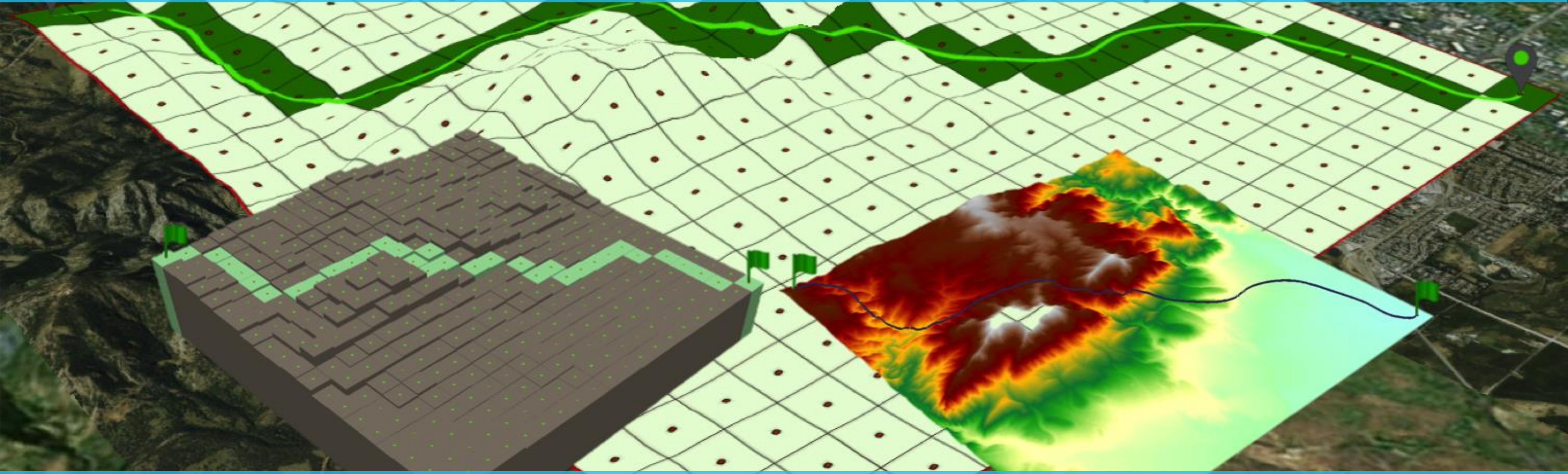
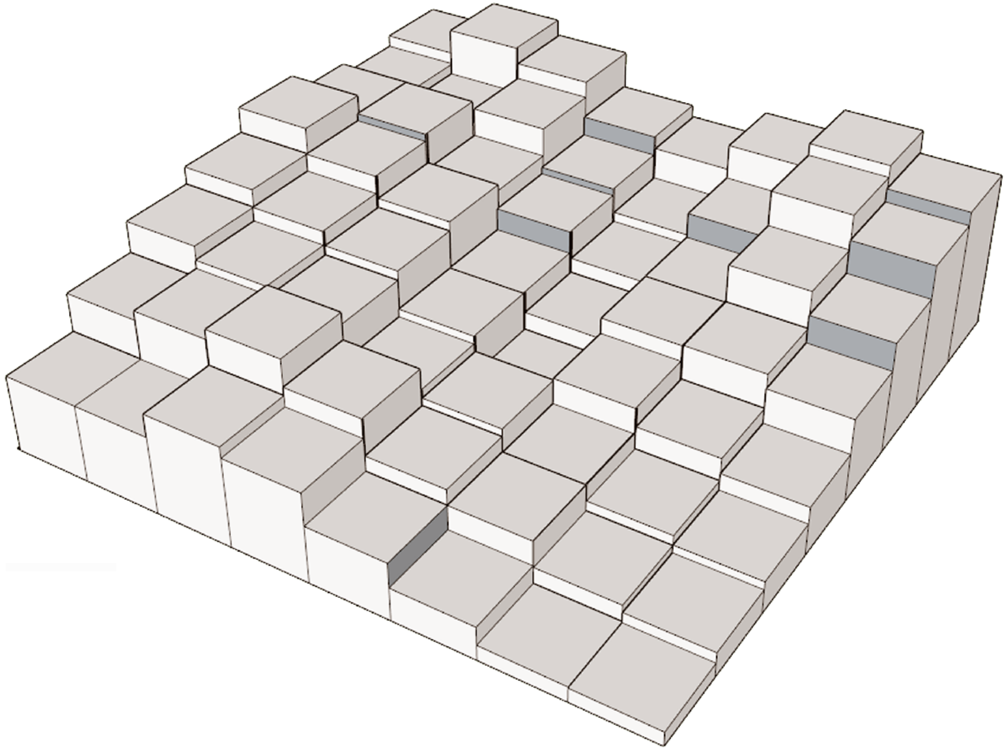
Aug 15th, 2017
Geomorphometry 2018
Boulder, Colorado

Slope-Adjusted Surface Area Computations in Digital Terrain

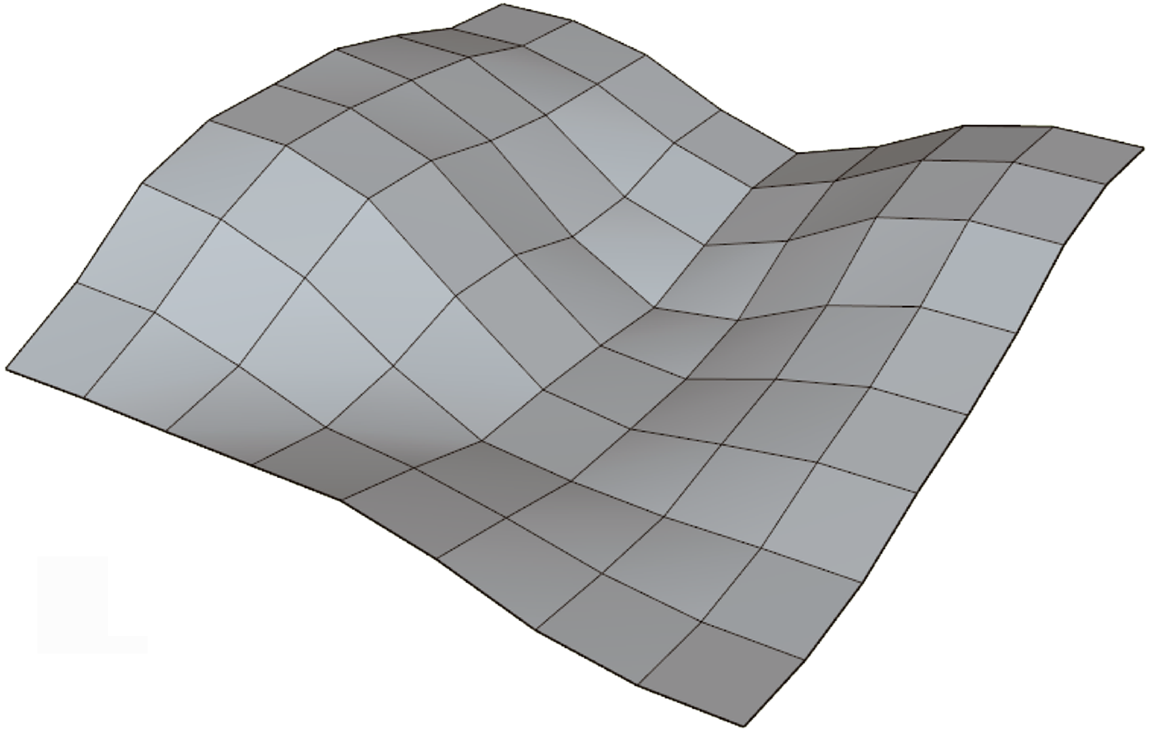


Mehran Ghandehari, and Barbara P. Buttenfield
Department of Geography, University of Colorado, Boulder
{mehran.ghandehari; babs} @colorado.edu

Introduction



Rigid Pixel Paradigm



Surface-Adjusted Paradigm

Surface Area Calculations

Slope Method:

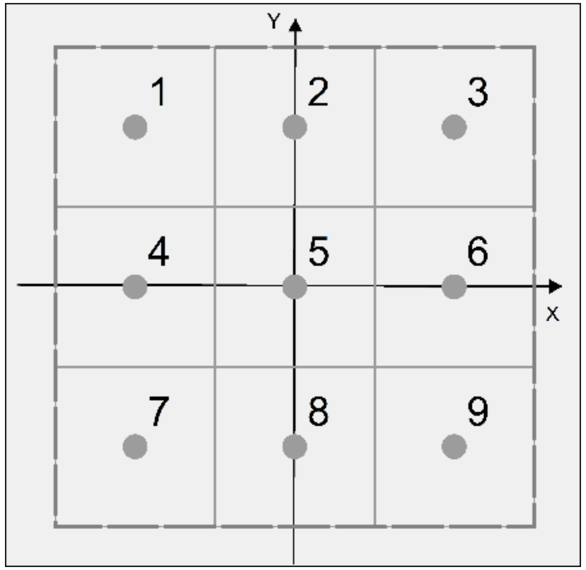
$$\mathit{Surace\ Area} = \frac{\text{Pixel size}^{\,2}}{\text{Cos}\ (\mathit{Slope})}$$

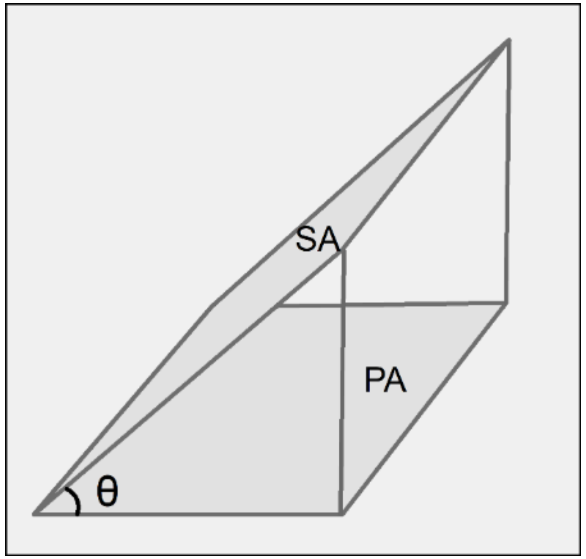
$$\mathit{Slope} = \left(\tan^{-1}\sqrt{\left(\frac{\mathrm{dz}}{\mathrm{d}x}\right)^2 + \left(\frac{\mathrm{dz}}{\mathrm{d}y}\right)^2}\ \right)$$

$$\frac{\mathrm{dz}}{\mathrm{d}x} = \frac{(H3 + 2 * H6 + H9) - (H1 + 2 * H4 + H7)}{8 * \text{Pixel size}}$$

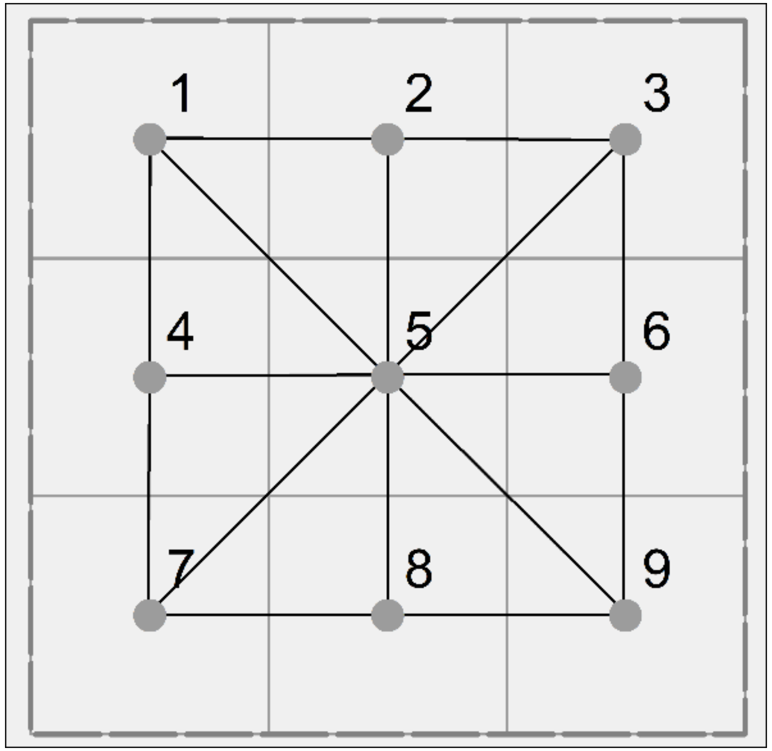
$$\frac{\mathrm{dz}}{\mathrm{d}x} = \frac{(H1 + 2 * H2 + H3) - (H7 + 2 * H8 + H9)}{8 * \text{Pixel size}}$$

Berry (2002)

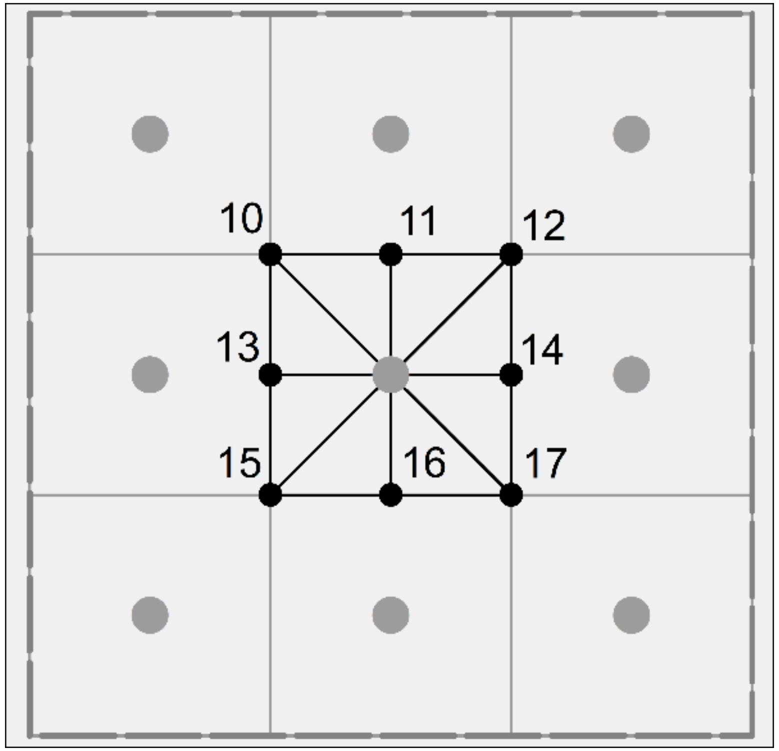




Surface Area Calculations



Jenness Method



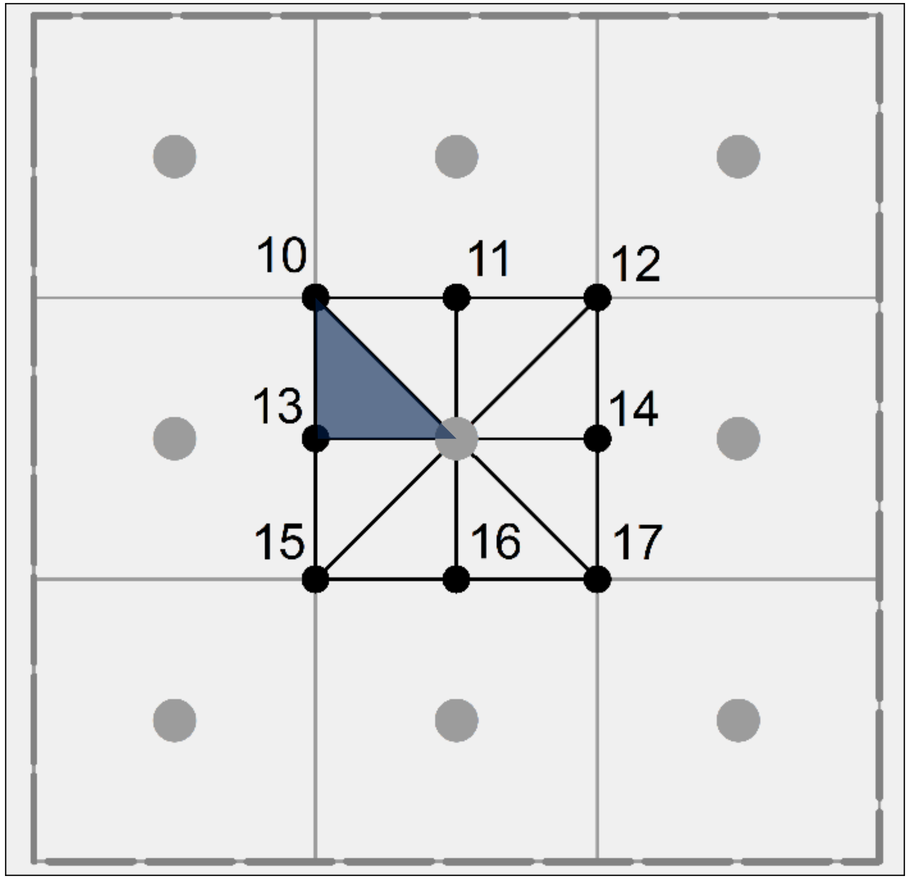
Modified Method

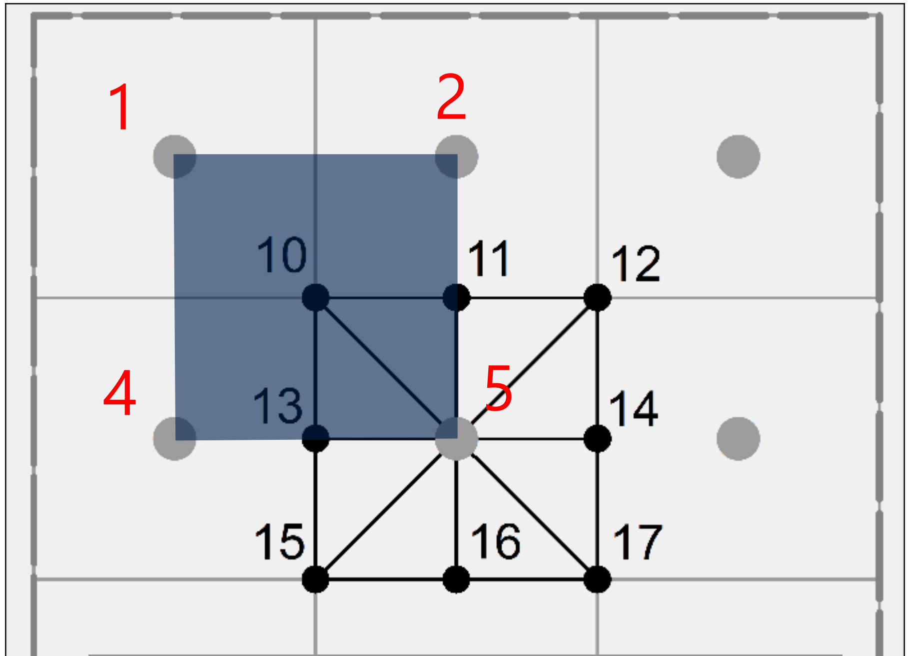
Jenness (2004)

Interpolation Methods

The modified Jenness method is tailored to each interpolation method. For example…

For the Bilinear Interpolation (4-pixel neighborhood), points 10, and13 are interpolated from the nearest four pixels (1,2,4,5).



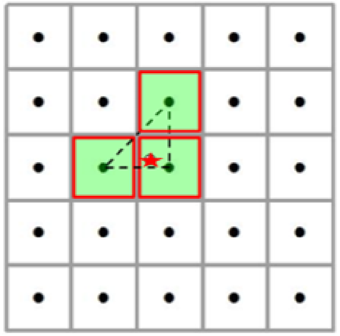


Bilinear 4 pixels:
$z = a_0 + a_1\,x + a_2\,y + a_3\,x\,y$

Interpolation Methods

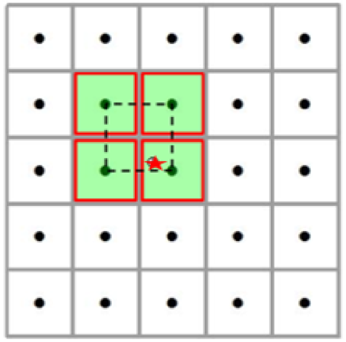
Linear 3:

$$z(x,y)$$
$$= a_0 + a_1\, x + a_2\, y$$



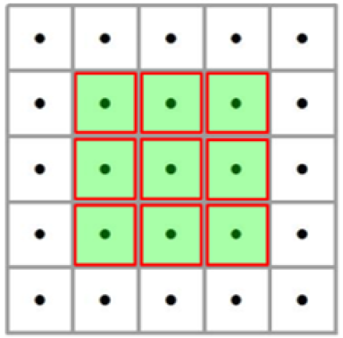
Bilinear 4:

$$z(x,y) = a_0 + a_1\, x + a_2\, y + a_3\, x\, y$$



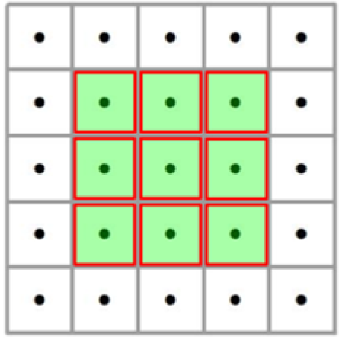
Weighted Average 9:

$$z(x,y) = \frac{\sum_{i=1}^{n} w_i\, z_i}{\sum_{i=1}^{n} w_i}\ ; \quad w_i = \frac{1}{d_i^2}$$



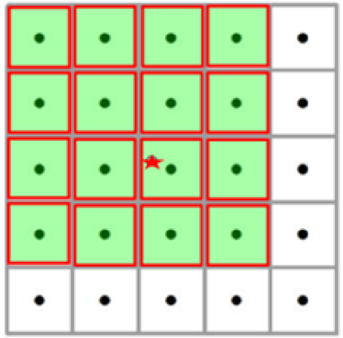
Biquadratic 9:

$$z(x,y) = a_0 + a_1\, x + a_2\, y + a_3\, x\, y + a_4\, x^2 + a_5\, y^2 + a_6\, x^2 y^2 + a_7\, x^2\, y + a_8\, xy^2$$



Bicubic 16:

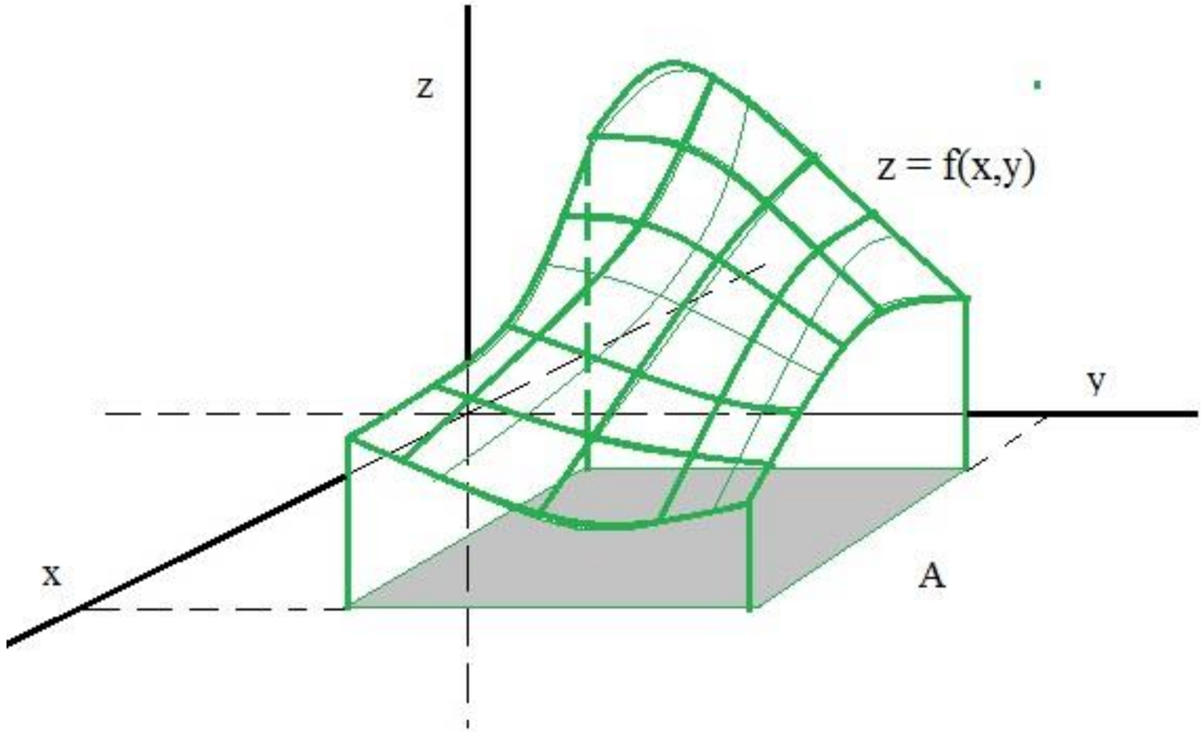
$$z(x,y) = a_0 + a_1\, x + a_2\, y + a_3\, x\, y + a_4\, x^2 + a_5\, y^2 + a_6\, x^2 y^2 + a_7\, x^2\, y + a_8\, xy^2 + a_9\, x^3 + a_{10}\, y^3 + a_{11}\, x^3 y^3 + a_{12}\, x^3 y^2 + a_{13} x^2 y^3 + a_{14}\, x^3 y + a_{15}\, xy^3$$



Surface Area Calculations

Double Integral Method:

$$SA = \iint_R \sqrt{1 + [f_x\,(x,y)]^2 + \left[f_y\,(x,y)\right]^2}\; dA$$



Surface Area Calculations

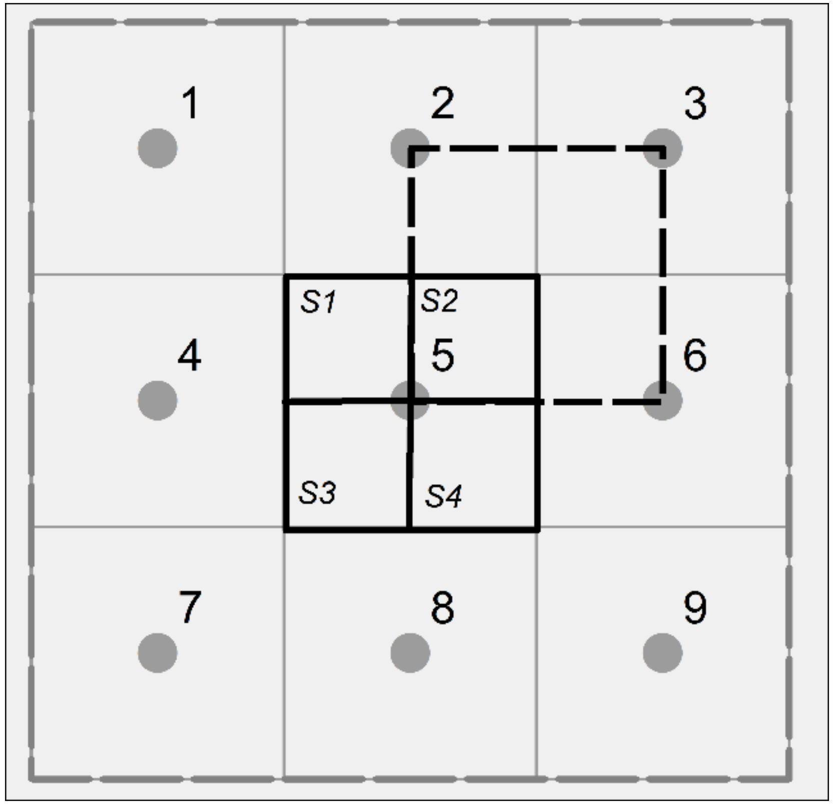
Double Integral Method:

$$SA = \iint_R \sqrt{1 + [f_x\,(x,y)]^2 + \left[f_y\,(x,y)\right]^2}\; dA$$

$$G(x,y) = \sqrt{1 + f_x^2 + f_y^2}$$

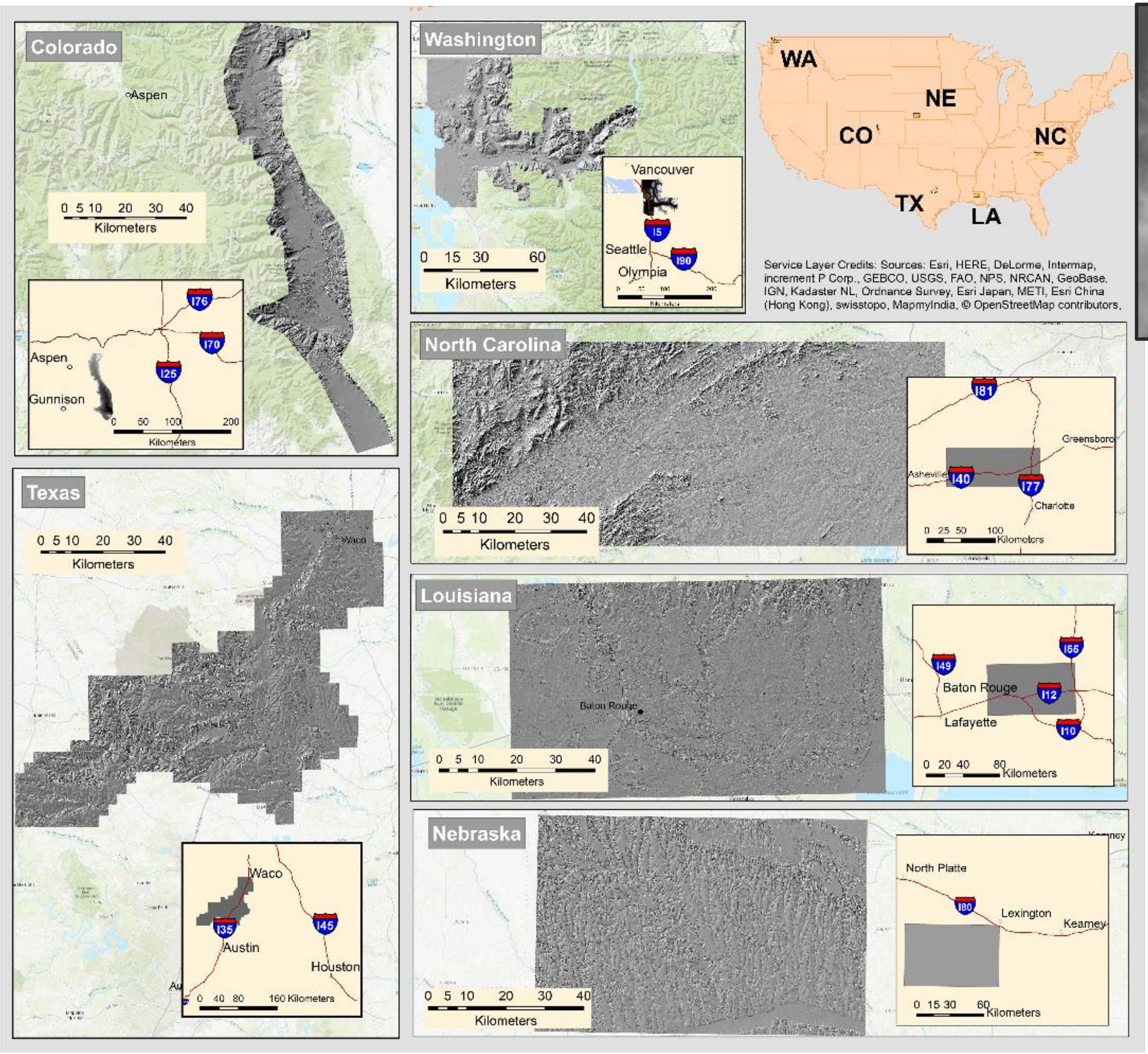
$$G(x_0 + \Delta x, y_0 + \Delta y) = G(x_0, y_0) + G_x(x_0, y_0)\Delta x + G_y(x_0, y_0)\Delta y$$

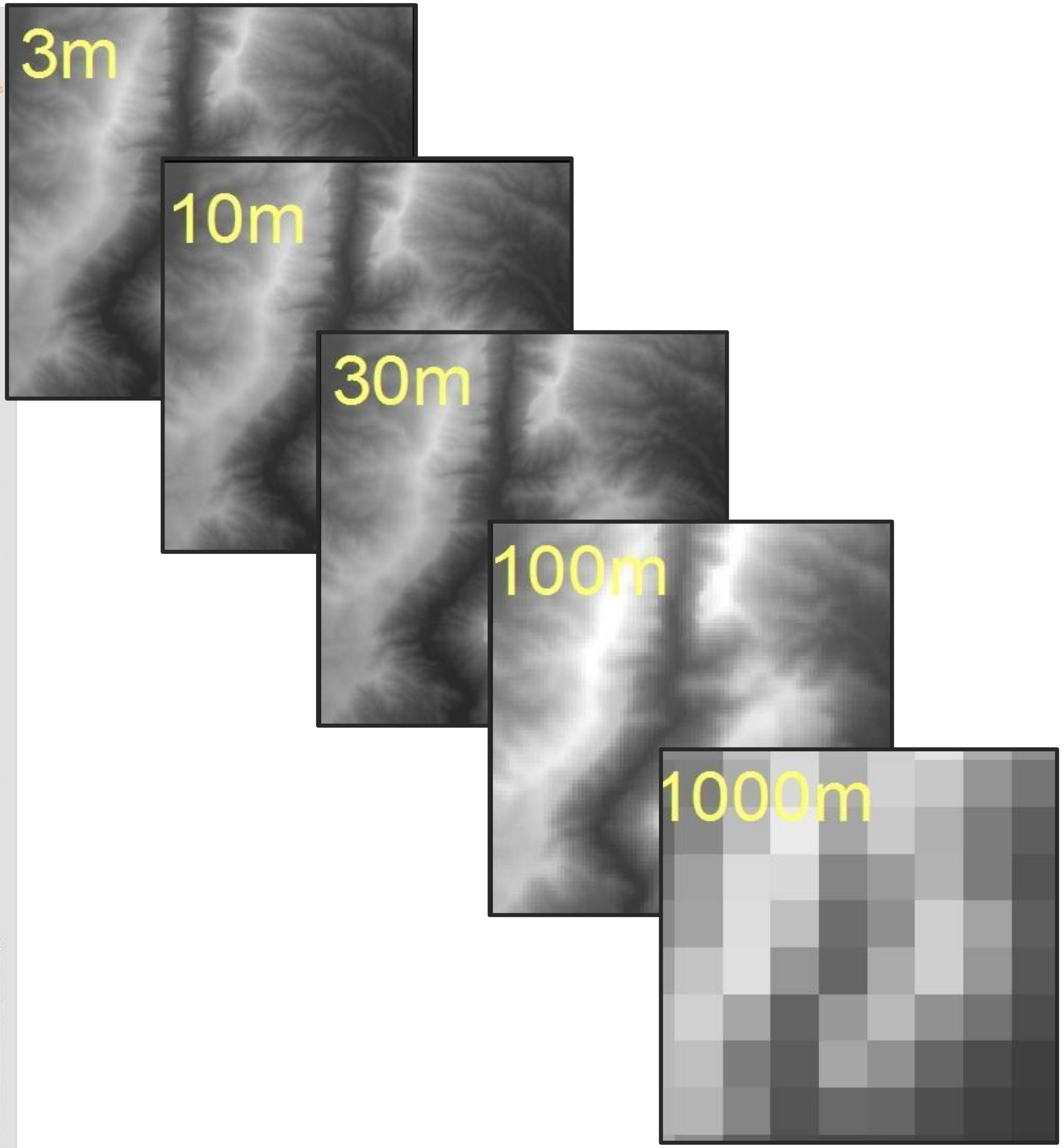
$$\mathrm{SA} \approx P^2 * G(x_0, y_0) + \frac{Pixel\ size^3}{2} G_x(x_0, y_0) + \frac{Pixel\ size^3}{2} G_y(x_0, y_0)$$



Xue et al. (2018) and Li et al. (2018)

Data Sets and Study Areas

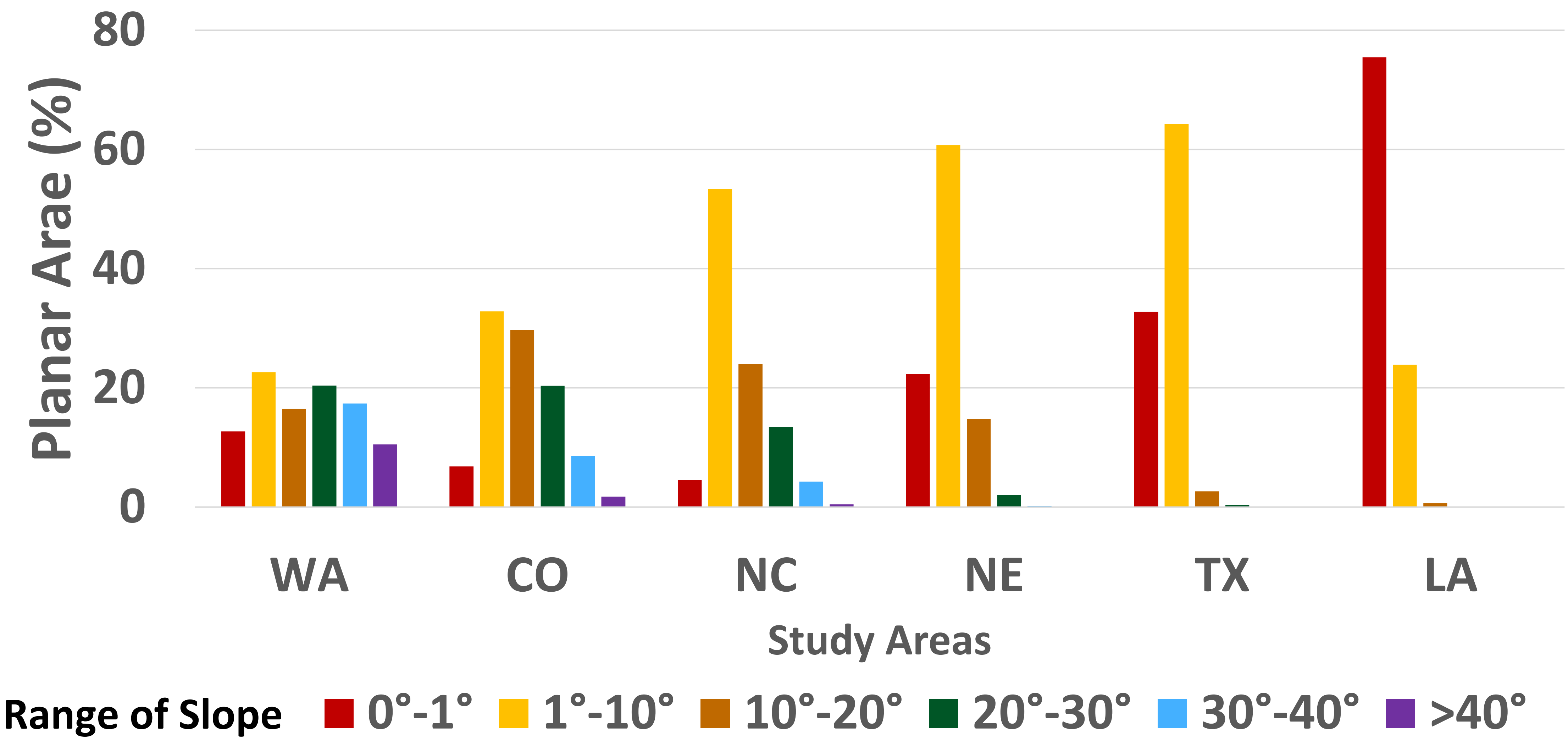




Data Sets and Study Areas

	Elevation (m)				Slope (°)		Roughness (m)		DEM size (Km^2)
	Maximum	Minimum	Mean	STD	Mean	STD	Mean	STD	
Washington	2341.15	-0.68	445.04	438.55	15.29	14.39	8.24	7.92	4533.35
Colorado	4225.39	2108.39	2818.50	366.51	12.31	10.35	5.68	4.73	2493.29
North Carolina	1616.97	192.59	442.18	223.07	11.01	9.11	4.87	3.97	7786.72
Nebraska	931.55	635.42	786.70	54.81	5.07	5.45	2.02	1.65	4709.26
Texas	471.28	99.18	225.84	77.94	2.44	3.03	1.09	1.13	5475.28
Louisiana	69.17	-4.87	13.81	12.26	0.92	1.53	0.35	0.44	5307.17

Data Sets and Study Areas



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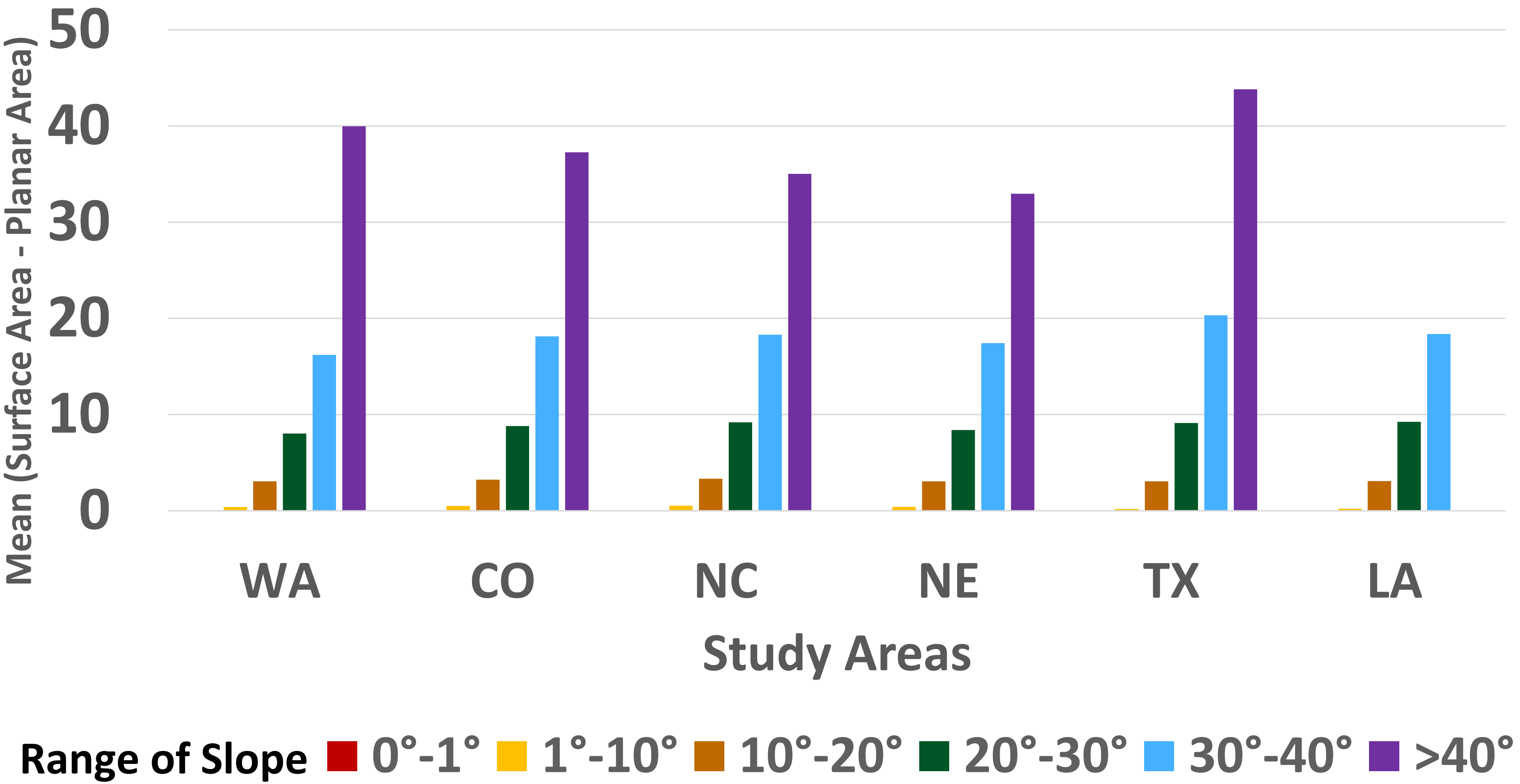

Workflow and Processing

- Systematically investigates the scale-, algorithm-, and topographic-dependence of surface area calculations in DEMs
- Four different DEM resolutions (from 10 meter to 1,000 meter pixel sizes), eight different methods, and six study areas across the conterminous United States
- Coding and statistical analysis are conducted in Python using open source modules (e.g., GDAL, Geopandas, numpy, scipy, and multiprocessing)
- A virtual server on Amazon Web Services (AWS) is used with 32 CPUs and 224 GB of RAM. This instance calculates surface area of 32 rows of DEM concurrently.

Summary Statistics of Surface Area Rasters for North Carolina

10 m	Mean Sq. m.	Std. Dev. Sq. m.	Sum Sq. km.
Benchmark	91.076	6.0618	8,374.753
Planar	87.259	0.000	8,023.207
Slope	90.284	4.926	8,301.949
Jenness	90.525	5.107	8,324.131
Linear3	90.525	5.187	8,324.136
Bilinear4	90.4937	5.115	8,321.188
Wtd Average9	89.043	2.852	8,187.846
Biquadratic9	90.402	5.066	8,312.787
Bicubic16	90.543	5.274	8,325.752

1000 m	Mean Sq. m.	Std. Dev. Sq. m.	Sum Sq. km.
Benchmark	712,923.750	10,346.761	7,575.528
Planar	706,837.062	0.000	7,510.850
Slope	707,758.000	2,225.832	7,520.636
Jenness	708,786.687	3,850.350	7,531.567
Linear3	708,787.812	4,185.199	7,531.579
Bilinear4	708,607.375	3,710.965	7,529.662
Wtd Average9	708,072.562	2,844.051	7,523.979
Biquadratic9	708,157.000	2,914.196	7,524.876
Bicubic16	708,754.750	4,091.762	7,531.228

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RMSE Values (sq. m.) for Different Methods at Different Resolutions

WASHINGTON	10 m	30 m	100 m	1000 m
Planar	13.695	104.784	790.141	43,265.816
Slope	4.297	34.359	394.108	29,894.168
Jenness	4.032	29.195	375.898	21,739.666
Linear3	3.854	27.765	388.559	22,343.145
Bilinear4	3.939	28.785	381.837	22,961.992
WtdAverage9	7.149	55.526	516.876	29,095.031
Biquadratic9	3.969	30.074	386.471	26,272.572
Bicubic16	3.693	26.118	385.719	22,067.41

COLORADO	10 m	30 m	100 m	1000 m
Planar	8.298	61.614	457.065	20,039.174
Slope	2.829	21.959	220.494	14,871.395
Jenness	2.507	17.682	193.281	11,866.118
Linear3	2.387	17.086	191.879	11,885.051
Bilinear4	2.464	17.711	195.622	12,284.397
WtdAverage9	4.526	33.634	296.526	14,807.516
Biquadratic9	2.537	19.146	205.202	13,521.696
Bicubic16	2.270	16.372	187.988	11,902.85

NORTH CAROLINA	10 m	30 m	100 m	1000 m
Planar	7.163	53.960	380.230	12,003.862
Slope	2.225	20.581	235.759	10,370.065
Jenness	1.924	15.043	207.241	8,652.469
Linear3	1.866	14.872	206.327	8,634.967
Bilinear4	1.922	15.455	210.426	8,899.853
WtdAverage9	3.992	30.097	277.748	9,802.203
Biquadratic9	2.016	17.242	220.189	9,660.688
Bicubic16	1.810	13.958	202.475	8,655.975

NEBRASKA	10 m	30 m	100 m	1000 m
Planar	2.773	16.372	78.373	639.683
Slope	1.372	11.447	70.379	605.736
Jenness	1.141	8.750	66.390	569.497
Linear3	1.064	8.593	66.364	569.904
Bilinear4	1.127	9.035	66.967	575.486
WtdAverage9	1.769	11.608	71.273	595.990
Biquadratic9	1.181	9.941	68.526	591.068
Bicubic16	0.969	8.178	65.944	569.425

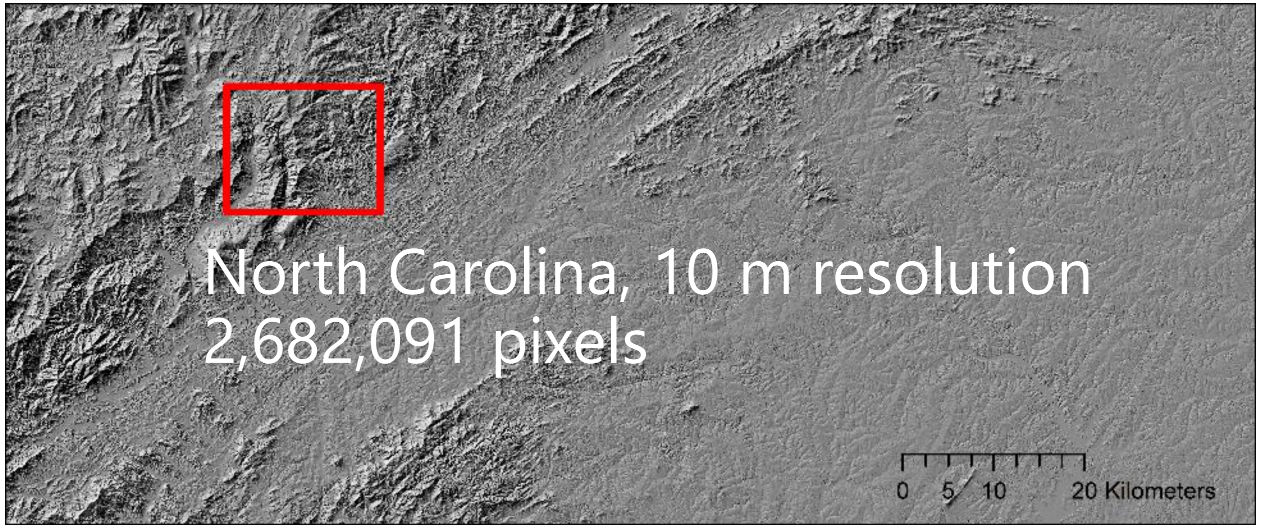
RMSE Values (sq. m.) for Different Methods at Different Resolutions

TEXAS	10 m	30 m	100 m	1000 m
Planar	1.987	11.763	65.334	1,029.190
Slope	1.020	6.767	49.179	938.384
Jenness	0.886	5.680	45.096	851.475
Linear3	0.824	5.348	44.206	844.650
Bilinear4	0.872	5.655	45.209	862.605
WtdAverage9	1.276	7.855	53.141	913.090
Biquadratic9	0.898	5.947	46.397	899.804
Bicubic16	0.782	5.137	43.096	848.091

LOUISIANA	10 m	30 m	100 m	1000 m
Planar	0.5830	2.674	8.718	42.616
Slope	0.4020	2.256	8.453	39.471
Jenness	0.330	1.867	9.445	36.458
Linear3	0.318	1.837	9.911	36.731
Bilinear4	0.333	1.915	9.360	37.029
WtdAverage9	0.422	2.147	8.917	38.817
Biquadratic9	0.355	2.061	8.770	38.123
Bicubic16	0.303	1.828	9.778	36.420

Processing Time Comparison

	Slope	Jenness	Linear3	Bilinear4	WtdAverage 9	Biquadratic 9	Bicubic16
Processing time in seconds	5.59	25.03	52.21	53.34	41.34	46.62	116.49
Relative processing time	X	4.47X	9.30X	9.54X	7.39X	8.33X	20.83X
Relative RMSE	1.27Y	1.12Y	1.06Y	1.10Y	2.49Y	1.14Y	Y



The Linear and Jenness methods balance improvement in accuracy with faster processing speed

Summary

Planar area ignores the characteristics of terrain surface and introduces discrepancies. This error can be neglected for individual pixels, but propagates dramatically for measurements that encompass many pixels or where pixel sizes become large.

Cross-scale analysis shows varying amounts of error and processing speed.

- Error magnitudes vary with DEM resolution and interpolation method and terrain uniformity.
- There is a general increase in the residuals at coarser resolutions

Summary

- The Bicubic polynomial has the lowest RMSE at fine resolutions
- The Linear interpolations perform slightly better than Jenness' method.
- Both Linear and Jenness perform better than Bilinear, Biquadratic and Bicubic at coarser resolutions
- How to choose?
  - If the ultimate goal is accuracy, choose Bicubic
  - If the ultimate goal is fast processing, choose Planar (Slope)
  - A balance between accuracy and processing time is achieved with Linear or Jenness.