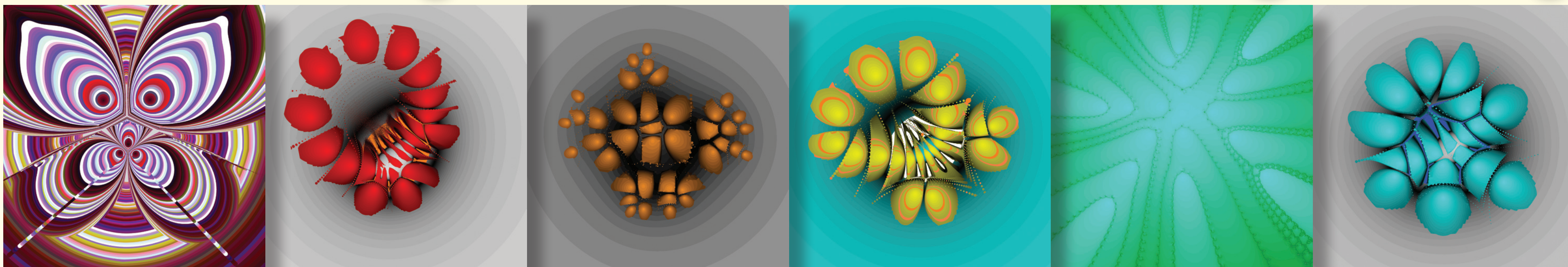
ISVD 2012

2012 Ninth International Symposium on
Voronoi Diagrams in Science and Engineering



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Proceedings of the

2012 Ninth International Symposium on Voronoi Diagrams in Science and Engineering

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2012 Ninth International Symposium on Voronoi Diagrams in Science and Engineering

27-29 June 2012 / Rutgers University, New Jersey, USA

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Chairs of ISVD 2012 and Proceedings Editors

Prof. Bahman Kalantari, Rutgers University, USA
Prof. Marina L. Gavrilova, University of Calgary, Canada

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Foreword from the Editors

The 9^{th} International Symposium on Voronoi Diagrams in Science and Engineering 2012 (ISVD 2012), will take place at Rutgers, The State University of New Jersey, Busch Campus in Piscataway, from June 27 to 29. Over the past few decades the subject of Voronoi diagrams has been flourishing through numerous research articles, turning it into a well-regarded area of computational geometry, with numerous theoretical and practical applications in diverse scientific areas. ISVD 2012 is the first time this significant international event is being held in the US.

The Proceedings contains an outstanding collection of twenty refereed articles: thirteen full articles and seven short ones, selected through expert refereeing process from forty submissions to ISVD 2012. There are also two invited lectures at the Symposium by distinguished speakers. We are grateful to the referees who have made this possible.

As one of the Chairs of this Symposium and Editors of its Proceedings, it is perhaps worthwhile to reflect on my own experiences with ISVD and why I have found the Symposium so valuable. I believe the strength of a conference lies in what it inspires, rather than its size. As a computer scientist and mathematician, initially with expertise in mathematical programming and optimization theory, and despite my general interest in computational geometry, ISVD Symposia would not have caught my attention.

However, due to my research interests in polynomial root-finding, when in 2005 I learned of the call for Voronoi art submissions, I had a communication with Deok-Soo Kim, Chair of the ISVD 2005 in Seoul, Korea. This resulted in my submission of a polynomiography artwork and I was pleased to see it appear in its art pamphlet of Voronoi art.

A few years later, when I had new theoretical results on the connections between complex polynomial root-finding and Voronoi diagrams, I contacted Francios Anton, a Chair of ISVD 2009 in Copenhagen and through communications that followed I attended the event, giving an invited talk on my work. Even though the work may have been considered as an isolated connection between seemingly unrelated areas, it caught the attention of Franz Aurenhammer, who brought to my attention the work of Asano, Matoušek, and Tokuyama on zone diagrams. Although zone diagrams are very different from polynomiography, there are visual resemblances.

This in turn inspired the introduction of a new variation of zone diagram, called mollified zone diagram, totally different from polynomial root-finding connections to Voronoi diagram. With co-authors Iraj Kalantari and my graduate student Sergio de Biasi we submitted an article on mollified zone diagrams to ISVD 2010 in Quebec City. Sergio presented the work, also delivering my invited talk on "Polynomiography as Voronoi Art," because I happened to be in Japan at the time.

As a research area, mollified zone diagrams was to become Sergio's Ph.D. thesis area. Excited by his experiences at the ISVD 2010 event, Sergio encouraged me to propose holding the ISVD 2012 Symposium at Rutgers. The Steering Committee of ISVD approved my proposal and this became a reality. Despite all the reasons for Sergio to have been present at this ISVD event, tragically on August 11, 2011 he ended his life for unknown reasons.

I would like to dedicate the ISVD 2012 Symposium to his memory and in a formal way, an article with three other co-authors is dedicated to Sergio's memory, "On Properties of Forbidden Zone of Polygons and Polytopes." I am convinced that the notion of forbidden zone, introduced earlier in the context of our mollified zone diagrams, will bring new and interesting lines of research and applications into computational geometry. Indeed, the concept of forbidden zone has already inspired me into developing a new algorithm for the convex hull decision problem and for linear programming, very distinct from my previous optimization-based algorithms for the same problems.

Summarizing my own research experiences with ISVD Symposia, Voronoi diagrams and related concepts are very significant in many areas that go beyond computational geometry. Personally, I am convinced that the distinctions between different disciplines of sciences is more man-made, than due to inherent or intrinsic differences in these disciplines.

Through polynomiography I have found ways to introduce undergraduate and graduate students, as well as middle school and high school students to a wide range of topics, from elementary to deep concepts related to polynomial root-finding algorithms, such as Newton's method, complex numbers, basins of attraction, Voronoi diagrams, convex sets and convex hulls, fractals, Fatou and Julia sets, and more. Indeed Newton's method itself, one of the most famous of all algorithms, can be viewed as a mechanism to approximate Voronoi diagram of roots of a complex polynomial.

One can argue that the notion of Voronoi diagrams was in the minds of Cayley and Schroder who pioneered complex polynomial root-finding in the late nineteenth century, before Georgy Voronoy took a formal interest in the concept that was to be named after him. In fact through polynomiography I have been able to introduce concepts such as basins of attraction of polynomial roots and Voronoi diagrams to artists and the general public making them realize that some artwork by the most famous artist bring to mind connections to these mathematical notions. My thanks go to the pioneers of ISVD, Kokichi Sugihara, Chair of ISVD 2004, and Deok-Soo Kim, Chair of ISVD 2005 for having created a platform to bring more visibility to Voronoi diagrams.

As in previous years, the goal of this Symposium is to foster synergy and exchange of ideas among researchers working in various areas who have found connections and applications of Voronoi diagram methodology and its generalizations or variations, and through this to facilitate the state-of-the-art research and collaboration between researchers of diverse areas. We hope that ISVD will continue to contribute to the further development of the theoretical foundations of computational geometry, bringing about new innovations and solutions to applied problems through the use of Voronoi methodology, and extending its boundaries by finding connections with other fields of science and art.

I would like to thank my colleague Bill Steiger who encouraged me to seek NSF funding for partial support of the conference. I would like to thank NSF itself and Dmitry Maslov, the NSF Program Director with whom I had very constructive consultations regarding my proposal for ISVD. I would like to thank DIMACS and its Director, Rebecca Wright and its staff for their expert administrative support. Specifically, I would like to thank the DIMACS Associate Director, Eugene Fiorini for his help and guidance regarding many details of the organization of the event, even for designing beautiful publicity posters.

Finally, we would like to thank all researchers who have expressed interest in this event through their submissions to ISVD 2012. We would like to thank the two Invited Speakers, Chee Yap and Prosenjit Bose, for accepting our invitation and for presenting their interesting work at this Symposium. We also would like to thank all conference attendants for making the ISVD 2012 Symposium one of the premium events on the scientific community scene.

We hope that the Voronoi diagram community will continue to grow and mature and the annual Voronoi Symposiums that follow ISVD 2012 will continue to facilitate exchange of ideas, foster new collaborations, and the development of new concepts and ideas in Computational Geometry, as well as their connections and applications in areas outside of Computational Geometry.

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ISVD 2012 Support

The 9th ISVD International Symposium on Voronoi Diagrams in Science and Engineering takes place at the Rutgers University, NJ, from June 27th to June 29th 2012 at CoRE Building auditorium on the Busch Campus.

ISVD 2012 would not have been possible without funding and support from the following organizations and institutions, for which all organizers and participants of ISVD 2012 express their sincere gratitude:

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2012 Ninth International Symposium on Voronoi Diagrams in Science and Engineering

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Short Papers

2012 Ninth International Symposium on Voronoi Diagrams in Science and Engineering

A Stable Voronoi-based Algorithm for Medial Axis Extraction through Labeling Sample Points

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***Abstract*—This paper presents a Voronoi-based algorithm to extract the medial axis through labeling sample points. A major issue of the medial axis is its inherent instability under small perturbations. The medial axis is very sensitive to small changes of the boundary, which produce many irrelevant branches in the medial axis. Filtering extraneous branches is a common solution to handle this issue; It may be applied as a pre-processing step through simplifying (smoothing) the boundary, or as a post-processing step through pruning, which eliminates the irrelevant branches of the extracted medial axis. However, filtering may alter the topological or geometrical structure of the medial axis. This paper proposes a modification to a Voronoi-based medial axis extraction algorithm to automatically avoid appearing irrelevant branches through labeling the sample points. The experimental results indicate that our method is stable, even in the presence of significant noises and perturbations.**

Keywords-Sample points; Medial axis extraction; Pruning; Voronoi diagram; Delaunay triangulation

I. INTRODUCTION

The Medial Axis (MA) was first introduced by Blum [1] to describe biological shapes, and it is used as a tool in image analysis. The MA is intuitively defined as follows: consider starting a fire at the same moment everywhere on the boundary of a shape in the plane. The fire propagates with homogeneous velocity in all directions. The MA is the set of points where the front of the fire collides with itself, or other fire front. Alternatively, in mathematical language, the medial axis is the set of points that are equidistant from at least two points on the boundary of the shape (Fig. 1).

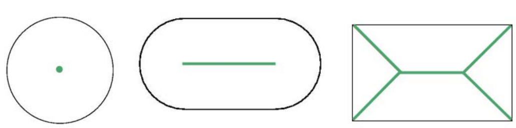

Figure 1. The MA of some 2D shapes

The MA is used in a variety of applications including pattern analysis and shape recognition [2, 3], image compression [4], surface fitting [5], font design [6], path planning [7], solid modeling [8, 9], feature extraction in geometric design [10, 11] and Geospatial Information System (GIS) [12-14].

The MA has some interesting properties, including: There is a unique MA for a given shape [15]; The MA is topologically equivalent to its shape [2, 16]; A shape and its MA are homotopically equivalent [17]; There is a one-to-one relation between a shape and its MA, which means that a shape can be reconstructed from its MA [18]; The dimensionality of a MA is lower than its shape [2].

The *skeleton* is a concept closely related to the MA. Some literatures consider the skeleton equivalent to the medial axis [19], while some others believe they are similar, but are not equal [20]. In this paper, we consider both the MA and the skeleton as equal terms and use the terms, interchangeably.

A major issue of the MA is its inherent instability under small perturbations. The MA is very sensitive to small changes of the boundary, which produce many irrelevant branches in the MA corresponding to non-significant parts of the boundary, so as two very similar shapes can have significantly different MAs (Fig. 2). Filtering extraneous branches is a common solution to handle this issue. Some of the filtering methods work as a pre-processing step through simplifying (smoothing) the boundary before computation of the MA; The others prune the irrelevant branches of the extracted MA as a post-processing step.

The purpose of the filtering methods is to remove irrelevant branches, in order to preserve only the meaningful parts of the MA. In general, however, they may alter the topological or geometrical structure of the MA.

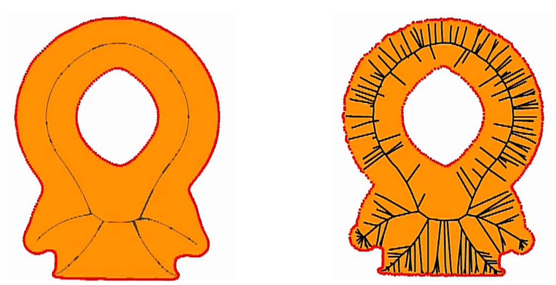

Figure 2. The MA is very sensitive to small changes of the boundary

In this paper, we propose a modification to a Voronoi-based MA extraction algorithm through labeling the sample points. Each main part of the shape is considered as a labeled curve segment. The results illustrate that our

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method is stable, easy to implement, robust and able to handle sharp corners and open curves, even in the presence of significant noise and perturbations.

The rest of the paper is structured as follows: Section 2 presents some related geometric definitions. In Section 3, an existing Voronoi-based algorithm for the medial axis extraction is described. In section 4, a modification to the algorithm presented in section 3 will be proposed based on labeling the sample points. Finally, section 5 concludes the paper and represents ideas for future work.

II. Geometric Definitions

This section represents some geometric preliminaries, including classification of Voronoi vertices and edges, the medial axis and two definitions related to sampling. In this section, $\mathcal{O}$ is a 2D object, $\partial\mathcal{O}$ is its boundary and $S \subset \partial\mathcal{O}$ is a dense sampling of $\partial\mathcal{O}$.

A. *Classification of Voronoi Vertices and Edges*

For Voronoi diagram of sample points S, the Voronoi vertices are classified into *inner* and *outer vertices*, which lie inside and outside $\mathcal{O}$, respectively. Then, the Voronoi edges are classified into three groups: edges between two inner vertices (*inner Voronoi edges*), edges between two outer vertices (*outer Voronoi edges*), and edges between an inner and an outer vertices (*mixed Voronoi edges*).

B. *Medial Axis*

Definition 1. The *medial axis* is (the closure of) the set of points in $\mathcal{O}$ that have at least two closest points on the object's boundary $\partial\mathcal{O}$ [21].

Another description defines the medial axis as the centers of the set of maximal disks contained in $\mathcal{O}$ (Fig. 3). A maximal disk is a disk contained in the shape not exactly covered by another disk contained in the shape.

C. *Local Feature Size and r-sampling*

The quality of sample points S has a direct effect on curve reconstruction. *Local feature size* is a quantitative measure to determine the level of details at a point on a curve, and the sampling density needed for curve reconstruction.

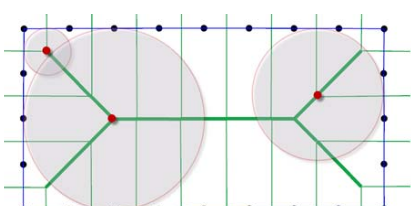


Figure 3. The MA of a 2D curve (rectangle)

Definition 2. The *local feature size* of a point $p \in \partial\mathcal{O}$, denoted as $LFS(p)$, is the distance from p to the nearest point m on the MA [21].

Note that $LFS(p)$ is different from radius of the medial circle, which is the tangent to curve at p (Fig. 4).

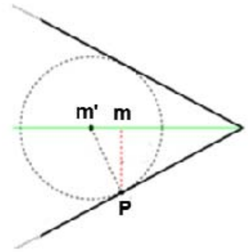


Figure 4. The local feature size of a point p (pm) is not necessarily the same as the smallest radius of the medial circle touching p (pm') [22]

Definition 3. The object $\mathcal{O}$ is *r-sampled* by a set of sample points S if for each point $p \in \partial\mathcal{O}$, there is at least one sample point $s \in S$ that $\| p\text{-}s \| \le r * LFS(p)$ [21].

The value of r is less than 1; and usually r=0.4 is considered a reasonably dense sampling [21]. Fig. 5 shows an example where sample points around the center are denser to provide a proper sampling.

(a) (b)

Figure 5. (a) A curve with its MA; (b) An r-sampling of the curve [22]

III. One-Step Crust and Skeleton Algorithm for Medial Axis Extraction

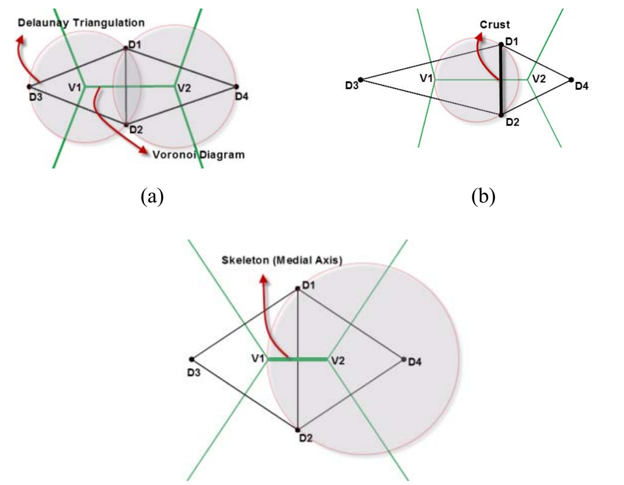
Amenta *et al.* [21] proposed a Voronoi-based algorithm (called crust algorithm) to reconstruct the boundary from a set of sample points forming the boundary of a shape. Gold and Snoeyink [13] improved this algorithm so that both the boundary (crust) and the MA (skeleton) are extracted, simultaneously; and named it "one-step crust and skeleton" algorithm.

In the one-step crust and skeleton algorithm, every Voronoi/Delaunay edge is either part of the crust (Delaunay) or the skeleton (Voronoi), which can be determined by a simple *inCircle* test. Each Delaunay edge (D_1D_2 in Fig. 6.a) belongs to two triangles ($D_1D_2D_3$ and $D_1D_2D_4$ in Fig. 6.a). For each Delaunay edge, there is a dual Voronoi edge (V_1V_2 in Fig. 6.a).

Suppose the two triangles $D_1D_2D_3$ and $D_1D_2D_4$ have a common edge D_1D_2 whose dual Voronoi edge is V_1V_2. The *InCircle*(D_1, D_2, V_1, V_2) determines the position of V_2 respect to the circle passes through D_1, D_2 and V_1. If V_2 is outside the circle, D_1D_2 belongs to the crust (Fig. 6.b). If V_2 is inside, however, V_1V_2 belongs to the skeleton (Fig. 6.c).

The value of $InCircle(D_1, D_2, V_1, V_2)$ test is calculated using the following determinant:

$$InCircle\ (D_1, D_2, V_1, V_2) = \begin{bmatrix} x_{D1} & y_{D1} & x_{D1}^2 + y_{D1}^2 & 1 \\ x_{D2} & y_{D2} & x_{D2}^2 + y_{D2}^2 & 1 \\ x_{V1} & y_{V1} & x_{V1}^2 + y_{V1}^2 & 1 \\ x_{V2} & y_{V2} & x_{V2}^2 + y_{V2}^2 & 1 \end{bmatrix} \quad (1)$$



(a) (b) (c)

Figure 6. One-step crust and skeleton extraction algorithm: (a) Delaunay triangulation and Voronoi diagram of four sample points D_1 to D_4; (b) V_2 is outside the circle passes through D_1, D_2 and V_1, so D_1D_2 belongs to the crust; (c) V_2 is inside the circle passes through D_1, D_2 and V_1, so V_1V_2 belongs to the skeleton.

D_1D_2 belongs to the crust if this determinant is negative, otherwise V_1V_2 belongs to the skeleton [13, 23].

The pseudo-code of the one-step crust and skeleton algorithm is as follows:

One-step crust and skeleton extraction
Input : Sample point S
Output: Crust and MA of the shape approximated by S

1. DT $\leftarrow$ Delaunay Triangulation of S
2. E $\leftarrow$ Edges of DT
3. For every $e \in$ E do
4. S_1, S_2 $\leftarrow$ triangles that contain e
5. D_1, D_2 $\leftarrow$ end points of e
6. V_1, V_2 $\leftarrow$ centers of the circum-circles of S_1 and S_2
7. H $\leftarrow$ $InCircle(D_1, D_2, V_1, V_2)$
8. If $H < 0$ then $D_1D_2 \in$ Crust
9. else $V_1V_2 \in$ Skeleton

IV. Proposed Approach for Medial Axis Extraction

In this section we propose an improvement to the one-step crust and skeleton algorithm through labeling the sample points, and show how our proposed approach improves the results.

Fig. 7.a illustrates the MA of a shape extracted using the one-step crust and skeleton algorithm. As this figure shows, this algorithm detects some extraneous edges as parts of the MA, which are filtered using simplification or pruning. However, we observed that such extraneous edges are the Voronoi edges created between the sample points that lie on the same segment of the curve. It led us to the idea of labeling the sample points in order to automatically avoid appearing such edges in the medial axis (Fig 7.b).

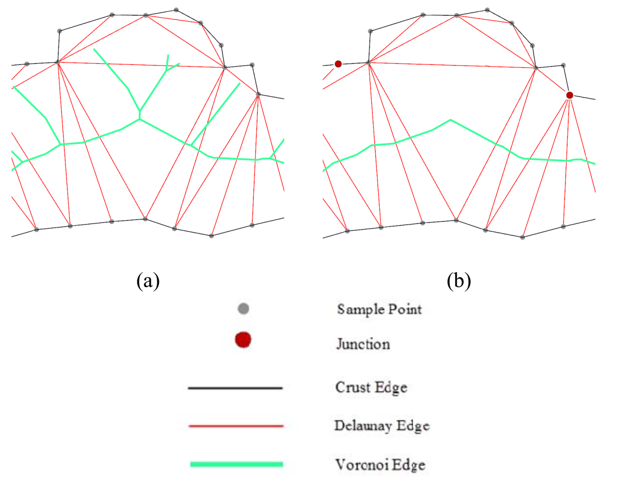


Figure 7. (a) One-step crust and skeleton algorithm detects some extraneous edges as parts of the MA. They are the Voronoi edges created between the sample points that lie on the same segment of the curve; (b) our proposed method automatically avoid such edges in the medial axis.

We consider the shape boundary as different curve segments $\partial\mathcal{O}_i$ and:

$$\partial O = \bigcup_{i=1}^{n} \partial O_i \quad (2)$$

Inner and outer Voronoi edges do not intersect with $\partial\mathcal{O}$, but mixed Voronoi edges do [24]. The same applied to the Delaunay edges: Delaunay edges of the sample points S are classified into three classes: *Mixed Delaunay edges* that join two consecutive points and belong to the crust; and *inner/outer Delaunay edges* that join two non-consecutive points and are completely inside/outside $\mathcal{O}$ (all Delaunay vertices lie on the $\partial\mathcal{O}$). Note that the inner/outer/mixed Voronoi edges are dual to the inner/outer/mixed Delaunay edges.

We observed that the extraneous MA edges are the inner Voronoi edges (or its dual inner Delaunay edges) whose both end points lie on the same curve segment. However, the dual of the main MA edges are the inner Voronoi edges (or its dual inner Delaunay edges) whose end points lie on two different curve segment. Therefore, the main idea of the proposed approach is to remove all the

MA edges whose corresponding Delaunay vertices lie on the same boundary curve.

We start with labeling the sample points: Each segment of the shape is assigned a unique label; and all of its sample points are assigned the same label. The points that are common between two curve segments are called *junctions*, which are assigned a unique negative label to distinguish them from other sample points.

Filtering in our proposed method is not a pre- or post-processing step, but it is performed simultaneously with the MA extraction. To extract the crust and MA, each Delaunay edge passes the *InCircle* test: If the determinant is negative and the corresponding Delaunay vertices have the same labels or one of them is a junction, that Delaunay edge is added to the crust. Otherwise, if the determinant is positive and the corresponding Delaunay vertices have different labels, its dual is added to the MA.

To apply our proposed approach in the one-step crust and skeleton algorithm, the lines 8 and 9 of the pseudo-code presented in section 3 are modified as follows:

8.	If $H < 0$ and label(D_1)=label(D_2) or label(D_1)*label(D_2)<0 then $D_1D_2 \in$ Crust
9.	else if label(D_1) ~= label(D_2) then $V_1V_2 \in$ Skeleton

Fig. 8 compares the result of one-step and crust algorithm and our proposed method.

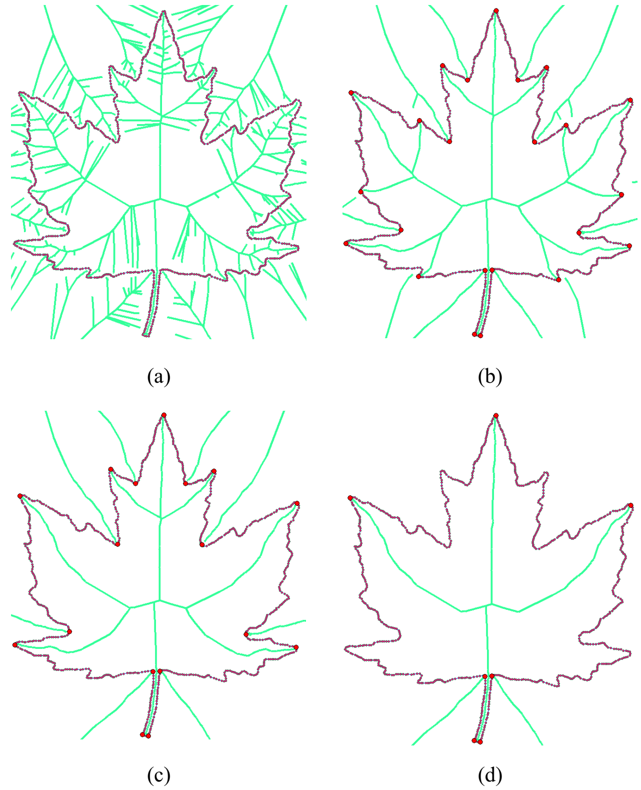


Figure 8. The MA extraction: (a) One-step crust and skeleton algorithm; (b), (c) and (d) our proposed method for different segmentations

A. Stability

Stability is important because few sources of data are ideal. For stable algorithms, the MA should not be sensitive to the small changes of the boundary; in other words, small perturbations in the input data should not lead to large changes in the MA. While existing methods apply a filtering process to remove the irrelevant branches, this issue is automatically solved in our labeling approach: The dual of proper edges in the MA are inner Delaunay edges whose end points lie on two different curve segments. Thus, if the end points of an inner Delaunay edge lie on the same curve segment, its dual inner Voronoi edge will be an irrelevant edge, which does not appear in the MA.

Fig. 9 illustrates the results of one-step and crust algorithm and our approach for an example, before and after addition of boundary perturbations. As this figure shows, the perturbations do not have any effects on the final results.

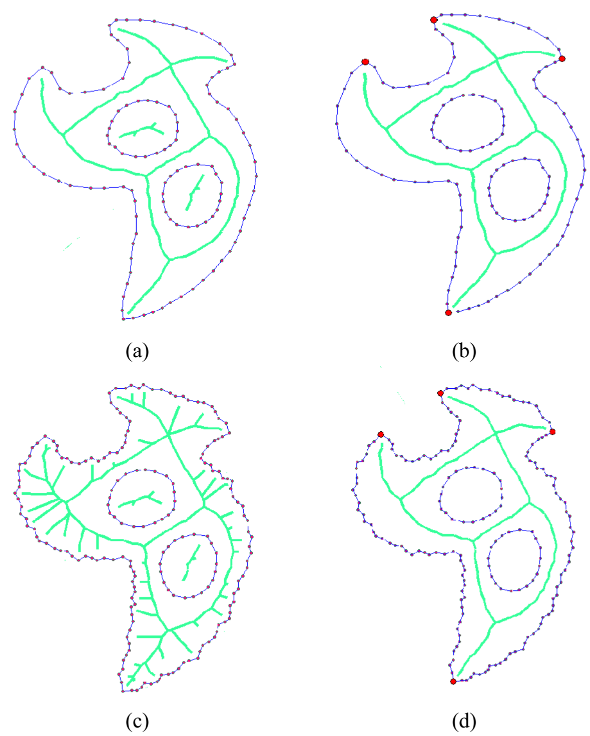


Figure 9. The MA extraction before and after the addition of boundary perturbations: (a) and (c) One-step crust and skeleton algorithm; (b) and (d) our proposed method.

B. Flexibility

Efficient algorithms for exact computation of the MA are only known for a few number of shapes [24]. Flexibility of proposed method increases the variety of shapes that can be used for the MA computation (Fig. 10). The shape and its complexity do not have any effect on the final results. This flexibility is due to labeling the sample points. Proposed algorithm can be used for open curve reconstruction and their MA computation, which is an issue for other Voronoi-based algorithms (Fig. 11).

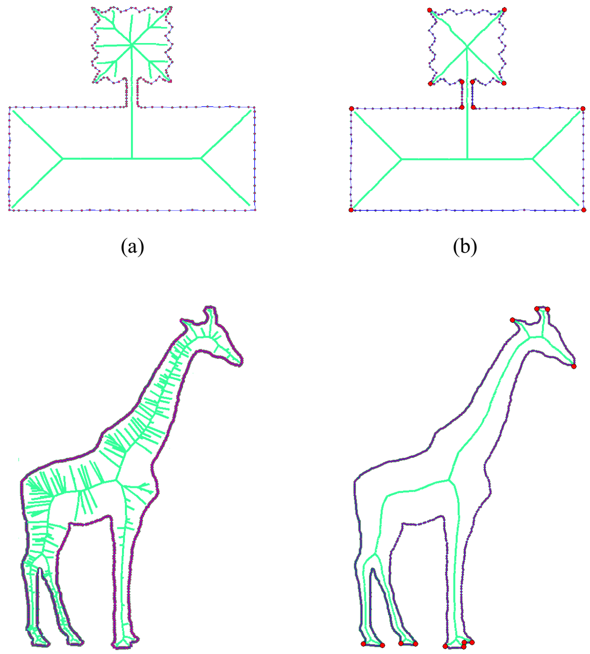


Figure 10. The MA extraction: (a) and (c) One-step crust and skeleton algorithm; (b) and (d) our proposed method

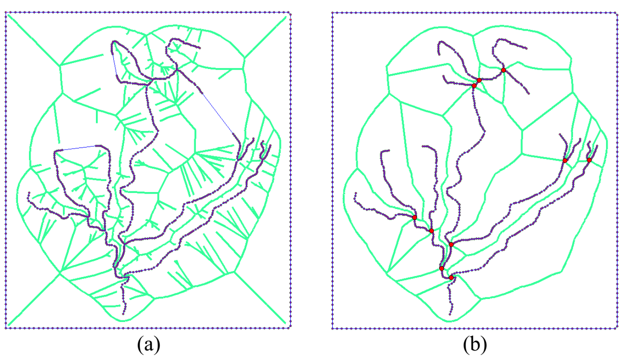


Figure 11. The MA extraction of open curves: (a) One-step crust and skeleton algorithm; (b) our proposed method

C. Accuracy and Precision

The MA computation methods are classified into discrete, semi-continuous and continuous methods. The accuracy of semi-continuous methods depends on the density of sample points: the more sample points, the more exact MA. The disadvantage of most existing methods is their low precision. Simplification and pruning which is done in most of the algorithms can alter the topological or geometrical structure of MA. This problem is solved in the proposed method.

D. Complexity

Pruning algorithms are complex and repetitive, and does not work automatically, which results in high complexity and low speed of the MA extraction algorithm. Filtering in our method is not a pre- or post- processing step, but is integrated in the MA computation as a simple check and does not affect the running speed of the algorithm. Furthermore, complexity of all algorithms depends on the amount of noise and perturbation, while the complexity of our proposed method only depends on the number of sample points.

E. Special issues

Conjunctions are usually a special case in medial axis extraction. Changing the sampling density may solve the problem. We used different densities (r=1, 0.5, 0.42, 0.24) in one-step crust and skeleton algorithm, and the result was only correct for r=0.24 (Fig. 12). It shows that increasing the density for the whole shape may not necessarily lead to a more accurate result.

In our proposed approach, such problems are automatically avoided, because a crust edge can be created between two points with the same label or two points with different labels if one of them is a junction.

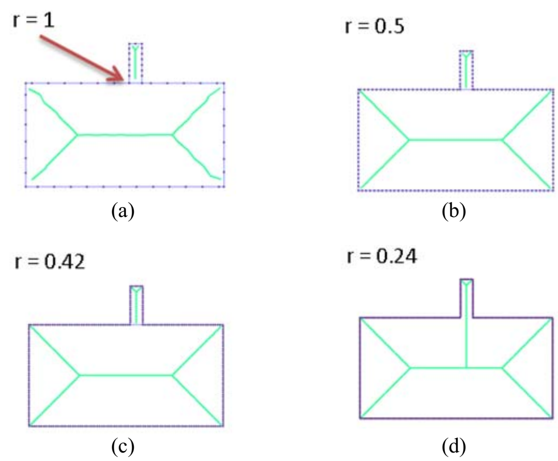


Figure 12. The medial axes extraxtion using the one-step crust and skeleton algorithm for different r-samplings (the result is only correct for r=0.24).

V. Conclusion and Future Works

In this paper, we improved one of the Voronoi-based MA extraction algorithms (i.e., one-step crust and skeleton algorithm) through labeling the sample points. It leads to a solution that is simple and easy to implement, robust to boundary perturbations, able to handle sharp corners and open curves. The results show that our proposed approach deals elegantly with different cases of sample points and solves the problems that may occur in other algorithms.

Simplification and pruning which is done in most of the algorithms can alter the topological or geometrical structure of the MA. The results illustrate that our method are stable, even in the presence of significant noise and perturbations, and have the same topology as the original MA.

In the future, we will extend the approach for surface reconstruction and 3D MA extraction. We will also study in more details the relationship between curve reconstruction and the MA extraction, as well as their applications in other fields.

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