Cross-Scale Analysis of Sub-pixel Variations in Digital Elevation Models

Mehran Ghandehari, Barbara P. Buttenfield and Carson J.Q. Farmer

Abstract Terrain is modeled on a grid of pixels, assuming that elevation values are constant within any single pixel of a Digital Elevation Model (DEM) (a 'rigid pixel' paradigm). This paradigm can generate imprecise measurements because it does not account for the slope and curvature of the terrain within each pixel, leading to precious information being lost. In order to improve the precision of interpolated points, this paper relaxes the rigid pixel assumption, allowing possible sub-pixel variations (a 'surface-adjusted' paradigm) to be used to interpolate elevation of arbitrary points given a regular grid. Tests based on interpolating elevation values for 5000 georeferenced random points from a DEM are presented, using the rigid pixel paradigm and different interpolation methods (e.g., Weighted Average, Linear, Bi-linear, Bi-quadratic, Bi-cubic, and best fitting polynomials) within different contiguity configurations (i.e., incorporating first and second order neighbors). The paper examines the sensitivity of surface adjustment to a progression of spatial resolutions (10, 30, 100, and 1000 m DEMs), evaluating sub-pixel variations that can be directly measured from 3 m resolution LIDAR data.

Keywords Digital elevation models · Rigid pixel paradigm · Sub-pixel estimation · Surface-adjusted analysis

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M. Ghandehari (✉) · B.P. Buttenfield · C.J.Q. Farmer
Department of Geography, University of Colorado, Boulder, CO 80309, USA
e-mail: mehran.ghandehari@colorado.edu

B.P. Buttenfield
e-mail: babs@colorado.edu

C.J.Q. Farmer
e-mail: carson.farmer@colorado.edu

M.P. Peterson (ed.), *Advances in Cartography and GIScience*, Lecture Notes in Geoinformation and Cartography, DOI 10.1007/978-3-319-57336-6_25

1 Introduction

Terrain models guide scientists and planners in multiple ways and have fundamental impacts on society, safety, and resource management. For example, digital terrain data is used by natural hazards scientists to model flooding inundation and debris flows (Griffin et al. 2015) and by hydrologists to model water flow direction and accumulation (Stanislawski et al. 2015). Ecologists use terrain models to delineate habitats and nesting territory or to predict the presence of certain species (Czarnecka and Chabudziński 2014). Avalanche risk is measured using solar insolation metrics based on slope and aspect (Jaboyedoff et al. 2012). Civil engineers plan construction of road and railroad networks on the basis of elevation and slope in conjunction with other factors (Zhao et al. 2005). This research contests a prevailing paradigm for terrain-based analyses, which forms the foundation for nearly all environmental models.

A terrain surface is commonly modeled as a regular grid of elevation values (a DEM). Modeling assumes that each pixel is a horizontal planar surface with a constant elevation. We refer to this modeling assumption as a *'rigid pixel'* paradigm (Fig. 1a). The slope and curvature of the terrain is ignored in this model, resulting in imprecise estimates of elevation, which in turn can generate errors in distance, area and volume. In addition, the assumption of constant elevation within each pixel implies an abrupt height change (i.e., discontinuity) at pixel boundaries.

In contrast, a '*surface-adjusted*' paradigm (Buttenfield et al. 2016) relaxes the assumption of the rigid pixel paradigm by acknowledging that sub-pixel variations may occur in elevation, slope and curvature (Fig. 1b). Other researchers utilize partial surface-adjustment methods that account for elevation, slope and aspect of the terrain surface. The *surface-adjusted* paradigm incorporates elevation, slope, aspect, and curvature of DEM pixels into computations.

This paper demonstrates interpolation methods as one possible strategy for the incorporation of sub-pixel variations. Interpolation methods can reconstruct the three-dimensional (3D) surface of each pixel using contextual information from adjacent pixels. For example, a bilinear polynomial estimates a surface at each data point using the four closest pixel centroids. Thus, a series of contiguous bilinear

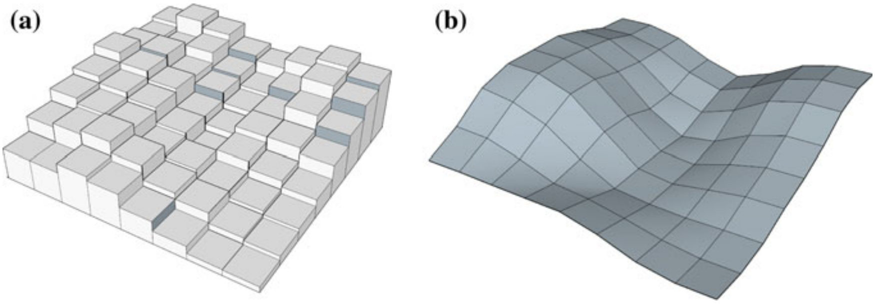

Fig. 1 (**a**) The rigid pixel paradigm and (**b**) the surface-adjusted paradigm

surfaces generate the whole terrain surface. Higher-order polynomials can be employed, but due to computational complexity, they are not normally used (though these higher-order polynomials are also evaluated in this paper). Initial results indicate that for any *individual* pixel, the improvement in measurements can be relatively small, however, the *additive* effect across the study area can become quite significant.

In the last decade, Light Detection and Ranging (LIDAR) has provided new opportunities for studying Earth surface processes, providing more precise measurements (Tarolli 2014). Although it is a fair assumption that elevation values are constant within any single pixel in LIDAR, this data provides some other challenges; LIDAR data needs to be filtered as a preprocessing step to remove artefacts and extract bare earth points. The filtering process reduces the accuracy of the original data and might eliminate or distort significant features (Passalacqua et al. 2015). LIDAR data has become more available but processing of this data is still a challenge due to data storage costs and long processing times. Furthermore, most GIS projects call for integration of various data layers. Ancillary data in the form of vegetation, precipitation, soil data, etc., is not generally available at fine resolutions, requiring filtering to harmonize LIDAR data with other data sources. Finally, and most importantly, the current availability of LIDAR data is far from comprehensive in developed nations, and non-existent in many rural and undeveloped regions.

The paper presents methods for cross-scale analysis of sub-pixel variations in DEMs. Different interpolation methods (Weighted Average, Linear, Bi-linear, Bi-quadratic, Bi-cubic, and best fitting polynomials) are compared, along with different contiguity configurations that incorporate first and second order neighbors. Methods and configurations are tested to interpolate the elevation of 5000 georeferenced random points from a DEM. This research examines the sensitivity of surface adjustments to a progression of spatial resolutions (10, 30, 100, and 1000 m DEMs), using sub-pixel variations that can be directly measured from a 3 m resolution LIDAR data benchmark. The analysis compares Root Mean Square Error (RMSE) between interpolated data and LIDAR to assess differences among the various methods and resolutions.

Discrepancies (error magnitudes) are expected to vary with DEM resolution, terrain roughness and landscape conditions (Buttenfield et al. 2016). With advances in computer processing speed and increased data availability, the scientific community must address limitations of existing modeling methods to provide decision-makers the best possible information about critical environmental issues including biophysical and anthropogenic factors and interactions.

2 Dataset and Study Area

The study area was chosen based on the availability of 3 m resolution LIDAR data to be used for validation. While other resolution terrain data are available for the continental US, coverage at 3 m resolution is much more limited. The study area in

North Carolina is centered on 35.798° N and 81.473° W. Its location at the southeast end of the Appalachian mountains provides elevations ranging from 209 to 1602 m. Its location where the Blue Ridge Mountains run down towards the coastal plains is a humid, hilly landscape, with annual precipitation averaging 51 in. (129.5 cm). The study area provides a mix of uninhabited land with smaller rural settlements.

DEM data will be tested at 10, 30, 100, and 1000 m resolutions, and compared against a 3 m LIDAR benchmark. The 3, 10, and 30 m resolutions are part of the USGS National Elevation Dataset (NED) and were downloaded from the Geospatial Data Gateway (https://gdg.sc.egov.usda.gov/). These are independently compiled DEMs generated according to the methodology outlined in Gesch (2007). The accuracy of NED varies spatially due to the diversity of data sources. The overall absolute vertical accuracy of this dataset has an RMSE of 2.44 m (Gesch et al. 2014). The source for 100 and 1000 m resolutions is the Shuttle Radar Topography Mission (SRTM) dataset (http://dds.cr.usgs.gov/srtm/version2_1/). The absolute vertical accuracy of this dataset has an RMSE of 4.01 m (Gesch et al. 2012). The DEMs are projected in the NAD 1983 UTM Zone 17 N coordinate system.

3 Methods

Tests will interpolate the elevation of 5000 georeferenced random points using different methods and contiguity configurations, across a progression of DEM resolutions. The tested methods all incorporate elevation, but differ in contextual information about surrounding pixels to gain a progression of surface adjustment methods. The methods are compared and validated against a 3 m LIDAR data benchmark (Fig. 2).

Figure 3 diagrams the research workflow that involves a three-factor research methodology. The first factor (along the x-axis) refers to tests of four spatial resolutions (10, 30, 100 and 1000 m). The second factor (along the y-axis) refers to tests for eight interpolation methods. The third factor (along the z-axis) indicates configurations of neighboring pixels to be tested. A range of contiguity configurations increases the number of pixels incorporated into elevation estimation. The second and third factors are not independent, since the interpolation method in some cases dictates the number of neighboring pixels. That is to say, tests for the polynomial fitting and Weighted Average interpolators utilize all configurations except the single pixel (1 neighbor). The other five interpolators use 1, 3, 4, 9, and 16 neighboring pixels, respectively. For the Weighted Average and polynomial fitting interpolation methods, all of the contiguity configurations are tested to find the optimum number of neighboring pixels.

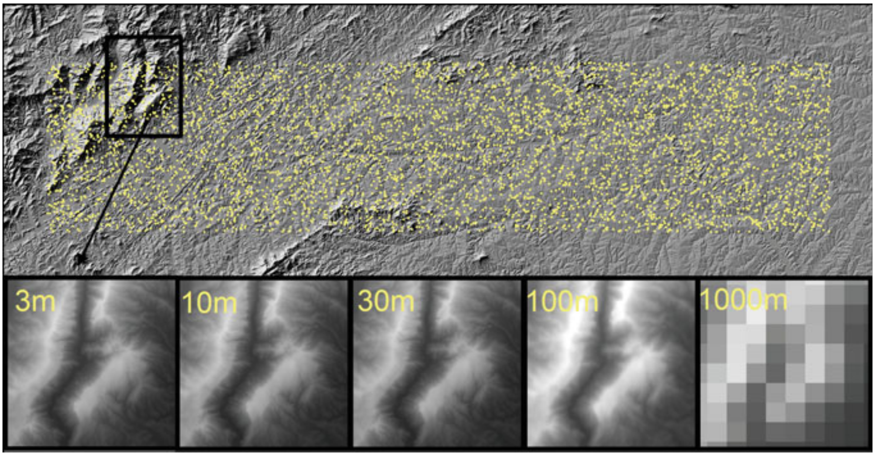

Fig. 2 3 m LIDAR DEM with four other resolutions. Dots show 5000 points sampled randomly within the study area

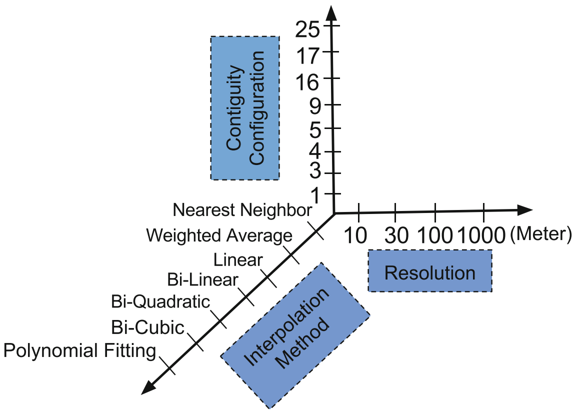

Fig. 3 The research methodology tests estimates of DEM elevations spanning four common spatial resolutions (*x-axis*), seven interpolation methods (*y-axis*) and eight contiguity configurations that incorporate information from a progression of neighboring pixels (*z-axis*). Nearest Neighbor, Linear, Bi-linear, Bi-quadratic, Bi-cubic interpolation methods are exact interpolation methods and utilize 1, 3, 4, 9, 16 neighboring pixels, respectively. The Weighted Average and polynomial fitting interpolation methods are tested in all but one contiguity configuration

3.1 Interpolation Methods

Given a regular elevation grid within the defined neighborhood, different interpolation techniques will generate differing elevation estimates. The methods compared in this paper include Nearest Neighbor, Weighted Average and exact polynomial surfaces (Linear, Bi-linear, Bi-quadratic, and Bi-cubic), all of which are exact interpolators, and least-squares polynomial fitting, an inexact interpolator.

• ***Nearest Neighbor (rigid pixel paradigm)***: This method is based upon the rigid pixel paradigm. Each point represents a small area, a "region of influence" (i.e., the pixel) surrounded by a horizontal planar surface. This interpolator is utilized in this paper as a "straw horse," a control method by which to compare the various surface adjusted interpolators. Estimation proceeds by assigning the elevation of the centroid for whatever pixel the sampled point lies within (Fig. 4a). Although this

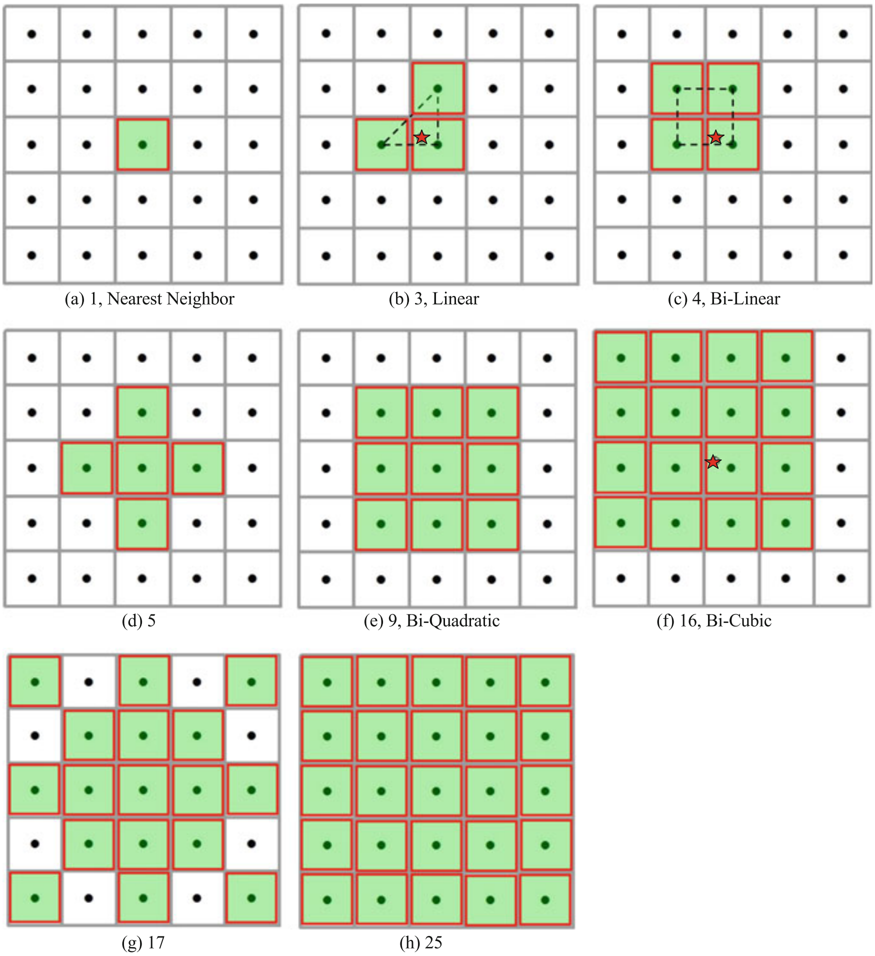

Fig. 4 Contiguity configurations; in (**b**), (**c**), and (**f**), the sample point is represented as a star and 3, 4, and 16 closest pixels are highlighted respectively. The Nearest Neighbor, Linear, Bi-linear, Bi-quadratic, and Bi-cubic interpolators are implemented based on 1, 3, 4, 9, and 16 neighboring pixels respectively

approach is quite simple and easy to implement, it does not incorporate terrain derivatives and thus there is no surface adjustment.

- ***Weighted Average***: This is a deterministic interpolator that computes an average elevation based on neighboring pixels. Distance to neighboring pixel centroids is commonly used as a weighting function, with shorter distances carrying larger weights. Weighted distance assumes that points that are closer together are more similar than those that are farther apart. Different distance functions can be used. The simplest and most common is the "Inverse Distance Weighting" (IDW) method, commonly used for continuous data such as terrain:

$$z(x,y) = \frac{\sum_{i=1}^{\mathrm{n}} w_i z_i}{\sum_{i=1}^{\mathrm{n}} w_i}; \quad w_i = \frac{1}{d_i^{\mathrm{p}}}$$

where w_i and z_i are the weight and elevation of each source point, respectively. Distance (d) is given an exponent (p); and a larger exponent leads to progressively smaller weights at larger distances.

- ***Exact Polynomial Surfaces***: This is a set of deterministic methods whose estimates are based on a polynomial equation with varying amounts of terms and powers. By definition, the exact polynomial will generate a precise estimate of any original point. Polynomial equations with more terms create a surface with more freedom to undulate (Li et al. 2004). Four different exact polynomial interpolators are used in this research:

Linear: A triangle is generated from a three-term polynomial. Linear interpolation needs the three closest points to solve the equation (Fig. 4b). The mathematical function creates a plane:

$$z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y}$$

Bi-linear: This is also a first order polynomial based upon four known points (Fig. 4c).

$$z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y} + a_3\mathrm{xy}$$

Bi-quadratic: This second order polynomial uses nine terms, allowing for more freedom of undulation. Nine closest neighbors are selected to solve the Bi-quadratic equations (Fig. 4e).

$$z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y} + a_3\mathrm{xy} + a_4x^2 + a_5y^2 + a_6x^2y^2 + a_7x^2y + a_8xy^2$$

Bi-cubic: This third order polynomial increases the surrounding pixels to sixteen closest neighbors (Fig. 4f). This method calculates a surface with additional local maxima and minima.

$$z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y} + a_3\mathrm{x\ y} + a_4x^2 + a_5y^2 + a_6x^2y^2 + a_7x^2y + a_8xy^2 + a_9x^3 + a_{10}y^3 + a_{11}x^3y^3 + a_{12}x^3y^2 + a_{13}x^2y^3 + a_{14}x^3\mathrm{y} + a_{15}xy^3$$

• ***Best Fitting Polynomials***: Terrain is a complex surface and using high-order polynomials in an exact fitting approach can lead to fluctuations in the resulting elevation estimate. Least-Squares Polynomial Best Fitting is a common way to resolve fluctuations by minimizing the sum of squared errors. This is an inexact interpolator, and is not necessarily a better approximation of the terrain surface. A best-fitting surface can be linear or a smoothed curved surface. Best fitting polynomials of order 1, 2, 3, and 4 are used as follows:

$$Order1: z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y}$$

$$Order2: z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y} + a_3\mathrm{x\ y} + a_4x^2 + a_5y^2$$

$$Oder3: z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y} + a_3\mathrm{x\ y} + a_4x^2 + a_5y^2 + a_6x^2y + a_7xy^2 + a_8x^3 + a_9y^3$$

$$Order4: z(x,y) = a_0 + a_1\mathrm{x} + a_2\mathrm{y} + a_3\mathrm{x\ y} + a_4x^2 + a_5y^2 + a_6x^2y + a_7xy^2 + a_8x^3 + a_9y^3 + a_{10}x^4 + a_{11}y^4 + a_{12}x^2y^2 + a_{13}x^3\mathrm{y} + a_{14}xy^3$$

Note that the first order polynomial shows an equation identical to the Linear exact polynomial described above. The difference is that the Linear exact method is tested based on the three-neighbor contiguity configuration, but the first order best fitting method is tested based on the four and higher neighbor contiguity configurations using the least square equation.

3.2 Contiguity Configuration

Different contiguity configurations can be investigated based upon the number of neighbors (here 3, 4, 5, 9, 16, 17, 25). In this research, first and second order neighboring pixels are combined in different ways to configure the contiguity structure of sub-pixel variations (Fig. 4). A 1-pixel contiguity configuration is used as a control to compare the various surface-adjusted methods with the rigid pixel "Nearest Neighbor" method. Some interpolation methods are an exact interpolation technique and can only be tested on one contiguity configuration. For example, the Bi-linear interpolation method is based upon four neighbors. On the other hand, the Weighted Average can be tested on multiple contiguity configurations. The Nearest Neighbor, Linear, Bi-linear, Bi-quadratic, and Bi-cubic interpolators use 1, 3, 4, 9, and 16 neighboring pixels, respectively. For the Weighted Average, all the contiguity configurations except the single pixel are tested to find the optimum number of neighboring pixels. For the best fitting polynomial, the single pixel and the 3-pixel configurations are left out to insure inclusion of sufficient pixels in the least squares equations.

3.3 Workflow and Processing

The workflow involves several steps. Each resolution DEM is input one at a time. The 3 m benchmark DEM is loaded and a random sample of 5000 points is generated, creating a shapefile containing coordinate positions in UTM meters, and elevations in meters extracted from the 3 m benchmark DEM. At this point, the LIDAR DEM is unloaded, and the shapefile is stored on disk. For each of the test DEMs, a nested loop processes each point for all DEMs and for all interpolation methods, then moves on to the next sampled point. Estimated elevations are added as new attributes in the original shapefile. Within the same nested loop, residuals are calculated for each elevation (each sampled point). Once all sampled elevations have been estimated for all DEMs at all resolutions, the residuals are summarized with RMSE standard deviation of residuals, 95% confidence interval and elevation extrema.

The overall sequence of operations is as follows:

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Cross-scale analysis of sub-pixel variation in DEMs using various interpolation methods

(1) Load 4 Input DEMs (DEM10, DEM30, DEM100, DEM1000)
(2) Load benchmark (DEM3)
(3) Generate sample of 5000 random points in the study area
(4) For i in 5,000 random points:
    For j in 4 Input DEMs:
        For k in 7 interpolation methods:
            - Interpolate the elevation of point i based on the DEM resolution j using interpolation method k ($Elev_{ijk}$)
            - Compute $Residual_{ijk} = Elev_{ijk} - Elev_{i}$ of benchmark
(5) Accuracy Assessment (RMSE, Standard Deviation, etc.)

3.4 Accuracy Assessment

Accuracy assessment is conducted on the residuals. Prior to accuracy assessment, the residuals are tested for normality. There are several ways to examine the accuracy of interpolated sample points. We compare the interpolated elevation of sample point to 3 m resolution LIDAR data. The residuals are calculated by subtracting the lidar value from the interpolated value. Next, we summarize standard deviation (STD) of residuals, Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and 95% confidence interval surrounding the RMSE values.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}}; \quad MAE = \frac{\sum_{i=1}^{n} |e_i|}{n}$$

4 Results and Discussion

4.1 Analysis of Residuals

Table 1 represents basic descriptive parameters of the residuals (mean, median, minimum, maximum, standard deviation, and 95th percentile–lower and upper bound) at different resolutions. These parameters are based on all sample points and for all of interpolation methods. The mean value is slightly positive (10, 100, and 1000 m) or negative (30 m) indicating overestimation and underestimation, respectively. Furthermore, the high values of minima and maxima are due to the occurrence of both positive and negative outliers. The standard deviation values are small, relative to the mean and data range.

Two standard normality tests are run on the residuals—the Kolmogorov–Smirnov test and the Anderson–Darling test. The results of these tests verify that residuals are not normally distributed. Based on the American Society for Photogrammetry and Remote Sensing (ASPRS) (Flood 2004) and the National Digital Elevation Program (NDEP) (NDEP 2004) guidelines, the 95th percentile can be used for reporting the vertical accuracy of elevation date when the error distribution is not normal.

Outliers are an indication to inspect the spatial pattern of residuals more closely. Local Moran's I (Anselin 1995) was calculated for the residuals in order to visualize clusters whether residuals with similar values (either high values or low values are clustered) and outliers. The inverse distance squared method was used and the neighborhood delineated that ensured every point has at least one neighbor. Residuals display spatial heterogeneity due to the varied character of terrain. The circles in Fig. 5 highlight the extreme residuals and illustrate that many are situated

Table 1 Summary statistics of residuals for different resolutions

Resolution	Mean	Median	Min	Max	Standard Deviation	95th percentile-lower bound	95th percentile-higher bound
10	0.01	0.0	−10.91	10.63	1.07	−1.53	1.60
30	−0.02	−0.12	−35.53	41.26	3.94	−5.60	6.28
100	5.11	3.71	−40.90	66.37	7.95	−4.75	19.52
1000	4.06	4.09	−399.81	381.55	41.89	−35.52	47.39

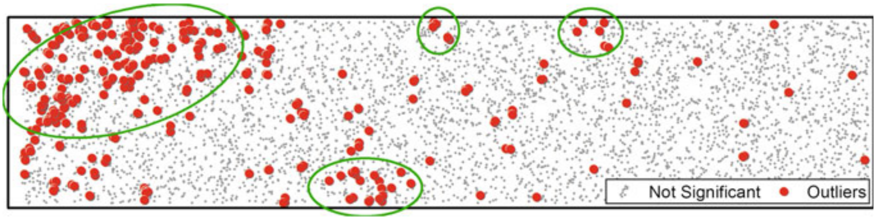

Fig. 5 Outliers of residuals in 10 m DEM using the Bi-linear interpolation

Table 2 The correlation and *p*-values in 95% confidence interval between residuals and elevation, slope, aspect, curvature, local relief, and roughness (30 m DEM is used here). *P*-values are so small that even at 4 or 5 decimal places they do not differ meaningfully from zero

Variable	Elevation	Slope	Aspect	Curvature	Local relief	Roughness
Correlation	0.38	0.62	–0.06	0.33	0.59	0.53
P-values	0.0000	0.0000	1.6e-05	0.0000	0.0000	0.0000

in rough terrain (in the northwest corner). These imply that terrain type is an important factor needing further investigation in future research.

Table 2 reports the correlation between the residuals and elevation, slope, aspect, curvature, local relief (the range of elevation in a 10 by 10 focal window), and roughness (the standard deviation of elevation in a 5 by 5 window) based on the 30 m DEM. All of the correlation coefficients are highly significant, though the relationship with aspect is weaker than with other terrain derivatives. The high significance levels could be a consequence of the large number of sampled pixels. Slope has the highest correlation with the residual, indicating the accuracy of DEM in terrain with high slopes is lower than with flatter slopes. In addition, there is a strong relationship between residuals and terrain roughness and local relief; and this also reflects the slope relationship.

4.2 Optimal Configuration for Weighted Average Interpolator

Table 3 shows the RMSEs resulting from the Weighted Average method for all contiguity configurations, and across all resolutions. The range of RMSEs in each resolution is quite small. The 16 neighboring pixel configuration is chosen as the optimum contiguity configuration as it shows the lowest RMSE in most resolutions for this interpolator, and is chosen for the further analysis in Sect. 4.4.

Table 3 RMSEs for the weighted average method using different number of neighbors in different resolutions; boldface indicates the lowest RMSE in each resolution

Contiguity Configuration	10 m	30 m	100 m	1000 m
WA3	1.09	4.01	**9.46**	42.57
WA4	**1.08**	3.97	**9.46**	42.24
WA5	1.10	4.02	9.51	42.53
WA9	**1.08**	3.95	9.50	41.56
WA16	**1.08**	**3.88**	9.53	**40.93**
WA17	**1.08**	3.91	9.54	41.15
WA25	**1.08**	3.89	9.61	40.80
Range of RMSEs (m)	*0.02*	*0.14*	*0.15*	*1.64*

4.3 Optimal Configuration for Best Fitting Polynomials

Table 4 shows the RMSEs for the best fitting polynomial method. Different orders and number of neighbors are tested. In the Table, polynomial order 1 shows the lowest RMSE. Reading down the columns for a given order, there is a general trend where RMSE increases with increasing numbers of neighboring pixels. At 1000 m resolution, the opposite occurs: polynomial order 1 with contiguity configuration 4 has the highest RMSE. Again, there is not an optimum method in all of the resolutions. Polynomial order 1 with 5 neighbors was selected for further analysis, having the lowest RMSE.

Table 4 RMSEs for the best fitting method using various polynomial orders and different number of neighbors in different resolutions. Boldface indicates the lowest RMSE in each resolution

Polynomial order	Contiguity Configuration	9 m	30 m	100 m	1000 m
1	4	**1.07**	3.94	**9.45**	42.09
	5	**1.07**	3.87	9.47	40.85
	9	1.08	**3.82**	9.65	**39.24**
	16	1.13	3.90	10.12	39.45
	25	1.22	4.14	10.76	40.75
2	9	1.08	**3.82**	9.65	39.26
	16	1.13	3.90	10.12	39.48
	17	1.18	4.02	10.44	40.17
	25	1.22	4.14	10.76	40.77
3	9	1.08	**3.82**	9.67	39.51
	16	1.13	3.90	10.12	39.50
	17	1.18	4.01	10.44	40.20
	25	1.22	4.14	10.76	40.80
4	16	1.14	3.91	10.18	39.58
	17	1.19	4.02	10.51	40.71
	25	1.22	4.14	10.76	40.82

4.4 Comparing Surface-Adjusted Elevations with the Rigid Pixel Paradigm

The comparison of interpolated elevations with the rigid pixel paradigm is accomplished by comparing results discussed so far with the Nearest Neighbor (NN) method. Recall that the NN method uses only a single pixel for estimating elevation. Table 5 shows the method comparison. Methods are ordered across the table columns by the number of pixels in each configuration. Only the optimal configurations selected for Weighted Average (WA) and for Best Fitting (BF) are included here, for comparison.

Error magnitudes vary with DEM resolution and with interpolation method. Accuracy assessment parameters show a general trend of increase in the residuals at coarser resolutions. The Nearest Neighbor method shows highest magnitude errors for all resolutions, except the 100 m in which the Bi-cubic method has the highest amount of error. Clearly, these results are mixed. Best Fitting Linear methods show the best performance (lowest RMSE) at 10 m resolution, with Linear, Bi-linear and Best Fitting Linear showing lowest residuals at 30 and 100 m. The Bi-quadratic RMSE is lowest at the coarsest 1000 m resolution. Readers should note very similar values for all metrics at each resolution indicating that this analysis should be repeated for other terrain surfaces before a definitive conclusion can be reached.

Table 5 Accuracy assessment parameters for different interpolation methods in different resolutions; Columns show interpolation methods (left to right): Nearest Neighbor, linear using 3 neighbors, Bi-linear using 4 neighbors, best fitting polynomial of order 1 using 5 neighbors, Bi-quadratic using 9 neighbors, Bi-cubic using 16 neighbors, weighted average using 16 neighbors. Boldface indicates the lowest RMSE in each resolution. STD = standard deviation of residuals

		NN	Li3	BiLi4	BF1_5	BiQ9	BiC16	WA16
10 m	*RMSE*	1.19	1.08	**1.07**	**1.07**	1.08	1.13	1.08
	RMSE (95%)	0.75	0.66	**0.65**	0.67	0.68	0.73	0.67
	MAE	0.72	0.65	0.65	0.65	0.66	0.71	0.66
	STD	1.19	1.08	1.07	1.07	1.08	1.13	1.09
30 m	*RMSE*	4.34	3.95	3.94	**3.87**	3.82	3.90	3.88
	RMSE (95%)	2.75	2.58	2.58	**2.53**	2.55	2.64	2.57
	MAE	2.70	2.53	2.53	2.50	2.50	2.61	2.51
	STD	4.34	3.95	3.95	3.87	3.82	3.90	3.88
100 m	*RMSE*	9.85	**9.45**	**9.45**	9.47	9.65	10.12	9.53
	RMSE (95%)	8.25	**7.97**	**7.97**	7.99	8.06	8.37	8.01
	MAE	6.87	6.61	6.61	6.63	6.75	7.08	6.68
	STD	8.42	7.94	7.95	7.99	8.19	8.72	8.037
1000 m	*RMSE*	45.08	42.20	42.09	40.85	**39.26**	39.50	40.93
	RMSE (95%)	23.77	22.22	22.34	22.04	**21.51**	21.84	21.87
	MAE	24.69	23.52	23.50	22.97	22.36	22.42	22.95
	STD	44.92	41.85	42.00	40.65	39.05	39.28	40.72

5 Summary

This work advances understanding of how spatial error propagates through resolutions, a cross-scale terrain analytics component that is at present not well understood. The current paradigm for estimating terrain elevations assumes that the DEM terrain surface is uniform within each pixel, ignoring slope and curvature. This research employs realistic surface geometries of terrain using different interpolation methods and the information from adjacent pixels, searching for a more accurate and precise interpolation of points. Sub-pixel variations in DEMs through different resolutions are investigated to develop a foundation of surface-adjusted computations on DEMs, resulting in more realistic analysis in terrain models.

Findings for this DEM, and for the interpolators tested here, indicates that different interpolators generate lowest RMSE values at various resolutions, with some type of linear polynomial performing well at finer and intermediate resolutions. RMSE values are so close for this analysis that it is difficult to conclude one method is preferable to the other. Ideally, one would prefer an interpolation method that carries the lowest computational intensity, such as the Weighted Average method. In this experiment, however, the Weighted Average RMSEs do not outperform the other methods of estimation.

The next stage of research will investigate how the results vary with geographic conditions (different types of terrain) and how terrain differences result in elevation estimations that affect modeling applications. Finally, the assumption of non-rigid pixels carries an additional computational load, as it must incorporate not only elevation and pixel size, but also slope and curvature, in effect, adjusting for the changing terrain surface at sub-pixel resolutions. The longer-term goal of this research addresses the balance between the increased computations needed to measure surface-adjusted elevation against the improvement in precision.

Acknowledgements This research is supported in part by the Grand Challenge Initiative "Earth Lab" (http://www.colorado.edu/grandchallenges/) funded by the University of Colorado. We acknowledge the USGS Center for Excellence in Geospatial Information Science (CEGIS) for analytic advice.

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