HOEK-BROWN VS. MOHR-COULOMB – RESULTS FROM A THREE-DIMENSIONAL OPEN-PIT/UNDERGROUND INTERACTION MODEL

HOEK-BROWN VS. MOHR-COULOMB – RESULTAT FRÅN 3D DAGBRÖTT/UNDERJORDISK INTERAKTION MODELL

David Saiang, Xander Gwynn, SRK Consulting (Sweden)
Neil Marshall, SRK Consulting (UK)

ABSTRACT

Since its introduction into numerical codes such as FLAC and Phase^2, the Hoek-Brown failure model has gained rapid popularity in its application in rock mechanics in the recent times. SRK Consulting has recently applied the Hoek-Brown and Mohr-Coulomb "models" concurrently to study the interaction between the open pit and underground mining of a large complex orebody. A large FLAC3D model was constructed and a short FISH program was written to permit automatic switching between Hoek-Brown and Mohr-Coulomb criteria, with appropriate inputs for the two models introduced. Experience from the mine was used to assess the accuracy of the FLAC3D model results. The results of Hoek-Brown and Mohr-Coulomb were clearly different and particularly the Hoek-Brown model results appeared questionable. Being aware of the difference between the two "models", their respective uses were based on the desired outcome to be achieved. The results from this simulation clearly show that the Mohr-Coulomb model captures the behavior very well, while Hoek-Brown needed a change of modelling approach and some tweaking to respond satisfactorily. SRK Consulting was nevertheless aware of the fact that Hoek-Brown is not a typical constitutive model. The results clearly demonstrate the limitation of the Hoek-Brown model for plastic analysis.

SAMMANFATTNING

Sedan introduktionen i numeriska koder som FLAC och Phase^2 har Hoek-Browns brottsmodell snabbt ökat i popularitet för tillämpningar inom bergmekanik. SRK Consulting har nyligen tillämpat Hoek-Brown och Mohr-Coulomb "modeller" samtidigt för att studera interaktionen mellan dagbrott och underjordsbrytning av en stor komplex malmkropp. En stor FLAC3D modell konstruerades och ett kort FISH program skrevs för att möjliggöra automatisk växling mellan Hoek-Browns och Mohr-Coulombs brottskriterier med lämpliga parametrar för de två modellerna som indata. Erfarenheter från gruvan användes för att bedöma riktigheten av FLAC3D resultaten. Resultaten från Hoek-Brown och Mohr-Coulomb var tydligt skilda och speciellt Hoek-Brown resultaten

verkade tveksamma. Medvetna om skillnaden mellan de två "modellerna" baserades deras respektive användningsområden på önskat resultat. Resultaten från den här simuleringen visar tydligt att Mohr-Coulomb modellen simulerar beteendet mycket väl medan Hoek-Brown modellen behövde en förändring i modelleringssätt och vissa justeringar för att reagera på ett tillfredsställande sätt. SRK Consulting var dock medvetna om att Hoek-Brown inte är en typisk konstitutiv modell. Resultaten visar tydligt begränsningarna för Hoek-Brown modellen för plastisk analys.

1   INTRODUCTION

Since its introduction into Itasca and Rocscience codes the Hoek-Brown failure criterion has been increasingly replacing the traditional Mohr-Coulomb model for plastic analysis in rock mechanics modelling. A primary reason is that, the Hoek-Brown parameters can now be directly used with the Hoek-Brown model without having to convert to the equivalent Mohr-Coulomb parameters to use with the Mohr-Coulomb model.

However, there are fundamental differences between the two models or criteria that are rather poorly understood or less appreciated. First and foremost is the fact that only the Mohr-Coulomb is a classic constitutive model, while the Hoek-Brown is a failure criterion. That means, the Hoek-Brown model cannot relate stress and strain in a general way as the Mohr-Coulomb model. Thus the plasticity results from the two models cannot be expected to be the same, which many mistakenly assume otherwise.

Numerical methods require full constitutive models, which relate stress to strain in a general way; in addition to a failure (or yield) criterion, a "flow rule" is also necessary, in order to provide a relation between the components of strain rate at failure. These formulations assume that the flow rule has some fixed relation to the failure criterion, and that the flow rule is isotropic. The Hoek-Brown model obviously lacks this flow rule and therefore cannot be considered as a typical constitutive model. Instead the Hoek-Brown incorporates a plasticity flow rule that varies as a function of the confining stress (see Itasca, 2013). That is, it is based on the relation between the major and minor principal stresses.

The next significant difference between Hoek-Brown and Mohr-Coulomb is the assumption about the yield and deformation characteristics of the rock mass. Hoek-Brown assumes that the rock mass is characterized by an elastic-brittle-plastic behaviour while Mohr-Coulomb assumes that it is characterized by an elastic-perfectly-plastic behaviour (see Figure 1). The models will give similar observations up to the point of yield or failure. Beyond yield the results will be different. Hoek-Brown will show much larger plastic straining than the Mohr-Coulomb for the same constant stress levels beyond yield. As also noted earlier, since the Hoek-Brown does not relate stress and strain in a general way as Mohr-Coulomb, the accuracy of the plasticity after the yield is hence questionable.
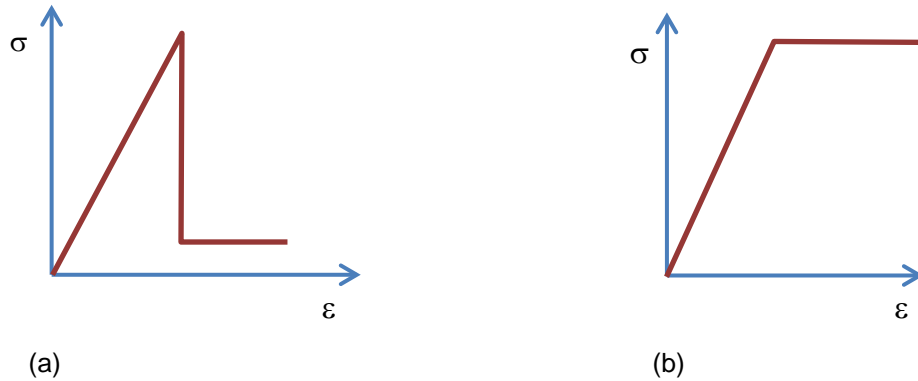
Figure 1:   Hoek-Brown assumes the rock mass is characterised by (a) elastic-brittle-plastic behaviour, while Mohr-Coulomb assumes it is characterised by (b) elastic-perfectly-plastic behaviour.

The results of the modelling presented in this paper clearly illustrate the scenarios described above. As soon as the rock mass started to yield the Hoek-Brown based models became unstable and the cycle time for one simulation consisting of 5 excavation stages took more than one week to complete, with frequent pauses and tweaks necessary to complete the cycling process. On the other hand the Mohr-Coulomb based models were very stable and took less than two days to complete the cycling with hardly any pauses and tweaks. This clearly illustrates the inability of the Hoek-Brown model to handle plasticity when large plastic straining occurs. If serious plastic analysis is considered then the Mohr-Coulomb criterion is recommended, but for the keen users of Hoek-Brown more caution and experience is required to cope with it when plastic straining occurs. On the other hand Hoek-Brown appears to show sensible results with small strains. This could be one reason for Hoek-Brown to work well in Phase^2 finite element program, since it is a small strain software, and not in large strain software packages, such as the Itasca codes.

2   MODELLING

2.1   Model setup

A large FLAC3D model (Figure 2) was setup to study the effect of concurrent open pit and underground mining on the stability of the open pit slopes and the cap pillar. Mining will occur concurrently from the open pit and underground over a period of 5 years (Figure 3). During this period the open pit will be deepened until a breakthrough occurs in 5th year, after which time mining will wholly occur underground.

The mine size FLAC3D model is 4500 m x 4500 m wide and 1500 m deep, and contains over 5.3 million zones, with the smallest zones being 10 to 15 m wide, whilst ensuring that the aspect ratio of 1:5 is maintained. Because of the need to model the open pit and underground excavation stages concurrently, a FISH script was written to automate the excavation process. One particular FISH script was written to permit switching between Mohr-Coulomb and Hoek-Brown criteria. There were two reasons for the addition of this script; (i) the client was keen to pursue strain analysis in the

future when field monitoring data becomes available, which suited the Mohr-Coulomb model and (ii) was to avoid using two sets of inputs; one for open pit and the other for underground, since the equivalent Mohr-Coulomb parameters are confining stress dependent when deriving from Hoek-Brown curve.
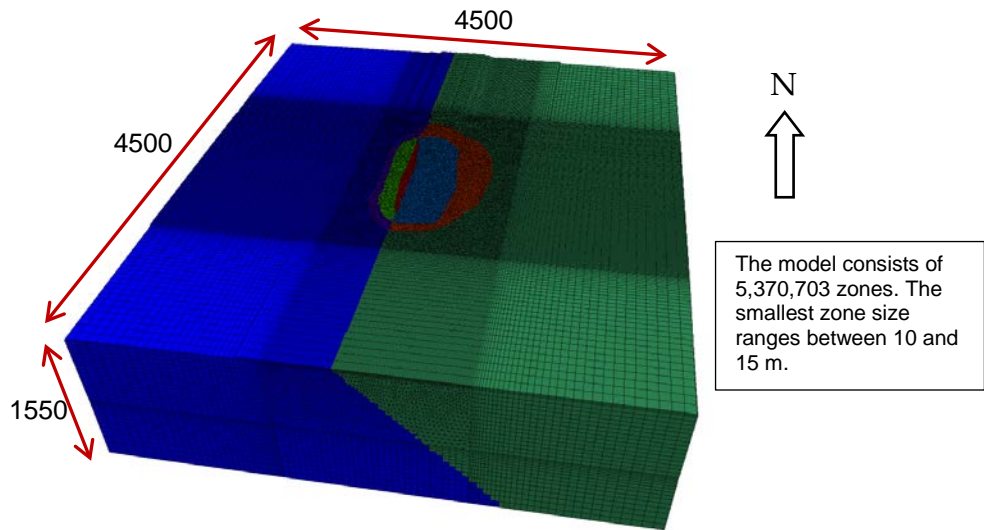
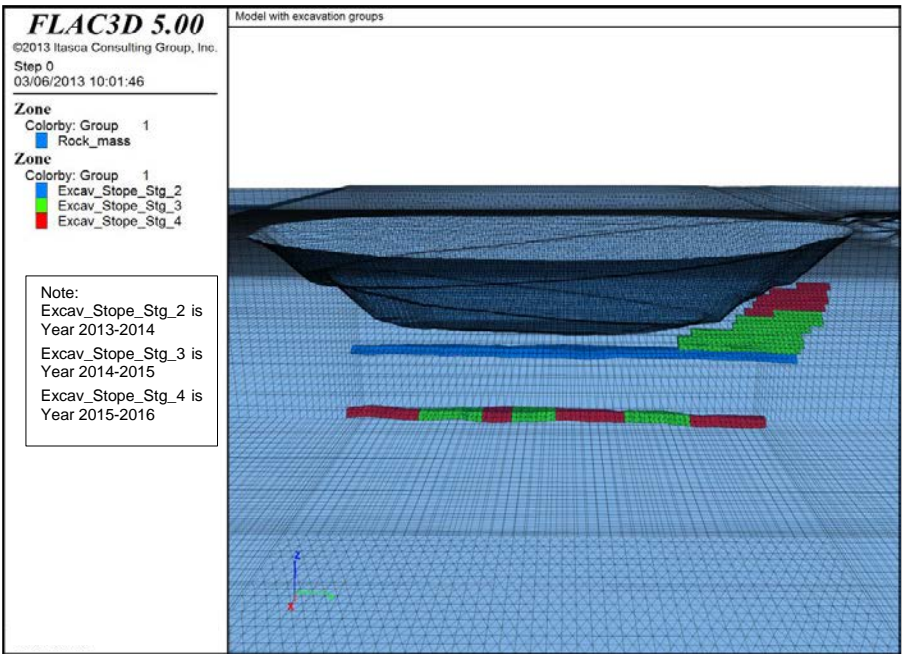

Figure 2:   Mine size FLAC3D model geometry.

Figure 3:   FLAC3D model of the open pit and underground excavation stages.

2.2 Inputs

Both the Hoek-Brown and the equivalent Mohr-Coulomb parameters were determined using RocData (Rocscience, 2012) based on rock mass data derived from rock mass characterization from borehole core logging and pit wall mapping. Since the equivalent Mohr-Coulomb parameters are derived from the Hoek-Brown parameters, the inputs for the two models are therefore considered compatible. The residual rock mass strength values were the same as the peak so that the perfectly plastic model is assumed for both the Hoek-Brown and Mohr-Coulomb models.

2.3 Simulation of mining stages

The simulations were staged to represent the concurrent open-pit and underground mining, see Figure 4. Prior to the initial excavation the model was fully enclosed as in Figure 2 and cycled to elastic equilibrium.

The stability of the open pit slopes, particularly the slopes where the haulage ramps are located, and the cap pillar that separates the underground from the open pit were of particular concern. Hence, measurement points or "markers" were located along these slopes and along the cap pillar to track displacements and strains. The year 2015-2016 is the most critical since the thickness of the cap pillar will be reduced to about 60 m (with a breakthrough in the following year). Hence the stability during this period (i.e. 2015-2016) is critical. This period is chosen to demonstrate the sensitivity of Hoek-Brown and Mohr-Coulomb models when increased plastic deformation was expected.
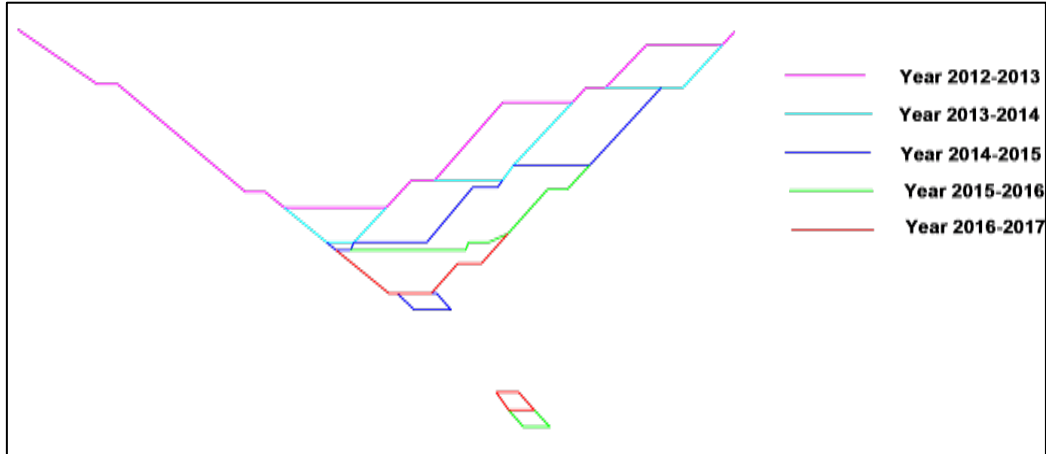

Figure 4:   Concurrent open-pit and underground mining stages planned: 2012-2017.

3 RESULTS

3.1 Results from modelling by usual approach

By usual approach it is meant that both the Hoek-Brown and Mohr-Coulomb based models were run using the same approach. That is, after each excavation stage the models were run to equilibrium using the SOLVE command. The Hoek-Brown based

model, however, was very sensitive to sudden excavations and thus became numerically unstable. Therefore the excavation was performed by gradually reducing the modulus until fully excavated. The SOLVE command was used to step to final equilibrium.

For the purpose of this paper only the results for the 2015-2016 mining stage are shown, see Table 1 to Table 4. The displacements and shear strain results are shown in order to compare the observation from Hoek-Brown and Mohr-Coulomb based models.

Table 1 and Table 2 show the displacements and shear strains for Hoek-Brown and Mohr-Coulomb models, for 2015-2016 mining stage, when the SOLVE command was used to run the models to equilibrium. The Hoek-Brown based models showed very large strains, leading to displacements as large as 116 cm in the center of the cap pillar. Elsewhere the displacements were very small relative to the 116 cm displacement, such that the contours of the small displacements became irrelevant. It is believed that such large displacements are not expected, when judging from the present experiences at the open pit.

The maximum displacement observed from Mohr-Coulomb based model was only 35 cm in the middle of the cap pillar, in the same location where the Hoek-Brown based model produced 116 cm. This displacement of 35 cm can be correlated to displacements experienced at the open pit. Furthermore, the Mohr-Coulomb based model showed the distribution of displacements and strain contours as expected.

3.2   Results from modification to modelling approach for Hoek-Brown models
Although the results from the Hoek-Brown based models do point to the areas of the mine that need attention the scale of the displacements and strains appeared unrealistic. Hence, the STEP command was used instead of SOLVE command. Here, the models were stepped after full excavation until the displacements of 30 to 50 cm were observed in the open pit, which are considered as realistic from experience at this mine. The results obtained from the approach using Hoek-Brown based models are shown in Table 3 and Table 4. Interestingly the results obtained by this approach for the Hoek-Brown based models show similar behaviour to Mohr-Coulomb based models shown in Table 1 and Table 2.

Table 1: Displacements obtained from Hoek-Brown and Mohr-Coulomb models. For E-W or N-S sections refer to Figure 1.
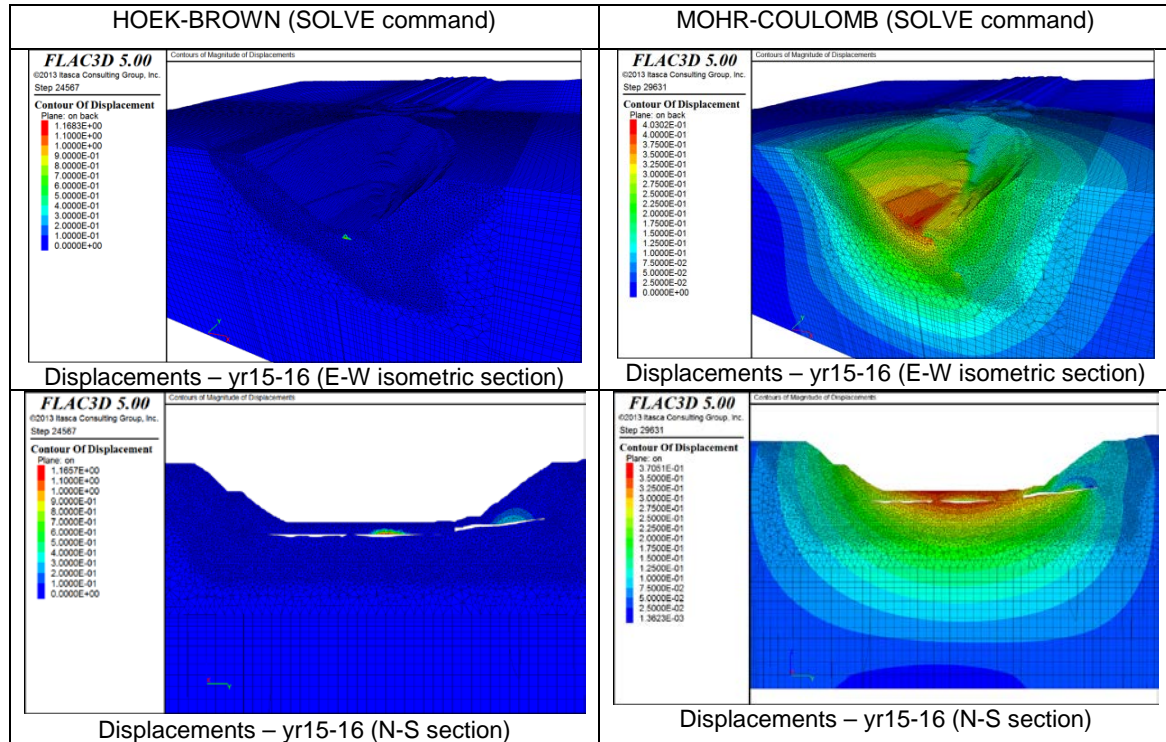
HOEK-BROWN (SOLVE command)	MOHR-COULOMB (SOLVE command)
 Displacements – yr15-16 (E-W isometric section)	 Displacements – yr15-16 (E-W isometric section)
 Displacements – yr15-16 (N-S section)	 Displacements – yr15-16 (N-S section)

Table 2: Shear strains obtained from Hoek-Brown and Mohr-Coulomb models.

HOEK-BROWN (SOLVE command)	MOHR-COULOMB (SOLVE command)
 Shear strain – yr15-16 (E-W isometric section)	 Shear train – yr15-16 (E-W isometric section)
 Shear strain – yr15-16 (N-S section)	 Shear strain – yr15-16 (N-S section)
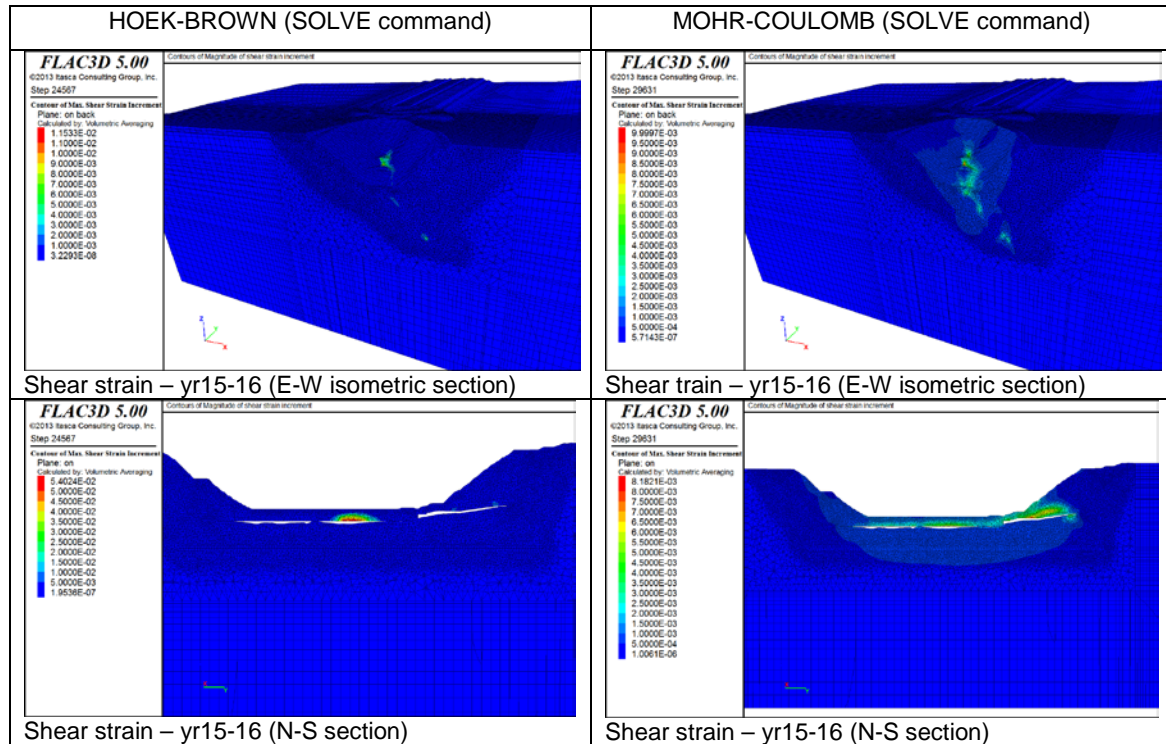
Table 3:   Displacements obtained from Hoek-Brown model simulated with STEP the command.

HOEK-BROWN (STEP command)	
Displacements – yr15-16 (E-W isometric section)	Displacements – yr15-16 (N-S isometric section)
Displacements – yr15-16 (E-W section)	Displacements – yr15-16 (N-S section)
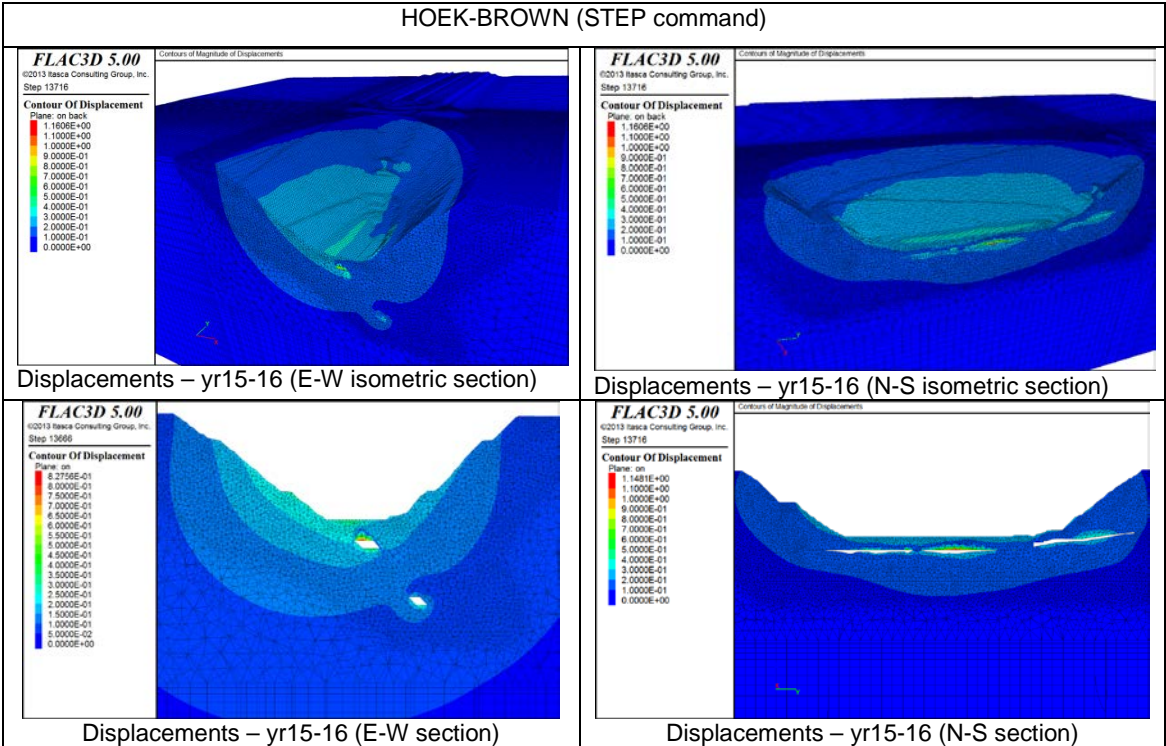
Table 4:   Shear strains obtained from Hoek-Brown simulated with STEP command.

HOEK-BROWN (STEP command)	
Shear strain – yr15-16 (E-W isometric section)	Shear strain – yr15-16 (N-S isometric section)
Shear strain – yr15-16 (E-W section)	Shear strain – yr15-16 (N-S section)
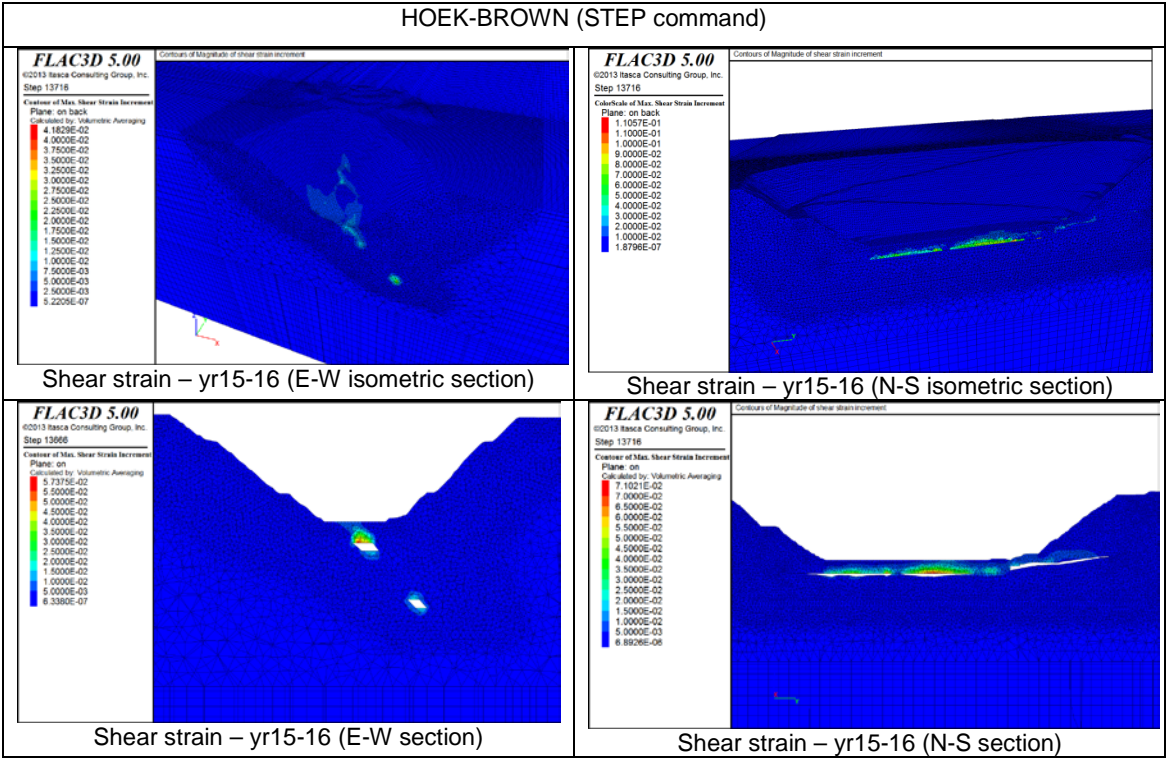
4 DISCUSSIONS

4.1 Adjustments to modelling approach for Hoek-Brown models

The Hoek-Brown based models immediately reflected numerical instability when excavations began. Therefore it was necessary to change the modelling approach. The modulus reduction approach was utilised and the STEP command was used instead of the SOLVE command. This enabled pauses during the running of the model to see from assessing the unbalanced force whether further modulus reduction is required (or not) until the mining stage is fully excavated. With the STEP command the models were terminated when the displacements (and strains) were judged to have reached the limits that reflected the experiences from the mine site. With the SOLVE command the models were ran to equilibrium, however, the final displacements were significantly overestimated.

4.2 Numerical results

Obviously the intriguing issue is the contrasting numerical results obtained from Hoek-Brown and Mohr-Coulomb models. With the usual approach of modelling, the Hoek-Brown based models over-estimated the displacements and strains. What is also interesting with the Hoek-Brown based model are the yielding and straining patterns. As seen in Table 1 and Table 2, once yielding starts at a point the concentration of further yielding and straining is concentrated around this point. This leads to unrealistically high displacements and strains at this point and very insignificant displacements and strains elsewhere. The Mohr-Coulomb based models however, show concentrations of displacements and strains evenly distributed as expected. Even more so the magnitudes of the displacements and strains are within the expected range.

4.3 Why the difference?

The difference in the observations in the two models certainly can be worked back to the theoretical basis of the two models. As noted all along the Hoek-Brown is a failure criterion and not a typical constitutive model. Therefore it cannot relate stress and strain in the same way as a classical constitutive model such as the Mohr-Coulomb. Hence, the Hoek-Brown model became immediately unstable when large plastic straining started to occur at or after yield. Because of the lack of a plastic flow rule to relate stress and strain after yield, the resulting displacements and strains from Hoek-Brown becomes unpredictable and eventually produces results that are questionable. The Mohr-Coulomb is a classic constitutive model with a plastic flow rule and therefore can relate stress and strain beyond failure, leading to results that are consistent and accurate.

On the other hand the Hoek-Brown model appears to respond reasonably well with small strains. This could be one reason why the Hoek-Brown model seems to work very well in the Phase^2 finite element program, since it is a small strain software, and not in FLAC for instance, where large strains are permitted. In this study the Phase^2 finite element software was used in the model calibration stages prior to the 3D modelling. Both the Hoek-Brown and Mohr-Coulomb models were tested as part of calibration. The results were similar, including the magnitudes of displacements and strains.

4.4 The common ground

In qualitative terms both Hoek-Brown and Mohr-Coulomb will indicate the same or similar areas of increased plasticity. This was clearly observed in the models discussed in this paper, where both models reveal the same locations where increased plasticity occurred. However, it is the quantitative assessment of the plasticity, that is, the magnitudes of the displacement and strains that the two models do not agree, except when the simulation approach for Hoek-Brown model is tweaked.

5   CONCLUSIONS

The main verdict from this paper is that, for serious plastic analysis the Mohr-Coulomb model is recommended over the Hoek-Brown, especially when large strains are expected. The Hoek-Brown can reveal areas of increased plasticity but the magnitudes of the plasticity could be misleading.

It is also shown here that the Hoek-Brown and Mohr-Coulomb models cannot be simulated using the same approach. The Hoek-Brown based models became numerically unstable and required a modulus reduction approach to perform the excavation to stabilize the models. Even so, the cycle times were averaging more than 2 days for each excavation stage compared to less than an hour for the Mohr-Coulomb models. When cycling to the final equilibrium the STEP command is recommended if Itasca codes are used for Hoek-Brown model. The models are to be terminated when the expected displacements (known from experience or measurements) are achieved.

It has to be emphasized again that the Hoek-Brown model is not a classic constitutive model like the Mohr-Coulomb. Hence, it cannot relate stress and strain the same way as the Mohr-Coulomb model. For this reason, Hoek-Brown will have the tendency to produce questionable results when large plastic strains occur. Itasca manuals (e.g. Itasca, 2013) are recommended for consultation regarding the derivation of the governing mathematical relations for the Mohr-Coulomb and Hoek-Brown models.

The Hoek-Brown model seems to produce sensible results when the simulations involve small strains. This is probably why it works reasonably well with the $Phase^2$ software, since it is a small strain program. The Itasca codes are large strain codes, which means the Hoek-Brown model may succumb if a proper modelling approach is not utilized.

6   REFERENCES

Rocscience, 2012, *RocData v.4*

Itasca, 2013, FLAC3D v.5 Manual