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A simplified three dimensional Hoek-Brown yield criterion

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ABSTRACT: Although it appears that the Hoek-Brown empirical failure criterion is still the most suitable and realistic criterion for predicting the strength of rocks or rock masses, difficulties have occured in applying the criterion to both laboratory and numerical modelling. It is shown that in three dimensional (3-D) principal stress space, the yield surface of the Hoek-Brown criterion is a combination of the components of six two dimensional parabolic surfaces. After a thorough investigation of the nature of the Hoek-Brown and some other yield surfaces, a modified 3-D Hoek-Brown criterion is suggested and tested in association with the finite element method. The proposed criterion has particular advantages in numerical analysis and also in predicting the strength of weak rock masses.

1 INTRODUCTION

An important phenomenon manifested by rock strata in the vicinity of underground openings is their non-linear response to induced stress. *In situ* observations and experimental results show that this is caused by a combination of pre-peak non-linear elasticity and post peak behaviour. After failure, a rock mass may exhibit strain softening, perfect plastic or strain hardening behaviour depending on confining pressure, rate of loading and rheologic characteristics of the material (Hudson et al. 1972a,b; Farmer 1983). For the mechanical description of non-linear post failure behaviour of a rock mass, a yield criterion based on the theory of plasticity is usually used.

To determine the conditions which govern the failure or yielding of a rock mass considered as an equivalent continuum, a 3-D strength criterion is generally required. Two of the classical strength criteria in incremental plasticity theory are the Mohr-Coulomb and Drucker-Prager criteria which define the simple yield surfaces of Fig. 1 in 3-D principal stress space.

A suitable shape of yield surfaces for rock materials has been considered to be a curved, pointed 'bullet' with curved cross sections in the octahedral planes, similar to those of Mohr-Coulomb but without sharp intersection points or corners (Serata et al. 1967; Akai and Mori 1970; Franklin 1971; Kim and Lade 1984). Recently, Michelis (1987) interpreted his truly triaxial test data in the meridian and deviatoric planes of yield surfaces and verified most of the above characteristics. Taking into account the practical application of a strength criterion in excavation design, not only the intact rock, but also the rock mass behaviour has to be considered in the development of a criterion(Hoek and Brown 1980). On the other hand, a rock strength criterion is at best only an approximate fit to the observed data, and thus it should be expressed in a simple way and involving the minimum number of parameters for practical excavation design.

2 REVIEW OF ROCK STRENGTH CRITERIA

Of the classical strength criteria, the Mohr-Coulomb criterion is generally considered the most acceptable for describing rock behaviour. However, through considering the strength criterion in 3-D principal stress space, the main discrepancies between the experimental results and those predicted by the Mohr-Coulomb criterion are the curvature of its yield surface. The 3-D Mohr-

Coulomb yield surface is composed of six planes, whereas true triaxial test data for rocks show that the surface is significantly curved. This discrepancy in two dimensions is shown in Fig. 2. The well known Griffith criterion predicts a parabolic relation between the stress components in plane compression but, for a number of reasons, often gives an inadequate fit to experimental data (Brady and Brown, 1985).

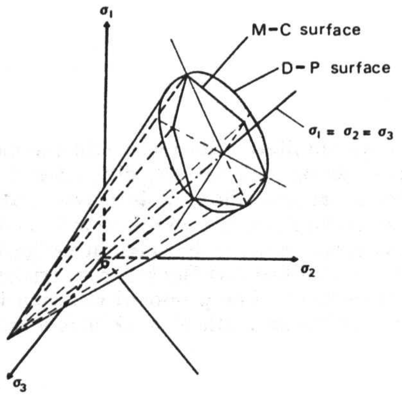

Fig.1 Yield surfaces in 3-D principal stress space.

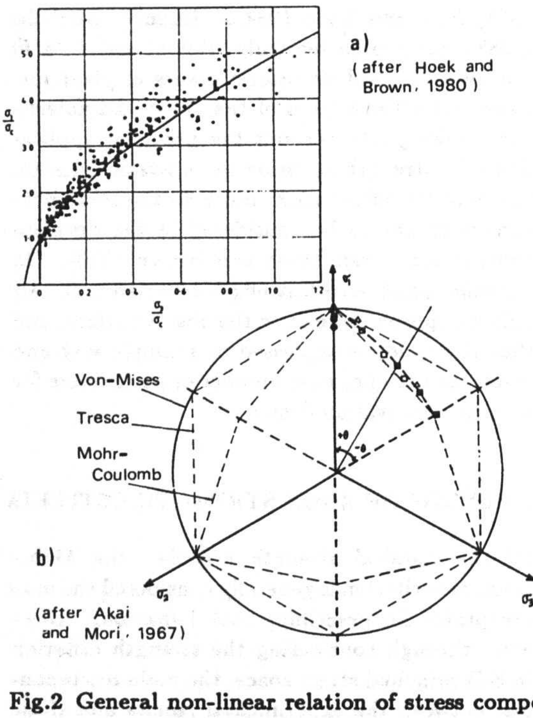

Fig.2 General non-linear relation of stress components of rock at failure. a) $\sigma_1 \sim \sigma_3$ plane, b) octahedral plane.

Since the 1960's, numerous investigations have been performed concerned with correcting these discrepancies and trying to fit a non-linear criterion with observed data (Murrell 1963; Fairhurst 1964; Hobbs 1966; Hoek 1968; Franklin 1971; Bieniawski 1974; Yoshinaka and Yamabe 1980; Hoek and Brown 1980a; Kim and Lade 1984; Johnston 1985; Desai and Salami 1987; Michelis 1987). A brief review of the available criteria is given in Table 1 in which the development of each criterion and the constants involved are outlined. Many of these criteria give good explanations of some aspects of rock behaviour, but fail to explain others.

It can be seen from Table 1 that most of the criteria were proposed on the assumption that rock strength is independent of the intermediate principal stress. This assumption implies that the yield surface is generated by lines parallel to the principal stress axes. Following experiments on hollow cylinders with external pressure and axial load, Obert and Stephenson(1965) suggested that the failure stresses are independent of σ_2. However, Handin et al. (1967) demonstrated quantitatively the influence of σ_2 on shear strength of Solenhofen limestone. Mogi (1967) pointed out that the effect of the intermediate principal stress did exist, but it was relatively small. Hoskins' (1969) hollow cylinder experiment on trachyte shows that σ_2 has a significant influence on rock strength. However, Akai and Mori's (1967) test data on sandstone manifests no such influence at all. Likewise, yield criteria for isotropic rock material have usually been based on empirical criteria independent of the intermediate principal stress. It is more acceptable to assume that the intermediate principal stress will have a negligible influence on yield conditions for intact rock than it is for jointed rock (Reik and Zacas 1978).

3 THE HOEK-BROWN CRITERION

Among those available criteria listed in Table 1, the only one which takes account of the strength of intact rock as well as jointed rock masses (excluding a regularly jointed rock mass such as that discussed by Amadei, 1988) is the Hoek-Brown criterion (Hoek and Brown,1980a). The assump-

Table 1 A review of rock strength criteria developed since the 1960's.

No.	Authors	The criteria	Constants involved	Development of the criteria
1	Murrell(1963)	$\tau_{oct}^2 = 8T_0\sigma_{oct}.$ or: $J_2 = 4T_0 I_1.$	One constant (3D criterion)	Extended 3D Griffith theory.
2	Fairhurst (1964)	if $m(2m-1)\sigma_1+\sigma_3 \geq 0$: $\sigma_1 = K,$ if $m(2m-1)\sigma_1+\sigma_3 < 0$: $\frac{(\sigma_1-\sigma_3)^2}{(\sigma_1+\sigma_3)} = -2(m-1)^2K\left[1+\frac{2K}{(\sigma_1+\sigma_3)}\left\{\left(\frac{m-1}{2}\right)^2-1\right\}\right].$	Two constants (2D criterion)	Empirical generalisation of 2D Griffith theory for intact rock.
3	Hobbs (1966)	$\sigma_1 = B\sigma_3^b + \sigma_3,$ or: $\tau = K_2\sigma_n^a.$	Two constants (2D criterion)	Empirical test data fitting for intact rocks.
4	Hoek (1968)	$\sigma_1 - \sigma_3 = 2C + A(\sigma_1+\sigma_3)^B,$ or: $\tau_{max} = \tau_{max_0} + A\sigma_m^b.$	Three parameters (2D criterion)	Empirical curve fitting for intact rock.
5	Franklin (1971)	$\sigma_1 - \sigma_3 = \sigma_c^{1-B}(\sigma_1+\sigma_3)^B.$	Two constants (2D criterion)	Empirical curve fitting for 500 rock specimens.
6	Bieniawski (1974)	$\sigma_1 = K'\sigma_3^A + \sigma_c,$ or: $\tau = B'\sigma_m^c + 0.1\sigma_c.$	Three constants (2D criterion)	Empirical curve fitting for 700 rock specimens. (5 types)
7	Yoshinaka & Yamabe (1980)	$\sigma_1 - \sigma_2 = \alpha K(q)(\sigma_1+\sigma_2+\sigma_3)^\beta.$	Three parameters (3D criterion)	Empirical test data analysis for soft rocks (mudstone, etc).
8	Hoek and Brown (1980)	$\sigma_1 - \sigma_3 = \sqrt{m\sigma_c\sigma_3 - s\sigma_c^2}.$ or: $\tau = A(\sigma_n + B)^C.$	Three parameters (2D cri. for rocks and rock masses)	Appl. of Griffith theory and empirical curve fitting for rock and rock mass.
9	Kim and Lade (1984)	$\left(\frac{(I_1')^3}{I_3'} - 27\right)\left(\frac{I_1'}{p_a}\right)^m = \eta_1.$	Three parameters (3D criterion)	Analytical examination on test data (originally for soil and concrete).
10	Johnston (1985)	$\frac{\sigma_1}{\sigma_c} = \left[\frac{M}{B}\frac{\sigma_3}{\sigma_c} + 1\right]^B.$	Three parameters (2D criterion)	Empirical curve fitting for soft rock specimens.
11	Desai and Salami (1987)	$J_2' = \left(-\frac{\alpha}{\alpha_0}(I_1')^n + \gamma(I_1')^2\right)(1-\beta S_r)^m.$	More than six parameters (3D criterion)	Polynominal expansion in terms of stress invariants to curve fitting.
12	Michelis (1987)	$\ln\left(\frac{q^2}{f_\theta^2} + a_1 p\frac{q}{f_\theta} + a_2 p^2\right) =$ $a_4 \ln\left(\frac{2q/pf_\theta + a_1 - a_3}{2q/pf_\theta + a_1 + a_3}\right) + \ln a_5.$	Four constants (3D criterion)	Analytical and experimental examination on yield surface (true triaxial test).

tions made in the criterion and some of its applications were discussed in detail by Hoek (1983). For intact rock, the Hoek-Brown criterion predicts a ratio $1:7 \sim 25$ for uniaxial tensile strength to uniaxial compressive strength which is much more realistic than that given by the Griffith theory from which it was developed.

The 3-D Hoek-Brown yield surface

It appears that the yield surface of the Hoek-Brown criterion in 3-D principal stress space has not been well understood. It was assumed that the only difference between the Hoek-Brown and Mohr-Coulomb criteria is that the sides of the conical hexagon yield surface are curved. However, investigation by means of computer graphics reveals that the Hoek-Brown yield surface satisfies another of the attributes of the ideal shape of a yield criterion for rock materials, namely, its cross section in the octahedral planes shows a curved hexagonal rather than a Mohr-Coulomb type cross section.

The yield function can be written in the general form of stress invariants as (Nayak and Zienkiewitz 1972): (compression positive)

$$F = 2J_2^{\frac{1}{2}}\cos\theta - \left[\frac{m\sigma_c J_2^{\frac{1}{2}}}{-\sqrt{3}}\left(\sin\theta - \sqrt{3}\cos\theta\right) + \frac{1}{3}I_1 m\sigma_c + s\sigma_c^2\right]^{\frac{1}{2}} = 0. \tag{1}$$

where I_1 is the first stress invariant, J_2 is the second deviatoric stress invariant and θ is related to the third stress invariant. According to Eqn. (1) the Hoek-Brown yield surface in principal stress space is constructed as shown in Fig. 3 in which no ordering with respect to magnitude of the principal stress σ_1, σ_2 and σ_3 is implied. Other yield surfaces, such as Mohr-Coulomb's can also be drawn in the same way, so that different yield criteria can be easily compared with each other. In Fig. 4 it is seen that the Hoek-Brown yield surface gives an acceptable approximation to the experimental data of Akai (1967) and Serata et al. (1972).

In Fig. 5, the 3-D Hoek-Brown yield surface is shown to be a combination of the components of six 'sigle curved' parabolic surfaces which can ex-

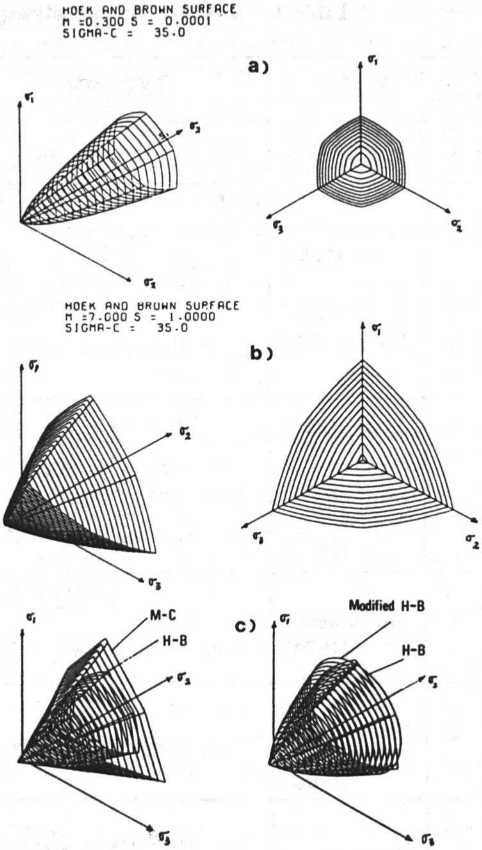

Fig.3 The Hoek-Brown yield surface in 3-D principal stress space. a) for rock masses, b) for intact rocks, and c) a comparison with the Mohr-Coulomb surface.

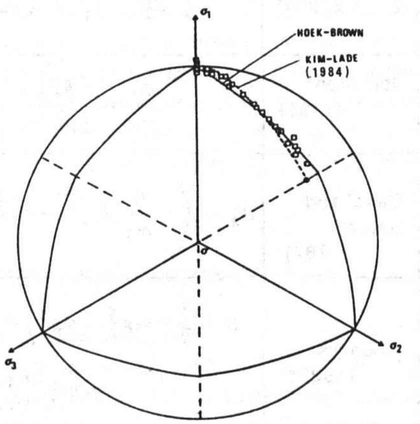

Fig.4 An interpretation of test data in octahedral plane using the Hoek-Brown criterion.

plain why the intermediate principal stress σ_2 has no influence on the yield criterion. As the stress point moves from point 1 to 3, passing through 2 on the yield surface, the locus is a straight line and the variable $J_2^{\frac{1}{2}}$ remains unchanged at points 1 and 3. This means that an increase of the intermediate principal stress σ_2 will lead to an increase of the first stress invariant I_1, but may not change the second deviatoric stress invariant J_2 of the criterion. This may be a common feature of most non-linear empirical criteria as long as they are intermediate principal stress independent.

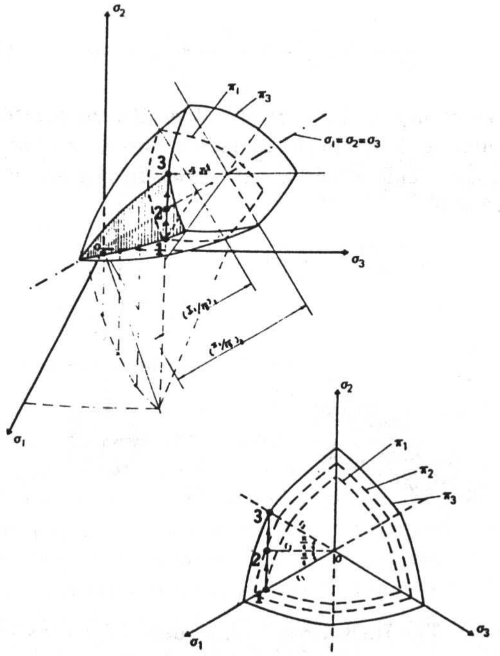

Fig.5 The construction of the Hoek-Brown yield surface in 3-D principal stress space.

The criterion in predicting soft rock strength

It is generally accepted that the Hoek-Brown criterion is suitable for most types of rock including soft rock. In applying the criterion, however, one must be very careful in choosing these material constants. Johnston (1986) compared the Hoek-Brown criterion with his own criterion, developed for soft rock by analysing triaxial test data for mudstone (Fig. 6). He found a large discrepancy and concluded that Hoek-Brown criterion would seem to be restricted to relatively hard rocks. Nevertheless, this comparison can be interpreted in an alternative way through consideration of the choice of parameters. In spite of one material parameter less, the test data would have been as well fitted by the Hoek-Brown criterion as by his own criterion (Fig. 7) if the parameter m were calculated as suggested by Hoek (1983). After conducting tests on a soft rock material, Yudhbir et al. (1983) also concluded that the Hoek-Brown criterion is suitable for soft rocks and predicts tests results very well in the brittle rock behaviour range.

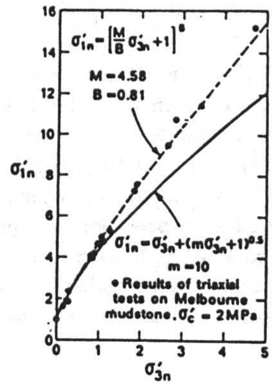

Fig.6 The comparison of the two criteria. (after Johnston, 1986).

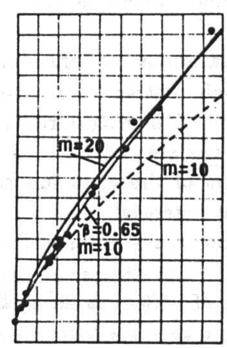

Fig.7 The alternative interpretations to the data using the Hoek-Brown criterion. (m = 20, or $\beta = 0.65$).

An extended form of the criterion

The discrepancies of the Hoek-Brown curve fitting to observed data are usually in the curvature of the yield surface because the Hoek-Brown criterion gives a quadratic approximation to this yield surface; however, many other criteria use more complicated empirical curved surfaces which usually involve more parameters. In fact, the Hoek-Brown criterion can be easily modified to take account of a more precise curve fitting by introducing a third empirical parameter β and, consequently, the criterion (7) in Table 1 becomes:

$$\frac{\sigma_1}{\sigma_c} - \frac{\sigma_3}{\sigma_c} = \left(m\frac{\sigma_3}{\sigma_c} + s\right)^{\beta}. \quad (2)$$

For instance, if $\beta = 0.65$ is chosen for the Melbourne mudstone of Johnston (1986), the Hoek-Brown criterion fits the test data very well using the original parameter $m = 10$ (see Fig. 7). Eqn. (2) can be used for applications in which a specific curve fitting or a higher precision is required. For modelling of a rock mass, however, the quadratic approximation seems more realistic and suitable because of its simplicity and relative accuracy.

4 A SIMPLIFIED 3-D CRITERION

Although it appears that the Hoek-Brown empirical strength criterion is still the most suitable and realistic criterion for predicting the strength of a rock mass, difficulties have often occured in applying the criterion in practical analysis or numerical modelling. These include the following points:

1. The consideration of the influence of the intermediate principal stress on the rock mass.
2. The treatment of the singularities on the Hoek-Brown yield surface.
3. Differentiation of the yield function (Eqn.(1)) or the plastic potential.
4. Determination of the empirical parameters m and s.

In modelling of soft rock mass behaviour, it was noticed that the Hoek-Brown yield surface is very close to a paraboloid surface if either the uniaxial compressive strength σ_c or the parameters m and s are small. In fact, as m and s decrease, the shape of the cross section in the octahedral plane changes from a curved triangular shape to a curved hexagon shape (see Fig. 8), which approximates to a circle. The criterion derived specifically for soft rocks by Yoshinaka and Yamabe (1980) exhibits a similar characteristic. It is, therefore, possible to extend the original Hoek-Brown criterion into a paraboloid surface of circular cross section with a central axis of $\sigma_1 = \sigma_2 = \sigma_3$ (Fig 9). Using this extended yield criterion, some of the above difficulties are overcome and resulting discrepancies are found to be negligible (Pan 1988). The proposed criterion is derived from Eqn. (1) as

$$\frac{3}{\sigma_c}J_2 + \frac{\sqrt{3}}{2}m\sqrt{J_2} - \frac{m}{3}I_1 = s\sigma_c. \quad (3)$$

where m,s and σ_c are the Hoek-Brown parameters. It is a 'mean' surface between the 'inner apices' and 'outer apices' Hoek-Brown surfaces (Fig.10).

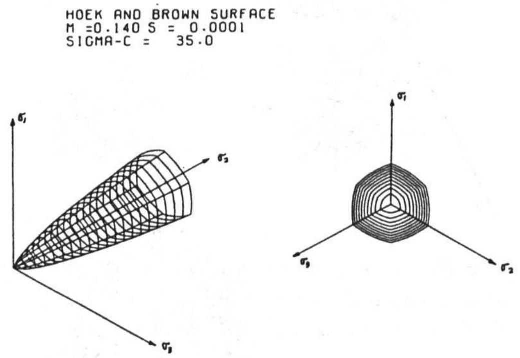

Fig.8 The Hoek-Brown yield surface for weak rock masses.

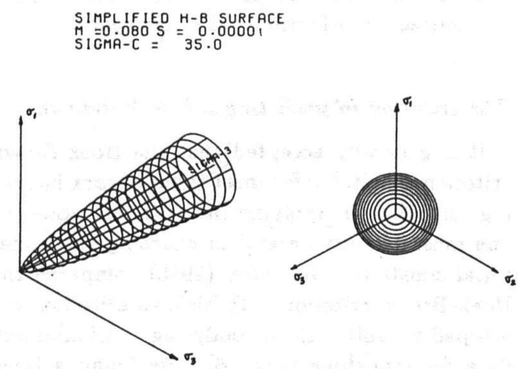

Fig.9 The simplified Hoek-Brown yield surface.

Thus, criterion (3) takes into account the influence of the intermediate principal stress in a similar way to the criteria of Drucker-Prager (1952) and Murrell (1963).

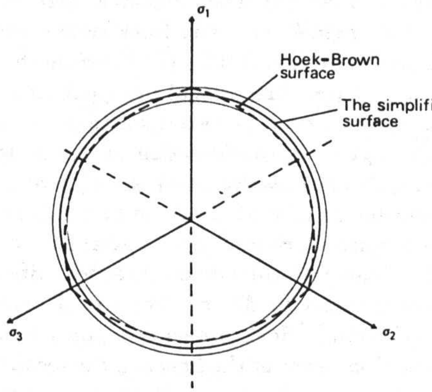

Fig.10 The mean surface between the 'inner apices' and 'outer apices' of the Hoek-Brown criterion.

5 APPLICATION

Incorporation of the criterion

In modelling the non-linear material behaviour of rock masses, finite element elasto-viscoplastic formulations (Zienkiewicz and Cormean 1974) have been well accepted and applied by many investigators . In applying these procedures, the decisions that must be made include the selection of a suitable yield surface for the rock mass, the post-failure behaviour such as strain hardening or softening and a viscoplastic flow rule, all of which depend on the initial choice of the strength (or yield) criterion used in the analysis. The Hoek-Brown criterion has been incorporated into the elasto-viscoplastic finite element analysis (Sharma et al. 1985; Reed 1987; Pan and Hudson 1988).

In evaluating the incremental viscoplastic strain using the implicit method of integration, a non-linear matrix such as

$$H^n = \frac{\partial^2 \epsilon^n_{vp}}{\partial t \partial \sigma} = \gamma \left[\Phi(F) \frac{\partial^2 Q}{\partial \sigma^2} + \frac{\partial \Phi}{\partial F} \frac{\partial Q}{\partial \sigma} \left(\frac{\partial F}{\partial \sigma} \right)^2 \right]. \quad (4)$$

has to be calculated (see Owen and Hinton 1980).

F and Q in Eqn.(4) are the yield function and plastic potential function which can be represented as Hoek-Brown yield surfaces with different material constants. However, the sharp corners on the Hoek-Brown surface must be 'rounded off ' to avoid singularities in the flow vector and furthermore, it is difficult to obtain the second order partial derivatives of Eqn.(1) in the case of associated flow. For non-associated flow which is recognised to be more realistic for rock masses (Brown 1987), a suitable plastic potential function and its derivatives are required. These problems can be easily solved by using the simplified Hoek-Brown surface, either for the yield surface to represent weak rock masses or, more generally, for the plastic potential function which can be written as:

$$Q(\sigma) = \frac{3}{\sigma_c} J_2 + \frac{\sqrt{3}m'}{2} \sqrt{J_2} - \frac{m'}{3} I_1. \quad (5)$$

where m' is the dilation parameter of the Hoek-Brown criterion. The flow rule in this case, shown in Fig. 11 has the following advantages:

i) dilatancy reduces continuously (non-linear) with increasing confining pressure,
ii) the numerical difficulties in dealing with singularities are avoided, and
iii) it is easy to calculate the second order partial derivatives of $Q(\sigma)$.

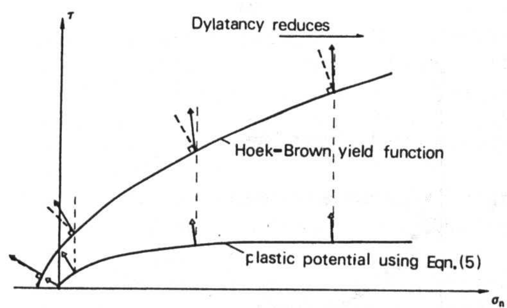

Fig.11 A flow rule using the Hoek-Brown yield surface and its simplified form as the yield function and the plastic potential function.

In the analysis of weak rock masses, the yield function can also be taken as Eqn. (3) and if the parameters are $m' = m, s' = s$, the associated flow rule is achieved.

Numerical example

A 4.0m diameter circular tunnel excavated in a continuous, homogeneous, isotropic, elastic perfectly plastic rock mass, complying with the Hoek-Brown yield criterion and the simplified yield surface as the plastic potential, is analysed. The *in situ* stress is assumed to be hydrostatic with a value of $10.0MPa$. The material constants used in the computation are: $E = 10GPa$, $\nu = 0.25$, $m = 7.0$, $s = 1.0$, $m' = 0$ and $\sigma_c = 8.6MPa$. Fig.12a shows the prediction of the stress distribution in the tunnel wall after tunnel excavation and Fig. 12b gives the displacement distribution. As expected, the analysis incorporating non-associated flow (zero dilation with $m' = 0$) predicts smaller displacements at the tunnel wall than for associated flow.

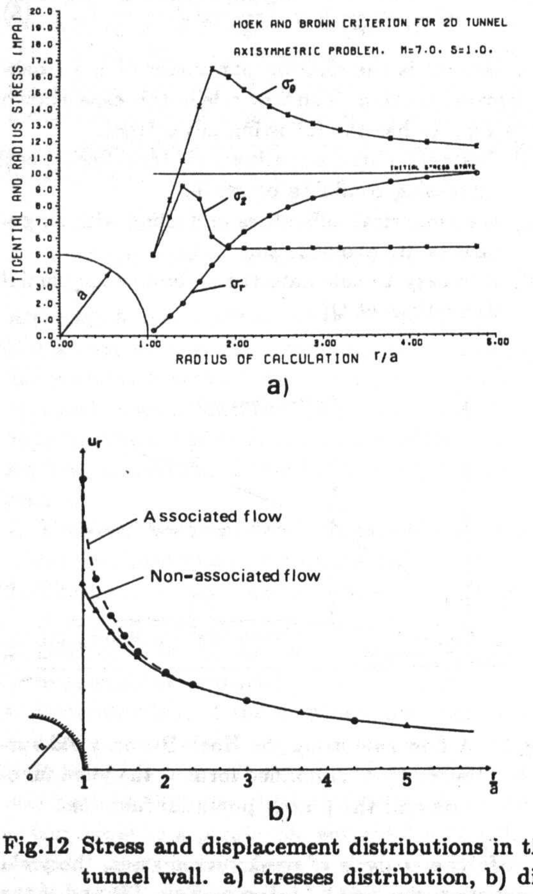

Fig.12 Stress and displacement distributions in the tunnel wall. a) stresses distribution, b) displacement distributions calculated by different flow rules.

6 CONCLUSIONS

Development of an improved and practical strength criterion for rocks and rock masses has always been an important research topic in geotechnical engineering. The Hoek-Brown empirical strength criterion and its yield surface characteristics in three dimensional principal stress space have been found to be satisfactory for most of rock types. In consideration of the intact rock strength, the criterion involves only two material parameters, so that it is one of the simplest of the empirical criteria. A new parameter β can be introduced to formulate an extended criterion for more accurate prediction. For a weak rock mass, the simplified Hoek-Brown criterion which is expressed in terms of the first and the second stress invariants may be more suitable for practical applications. It takes into account the influence of the intermediate principal stress on failure and has the same parameters as the Hoek-Brown criterion. In the general non-linear plasticity analysis of rock masses, the simplified criterion has been found particularly suitable for representing the plastic potential function. Both the Hoek-Brown and the simplified criterion have been incorporated into a non-linear finite element analysis and proved very satisfactory.

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