Novel 3D Failure Criterion for Rock Materials

Feng Gao[1]; Yugui Yang[2]; Hongmei Cheng[3]; and Chengzheng Cai[4]

Abstract: The failure criterion of rock material has been used in geotechnical engineering to assess the failure behavior of rocks under complicated stress conditions. In the present study, a new failure criterion is proposed; it takes into consideration the influence of intermediate principal stress for rock materials. The general characteristics of failure surface were analyzed, showing that the failure envelopes are convex and smooth everywhere except at the triaxial compression and triaxial tension conditions. A new shape function is presented to approach the failure criterion in the deviatoric plane, and the smoothness and convexity of the criterion is discussed. The new failure criterion is compared with some classic criteria by cross-sectional failure envelopes in the π plane and meridian failure envelopes in the meridian plane, and the advantages of the new three-dimensional (3D) failure criterion are presented. The results show that the new criterion not only inherits the strength characteristic of the Hoek-Brown failure criterion at the triaxial compression condition, but it also predicts failure stress better than other 3D failure criteria for rocks under complicated stress states. **DOI:** 10.1061/(ASCE)GM.1943-5622.0001421.

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Introduction

It is generally difficult to predict failure behavior with precision in many engineering activities because different construction schemes and support methods will result in different complicated stress states. The failure criterion is generally used to describe the strength capacity of material under particular conditions. Much research on failure theory for rock materials has been conducted (Hoek and Brown 1980; Kim and Lade 1984; Zhang and Zhu 2007; Mortara 2008; Melkoumian et al. 2009; Liu and Indraratna 2011; Jiang 2017). To overcome the limitation of the linear Mohr-Coulomb criterion, the empirical Hoek-Brown failure criterion is presented for analysis of the nonlinear strength behavior of rock materials (Hoek and Brown 1980; Hoek et al. 1998), and the parameters can be obtained from uniaxial compressive tests and a geological strength index (Hoek et al. 2002). The original purpose in establishing the Hoek-Brown failure criterion was to start with the intact rock. The expression can be given as follows:

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(m_b \frac{\sigma_3}{\sigma_c} + s\right)^a \tag{1}$$

where m_b, s, and a = material parameters for rock mass.

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The Hoek-Brown criterion, which was obtained by fitting triaxial test data, describes a nonlinear empirical relationship between the first principal and third principal stresses. The Hoek-Brown criterion is more suitable for describing the strength characteristic of rock materials than is the linear Mohr-Coulomb criterion. However, it neglects the effect of intermediate principal stress, σ_2, although many researchers have shown that intermediate principal stress greatly influences the strength characteristic of rock materials (Singh et al. 1998; Chang and Haimson 2000; Colmenares and Zoback 2002; Al-Ajmi and Zimmerman 2005; Cai 2008). In fact, the influence of intermediate principal stress on the failure behavior of geomaterials is important during underground engineering excavation. Several three-dimensional (3D) failure criteria have been proposed in terms of principal stresses to reflect the effect of intermediate principal stress. However, according to related comparative studies (Benz and Schwab 2008; Priest 2012), none of the existing 3D failure criteria has a significant advantage over the others from both a mechanical mechanism and a mathematical form. Thus, there is a practical need to establish a new 3D failure criterion to offer improved and convenient calculations on the failure of rock materials in the engineering design process.

In this study, we propose a new 3D failure criterion for rocks that takes the influence of intermediate principal stress into account. To meet the requirements of smoothness and convexity for the numerical implementation, a new shape function is proposed to approach the failure criterion, and the convexity of the function is discussed. The new failure criterion should not only inherit the characteristic of the original Hoek-Brown criterion in describing the strength of rocks under the triaxial compression condition but also take into consideration the effect of intermediate principal stress. Comparing the prediction of the proposed failure criterion with polyaxial testing data demonstrated that the predictions using the proposed criterion agreed well with experimental results for a variety of rocks.

Geometric Representation of Stress Space

A point in the Haigh-Westergaard stress space is representative of tensors, σ_{ij}. The stress tensor σ_{ij} is difficult to deal with because it has six independent components. This can be shown graphically using the principal stresses as coordinates and regarding the stress state as a point in this space, as shown in Fig. 1. In the principal stress space, the straight line $O_1O_2(\sigma_1 = \sigma_2 = \sigma_3)$ has equal angles with the three coordinate axes and passes through the origin. The deviatoric plane is perpendicular to the line, O_1O_2, and to the lengths, $OO_1 = \sigma_{ii}/\sqrt{3}$ and $O_1P = \sqrt{s_{ij}s_{ij}}$. The three principal stresses can be determined from the stress invariants as follows:

$$\sigma_1 = \frac{2}{\sqrt{3}}\sqrt{J_2}\sin\left(\theta + \frac{2\pi}{3}\right) + \frac{I_1}{3} \tag{2}$$

$$\sigma_2 = \frac{2}{\sqrt{3}}\sqrt{J_2}\sin\theta + \frac{I_1}{3} \tag{3}$$

$$\sigma_3 = \frac{2}{\sqrt{3}}\sqrt{J_2}\sin\left(\theta - \frac{2\pi}{3}\right) + \frac{I_1}{3} \tag{4}$$

where I_1 = first invariant of the stress tensor; $I_1 = \sigma_{ii}$; J_2 = second invariant of the stress deviator tensor; $J_2 = s_{ij}s_{ij}/2$; and $s_{ij} = \sigma_{ij} - \sigma_{ii}\delta_{ij}/3$.

The Lode angle, θ, can be presented by the second and third principal stress deviator invariants as follows:

$$\sin 3\theta = -\frac{3\sqrt{3}}{2}\frac{J_3}{(J_2)^{3/2}} \tag{5}$$

where J_3 = third invariant of stress deviator tensor, $J_3 = s_{ij}s_{jk}s_{ki}/3$.

The generalized mechanics present a convenient way to investigate the strength theory for various materials. In this theory, the mean stress, p, is often used in conjunction with a generalized shear stress, q, defined below

$$p = I_1/3, \qquad q = \sqrt{3J_2} \tag{6}$$

In terms of p and q, the principal stresses are

$$\sigma_1 = \frac{2}{3}q\sin\left(\theta + \frac{2\pi}{3}\right) + p \tag{7}$$

$$\sigma_2 = \frac{2}{3}q\sin(\theta) + p \tag{8}$$

$$\sigma_3 = \frac{2}{3}q\sin\left(\theta - \frac{2\pi}{3}\right) + p \tag{9}$$

Novel Failure Criterion for Rock Material

With the development of failure criteria, the influence of intermediate principal stress, σ_2, is more often taken into account in solving geotechnical engineering problems. Among the failure criteria, the failure criteria developed based on the Hoek-Brown criterion have advantages over other 3D failure criteria even though they use the same strength parameters (Lee et al. 2012). To better reflect the effect of intermediate principal stress, a new failure criterion based on the Hoek-Brown criterion is presented in this study for rocks as follows:

$$\sigma_1 = \sigma_3 + \sigma_c\left[m_b\frac{2\sigma_3 + \alpha(\sigma_2 - \sigma_3)}{2\sigma_c} + s\right]^a \tag{10}$$

where m_b and s = Hoek-Brown parameters; and α = parameter related to intermediate principal stress, $0 \le \alpha \le 1$.

Substituting Eqs. (7)–(9) into Eq. (10), the proposed failure criterion can be obtained in terms of generalized stress p,q, and θ as follows:

$$\frac{2q\cos\theta}{\sqrt{3}\sigma_c} = \left[\frac{(3\alpha - 2)\sin\theta + \sqrt{3}(\alpha - 2)\cos\theta}{6\sigma_c}m_bq + \frac{m_bp}{\sigma_c} + s\right]^a \tag{11}$$

When $\sigma_2 = \sigma_3 < \sigma_1$, from Eq. (11), it follows that $\theta = -\pi/6$. Substituting θ as $-\pi/6$ into Eq. (11), the triaxial compression meridian equation can be obtained as

$$\frac{q_c}{\sigma_c} = \left[\frac{-m_bq_c}{3\sigma_c} + \frac{m_bp}{\sigma_c} + s\right]^a \tag{12}$$

and $\theta = \pi/6$ responds to the triaxial tension meridian for the condition $\sigma_3 < \sigma_2 = \sigma_1$. Thus, the failure envelope equation can expressed as

$$\frac{q_t}{\sigma_c} = \left[\frac{(3\alpha - 4)m_bq_t}{6\sigma_c} + \frac{m_bp}{\sigma_c} + s\right]^a \tag{13}$$

From Eqs. (12) and (13), it can be seen that the triaxial compressive meridians of the new failure criterion are the same as those of the Hoek-Brown criterion. The triaxial tension meridians of the proposed criterion give different strength envelopes for different values of the parameter α, which improves the suitability of the proposed failure criterion for various rocks.

The failure criterion for the intact rocks, in which the parameters $s = 1$ and $a = 0.5$, can be written

$$\sigma_1 = \sigma_3 + \sigma_c\left[m_b\frac{2\sigma_3 + \alpha(\sigma_2 - \sigma_3)}{2\sigma_c} + 1\right]^{0.5} \tag{14}$$

Rearranging Eq. (14), the failure criterion can be written in terms of p,q, and θ

$$\begin{aligned} &8q^2\cos^2\theta - m_b\sigma_c\left[(3\alpha - 2)\sin\theta + \sqrt{3}(\alpha - 2)\cos\theta\right]q \\ &\quad - 6\sigma_c(m_bp + \sigma_c) = 0 \end{aligned} \tag{15}$$

Consequently, solving Eq. (15) gives the following failure criterion:

$$q = \frac{m_b\sigma_c\left[(3\alpha - 2)\sin\theta + \sqrt{3}(\alpha - 2)\cos\theta\right] + \sqrt{m_b^2\sigma_c^2\left[(3\alpha - 2)\sin\theta + \sqrt{3}(\alpha - 2)\cos\theta\right]^2 + 192\sigma_c(m_bp + \sigma_c)\cos^2\theta}}{16\cos^2\theta} \tag{16}$$

To discuss the effect of parameter α on the strength characteristics of rock materials, the parameters of the proposed failure criterion for Trachyte rock, σ_c = 99.13 MPa, m = 11.4 can be obtained from conventional triaxial compression tests. The failure envelopes of the proposed criterion in the deviatoric and meridian planes with a different parameter α are presented in Figs. 2 and 3, respectively.

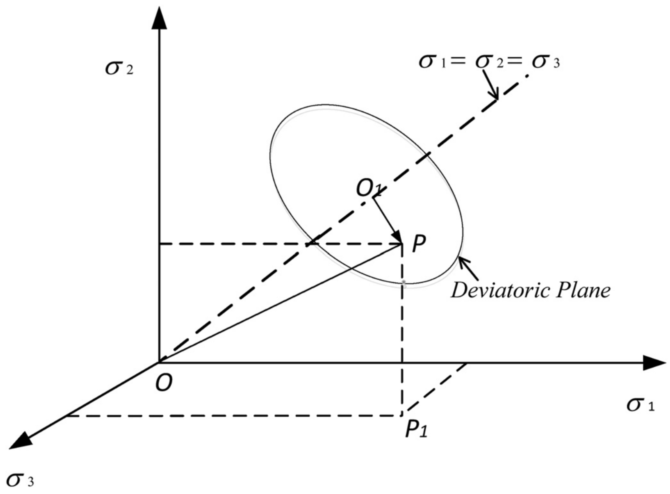

Fig. 1. Geometric representation of principal stress space and deviatoric plane.

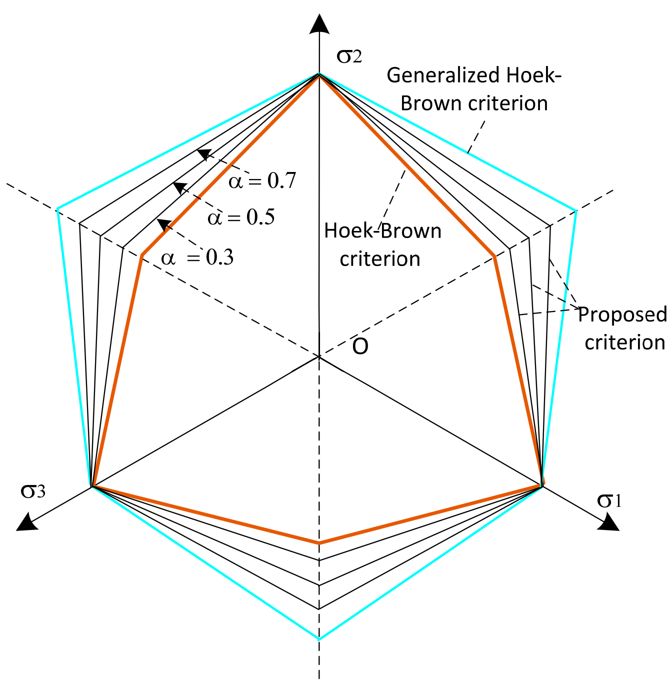

Fig. 2. Failure envelopes of proposed criterion in deviatoric plane.

From Fig. 2, it can be seen that the cross-sectional failure shapes of the new failure criterion in the deviatoric plane are irregular hexagons. The failure envelope distributions range from those of the Hoek-Brown criterion to those of the B. Singh criterion. This means that the Hoek-Brown criterion is the lower bound of the proposed criterion, and the B. Singh criterion can be regarded as the upper bound in the octahedral plane. The failure envelope allowed the transition from the lower bound to the upper bound with the change of coefficient α from 0 to 1. From Fig. 3, it can be seen that the change of parameter α will greatly change the failure envelope for the triaxial tension condition. The transition possibility of the new failure criterion for the different value of parameter α is the significant character and the important attribute of the proposed failure criterion.

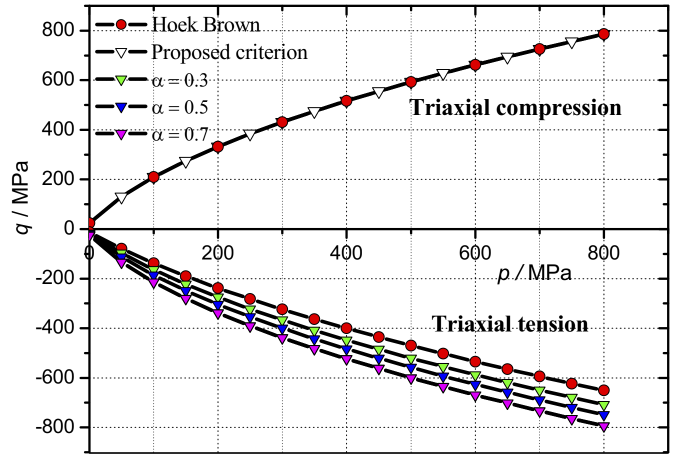

Fig. 3. Failure envelopes of proposed criterion in meridian plane.

Figs. 4(a–d) and Figs. 5(a–d) show polyaxial test results with the failure envelopes of the proposed criterion. It can be observed that the proposed failure criterion has a significant advantage and can well capture the polyaxial test results, whereas the Hoek-Brown failure criterion is only accurate under the conventional triaxial compression condition. The predicted results for rocks show the difference between the proposed failure criterion and the Hoek-Brown failure criterion in handling the influence of intermediate principal stress in a polyaxial loading condition. It can be seen that the proposed criterion can reasonably describe the effect of intermediate principal stress. The original Hoek-Brown criterion does not consider the influence of intermediate principal stress, which causes the principal stress σ_1 to be a constant value, with the change of principal stress, σ_2, at a fixed value of principal stress, σ_3. So, the failure envelope is a horizon line in, σ_2–σ_1, rectangular coordinate system. The influence of intermediate principal stress has been taken into account in the equations of the proposed criterion, and the failure envelope inclines in the σ_2–σ_1 plane to better fit the polyaxial data. The proposed criterion has an obvious advantage in capturing the influence of intermediate principal stress.

By studying the role of the parameter, the advantage and flexibility of the proposed function has been explored. It can be concluded that the envelope of the failure criterion is smooth everywhere except at the triaxial tension and compression conditions, and the proposed criterion has a noncircular convex failure envelope in the deviatoric plane. The existence of a corner in the failure surface causes difficulties in the numerical calculation; this is because the gradient of the failure surface is not uniquely defined at the corner. When this limitation was judged to be important, the use of an alternative shape function, which closely smooths the proposed criterion, was developed to overcome the limitation.

Smooth Approximations to Proposed Failure Criterion

A strength criterion can be expressed in terms of the invariant of stress tensor as follows (Zienkiewicz and Pande 1977; Al-Ajmi and Zimmerman 2005)

$$F\left(I_1, \sqrt{J_2}, \theta\right) = f\left(I_1, \frac{\sqrt{J_2}}{g(\theta)}\right) \tag{17}$$

where $g(\theta)$ = shape function in the deviatoric plane; I_1 = invariant of stress tensor; and J_2 = invariant of stress deviatoric tensor.

Int. J. Geomech., 2019, 19(6): 04019046

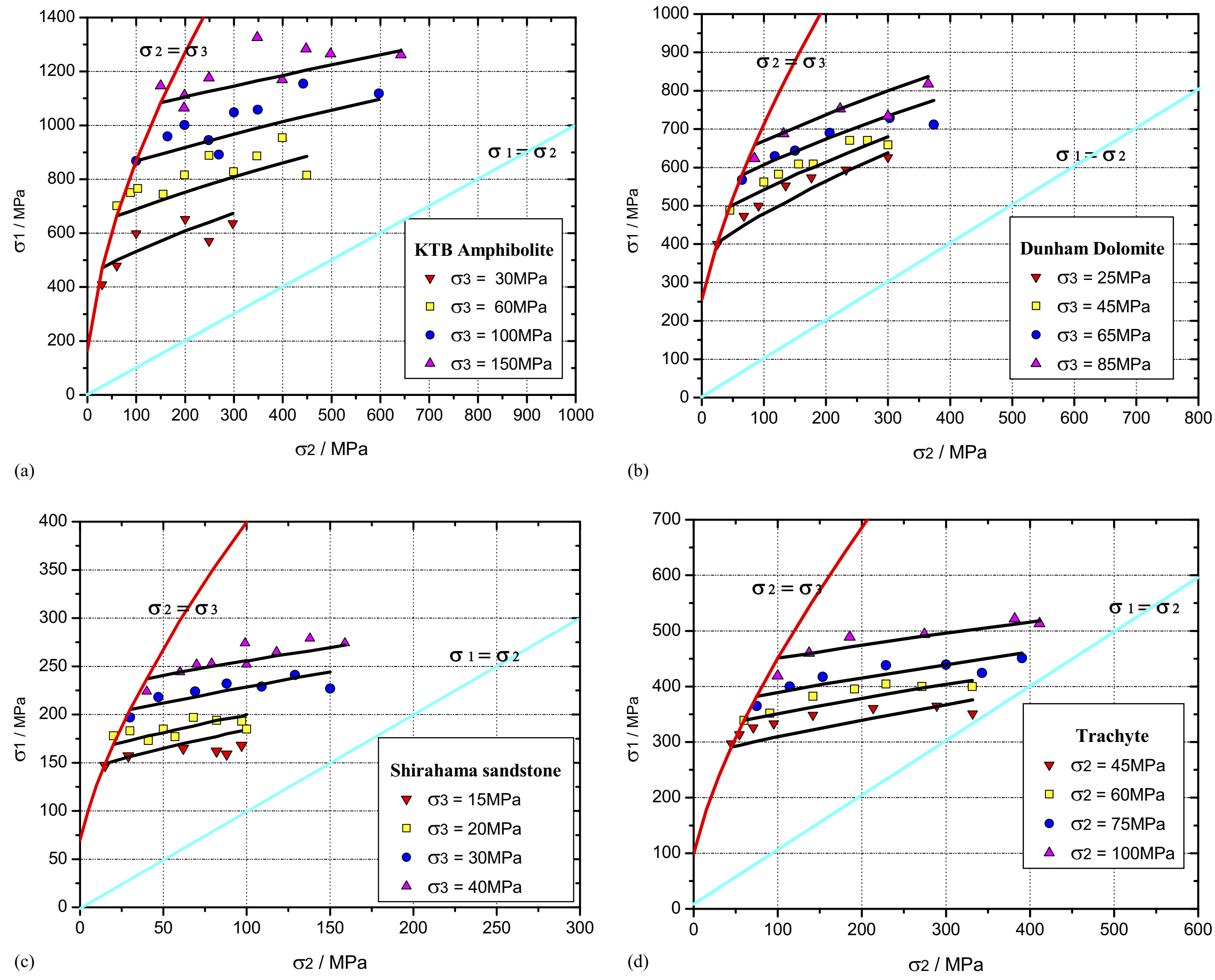

Fig. 4. Comparison of predictions with test data for various rocks in deviatioric plane: (a) KTB Amphibolite; (b) Dunham Dolomite; (c) Shirahama sandstone; and (d) Trachyte. KTB = German Continental Deep Drilling Program.

According to the failure characteristic of frictional material and the requirement of smoothness and convexity for the failure envelope, the shape function in the deviatoric plane should satisfy the following conditions (Su et al. 2009):

$$g(\theta)|_{\theta=-\pi/6} = 1,\ g(\theta)|_{\theta=\pi/6} = K \tag{18}$$

$$\left.\frac{dg(\theta)}{d\theta}\right|_{\theta=-\pi/6} = 0,\ \left.\frac{dg(\theta)}{d\theta}\right|_{\theta=\pi/6} = 0 \tag{19}$$

Several different generalizations of shape functions have been suggested by researchers (Piccolroaz and Bigoni 2009). In this study, the classic expressions of the Gudehus and Argyris, Willam and Warnke, and Lade-Duncan criteria were selected to assess the applicability of the shape function (Lee et al. 2012).

(1) Gudehus and Argyris function
The first, and the simplest, form of the shape function, which presents a smooth cross section and three axes of symmetry, was proposed simultaneously by Gudehus (1973) and Argyris and Schön (1974). The formula can be expressed in the following form:

$$g_{G-A}(\theta) = \frac{2K}{(1+K)+(1-K)\cos 3\theta} \tag{20}$$

where K is the constant related to the failure stress ratio, $K = q_t/q_c$.

(2) Willam and Warnke function
To describe the failure of concrete, an elliptic shape function is presented based on the original elliptic function proposed by Willam and Warnke (1975) as follows:

$$g_{W-W}(\theta) = \frac{2(1-K^2)\cos\theta + (2K-1)\left[4(1-K^2)\cos^2\theta + 5K^2 - 4K\right]^{1/2}}{4(1-K^2)\cos^2\theta + (2K-1)^2} \tag{21}$$

To ensure the smooth and convex failure surface in deviatoric plane, the value range of parameter K is $1/2 \le K \le 1$. The

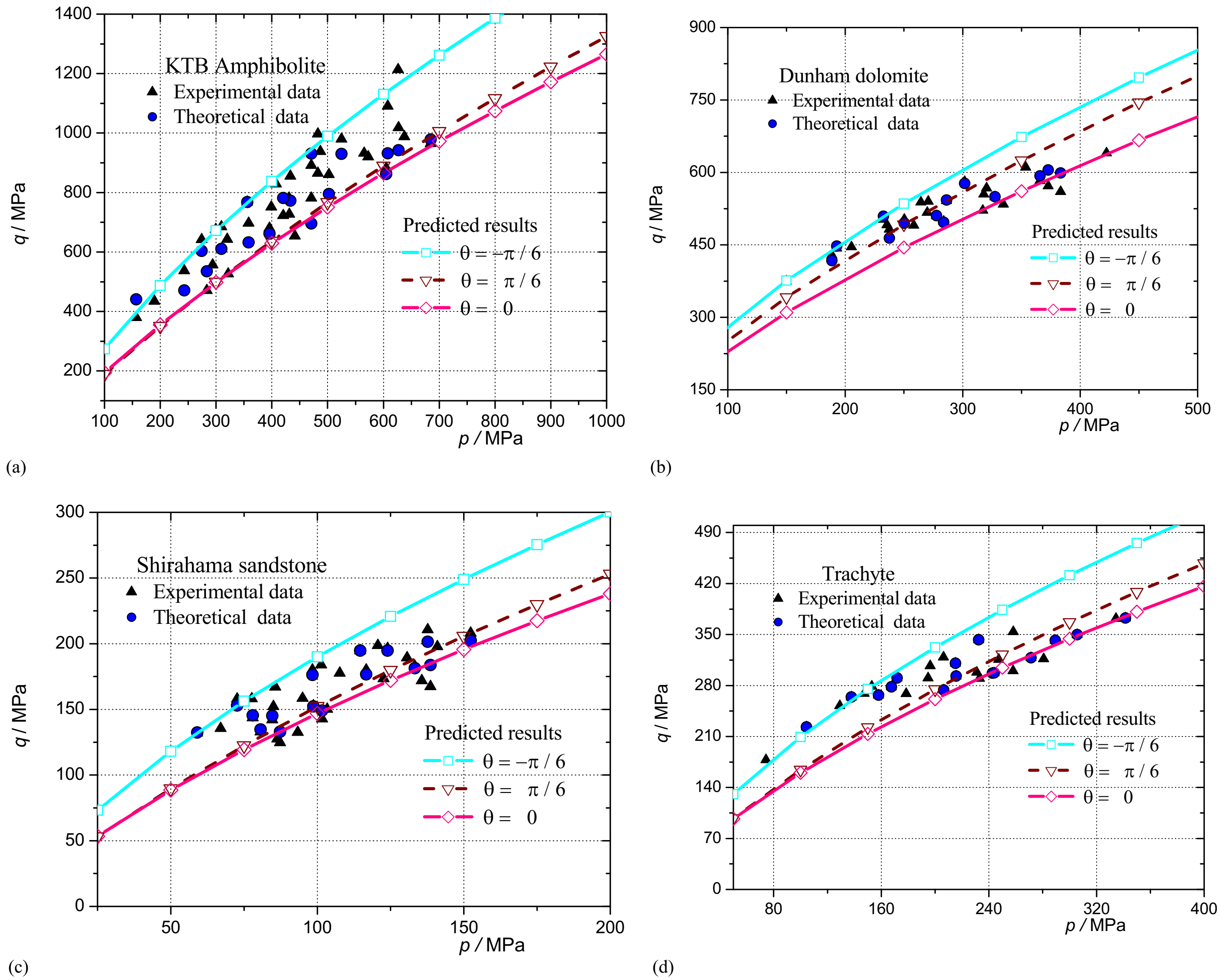

Fig. 5. Comparisons of predictions with test data for various rocks in meridian plane: (a) KTB Amphibolite; (b) Dunham Dolomite; (c) Shirahama sandstone; and (d) Trachyte. KTB = German Continental Deep Drilling Program.

failure envelope is a circle in deviatioric plane when $K = 1$. The shape function represents an equilateral triangle with non-smooth corners in deviatioric plane when $K = 1/2$.

(3) Lade-Duncan function

The Lade-Duncan strength criterion (Lade and Duncan 1975) is proposed in terms of stress invariants or principal stress. It is widely used to describe the failure behavior of frictional materials under complicated stress states. The criterion is given by

$$I_1^3/I_3 = k \tag{22}$$

where k = constant that determines the shapes of strength curves in the deviatoric plane.

The shape function can be obtained as follows:

$$g_{L-D}(\theta) = \frac{\sin\left(\dfrac{\pi}{3} - \dfrac{1}{3}\sin^{-1}\sqrt{\dfrac{k-27}{k}}\right)}{\sin\left(\dfrac{\pi}{3} + \dfrac{1}{3}\sin^{-1}\left(\sqrt{\dfrac{k-27}{k}}\sin 3\theta\right)\right)} \tag{23}$$

Fig. 6 presents the failure envelopes of the different failure shape functions and compares them with the proposed criterion. It can be seen that the failure envelopes of the Gudehus and Argyris, the Willam and Warnke, and the Lade-Duncan shape functions almost completely coincide when $-\pi/6 \le \theta \le \pi/6$, and the Hoek-Brown criterion is circumscribed by the failure criteria. The criteria predict obviously higher values than the proposed criterion except for triaxial compression and extension conditions. In order to obtain a shape function, which could more closely approach the proposed criterion, a new shape function should be presented in the following content.

A new shape function is suggested to smooth approximation to the proposed failure criterion, so that the failure criterion not only well reflects the influence of the intermediate principal stress but also meets the smoothness and convexity. In this study, the following formulation is suggested to describe the flexibility of the shape function:

$$g_\pi(\theta) = \frac{2\alpha_1(1+\sin 3\theta) + \beta(1-\sin 3\theta)}{2\alpha_1(1+\sin 3\theta)/K + \beta(1-\sin 3\theta)} \tag{24}$$

where α_1 and β = constants that determine the shapes of strength curves in deviatoric plane.

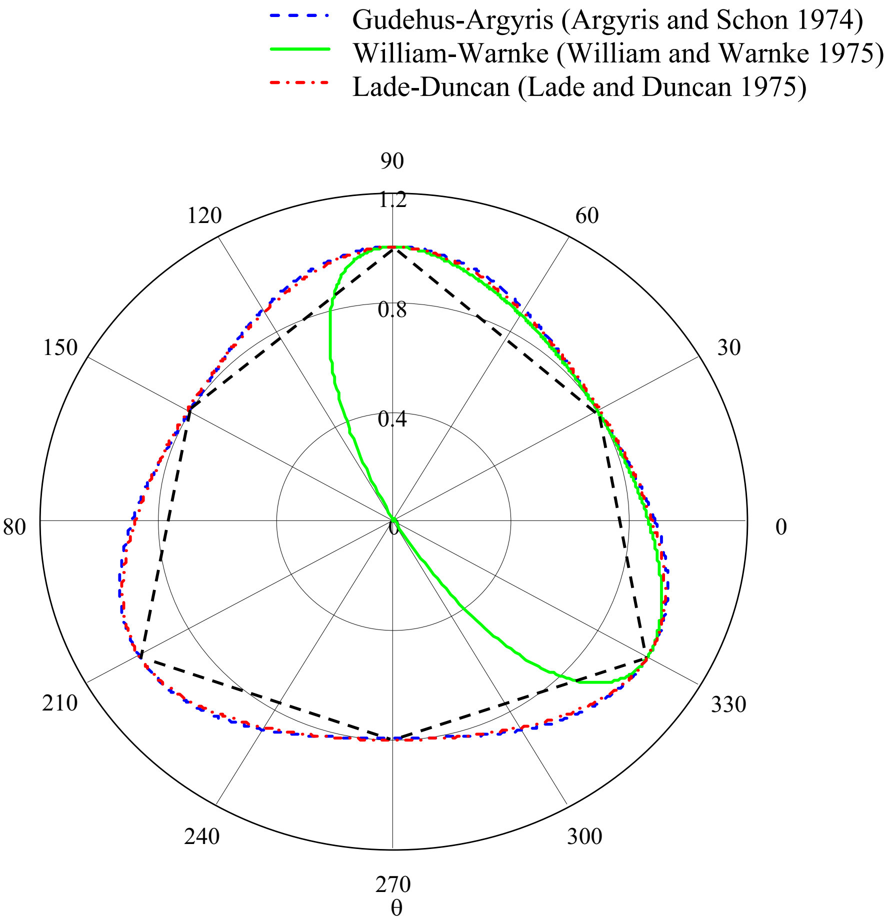

Fig. 6. Comparisons of common shape functions in deviatoric plane.

The following comparisons illustrate how the proposed shape function may be applied to describe the failure envelopes of rock materials with different parameters. After specifying the parameters $\alpha_1 = 1$ and $\beta = 1$, the variations of the envelopes influenced by parameter K between 0.6 and 0.9 are presented in Fig. 7. It can be seen by the envelope of the proposed shape function that there is transition from an irregular smooth triangle to a circle, demonstrating the great versatility in describing the failure surface of various rock materials under complicated stress states. Fig. 7 also reveals that the envelope becomes convex in the deviatoric plane when K is small. Because the parameter K is related to the rock properties and determined by the ratio of triaxial tension and compression strengths, the shape function requires the parameters α_1 and β in order to meet the convex conditions. The variations of the envelope influenced by parameters α_1 and β are presented in Figs. 8 and 9, respectively. Figs. 8 and 9 indicate that the proposed criterion can take different forms, depending on the values of the parameters used. Compared with the aforementioned shape functions, the proposed criterion offers more flexibility in its smoothing ability for the failure criterion.

The convexity of the failure surface, which is important for numerical applications (Lin and Bažant 1986), is addressed in the following content. To ensure the convexity of the shape function, the shape function should follow the equation as (Jiang and Pietruszczak 1988)

$$g^2 + 2g'^2 - gg'' \geq 0 \tag{25}$$

Assuming $\rho = 1/g$, the formulation Eq. (25) is equal to the following formulation:

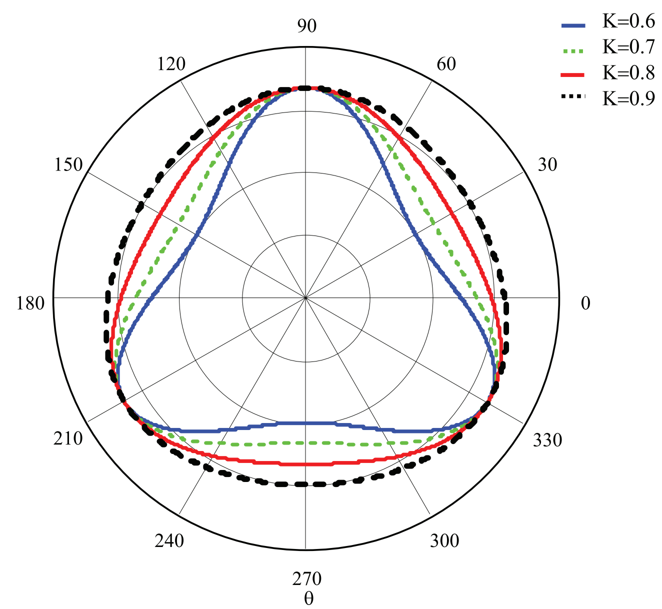

Fig. 7. Envelopes of proposed shape function with different K.

$$\rho + \rho'' = \frac{1}{g} + \left(\frac{1}{g}\right)'' = \frac{g^2 + 2g'^2 - gg''}{g^3} \geq 0 \tag{26}$$

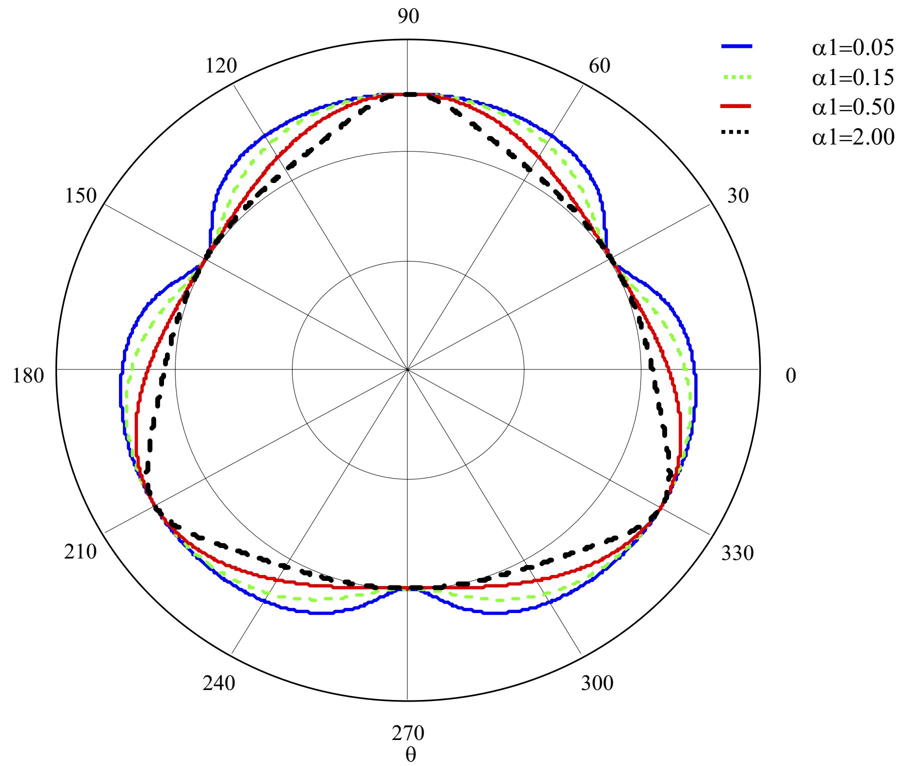

Fig. 8. Envelopes of proposed shape function with different $\alpha 1$.

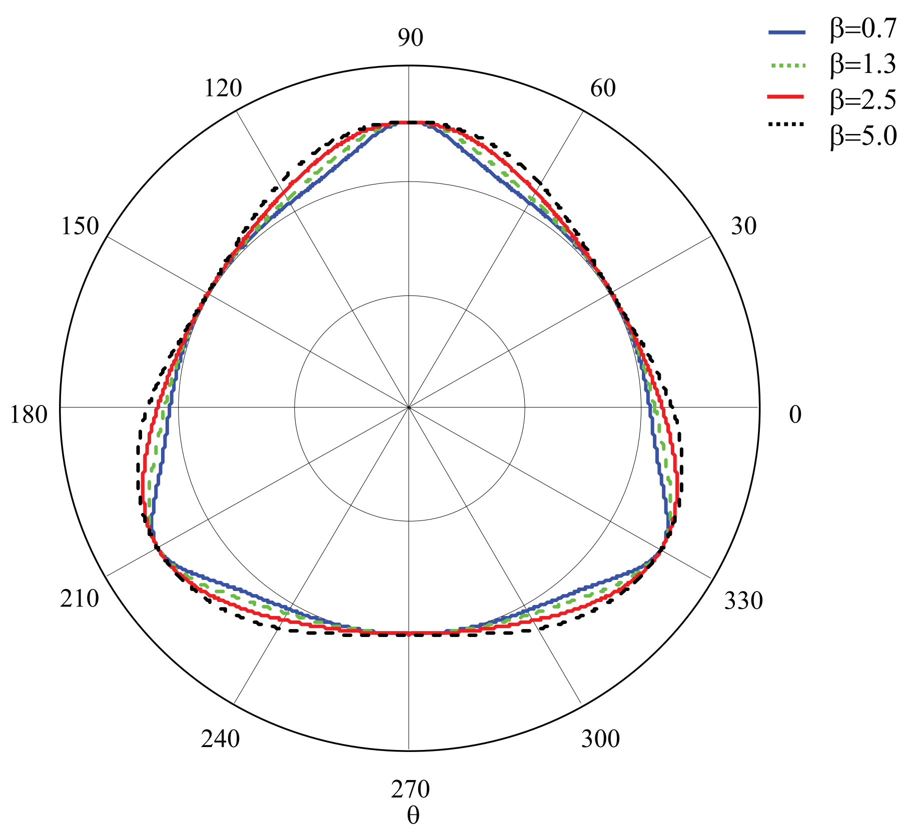

Fig. 9. Envelopes of proposed shape function with different β.

Int. J. Geomech., 2019, 19(6): 04019046

To provide for convenient use of the proposed shape function, it can be rewritten

$$g(\theta) = \frac{k'(a + \sin 3\theta)}{b + \sin 3\theta} \tag{27}$$

where the parameters can be expressed as

$$k' = \frac{(2\alpha - \beta)K}{2\alpha - \beta K} \tag{28a}$$

$$a = \frac{2\alpha + \beta}{2\alpha - \beta} \tag{28b}$$

$$b = \frac{2\alpha + \beta K}{2\alpha - \beta K} \tag{28c}$$

Substituting Eq. (27) into Eq. (26), the following equation can be obtained

$$\rho + \rho'' = \frac{1}{k'}\frac{b + \sin 3\theta}{a + \sin 3\theta} + \frac{9(a-b)(\sin^2 3\theta - a\sin 3\theta - 2)}{k'(a + \sin 3\theta)^3} \tag{29}$$

Combining Eqs. (26) and (29), the parameters should meet the conditions listed in the Appendix to ensure the convexity of smooth function.

The failure envelope of the proposed shape function is presented in Fig. 10. From Fig. 10, it can be seen that the envelope of the proposed shape function not only coincides with that of the proposed failure criterion in triaxial compression and triaxial extension conditions but that it also smooths the failure criterion better than the Gudehus and Argyris, Willam and Warnke, and Lade-Duncan criteria. This means the proposed shape function can more closely reproduce the characteristics of the proposed criterion.

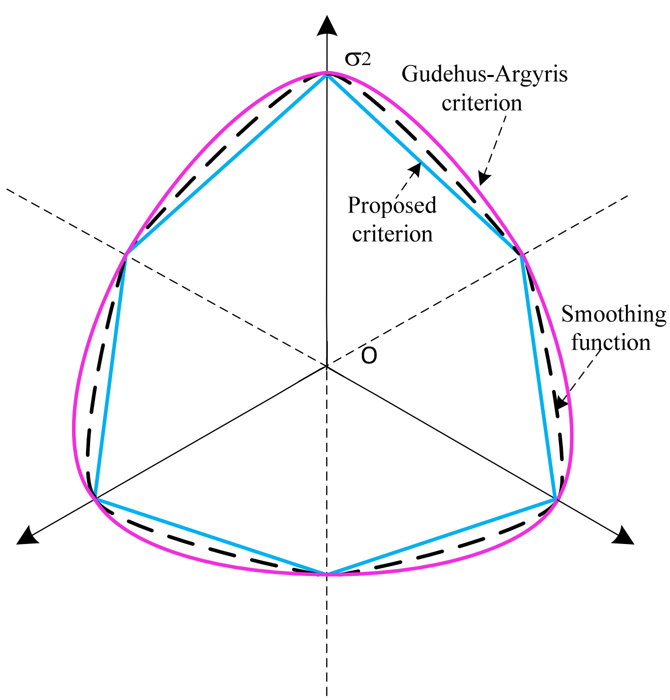

Fig. 10. Envelopes of proposed shape function smoothing the failure criterion.

By way of Eq. (17), the shape function can now be assigned to the proposed failure criterion, and the 3D failure criterion can be expressed as

$$\frac{q}{\sigma_c g_N(\theta)} = \left[\frac{-m_b q}{3\sigma_c g_N(\theta)} + \frac{m_b p}{\sigma_c} + s\right]^a \tag{30}$$

For the intact rock, $s = 1$, $a = 0.5$ can be obtained, then Eq. (30) is simplified to the following form:

$$\frac{q}{g_N(\theta)} = -\frac{m_b}{6}\sigma_c + \sqrt{\frac{m_b^2\sigma_c^2}{36} + \sigma_c(m_b p + \sigma_c)} \tag{31}$$

Experimental Verification of the Proposed Criterion

To investigate the performance of the proposed 3D failure criterion, it is necessary to use reliable experimental data to validate the criterion for true triaxial tests. In this study, the first two sets of experimental data were obtained from polyaxial tests on Trachyte and Dunham Dolomite (Zhou et al. 2014). The other three data sets for Shirahama sandstone, Westerly granite, and German Continental Deep Drilling Program (KTB) Amphibolite were obtained from Lee et al. (2012).

The proposed failure criterion has a convenient feature in that it uses the same parameters m and s (as for the Hoek-Brown criterion) with the assumption of intact rock. The parameters can be obtained by using the least-squares technique, and the object function can be defined as follows:

$$\delta_i = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\sigma_i^{\text{calc}} - \sigma_i^{\text{test}}\right)^2} \tag{32}$$

where n = number of experimental data; σ_i^{calc} = predicted results; and σ_i^{test} = experimental data.

Fig. 11 presents the application of the new 3D criterion to describe the strength of various rocks under true triaxial stress states. In all of these cases, the criterion is in good agreement with the test results. The comparisons show that the 3D criterion can well reflect the influence of intermediate principal stress on the strength for rocks. Fig. 12 compares the new 3D failure criterion with the classic criteria for Trachyte rock under different σ_3 values. The experimental results are compared with those corresponding to the Hoek-Brown, B. Singh, Pan-Hudson (Pan and Hudson 1988), and Zhang-Zhu criteria. It is observed that the Hoek-Brown strength criterion fit very well to the experimental results under the triaxial compression condition. However, with the increase of intermediate principal stress, the discrepancies between predicted results and experimental data became obvious. The Hoek-Brown strength criterion, which does not take into account the intermediate principal stress effect, constantly predicted values that were lower than experimental data. The comparisons confirmed that the original Hoek-Brown strength criterion could not well predict the failure of rocks at the true triaxial stress state, particularly for relatively high values of the intermediate principal stress. The B. Singh criterion also fit very well to the experimental data under triaxial compression condition, but it predicted values that were much higher than experimental results under increasing intermediate principal stress. The Pan-Hudson criterion predicted identical strength values for both the triaxial compression and extension conditions; however, its predictions did not fit well with the experimental results at the triaxial tension stress state. The Zhang–Zhu criterion captured the experimental data

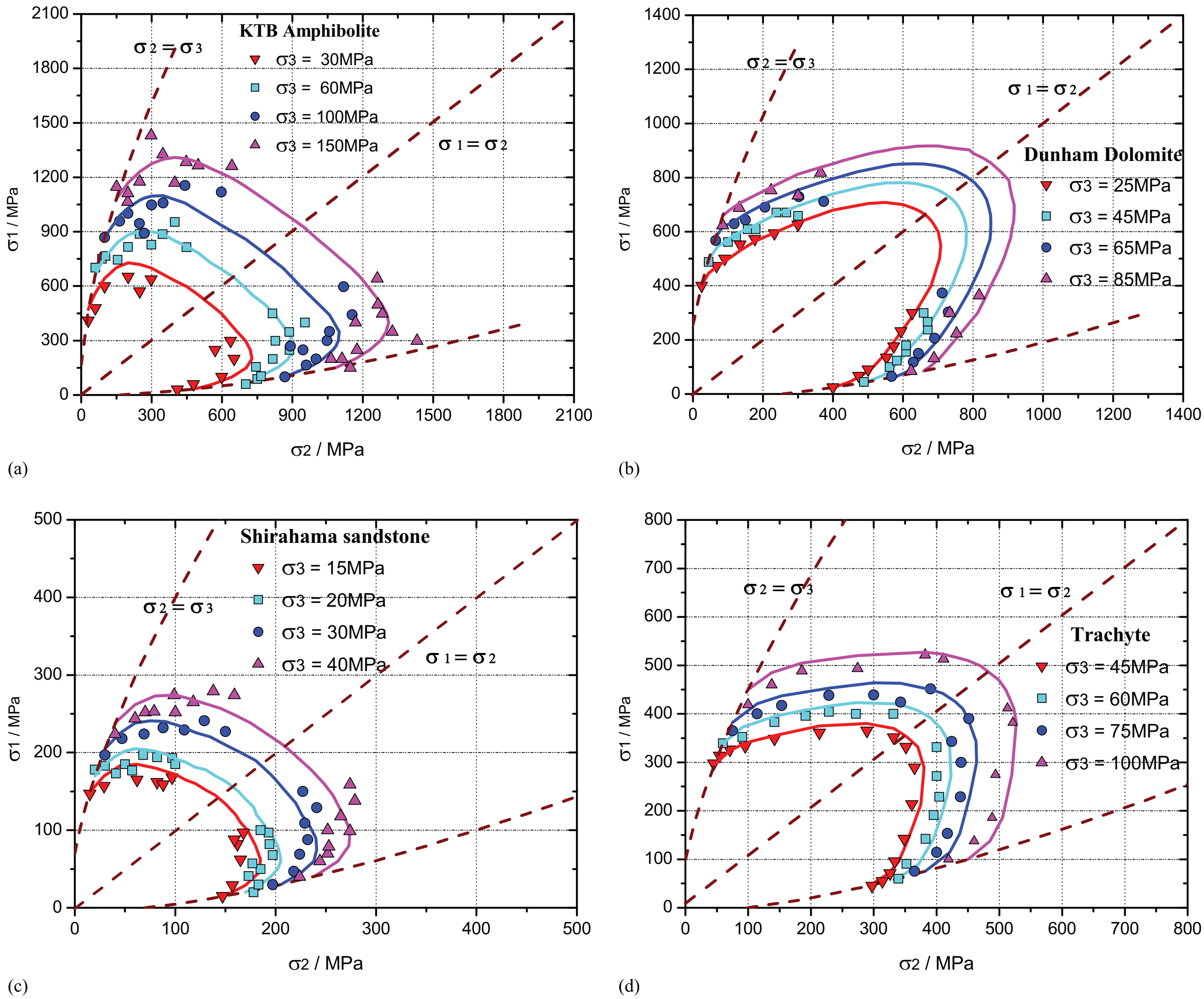

Fig. 11. Comparisons of smoothing failure criterion with test data for various rocks: (a) KTB Amphibolite; (b) Dunham Dolomite; (c) Shirahama sandstone; and (d) Trachyte. KTB = German Continental Deep Drilling Program.

by taking the influence of intermediate principal stress into account. However, the Zhang–Zhu failure criterion had difficulties in numerical application at the triaxial stress state because the failure envelope of the Zhang–Zhu failure criterion was not smooth at both the triaxial compression and extension states. The new proposed 3D failure criterion captured the experimental data better than the Hoek-Brown, B. Singh, or Pan-Hudson criteria under true triaxial stress states, and the envelope of the failure criterion was convex and smooth in the principal stress space.

Summary and Conclusion

In order to asses the influence of the intermediate principal stress on the failure behavior of rock materials, a new 3D failure criterion—in which the Hoek-Brown criterion can be regarded as the lower bound and the B. Singh failure criterion as the upper bound in the deviatoric plane—is proposed in this study. The strength parameters of the proposed criterion can be easily obtained by using the triaxial compression and tension tests. The proposed criterion not only inherits the advantages of the Hoek-Brown failure criterion at triaxial compression state, but reflect the influence of intermediate principal stress well in complicated stress states. The criterion is smoothed by presenting a new Lode dependence function with characteristics of both smoothness and convexity. The general characteristics of the failure surface were analyzed and compared with some widely used failure criteria with its cross-sectional envelopes in the π plane and its meridian shape in the meridian plane. The main characteristics of the new 3D criterion have been described in detail, and the advantages of this criterion are analyzed from every aspect.

Appendix. Conditions for convex problems

(1) When $a > \frac{23}{4}$, the parameter b should have

$$\frac{8a-1}{a+10} < b < \frac{8a^2-22a-3}{11a-16} \tag{33}$$

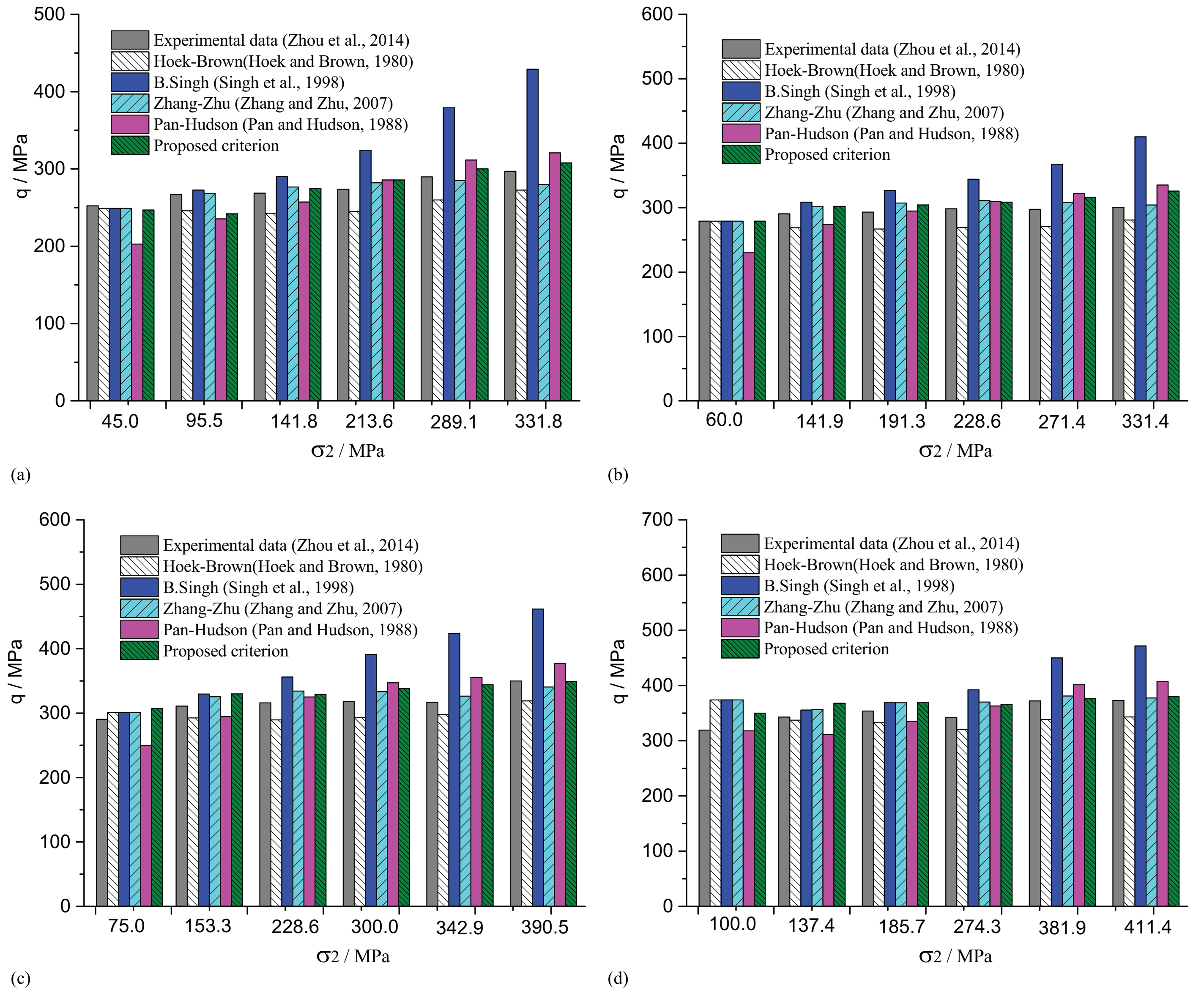

Fig. 12. Comparisons of failure criteria and test data of Trachyte rock: (a) $\sigma_3 = 45$ MPa; (b) $\sigma_3 = 60$ MPa; (c) $\sigma_3 = 75$ MPa; and (d) $\sigma_3 = 100$ MPa.

$$\text{or} \quad \frac{8a^2 + 22a - 3}{11a + 16} < b < a \tag{34}$$

$$\text{or} \quad \frac{8a^2 - 22a - 3}{11a - 16} < b < \frac{8a^2 + 22a - 3}{11a + 16} \quad \text{and} \quad f(t_0) > 0 \tag{35}$$

where

$$f(t) = \frac{b + t}{a + t} + \frac{9(a - b)(t^2 - at - 2)}{(a + t)^3} \qquad t \in [-1, 1] \tag{36}$$

$$t_0 = -\frac{(11a - 8b)}{3} + \sqrt{\frac{145a^2 - 209ab + 64b^2}{9}} \tag{37}$$

(2) When $\frac{11}{7} \le a \le \frac{23}{4}$, the parameter b should have

$$\frac{8a - 1}{a + 10} < b < \frac{8a^2 + 22a - 3}{11a + 16} \quad \text{and} \quad f(t_0) > 0 \tag{38}$$

$$\text{or} \quad \frac{8a^2 + 22a - 3}{11a + 16} < b < a \tag{39}$$

(3) When $1 < a < \frac{11}{7}$, the parameter b should have

$$1 < b < \frac{8a^2 + 22a - 3}{11a + 16} \quad \text{and} \quad f(t_0) > 0 \tag{40}$$

$$\text{or} \quad \frac{8a^2 + 22a - 3}{11a + 16} < b < a \tag{41}$$

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