Theoretical framework of the friction and wear phenomena through of thermodynamics approaches: application to aeronautical braking

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Theoretical framework of the friction and wear phenomena through of thermodynamics approaches : application to aeronautical braking

Numerical implementation with **MFRONT** and **Code-Aster**

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Summary

Problem statement

Archard law :

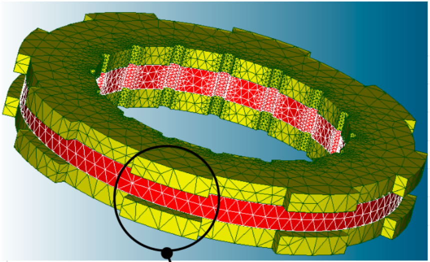
$$V = k\frac{F_n D}{H} \tag{1}$$

Limitations of this law :

- Wear is described as the total loss of material from the surfaces in contact : a valid assumption that when the quantity of material produced during friction is equal to that ejected out of contact.
- The hypothesis of a 3rd body in contact is therefore neglected.

Goal : setup a degradation criterion on the finite element scale which takes into account the rheology of the material liable to be trapped in the contact ...

The general idea of the model



Mettre en place une loi thermo-élasto-viscoplastique avec MFRONT

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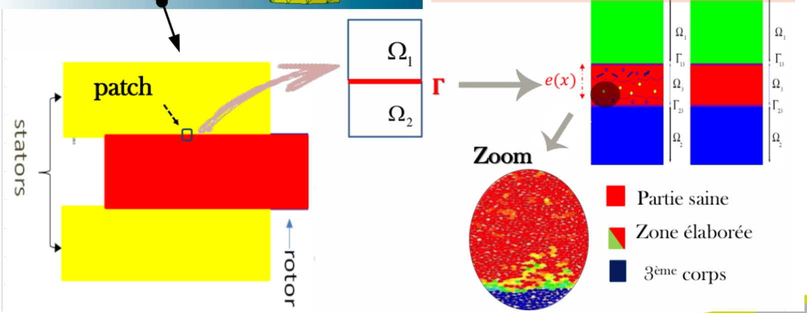
Cas-test de validation : essai de cisaillement avec Code_aster

Modèle continu [Stolz , 2010]



Concretely, it is about :

- Implement a thermo-elasto-viscoplastic law with the Mfront tool. The goal is to reproduce at finite element scale the thermo-mechanical behaviour of an aeronautical braking process.
- The model is validated with a benchmark of shearing with Code-Aster.

Rheology of 1st bodies - Assumption

The bodies in contact have a thermo-elastic behaviour far from the contact zone and susceptible to damage when they are in friction.

- The behaviour of the 1st body is assumed to be elastic and fragile.
- The stiffness decreases irreversibly when the strain energy becomes high.
- This loss of rigidity is measured by a variable of isotropic damage noted D. It mainly depends on the energy of deformation of the bodies in contact (1st body) :

$$D = \left\langle \frac{\sqrt{\mathcal{Y}_{\max}} - \sqrt{\mathcal{Y}_D}}{S} \right\rangle . \qquad (2)$$

- The stress, $\underline{\underline{\sigma}}$ cannot exceed a threshold which also decreases with the level of damage to reach 0 when the material is totally damaged.

Rheology of 1st bodies - Assumption

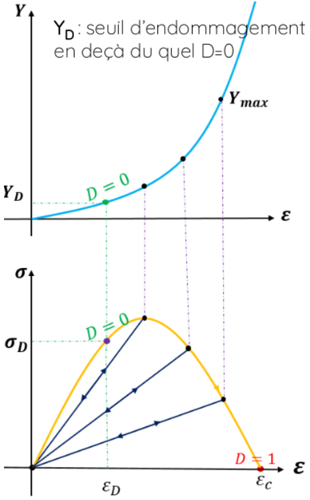


Figure – Deformation energy and stresses of contacting bodies.

Material properties

C/C : material with transverse isotropic properties.
The stiffness matrix at given temperature is written :

$$\boldsymbol{\mathcal{A}} = \begin{pmatrix} \lambda_\perp + 2\mu_\perp & \lambda_\perp & \lambda & 0 & 0 & 0 \\ \lambda_\perp & \lambda_\perp + 2\mu_\perp & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda_\perp + 2\mu_\perp & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_\perp \end{pmatrix} \qquad (3)$$

First body law - Thermoelastic behaviour

These material parameters are actually a function of the contact temperature, i.e. : $E, E_\perp = f\,(T)$.

- Thermoelasticity of the 1st bodies in contact :

$$\underline{\underline{\sigma}} = (1 - D)\,\boldsymbol{\mathcal{A}} : \underline{\underline{\varepsilon}}^{e} = (1 - D)\,\underline{\underline{\sigma}}^{nd} \tag{4}$$

with : $\underline{\underline{\varepsilon}}^{e} = \underline{\underline{\varepsilon}}^{to} - \underline{\underline{\varepsilon}}^{th} - \underline{\underline{\varepsilon}}^{vp}$ and where : $\underline{\underline{\varepsilon}}^{th} = \underline{\underline{\alpha}}\,(T)\,(T - T_0)$

$$\underline{\underline{\alpha}}\,(T) = \begin{pmatrix} \alpha_\perp & 0 & 0 \\ 0 & \alpha_\perp & 0 \\ 0 & 0 & \alpha \end{pmatrix} \tag{5}$$

$\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{A}}\,(T_0)$: the stiffness matrix decreases with the levels of damage ;
$\alpha_\perp$: thermal expansion in the normal direction to the fibers ;
α : thermal expansion in direction along fibers ;

First body law - damage description

- **Damage law : 1st bodies in contact**

$$D = \left\langle \frac{\sqrt{\mathcal{Y}_{\text{max}}} - \sqrt{\mathcal{Y}_D}}{S} \right\rangle \tag{6}$$

where :

$$\overset{\bullet}{D} = \begin{cases} 0 & \text{si } \mathcal{Y} < \mathcal{Y}_{max} \\ \dfrac{\overset{\bullet}{\mathcal{Y}}_{\text{max}}}{2S\sqrt{\mathcal{Y}_{\text{max}}}} & \text{si } \mathcal{Y} = \mathcal{Y}_{\text{max}} \text{ et } \dfrac{d\mathcal{Y}}{dt} > 0 \end{cases} \tag{7}$$

whith :
$\mathcal{Y}_{\text{max}}$: Maximum damage driven force ;
$\mathcal{Y}_D$: Threshold damaging energy of friction material ;
S : Breaking threshold of bodies in contact.

Example of rheology of 3rd body

- The variation of the volume of 3rd body in the contact is due on the one hand to the damage on the other hand to the visco-plastic deformations generated. These properties are defined empirically from a discrete element model. σ_y is the elastic limit in shear of the 1st bodies.

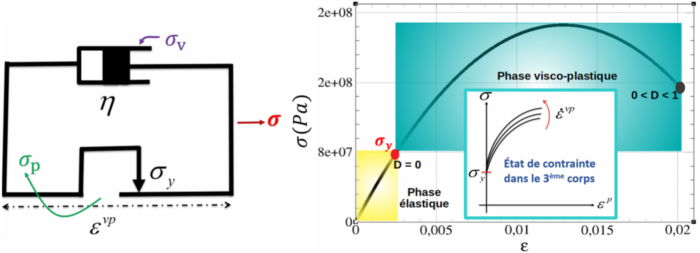


Figure – Rheology of 3rd body

Rheology of 3rd body - equations

- **Visco-plastic evolution :** 3rd body

$$\frac{d\underline{\underline{\varepsilon}}^{vp}}{dt} = \frac{dp}{dt}\underline{\underline{n}} = \left\langle \frac{\sigma_{eq} - \sigma_y(T)}{\eta(T)} \right\rangle \underline{\underline{n}} \tag{8}$$

$\underline{\underline{n}} = \frac{3}{2}\frac{\underline{\underline{\sigma}}'}{\sigma_{eq}}$: von Mises normal

$\sigma_{eq} = \sqrt{\frac{3}{2}\underline{\underline{\sigma}}' : \underline{\underline{\sigma}}'}$: von Mises' deviatoric stress

$\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} - \frac{1}{3}\mathrm{Tr}\left(\underline{\underline{\sigma}}\right)\underline{\underline{\mathbb{I}}}$: the deviatoric part of the stress tensor.

$\sigma_y(T) = \sigma_0 \exp\left(-BT\right)$: the shear yield strength of bodies in contact. It decreases by assumption with T;

$\eta(T) = \eta_0 \exp\left(-AT\right)$: viscosity of the 3rd body (data resulting from the model by discrete element).

Implicit scheme – equations to solve

If : $\vec{H} = \left(\Delta\underline{\underline{\varepsilon}}^e, \Delta p\right)^t$, the vector which groups the evolution of the variables state. The implicit system of equations to be solved by the Newton-Raphson method, is written :

$$F\left(\Delta\vec{H}\right) = 0 \iff \begin{cases} f_{\underline{\underline{\varepsilon}}^e} = \Delta\underline{\underline{\varepsilon}}^e - \Delta t g_{\underline{\underline{\varepsilon}}^e}\left(H\Big|_{t+\theta\Delta t}, t\right) = 0 \\ \\ f_p = \Delta p - \Delta t g_p\left(H\Big|_{t+\theta\Delta t}, t\right) = 0 \end{cases} \tag{9}$$

Implicit scheme – discretization

The non-linear system to be solved is thus :

$$F\left(\Delta\vec{H}\right) = \begin{cases} f_{\underline{\underline{\varepsilon}}^e} = \Delta\underline{\underline{\varepsilon}}^e - \left(\Delta\underline{\underline{\varepsilon}}^{to} - \Delta\underline{\underline{\varepsilon}}^{th}\right) + \Delta p \underline{\underline{n}}\Big|_{t+\theta\Delta t} = 0 \\ f_p = \Delta p - \Delta t \left\langle \dfrac{\sigma_{eq}\Big|_{t+\theta\Delta t} - \sigma_0 \exp\left(-BT\big|_{t+\theta\Delta t}\right)}{\eta_0 \exp\left(-AT\big|_{t+\theta t}\right)} \right\rangle = 0 \end{cases} \tag{10}$$

One thus obtains a system of 7 equations : 6 equations relating to the additive decomposition of the strain tensor (in 3D), and an equation relating to the visco-plastic flow of the 3th body.

Implicit integration - Jacobian matrix

If we set $\mathbb{J}$, the Jacobian matrix of the system defined above. Its expression according to internal variables $\Delta\underline{\underline{\varepsilon}}^{\mathrm{e}}, \Delta p$ is defined as :

$$\mathbb{J} = \frac{\partial F}{\partial \Delta \vec{H}} = \begin{pmatrix} \dfrac{\partial f_{\underline{\underline{\varepsilon}}^e}}{\partial \Delta \underline{\underline{\varepsilon}}^e} & \dfrac{\partial f_p}{\partial \Delta \underline{\underline{\varepsilon}}^e} \\ \\ \dfrac{\partial f_{\underline{\underline{\varepsilon}}^e}}{\partial \Delta p} & \dfrac{\partial f_p}{\partial \Delta p} \end{pmatrix} \tag{11}$$

One of the keys of the numerical model is to determine the exact values of the components of this Jacobian matrix according to the state variables.

Consistent Tangent Operator

By definition, the consistent tangent operator is written as :

$$\frac{d\underline{\underline{\sigma}}}{d\Delta\underline{\underline{\varepsilon}}^{\mathsf{to}}} = \frac{\partial\underline{\underline{\sigma}}}{\partial\Delta\underline{\underline{\varepsilon}}^{\mathsf{to}}} + \frac{\partial\underline{\underline{\sigma}}}{\partial\Delta\vec{H}}\frac{\partial\Delta\vec{H}}{\partial\Delta\underline{\underline{\varepsilon}}^{\mathsf{to}}} \qquad (12)$$

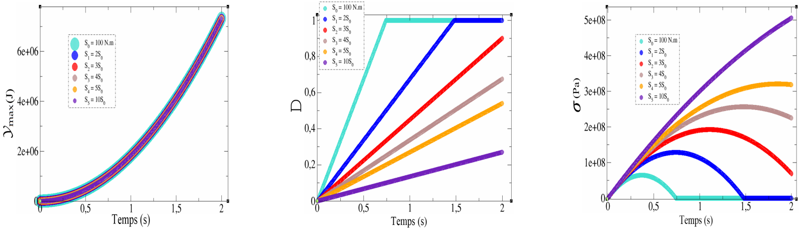
$$\iff \left.\frac{d\underline{\underline{\sigma}}}{d\Delta\underline{\underline{\varepsilon}}^{\mathsf{to}}}\right|_{t+\Delta t} = \left.\frac{\partial\underline{\underline{\sigma}}}{\partial\Delta\underline{\underline{\varepsilon}}^{\mathsf{e}}}\frac{d\Delta\underline{\underline{\varepsilon}}^{\mathsf{e}}}{d\Delta\underline{\underline{\varepsilon}}^{\mathsf{to}}}\right|_{t+\Delta t} \qquad (13)$$

From Hooke's law we prove that :

$$\left.\frac{\partial\underline{\underline{\sigma}}}{\partial\Delta\underline{\underline{\varepsilon}}^{\mathsf{e}}}\right|_{t+\Delta t} = \left(1 - \left.D\right|_{t+\Delta t}\right)\boldsymbol{\mathcal{A}} - \left.\frac{\theta\Delta\underline{\underline{\sigma}}^{\mathsf{nd}}}{2S\sqrt{\mathcal{Y}_{\mathsf{max}}}} : \underline{\underline{\sigma}}^{\mathsf{eff}}\right|_{t+\Delta t} \qquad (14)$$

Some studies : Primary creep in compression - shear

$\varepsilon_{zz} = -2\%$ et $\varepsilon_{xx} = 2\%$ $E_\perp = 3,5.10^{10} Pa$; $\nu_\perp = 0,25$;
$E_{||} = 2,5.10^{9} Pa$ et $\nu_{||} = 0,2$ à $T = 298.15K$.



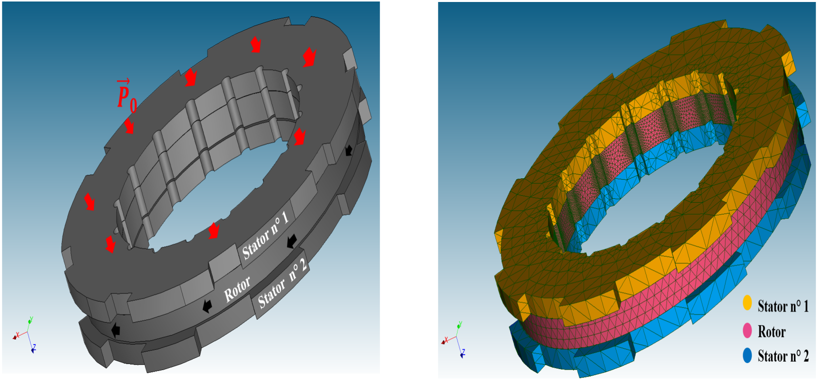
(a) $\mathcal{Y}_{max}$=f(S) (b) endo = f(S) (c) $\sigma = f\,(S)$

Figure – Influence of breaking threshold

Some studies : Primary creep in compression - shear

- The damage energy is identical in all the cases of studies whatever the value of breaking threshold.
- The damage to bodies in contact is delayed with the increase in the breaking point of the material. This has the effect of increasing the elasticity range.
- For the low values of the breaking threshold, one observes a lower stress state in the first moments of the simulation.
- In each case, the visco-plastic strains for the same energy $\mathcal{Y}_{\mathsf{max}}$, take more time to develop with the increase in the breaking point.

Some studies : Rotor-stator interface in 3D



(a) (b)

Figure – Geometry, boundary conditions and mesh

Algorithme 1 : Contact and friction conditions

Entrées : $\underline{\xi}_0$; ε_n ;$\varepsilon_t(i=0)$; $\underline{\lambda} = \underline{t}_c$

1 Get the contact conditions ($\varepsilon_n, \varepsilon_t$ et $\underline{t}_c$)

2 Computation of the incremental correction, $\Delta\underline{\xi}_i$:

$\underline{R}(\underline{\xi}_i) + \underline{K}_T(\underline{\xi}_i)\Delta\underline{\xi}_i = 0$

3 Computation of the displacements at iteration $i+1$:

$\underline{\xi}_{i+1} = \underline{\xi}_{i+1} + \Delta\underline{\xi}_i$

4 **if** *(Convergence)* **then**

5   **if** *(penetration and relative slip are less than tolerance)* **then**

6         $\underline{\xi}_{i+1}$ = equilibrium state

7   **else**

8   Update of the Lagrange multipliers ($\underline{\lambda} = \underline{t}_c$)

9   Back to step 1 with $i = i+1$

10   **end**

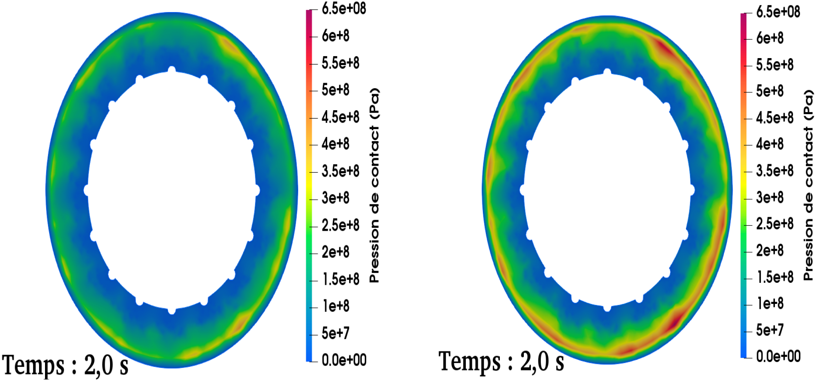
11 **else**

12   Back to step 1 with $i = i+1$

13 **end**

Some studies

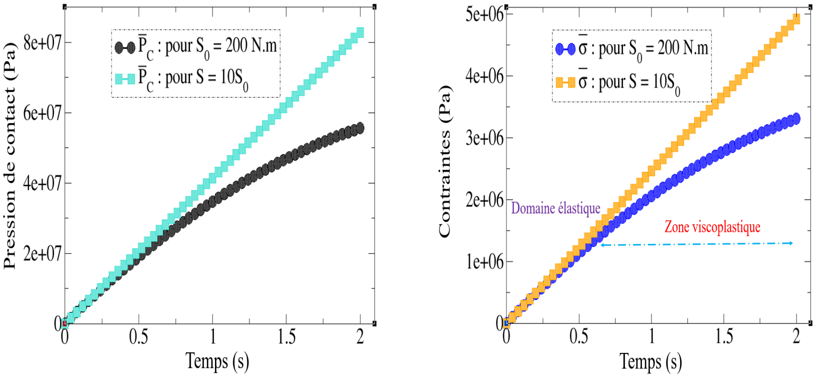
- Contact pressure distribution after 2s



(a) $S_0 = 200$ N.m (b) $S = 10S_0$

Figure – Influence of breaking threshold

Some studies



(a) Contact pressure (b) Final stress

Figure – Influence of breaking threshold

Some studies

- Contact pressure distribution after 2s

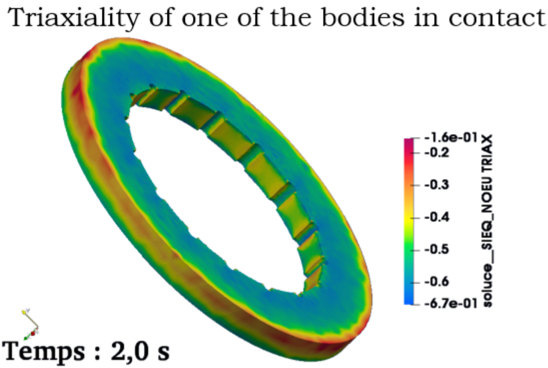


Figure – Triaxiality of the stresses

Future works

- Writing of dependencies of material properties and friction with temperature
- Adjustment the parameters of the 3rd body from the braking test databases and data from local models
- Model formulation in large deformations.

Références I

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