Optimization Methods

Until now, you've always used Gradient Descent to update the parameters and minimize the cost. In this notebook, you will learn more advanced optimization methods that can speed up learning and perhaps even get you to a better final value for the cost function. Having a good optimization algorithm can be the difference between waiting days vs. just a few hours to get a good result.

Gradient descent goes "downhill" on a cost function
J. Think of it as trying to do this:

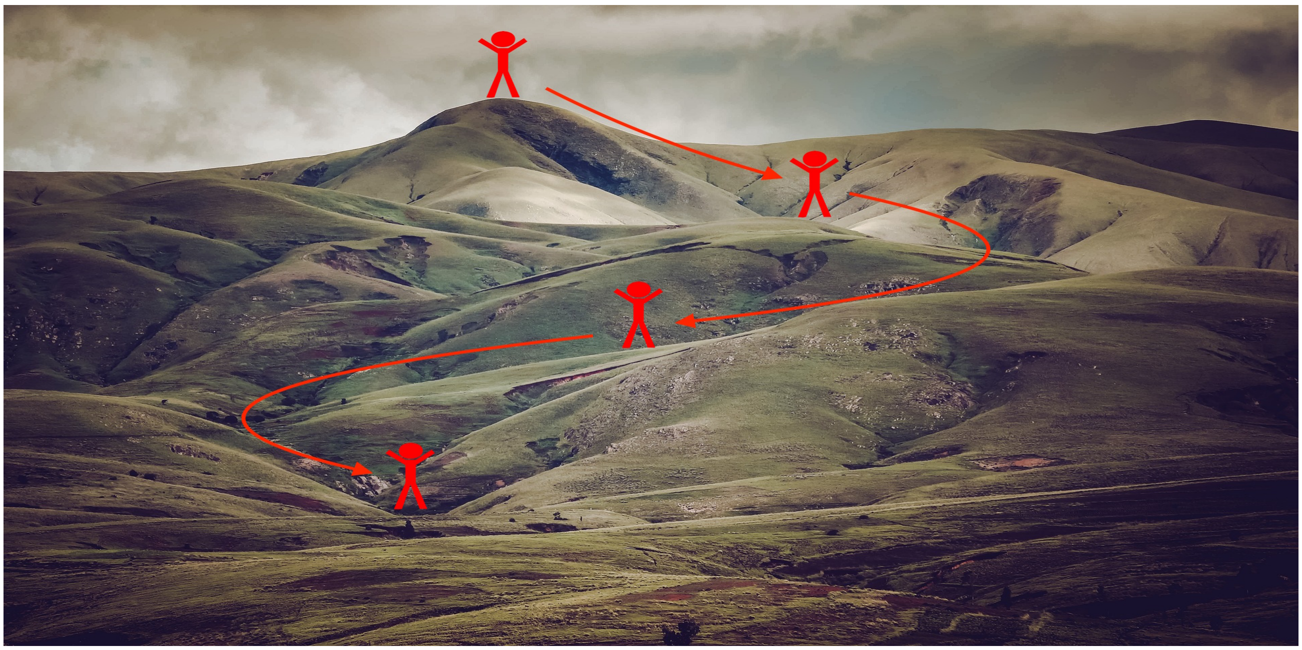


Figure 1 : **Minimizing the cost is like finding the lowest point in a hilly landscape**
At each step of the training, you update your parameters following a certain direction to try to get to the lowest possible point.

Notations: As usual, $\frac{\partial J}{\partial a} = \mathtt{da}$ for any variable `a`.

To get started, run the following code to import the libraries you will need.

Updates to Assignment

If you were working on a previous version

- The current notebook filename is version "Optimization_methods_v1b".
- You can find your work in the file directory as version "Optimization methods'.
- To see the file directory, click on the Coursera logo at the top left of the notebook.

List of Updates

- op_utils is now opt_utils_v1a. Assertion statement in `initialize_parameters` is fixed.
- opt_utils_v1a: `compute_cost` function now accumulates total cost of the batch without taking the average (average is taken for entire epoch instead).
- In `model` function, the total cost per mini-batch is accumulated, and the average of the entire epoch is taken as the average cost. So the plot of the cost function over time is now a smooth downward curve instead of an oscillating curve.
- Print statements used to check each function are reformatted, and 'expected output` is reformatted to match the format of the print statements (for easier visual comparisons).

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.io
import math
import sklearn
import sklearn.datasets

from opt_utils_v1a import load_params_and_grads, initialize_parameters, f
from opt_utils_v1a import compute_cost, predict, predict_dec, plot_decisi
from testCases import *

%matplotlib inline
plt.rcParams['figure.figsize'] = (7.0, 4.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'
```

1 - Gradient Descent

A simple optimization method in machine learning is gradient descent (GD). When you take gradient steps with respect to all m examples on each step, it is also called Batch Gradient Descent.

Warm-up exercise: Implement the gradient descent update rule. The gradient descent rule is, for $l = 1, \ldots, L$:

$$W^{[l]} = W^{[l]} - \alpha \, dW^{[l]} \tag{1}$$
$$b^{[l]} = b^{[l]} - \alpha \, db^{[l]} \tag{2}$$

where L is the number of layers and
α is the learning rate. All parameters should be stored in the `parameters` dictionary. Note that the iterator `l` starts at 0 in the `for` loop while the first parameters are
$W^{[1]}$ and
$b^{[1]}$. You need to shift `l` to `l+1` when coding.

In [4]:

```
# GRADED FUNCTION: update_parameters_with_gd

def update_parameters_with_gd(parameters, grads, learning_rate):
    """
    Update parameters using one step of gradient descent

    Arguments:
    parameters -- python dictionary containing your parameters to be upda
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl
    grads -- python dictionary containing your gradients to update each p
                    grads['dW' + str(l)] = dWl
                    grads['db' + str(l)] = dbl
    learning_rate -- the learning rate, scalar.

    Returns:
    parameters -- python dictionary containing your updated parameters
    """

    L = len(parameters) // 2 # number of layers in the neural networks

    # Update rule for each parameter
    for l in range(L):
        ### START CODE HERE ### (approx. 2 lines)
        parameters["W" + str(l+1)] = parameters["W" + str(l+1)] - learnin
        parameters["b" + str(l+1)] = parameters["b" + str(l+1)] - learnin
        ### END CODE HERE ###

    return parameters
```

In [5]:
```
parameters, grads, learning_rate = update_parameters_with_gd_test_case()

parameters = update_parameters_with_gd(parameters, grads, learning_rate)
print("W1 =\n" + str(parameters["W1"]))
print("b1 =\n" + str(parameters["b1"]))
print("W2 =\n" + str(parameters["W2"]))
print("b2 =\n" + str(parameters["b2"]))
```

```
W1 =
[[ 1.63535156 -0.62320365 -0.53718766]
 [-1.07799357  0.85639907 -2.29470142]]
b1 =
[[ 1.74604067]
 [-0.75184921]]
W2 =
[[ 0.32171798 -0.25467393  1.46902454]
 [-2.05617317 -0.31554548 -0.3756023 ]
 [ 1.1404819  -1.09976462 -0.1612551 ]]
b2 =
[[-0.88020257]
 [ 0.02561572]
 [ 0.57539477]]
```

Expected Output:


```
W1 =
[[ 1.63535156 -0.62320365 -0.53718766]
 [-1.07799357  0.85639907 -2.29470142]]
b1 =
[[ 1.74604067]
 [-0.75184921]]
W2 =
[[ 0.32171798 -0.25467393  1.46902454]
 [-2.05617317 -0.31554548 -0.3756023 ]
 [ 1.1404819  -1.09976462 -0.1612551 ]]
b2 =
[[-0.88020257]
 [ 0.02561572]
 [ 0.57539477]]
```

A variant of this is Stochastic Gradient Descent (SGD), which is equivalent to mini-batch gradient descent where each mini-batch has just 1 example. The update rule that you have just implemented does not change. What changes is that you would be computing gradients on just one training example at a time, rather than on the whole training set. The code examples below illustrate the difference between stochastic gradient descent and (batch) gradient descent.

- **(Batch) Gradient Descent**:

```
X = data_input
Y = labels
parameters = initialize_parameters(layers_dims)
for i in range(0, num_iterations):
    # Forward propagation
    a, caches = forward_propagation(X, parameters)
    # Compute cost.
    cost += compute_cost(a, Y)
    # Backward propagation.
    grads = backward_propagation(a, caches, parameters)
    # Update parameters.
    parameters = update_parameters(parameters, grads)
```

- **Stochastic Gradient Descent**:

```
X = data_input
Y = labels
parameters = initialize_parameters(layers_dims)
for i in range(0, num_iterations):
    for j in range(0, m):
        # Forward propagation
        a, caches = forward_propagation(X[:,j], parameters)
        # Compute cost
        cost += compute_cost(a, Y[:,j])
        # Backward propagation
        grads = backward_propagation(a, caches, parameters)
        # Update parameters.
        parameters = update_parameters(parameters, grads)
```

In Stochastic Gradient Descent, you use only 1 training example before updating the gradients. When the training set is large, SGD can be faster. But the parameters will "oscillate" toward the minimum rather than converge smoothly. Here is an illustration of this:

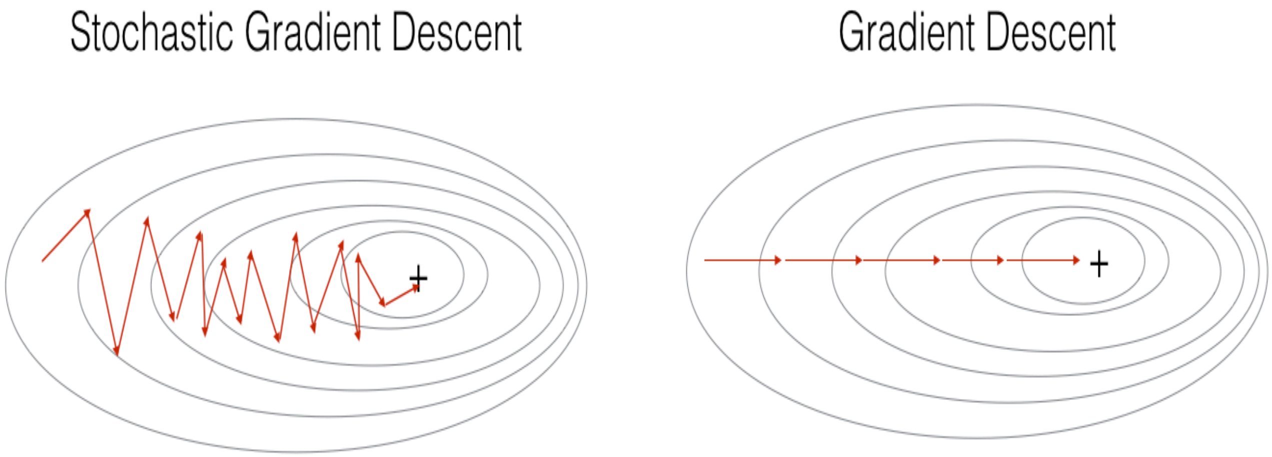


Figure 1 : **SGD vs GD**
"+" denotes a minimum of the cost. SGD leads to many oscillations to reach convergence. But each step is a lot faster to compute for SGD than for GD, as it uses only one training example (vs. the whole batch for GD).

Note also that implementing SGD requires 3 for-loops in total:

1. Over the number of iterations
2. Over the m training examples
3. Over the layers (to update all parameters, from $(W^{[1]}, b^{[1]})$ to $(W^{[L]}, b^{[L]})$)

In practice, you'll often get faster results if you do not use neither the whole training set, nor only one training example, to perform each update. Mini-batch gradient descent uses an intermediate number of examples for each step. With mini-batch gradient descent, you loop over the mini-batches instead of looping over individual training examples.

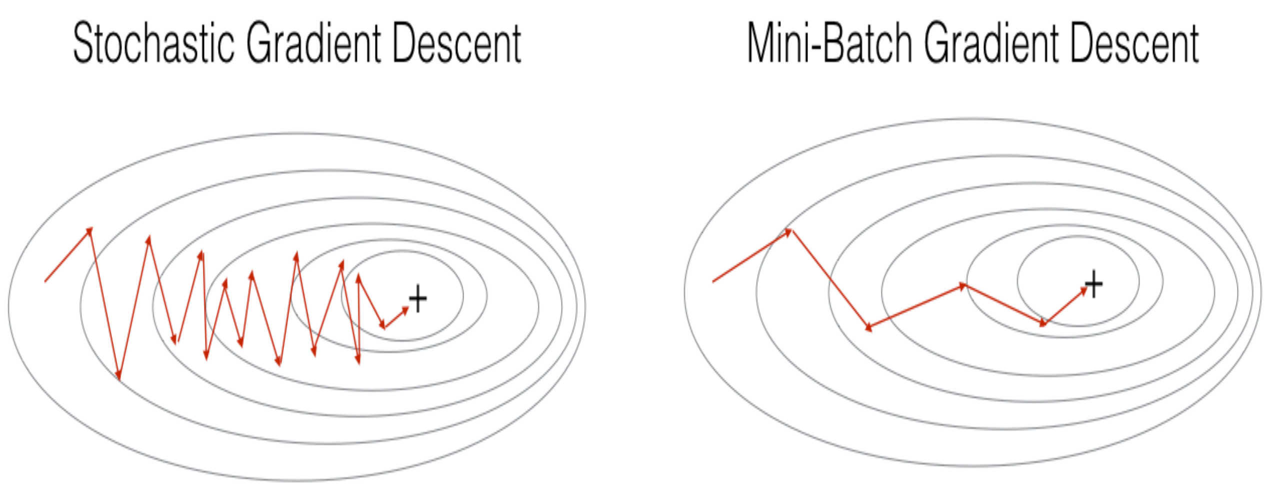


Figure 2 : **SGD vs Mini-Batch GD**
"+" denotes a minimum of the cost. Using mini-batches in your optimization algorithm often leads to faster optimization.

What you should remember:

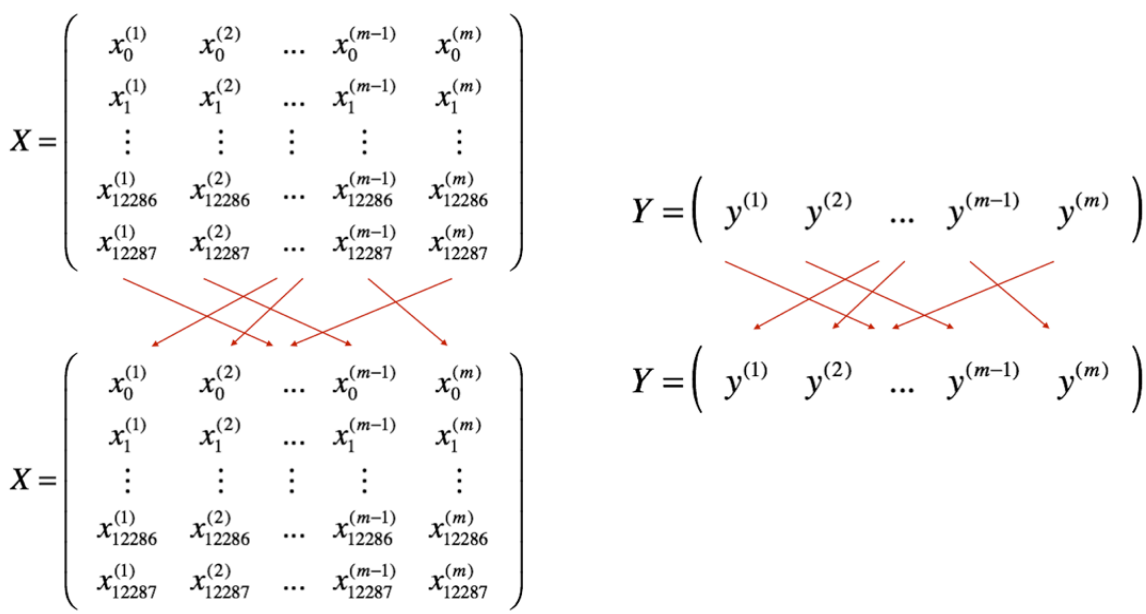
- The difference between gradient descent, mini-batch gradient descent and stochastic gradient descent is the number of examples you use to perform one update step.
- You have to tune a learning rate hyperparameter α.
- With a well-turned mini-batch size, usually it outperforms either gradient descent or stochastic gradient descent (particularly when the training set is large).

2 - Mini-Batch Gradient descent

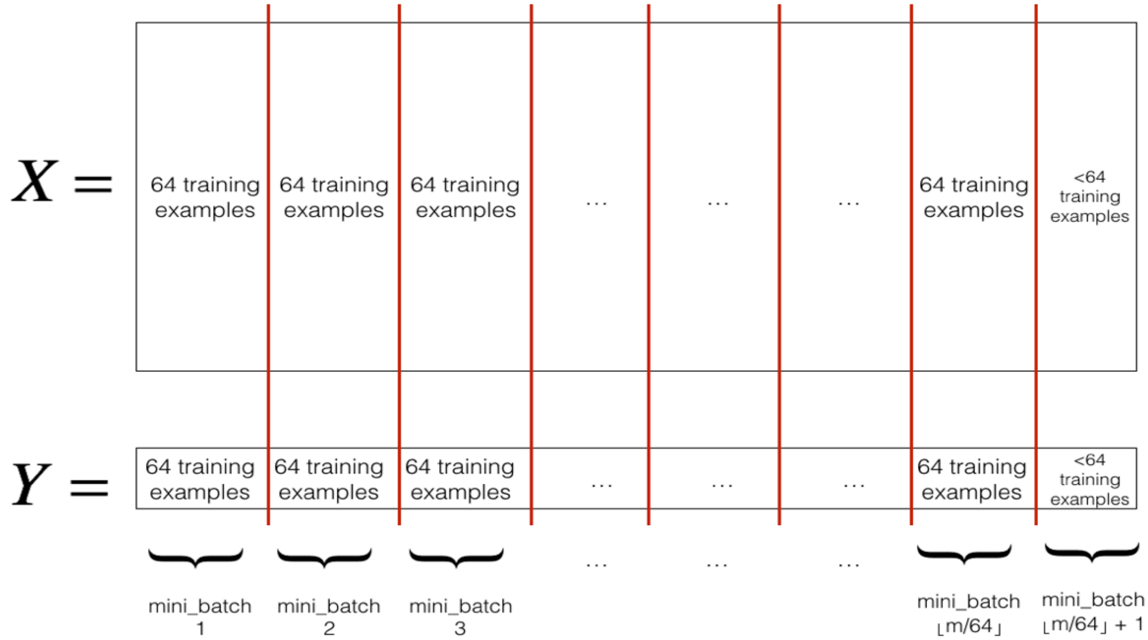
Let's learn how to build mini-batches from the training set (X, Y).

There are two steps:

- **Shuffle**: Create a shuffled version of the training set (X, Y) as shown below. Each column of X and Y represents a training example. Note that the random shuffling is done synchronously between X and Y. Such that after the shuffling the i^{th} column of X is the example corresponding to the i^{th} label in Y. The shuffling step ensures that examples will be split randomly into different mini-batches.



- **Partition**: Partition the shuffled (X, Y) into mini-batches of size `mini_batch_size` (here 64). Note that the number of training examples is not always divisible by `mini_batch_size`. The last mini batch might be smaller, but you don't need to worry about this. When the final mini-batch is smaller than the full `mini_batch_size`, it will look like this:



Exercise: Implement `random_mini_batches`. We coded the shuffling part for you. To help you with the partitioning step, we give you the following code that selects the indexes for the 1^{st} and 2^{nd} mini-batches:

```
first_mini_batch_X = shuffled_X[:, 0 : mini_batch_size]
second_mini_batch_X = shuffled_X[:, mini_batch_size : 2 * mini_ba
tch_size]
...
```

Note that the last mini-batch might end up smaller than `mini_batch_size=64`. Let $\lfloor s \rfloor$ represents s rounded down to the nearest integer (this is `math.floor(s)` in Python). If the total number of examples is not a multiple of `mini_batch_size=64` then there will be $\lfloor \frac{m}{mini\_batch\_size} \rfloor$ mini-batches with a full 64 examples, and the number of examples in the final mini-batch will be ($m - mini\_batch\_size \times \lfloor \frac{m}{mini\_batch\_size} \rfloor$).

In [26]:
```
# GRADED FUNCTION: random_mini_batches

def random_mini_batches(X, Y, mini_batch_size = 64, seed = 0):
    """
    Creates a list of random minibatches from (X, Y)

    Arguments:
    X -- input data, of shape (input size, number of examples)
    Y -- true "label" vector (1 for blue dot / 0 for red dot), of shape (
    mini_batch_size -- size of the mini-batches, integer

    Returns:
    mini_batches -- list of synchronous (mini_batch_X, mini_batch_Y)
    """

    np.random.seed(seed)            # To make your "random" minibatches t
    m = X.shape[1]                  # number of training examples
    mini_batches = []

    # Step 1: Shuffle (X, Y)
    permutation = list(np.random.permutation(m))
    shuffled_X = X[:, permutation]
    shuffled_Y = Y[:, permutation].reshape((1,m))

    # Step 2: Partition (shuffled_X, shuffled_Y). Minus the end case.
    num_complete_minibatches = math.floor(m/mini_batch_size) # number of
    for k in range(0, num_complete_minibatches):
        ### START CODE HERE ### (approx. 2 lines)
        mini_batch_X = shuffled_X[:, k * mini_batch_size:(k+1)*mini_batch
        mini_batch_Y = shuffled_Y[:, k * mini_batch_size:(k+1)*mini_batch
        ### END CODE HERE ###
        mini_batch = (mini_batch_X, mini_batch_Y)
        mini_batches.append(mini_batch)

    # Handling the end case (last mini-batch < mini_batch_size)
    if m % mini_batch_size != 0:
        ### START CODE HERE ### (approx. 2 lines)
        mini_batch_X[:,mini_batch_size*num_complete_minibatches:]
        mini_batch_Y[:,mini_batch_size*num_complete_minibatches:]
        ### END CODE HERE ###
        mini_batch = (mini_batch_X, mini_batch_Y)
        mini_batches.append(mini_batch)

    return mini_batches
```

In [27]:
```
X_assess, Y_assess, mini_batch_size = random_mini_batches_test_case()
mini_batches = random_mini_batches(X_assess, Y_assess, mini_batch_size)

print ("shape of the 1st mini_batch_X: " + str(mini_batches[0][0].shape))
print ("shape of the 2nd mini_batch_X: " + str(mini_batches[1][0].shape))
print ("shape of the 3rd mini_batch_X: " + str(mini_batches[2][0].shape))
print ("shape of the 1st mini_batch_Y: " + str(mini_batches[0][1].shape))
print ("shape of the 2nd mini_batch_Y: " + str(mini_batches[1][1].shape))
print ("shape of the 3rd mini_batch_Y: " + str(mini_batches[2][1].shape))
print ("mini batch sanity check: " + str(mini_batches[0][0][0][0:3]))
```

```
shape of the 1st mini_batch_X: (12288, 64)
shape of the 2nd mini_batch_X: (12288, 64)
shape of the 3rd mini_batch_X: (12288, 64)
shape of the 1st mini_batch_Y: (1, 64)
shape of the 2nd mini_batch_Y: (1, 64)
shape of the 3rd mini_batch_Y: (1, 64)
mini batch sanity check: [ 0.90085595 -0.7612069   0.2344157 ]
```

Expected Output:

shape of the 1st mini_batch_X	(12288, 64)
shape of the 2nd mini_batch_X	(12288, 64)
shape of the 3rd mini_batch_X	(12288, 20)
shape of the 1st mini_batch_Y	(1, 64)
shape of the 2nd mini_batch_Y	(1, 64)
shape of the 3rd mini_batch_Y	(1, 20)
mini batch sanity check	[0.90085595 -0.7612069 0.2344157]

What you should remember:

- Shuffling and Partitioning are the two steps required to build mini-batches
- Powers of two are often chosen to be the mini-batch size, e.g., 16, 32, 64, 128.

3 - Momentum

Because mini-batch gradient descent makes a parameter update after seeing just a subset of examples, the direction of the update has some variance, and so the path taken by mini-batch gradient descent will "oscillate" toward convergence. Using momentum can reduce these oscillations.

Momentum takes into account the past gradients to smooth out the update. We will store the 'direction' of the previous gradients in the variable v. Formally, this will be the exponentially weighted average of the gradient on previous steps. You can also think of v as the "velocity" of a ball rolling downhill, building up speed (and momentum) according to the direction of the gradient/slope of the hill.

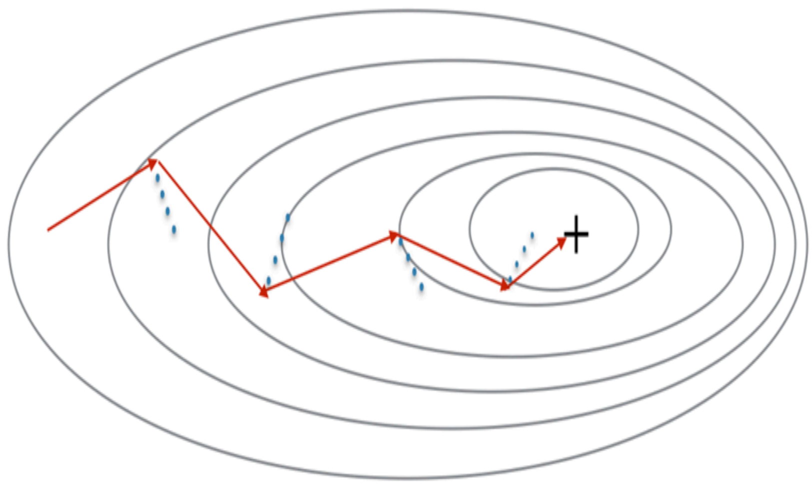


Figure 3: The red arrows shows the direction taken by one step of mini-batch gradient descent with momentum. The blue points show the direction of the gradient (with respect to the current mini-batch) on each step. Rather than just following the gradient, we let the gradient influence v and then take a step in the direction of v.

Exercise: Initialize the velocity. The velocity, v, is a python dictionary that needs to be initialized with arrays of zeros. Its keys are the same as those in the `grads` dictionary, that is: for $l = 1, \ldots, L$:

```
v["dW" + str(l+1)] = ... #(numpy array of zeros with the same shape as parameters["W" + str(l+1)])
v["db" + str(l+1)] = ... #(numpy array of zeros with the same shape as parameters["b" + str(l+1)])
```

Note that the iterator l starts at 0 in the for loop while the first parameters are v["dW1"] and v["db1"] (that's a "one" on the superscript). This is why we are shifting l to l+1 in the `for` loop.

In [28]:
```python
# GRADED FUNCTION: initialize_velocity

def initialize_velocity(parameters):
    """
    Initializes the velocity as a python dictionary with:
                - keys: "dW1", "db1", ..., "dWL", "dbL"
                - values: numpy arrays of zeros of the same shape as the
    Arguments:
    parameters -- python dictionary containing your parameters.
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl

    Returns:
    v -- python dictionary containing the current velocity.
                    v['dW' + str(l)] = velocity of dWl
                    v['db' + str(l)] = velocity of dbl
    """

    L = len(parameters) // 2 # number of layers in the neural networks
    v = {}

    # Initialize velocity
    for l in range(L):
        ### START CODE HERE ### (approx. 2 lines)
        v["dW" + str(l + 1)] = np.zeros_like(parameters["W" + str(l + 1)]
        v["db" + str(l + 1)] = np.zeros_like(parameters["b" + str(l + 1)]
        ### END CODE HERE ###

    return v
```

In [29]:
```python
parameters = initialize_velocity_test_case()

v = initialize_velocity(parameters)
print("v[\"dW1\"] =\n" + str(v["dW1"]))
print("v[\"db1\"] =\n" + str(v["db1"]))
print("v[\"dW2\"] =\n" + str(v["dW2"]))
print("v[\"db2\"] =\n" + str(v["db2"]))
```

```
v["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Expected Output:

```
v["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Exercise: Now, implement the parameters update with momentum. The momentum update rule is, for $l = 1, \dots, L$:

$$\begin{cases} v_{dW^{[l]}} = \beta v_{dW^{[l]}} + (1 - \beta) dW^{[l]} \\ W^{[l]} = W^{[l]} - \alpha v_{dW^{[l]}} \end{cases} \tag{3}$$

$$\begin{cases} v_{db^{[l]}} = \beta v_{db^{[l]}} + (1 - \beta) db^{[l]} \\ b^{[l]} = b^{[l]} - \alpha v_{db^{[l]}} \end{cases} \tag{4}$$

where L is the number of layers,
β is the momentum and
α is the learning rate. All parameters should be stored in the `parameters` dictionary. Note that the iterator `l` starts at 0 in the `for` loop while the first parameters are
$W^{[1]}$ and
$b^{[1]}$ (that's a "one" on the superscript). So you will need to shift `l` to `l+1` when coding.

In [30]:

```
# GRADED FUNCTION: update_parameters_with_momentum

def update_parameters_with_momentum(parameters, grads, v, beta, learning_
    """
    Update parameters using Momentum

    Arguments:
    parameters -- python dictionary containing your parameters:
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl
    grads -- python dictionary containing your gradients for each paramet
                    grads['dW' + str(l)] = dWl
                    grads['db' + str(l)] = dbl
    v -- python dictionary containing the current velocity:
                    v['dW' + str(l)] = ...
                    v['db' + str(l)] = ...
    beta -- the momentum hyperparameter, scalar
    learning_rate -- the learning rate, scalar

    Returns:
    parameters -- python dictionary containing your updated parameters
    v -- python dictionary containing your updated velocities
    """

    L = len(parameters) // 2 # number of layers in the neural networks

    # Momentum update for each parameter
    for l in range(L):

        ### START CODE HERE ### (approx. 4 lines)
        # compute velocities
        v["dW" + str(l+1)] = beta * v["dW" + str(l+1)] + (1 - beta) * gra
        v["db" + str(l+1)] = beta * v["db" + str(l+1)] + (1 - beta) * gra
        # update parameters
        parameters["W" + str(l+1)] = parameters["W" + str(l+1)] - learnin
        parameters["b" + str(l+1)] = parameters["b" + str(l+1)] - learnin
        ### END CODE HERE ###

    return parameters, v
```

In [31]:
```
parameters, grads, v = update_parameters_with_momentum_test_case()

parameters, v = update_parameters_with_momentum(parameters, grads, v, bet
print("W1 = \n" + str(parameters["W1"]))
print("b1 = \n" + str(parameters["b1"]))
print("W2 = \n" + str(parameters["W2"]))
print("b2 = \n" + str(parameters["b2"]))
print("v[\"dW1\"] = \n" + str(v["dW1"]))
print("v[\"db1\"] = \n" + str(v["db1"]))
print("v[\"dW2\"] = \n" + str(v["dW2"]))
print("v[\"db2\"] = v" + str(v["db2"]))
```

```
W1 =
[[ 1.62544598 -0.61290114 -0.52907334]
 [-1.07347112  0.86450677 -2.30085497]]
b1 =
[[ 1.74493465]
 [-0.76027113]]
W2 =
[[ 0.31930698 -0.24990073  1.4627996 ]
 [-2.05974396 -0.32173003 -0.38320915]
 [ 1.13444069 -1.0998786  -0.1713109 ]]
b2 =
[[-0.87809283]
 [ 0.04055394]
 [ 0.58207317]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] = v[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
```

Expected Output:

```
W1 =
[[ 1.62544598 -0.61290114 -0.52907334]
 [-1.07347112  0.86450677 -2.30085497]]
b1 =
[[ 1.74493465]
 [-0.76027113]]
W2 =
[[ 0.31930698 -0.24990073  1.4627996 ]
 [-2.05974396 -0.32173003 -0.38320915]
 [ 1.13444069 -1.0998786  -0.1713109 ]]
b2 =
[[-0.87809283]
 [ 0.04055394]
 [ 0.58207317]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] = v[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
```

Note that:

- The velocity is initialized with zeros. So the algorithm will take a few iterations to "build up" velocity and start to take bigger steps.
- If $\beta = 0$, then this just becomes standard gradient descent without momentum.

How do you choose β?

- The larger the momentum β is, the smoother the update because the more we take the past gradients into account. But if β is too big, it could also smooth out the updates too much.
- Common values for β range from 0.8 to 0.999. If you don't feel inclined to tune this, $\beta = 0.9$ is often a reasonable default.
- Tuning the optimal β for your model might need trying several values to see what works best in term of reducing the value of the cost function J.

What you should remember:

- Momentum takes past gradients into account to smooth out the steps of gradient descent. It can be applied with batch gradient descent, mini-batch gradient descent or stochastic gradient descent.
- You have to tune a momentum hyperparameter β and a learning rate α.

4 - Adam

Adam is one of the most effective optimization algorithms for training neural networks. It combines ideas from RMSProp (described in lecture) and Momentum.

How does Adam work?

1. It calculates an exponentially weighted average of past gradients, and stores it in variables v (before bias correction) and $v^{corrected}$ (with bias correction).
2. It calculates an exponentially weighted average of the squares of the past gradients, and stores it in variables s (before bias correction) and $s^{corrected}$ (with bias correction).
3. It updates parameters in a direction based on combining information from "1" and "2".

The update rule is, for $l = 1, \ldots, L$:

$$\begin{cases}
v_{dW^{[l]}} = \beta_1 v_{dW^{[l]}} + (1 - \beta_1) \frac{\partial \mathcal{J} }{ \partial W^{[l]} } \\
v^{corrected}_{dW^{[l]}} = \frac{v_{dW^{[l]}}}{1 - (\beta_1)^t} \\
s_{dW^{[l]}} = \beta_2 s_{dW^{[l]}} + (1 - \beta_2) (\frac{\partial \mathcal{J} }{\partial W^{[l]} })^2 \\
s^{corrected}_{dW^{[l]}} = \frac{s_{dW^{[l]}}}{1 - (\beta_2)^t} \\
W^{[l]} = W^{[l]} - \alpha \frac{v^{corrected}_{dW^{[l]}}}{\sqrt{s^{corrected}_{dW^{[l]}}} + \varepsilon}
\end{cases}$$

where:

- t counts the number of steps taken of Adam
- L is the number of layers
- β_1 and β_2 are hyperparameters that control the two exponentially weighted averages.
- α is the learning rate
- ε is a very small number to avoid dividing by zero

As usual, we will store all parameters in the `parameters` dictionary

Exercise: Initialize the Adam variables v, s which keep track of the past information.

Instruction: The variables v, s are python dictionaries that need to be initialized with arrays of zeros. Their keys are the same as for `grads`, that is: for $l = 1, \ldots, L$:

```
v["dW" + str(l+1)] = ... #(numpy array of zeros with the same shape as parameters["W" + str(l+1)])
v["db" + str(l+1)] = ... #(numpy array of zeros with the same shape as parameters["b" + str(l+1)])
s["dW" + str(l+1)] = ... #(numpy array of zeros with the same shape as parameters["W" + str(l+1)])
s["db" + str(l+1)] = ... #(numpy array of zeros with the same shape as parameters["b" + str(l+1)])
```

In [34]:
```
# GRADED FUNCTION: initialize_adam

def initialize_adam(parameters) :
    """
    Initializes v and s as two python dictionaries with:
                - keys: "dW1", "db1", ..., "dWL", "dbL"
                - values: numpy arrays of zeros of the same shape as the

    Arguments:
    parameters -- python dictionary containing your parameters.
                    parameters["W" + str(l)] = Wl
                    parameters["b" + str(l)] = bl

    Returns:
    v -- python dictionary that will contain the exponentially weighted a
                    v["dW" + str(l)] = ...
                    v["db" + str(l)] = ...
    s -- python dictionary that will contain the exponentially weighted a
                    s["dW" + str(l)] = ...
                    s["db" + str(l)] = ...

    """

    L = len(parameters) // 2 # number of layers in the neural networks
    v = {}
    s = {}

    # Initialize v, s. Input: "parameters". Outputs: "v, s".
    for l in range(L):
    ### START CODE HERE ### (approx. 4 lines)
        v["dW" + str(l+1)] = np.zeros_like(parameters["W" + str(l + 1)])
        v["db" + str(l+1)] = np.zeros_like(parameters["b" + str(l + 1)])
        s["dW" + str(l+1)] = np.zeros_like(parameters["W" + str(l + 1)])
        s["db" + str(l+1)] = np.zeros_like(parameters["b" + str(l + 1)])
    ### END CODE HERE ###

    return v, s
```

In [35]:

```
parameters = initialize_adam_test_case()

v, s = initialize_adam(parameters)
print("v[\"dW1\"] = \n" + str(v["dW1"]))
print("v[\"db1\"] = \n" + str(v["db1"]))
print("v[\"dW2\"] = \n" + str(v["dW2"]))
print("v[\"db2\"] = \n" + str(v["db2"]))
print("s[\"dW1\"] = \n" + str(s["dW1"]))
print("s[\"db1\"] = \n" + str(s["db1"]))
print("s[\"dW2\"] = \n" + str(s["dW2"]))
print("s[\"db2\"] = \n" + str(s["db2"]))
```

```
v["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
s["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db1"] =
[[ 0.]
 [ 0.]]
s["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Expected Output:

```
v["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
s["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db1"] =
[[ 0.]
 [ 0.]]
s["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Exercise: Now, implement the parameters update with Adam. Recall the general update rule is, for $l = 1, \dots, L$:

$$\begin{cases} v_{W^{[l]}} = \beta_1 v_{W^{[l]}} + (1 - \beta_1) \frac{\partial J}{\partial W^{[l]}} \\ v^{corrected}_{W^{[l]}} = \frac{v_{W^{[l]}}}{1 - (\beta_1)^t} \\ s_{W^{[l]}} = \beta_2 s_{W^{[l]}} + (1 - \beta_2) (\frac{\partial J}{\partial W^{[l]}})^2 \\ s^{corrected}_{W^{[l]}} = \frac{s_{W^{[l]}}}{1 - (\beta_2)^t} \\ W^{[l]} = W^{[l]} - \alpha \frac{v^{corrected}_{W^{[l]}}}{\sqrt{s^{corrected}_{W^{[l]}}} + \varepsilon} \end{cases}$$

Note that the iterator `l` starts at 0 in the `for` loop while the first parameters are $W^{[1]}$ and $b^{[1]}$. You need to shift `l` to `l+1` when coding.

In [38]:
```
# GRADED FUNCTION: update_parameters_with_adam

def update_parameters_with_adam(parameters, grads, v, s, t, learning_rate
                                beta1 = 0.9, beta2 = 0.999,  epsilon = 1e
    """
    Update parameters using Adam

    Arguments:
    parameters -- python dictionary containing your parameters:
                    parameters['W' + str(l)] = Wl
                    parameters['b' + str(l)] = bl
    grads -- python dictionary containing your gradients for each paramet
                    grads['dW' + str(l)] = dWl
                    grads['db' + str(l)] = dbl
    v -- Adam variable, moving average of the first gradient, python dict
    s -- Adam variable, moving average of the squared gradient, python di
    learning_rate -- the learning rate, scalar.
    beta1 -- Exponential decay hyperparameter for the first moment estima
    beta2 -- Exponential decay hyperparameter for the second moment estim
    epsilon -- hyperparameter preventing division by zero in Adam updates

    Returns:
    parameters -- python dictionary containing your updated parameters
    v -- Adam variable, moving average of the first gradient, python dict
    s -- Adam variable, moving average of the squared gradient, python di
    """

    L = len(parameters) // 2                 # number of layers in the ne
    v_corrected = {}                         # Initializing first moment
    s_corrected = {}                         # Initializing second moment

    # Perform Adam update on all parameters
    for l in range(L):
        # Moving average of the gradients. Inputs: "v, grads, beta1". Out
```

```
        ### START CODE HERE ### (approx. 2 lines)
        v["dW" + str(l+1)] = beta1*v["dW" + str(l+1)] + (1-beta1) * grads
        v["db" + str(l+1)] = beta1*v["db" + str(l+1)] + (1-beta1) * grads
        ### END CODE HERE ###

        # Compute bias-corrected first moment estimate. Inputs: "v, beta1
        ### START CODE HERE ### (approx. 2 lines)
        v_corrected["dW" + str(l+1)] = v["dW" + str(l+1)] / (1-np.power(b
        v_corrected["db" + str(l+1)] = v["db" + str(l+1)] / (1-np.power(b
        ### END CODE HERE ###

        # Moving average of the squared gradients. Inputs: "s, grads, bet
        ### START CODE HERE ### (approx. 2 lines)
        s["dW" + str(l+1)] = beta2*s["dW"+str(l+1)] +(1-beta2) * np.power
        s["db" + str(l+1)] = beta2*s["db"+str(l+1)] +(1-beta2) * np.power
        ### END CODE HERE ###

        # Compute bias-corrected second raw moment estimate. Inputs: "s,
        ### START CODE HERE ### (approx. 2 lines)
        s_corrected["dW" + str(l+1)] = s["dW" + str(l+1)] / (1-np.power(b
        s_corrected["db" + str(l+1)] = s["db" + str(l+1)] / (1-np.power(b
        ### END CODE HERE ###

        # Update parameters. Inputs: "parameters, learning_rate, v_correc
        ### START CODE HERE ### (approx. 2 lines)
        parameters["W" + str(l+1)] = parameters["W" +str(l+1)] - learning
        parameters["b" + str(l+1)] = parameters["b" +str(l+1)] - learning
        ### END CODE HERE ###

    return parameters, v, s
```

In [39]:

```
parameters, grads, v, s = update_parameters_with_adam_test_case()
parameters, v, s  = update_parameters_with_adam(parameters, grads, v, s,

print("W1 = \n" + str(parameters["W1"]))
print("b1 = \n" + str(parameters["b1"]))
print("W2 = \n" + str(parameters["W2"]))
print("b2 = \n" + str(parameters["b2"]))
print("v[\"dW1\"] = \n" + str(v["dW1"]))
print("v[\"db1\"] = \n" + str(v["db1"]))
print("v[\"dW2\"] = \n" + str(v["dW2"]))
print("v[\"db2\"] = \n" + str(v["db2"]))
print("s[\"dW1\"] = \n" + str(s["dW1"]))
print("s[\"db1\"] = \n" + str(s["db1"]))
print("s[\"dW2\"] = \n" + str(s["dW2"]))
print("s[\"db2\"] = \n" + str(s["db2"]))
```

```
W1 = 
[[ 1.63178673 -0.61919778 -0.53561312]
 [-1.08040999  0.85796626 -2.29409733]]
b1 = 
[[ 1.75225313]
 [-0.75376553]]
W2 = 
```

```
[[ 0.32648046 -0.25681174  1.46954931]
 [-2.05269934 -0.31497584 -0.37661299]
 [ 1.14121081 -1.09245036 -0.16498684]]
b2 =
[[-0.88529978]
 [ 0.03477238]
 [ 0.57537385]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] =
[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
s["dW1"] =
[[ 0.00121136  0.00131039  0.00081287]
 [ 0.0002525   0.00081154  0.00046748]]
s["db1"] =
[[  1.51020075e-05]
 [  8.75664434e-04]]
s["dW2"] =
[[  7.17640232e-05   2.81276921e-04   4.78394595e-04]
 [  1.57413361e-04   4.72206320e-04   7.14372576e-04]
 [  4.50571368e-04   1.60392066e-07   1.24838242e-03]]
s["db2"] =
[[  5.49507194e-05]
 [  2.75494327e-03]
 [  5.50629536e-04]]
```

Expected Output:

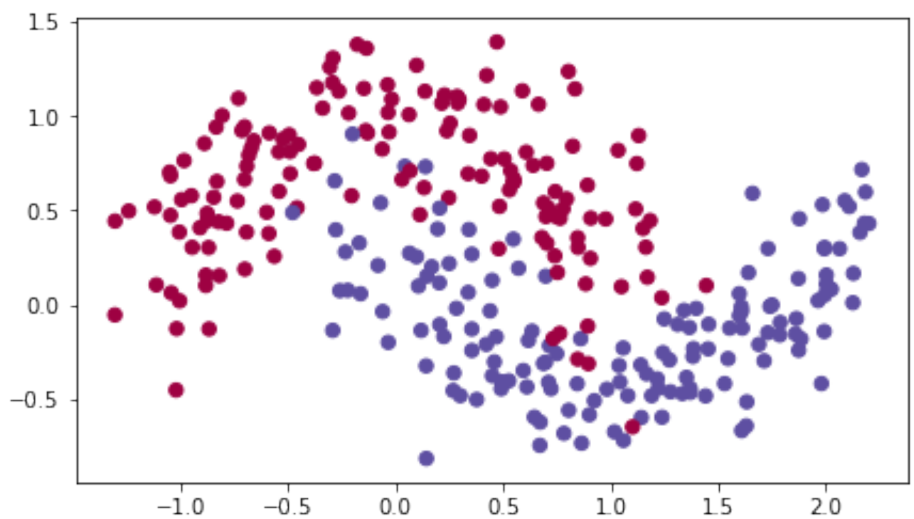
```
W1 =
[[ 1.63178673 -0.61919778 -0.53561312]
 [-1.08040999  0.85796626 -2.29409733]]
b1 =
[[ 1.75225313]
 [-0.75376553]]
W2 =
[[ 0.32648046 -0.25681174  1.46954931]
 [-2.05269934 -0.31497584 -0.37661299]
 [ 1.14121081 -1.09245036 -0.16498684]]
b2 =
[[-0.88529978]
 [ 0.03477238]
 [ 0.57537385]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] =
[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
s["dW1"] =
[[ 0.00121136  0.00131039  0.00081287]
 [ 0.0002525   0.00081154  0.00046748]]
s["db1"] =
[[  1.51020075e-05]
 [  8.75664434e-04]]
s["dW2"] =
[[  7.17640232e-05   2.81276921e-04   4.78394595e-04]
 [  1.57413361e-04   4.72206320e-04   7.14372576e-04]
 [  4.50571368e-04   1.60392066e-07   1.24838242e-03]]
s["db2"] =
[[  5.49507194e-05]
 [  2.75494327e-03]
 [  5.50629536e-04]]
```

You now have three working optimization algorithms (mini-batch gradient descent, Momentum, Adam). Let's implement a model with each of these optimizers and observe the difference.

5 - Model with different optimization algorithms

Lets use the following "moons" dataset to test the different optimization methods. (The dataset is named "moons" because the data from each of the two classes looks a bit like a crescent-shaped moon.)

In [40]:
```
train_X, train_Y = load_dataset()
```



We have already implemented a 3-layer neural network. You will train it with:

- Mini-batch **Gradient Descent**: it will call your function:
    - `update_parameters_with_gd()`
- Mini-batch **Momentum**: it will call your functions:
    - `initialize_velocity()` and `update_parameters_with_momentum()`
- Mini-batch **Adam**: it will call your functions:
    - `initialize_adam()` and `update_parameters_with_adam()`

In [41]:
```
def model(X, Y, layers_dims, optimizer, learning_rate = 0.0007, mini_batc
          beta1 = 0.9, beta2 = 0.999,  epsilon = 1e-8, num_epochs = 10000
    """
    3-layer neural network model which can be run in different optimizer

    Arguments:
    X -- input data, of shape (2, number of examples)
    Y -- true "label" vector (1 for blue dot / 0 for red dot), of shape (
    layers_dims -- python list, containing the size of each layer
    learning_rate -- the learning rate, scalar.
```

```
mini_batch_size -- the size of a mini batch
beta -- Momentum hyperparameter
beta1 -- Exponential decay hyperparameter for the past gradients esti
beta2 -- Exponential decay hyperparameter for the past squared gradie
epsilon -- hyperparameter preventing division by zero in Adam updates
num_epochs -- number of epochs
print_cost -- True to print the cost every 1000 epochs

Returns:
parameters -- python dictionary containing your updated parameters
"""

L = len(layers_dims)             # number of layers in the neural net
costs = []                       # to keep track of the cost
t = 0                            # initializing the counter required
seed = 10                        # For grading purposes, so that your
m = X.shape[1]                   # number of training examples

# Initialize parameters
parameters = initialize_parameters(layers_dims)

# Initialize the optimizer
if optimizer == "gd":
    pass # no initialization required for gradient descent
elif optimizer == "momentum":
    v = initialize_velocity(parameters)
elif optimizer == "adam":
    v, s = initialize_adam(parameters)

# Optimization loop
for i in range(num_epochs):

    # Define the random minibatches. We increment the seed to reshuff
    seed = seed + 1
    minibatches = random_mini_batches(X, Y, mini_batch_size, seed)
    cost_total = 0

    for minibatch in minibatches:

        # Select a minibatch
        (minibatch_X, minibatch_Y) = minibatch

        # Forward propagation
        a3, caches = forward_propagation(minibatch_X, parameters)

        # Compute cost and add to the cost total
        cost_total += compute_cost(a3, minibatch_Y)

        # Backward propagation
        grads = backward_propagation(minibatch_X, minibatch_Y, caches

        # Update parameters
        if optimizer == "gd":
            parameters = update_parameters_with_gd(parameters, grads,
```

```
            elif optimizer == "momentum":
                parameters, v = update_parameters_with_momentum(parameter
            elif optimizer == "adam":
                t = t + 1 # Adam counter
                parameters, v, s = update_parameters_with_adam(parameters
                                                               t, learnin
        cost_avg = cost_total / m

        # Print the cost every 1000 epoch
        if print_cost and i % 1000 == 0:
            print ("Cost after epoch %i: %f" %(i, cost_avg))
        if print_cost and i % 100 == 0:
            costs.append(cost_avg)

    # plot the cost
    plt.plot(costs)
    plt.ylabel('cost')
    plt.xlabel('epochs (per 100)')
    plt.title("Learning rate = " + str(learning_rate))
    plt.show()

    return parameters
```

You will now run this 3 layer neural network with each of the 3 optimization methods.

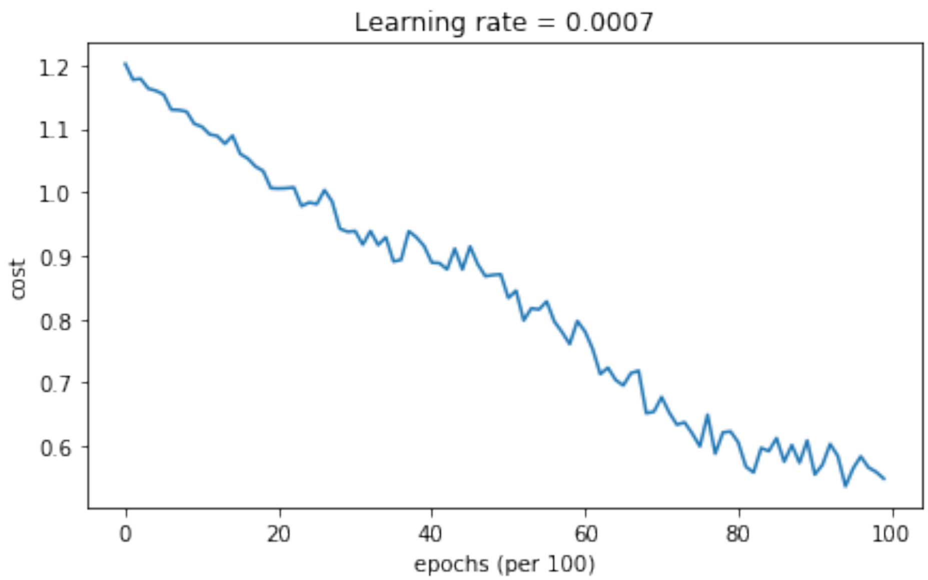
5.1 - Mini-batch Gradient descent

Run the following code to see how the model does with mini-batch gradient descent.

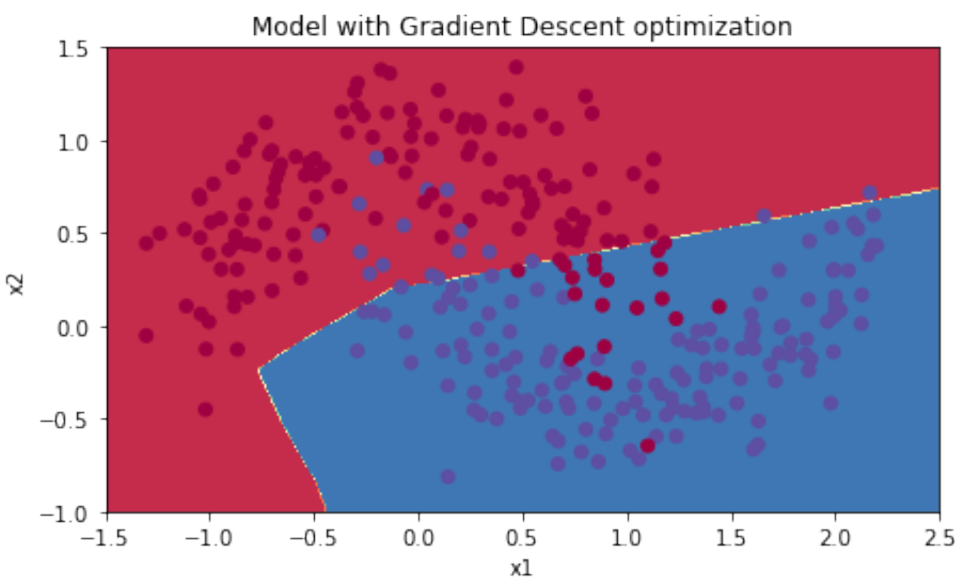
In [42]:
```
# train 3-layer model
layers_dims = [train_X.shape[0], 5, 2, 1]
parameters = model(train_X, train_Y, layers_dims, optimizer = "gd")

# Predict
predictions = predict(train_X, train_Y, parameters)

# Plot decision boundary
plt.title("Model with Gradient Descent optimization")
axes = plt.gca()
axes.set_xlim([-1.5,2.5])
axes.set_ylim([-1,1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, t
```

```
Cost after epoch 0: 1.202164
Cost after epoch 1000: 1.103201
Cost after epoch 2000: 1.005794
Cost after epoch 3000: 0.938804
Cost after epoch 4000: 0.889158
Cost after epoch 5000: 0.833325
Cost after epoch 6000: 0.780598
Cost after epoch 7000: 0.677254
Cost after epoch 8000: 0.605953
```

```
Cost after epoch 9000: 0.555257
```




```
Accuracy: 0.87
```



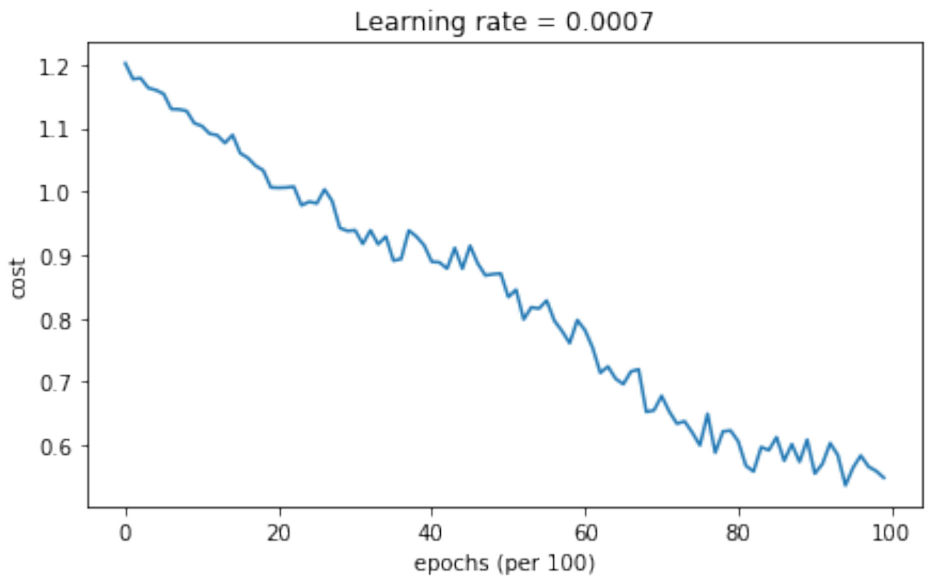
5.2 - Mini-batch gradient descent with momentum

Run the following code to see how the model does with momentum. Because this example is relatively simple, the gains from using momemtum are small; but for more complex problems you might see bigger gains.

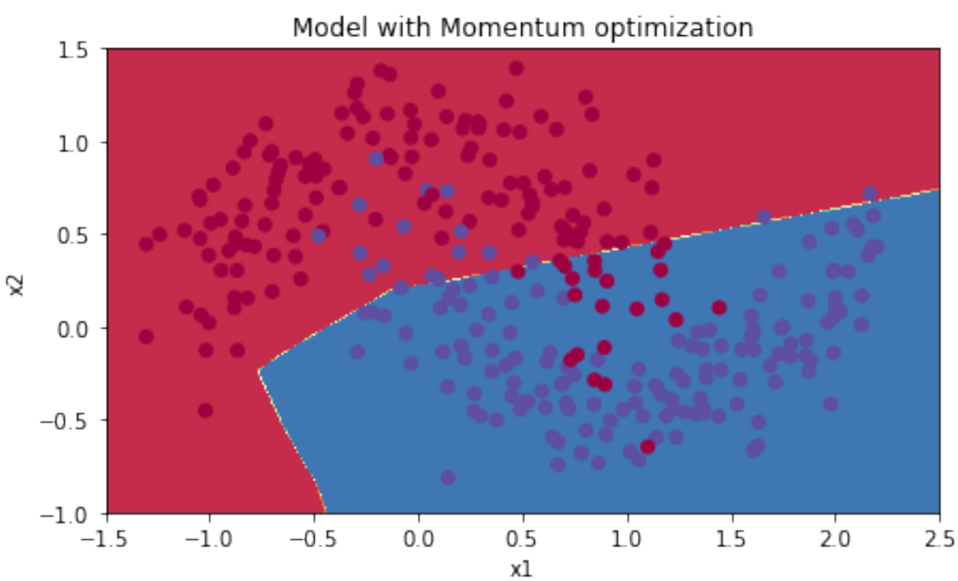
In [43]:
```
# train 3-layer model
layers_dims = [train_X.shape[0], 5, 2, 1]
parameters = model(train_X, train_Y, layers_dims, beta = 0.9, optimizer =

# Predict
predictions = predict(train_X, train_Y, parameters)

# Plot decision boundary
```

```
plt.title("Model with Momentum optimization")
axes = plt.gca()
axes.set_xlim([-1.5,2.5])
axes.set_ylim([-1,1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, t
```

```
Cost after epoch 0: 1.202200
Cost after epoch 1000: 1.103325
Cost after epoch 2000: 1.005985
Cost after epoch 3000: 0.938974
Cost after epoch 4000: 0.889361
Cost after epoch 5000: 0.833650
Cost after epoch 6000: 0.781414
Cost after epoch 7000: 0.677861
Cost after epoch 8000: 0.606174
Cost after epoch 9000: 0.555373
```




```
Accuracy: 0.87
```

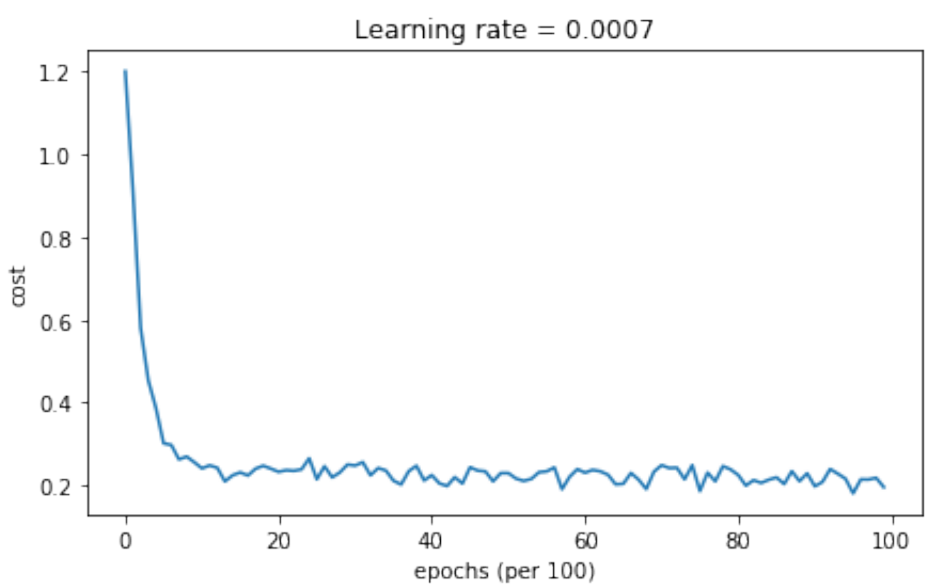


5.3 - Mini-batch with Adam mode

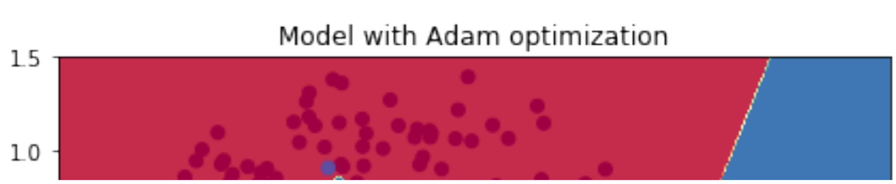
Run the following code to see how the model does with Adam.

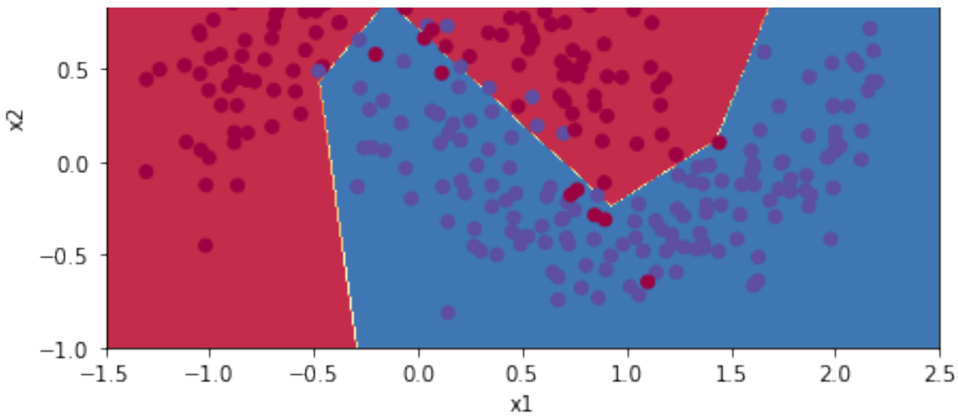
In [44]:
```
# train 3-layer model
layers_dims = [train_X.shape[0], 5, 2, 1]
parameters = model(train_X, train_Y, layers_dims, optimizer = "adam")

# Predict
predictions = predict(train_X, train_Y, parameters)

# Plot decision boundary
plt.title("Model with Adam optimization")
axes = plt.gca()
axes.set_xlim([-1.5,2.5])
axes.set_ylim([-1,1.5])
plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X, t
```

```
Cost after epoch 0: 1.201077
Cost after epoch 1000: 0.241020
Cost after epoch 2000: 0.232754
Cost after epoch 3000: 0.247820
Cost after epoch 4000: 0.224771
Cost after epoch 5000: 0.229633
Cost after epoch 6000: 0.230636
Cost after epoch 7000: 0.248728
Cost after epoch 8000: 0.225002
Cost after epoch 9000: 0.197702
```




```
Accuracy: 0.94
```





5.4 - Summary

optimization method	accuracy	cost shape
Gradient descent	79.7%	oscillations
Momentum	79.7%	oscillations
Adam	94%	smoother

Momentum usually helps, but given the small learning rate and the simplistic dataset, its impact is almost negligeable. Also, the huge oscillations you see in the cost come from the fact that some minibatches are more difficult thans others for the optimization algorithm.

Adam on the other hand, clearly outperforms mini-batch gradient descent and Momentum. If you run the model for more epochs on this simple dataset, all three methods will lead to very good results. However, you've seen that Adam converges a lot faster.

Some advantages of Adam include:

- Relatively low memory requirements (though higher than gradient descent and gradient descent with momentum)
- Usually works well even with little tuning of hyperparameters (except α)

References:

- Adam paper: https://arxiv.org/pdf/1412.6980.pdf (https://arxiv.org/pdf/1412.6980.pdf)