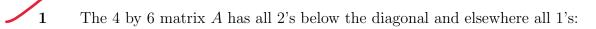
Your name is:	

Please circle your recitation:

1)	M2	2-131	PO. Persson	2-088	2-1194	persson
2)	M2	2-132	I. Pavlovsky	2-487	3-4083	igorvp
3)	M3	2-131	I. Pavlovsky	2-487	3-4083	igorvp
4)	T10	2-132	W. Luo	2-492	3-4093	luowei
5)	T10	2-131	C. Boulet	2-333	3-7826	cilanne
6)	T11	2-131	C. Boulet	2-333	3-7826	cilanne
7)	T11	2-132	X. Wang	2-244	8-8164	xwang
8)	T12	2-132	P. Clifford	2-489	3-4086	peter
9)	T1	2-132	X. Wang	2-244	8-8164	xwang
10)	T1	2-131	P. Clifford	2-489	3-4086	peter
11)	T2	2-132	X. Wang	2-244	8-8164	xwang

The ten questions are worth 10 points each. Thank you for taking 18.06!



- (a) By elimination factor A into L (4 by 4) times U (4 by 6).
- (b) Find the rank of A and a basis for its nullspace (the special solutions would be good).



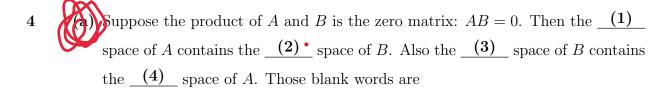
Suppose you know that the 3 by 4 matrix A has the vector $\mathbf{s} = (2, 3, 1, 0)$ as a basis for its nullspace.

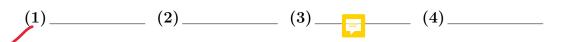
- (a) What is the rank of A and the complete solution to Ax = 0?
- (b) What is the exact row reduced echelon form R of A?

3 The following matrix is a projection matrix:

$$P = \frac{1}{21} \left[\begin{array}{rrr} 1 & 2 & -4 \\ 2 & 4 & -8 \\ -4 & -8 & 16 \end{array} \right].$$

- (a) What subspace does P project onto?
 (b) What is the distance from that subspace to b = (1,1,1)?
- What are the three eigenvalues of P? Is P diagonalizable?





(b) Suppose that matrix A is 5 by 7 with rank r, and B is 7 by 9 of rank s. What are the dimensions of spaces (1) and (2)? From the fact that space (1) contains space (2), what do you learn about r + s?

- $\mathbf{5}$ Suppose the 4 by 2 matrix Q has orthonormal columns.
 - (a) Find the least squares solution \hat{x} to Qx = b.
 - (b) Explain why QQ^{T} is not positive definite. What are the (nonzero) singular values of Q, and why?

- 6 Let S be the subspace of \mathbb{R}^3 spanned by $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix}$.
 - (a) Find an orthonormal basis $\boldsymbol{q}_1, \boldsymbol{q}_2$ for S by Gram-Schmidt.
 - **(b)** Write down the 3 by 3 matrix P which projects vectors perpendicularly onto S.
 - (c) Show how the properties of P (what are they?) lead to the conclusion that $P\boldsymbol{b}$ is orthogonal to $\boldsymbol{b} P\boldsymbol{b}$.

7 (a) If v_1, v_2, v_3 form a basis for \mathbb{R}^3 then the matrix with those three columns is _____.

(b) If v_1, v_2, v_3, v_4 span \mathbb{R}^3 , give all possible ranks for the matrix with those four columns. ______.

of q_1, q_2, q_3 form an orthonormal basis for \mathbb{R}^3 , and T is the transformation that projects every vector \boldsymbol{v} onto the plane of q_1 and q_2 , what is the matrix for T in this basis? Explain.



Suppose the n by n matrix A_n has 3's along its main diagonal and 2's along the diagonal below and the (1, n) position:

$$A_4 = \left[\begin{array}{cccc} 3 & 0 & 0 & 2 \\ 2 & 3 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 2 & 3 \end{array} \right].$$

Find by cofactors of row 1 or otherwise the determinant of A_4 and then the determinant of A_n for n > 4.

- **9** There are six 3 by 3 permutation matrices P.
 - (a) What numbers can be the determinant of P? What numbers can be pivots?
 - What numbers can be the trace of P? What four numbers can be eigenvalues of P?

- Suppose A is a 4 by 4 upper triangular matrix with 1, 2, 3, 4 on its main diagonal. (You could put all 1's above the diagonal.)
- For A 3I, which columns have pivots? Which components of the eigenvector \boldsymbol{x}_3 (the special solution in the nullspace) are definitely zero?
 - Using part (a), show that the eigenvector matrix S is also upper triangular.