Please PRINT your name \_\_\_\_\_

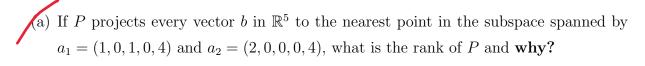
2.

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r1	Т	10	36-156	Russell Hewett	r7	Т	1	36-144	Vinoth Nandakumar
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r6	Τ	1	36 - 153	Stephen Curran					

## (1) **(40 pts)**



(b) If these two vectors are the columns of the 5 by 2 matrix A, which of the four fundamental subspaces for A is the nullspace of P?

(c) By Gram-Schmidt find an orthonormal basis for the column space of A (spanned by  $a_1$  and  $a_2$ ).

If P is any (symmetric) projection matrix, show that Q = I - 2P is an orthogonal matrix.

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- (2) **(30 pts.)** 
  - (a) Find the determinant of the matrix A

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 0 & 2 & 3 & 1 \\ \hline 0 & 0 & 3 & 4 \end{bmatrix}.$$

(b) The absolute value of det A tells you the volume of a box in  $\mathbb{R}^4$ . Describe that box (2 points – describe a different box with the same volume).

Suppose you remove row 3 and column 4 of an invertible 5 by 5 matrix A. If that reduced matrix is not invertible, what fact does that tell you about  $A^{-1}$ ?

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(3) (30 pts.) This 4 by 4 Hadmard matrix is an orthogonal matrix. Its columns are orthogonal unit vectors.

- (a) What projection matrix  $P_4$  (give numbers) will project every b in  $\mathbb{R}^4$  onto the line through  $q_4$ ?
- What projection matrix  $P_{123}$  will project every b in  $\mathbb{R}^4$  onto the subspace spanned by  $q_1, q_2$ , and  $q_3$ ? Remember that those columns are orthogonal.

(c) Suppose A is the 4 by 3 matrix whose columns are  $q_1$ ,  $q_2$ ,  $q_3$ . Find the least-squares solution  $\hat{x}$  to the four equations

What is the error vector e?

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