18.06 Professor Strang Quiz 3 December 6, 2000

**Your name is:** ______________________________

**Please circle your recitation:**

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|---|---|---|---|---|---|---|
| 1) | M2 | 2-131 | Holm | 2-181 | 3-3665 | `tsh@math` |
| 2) | M2 | 2-132 | Dumitriu | 2-333 | 3-7826 | `dumitriu@math` |
| 3) | M3 | 2-131 | Holm | 2-181 | 3-3665 | `tsh@math` |
| 4) | T10 | 2-132 | Ardila | 2-333 | 3-7826 | `fardila@math` |
| 5) | T10 | 2-131 | Czyz | 2-342 | 3-7578 | `czyz@math` |
| 6) | T11 | 2-131 | Bauer | 2-229 | 3-1589 | `bauer@math` |
| 7) | T11 | 2-132 | Ardila | 2-333 | 3-7826 | `fardila@math` |
| 8) | T12 | 2-132 | Czyz | 2-342 | 3-7578 | `czyz@math` |
| 9) | T12 | 2-131 | Bauer | 2-229 | 3-1589 | `bauer@math` |
| 10) | T1 | 2-132 | Ingerman | 2-372 | 3-4344 | `ingerman@math` |
| 11) | T1 | 2-131 | Nave | 2-251 | 3-4097 | `nave@math` |
| 12) | T2 | 2-132 | Ingerman | 2-372 | 3-4344 | `ingerman@math` |
| 13) | T2 | 1-150 | Nave | 2-251 | 3-4097 | `nave@math` |

**1 (30 pts.)** (a) Find the diagonalization $A = S\Lambda S^{-1}$ of

$$A = \begin{bmatrix} 0.5 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) What is the limit of $A^k$ as $k \to \infty$?

(c) Suppose $B^k$ approaches $I$ (the 2 by 2 identity) as $k \to \infty$. How do you know that $B = I$? Explain using eigenvalues and Jordan forms like

$$J = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

**2 (40 pts.)** (a) Suppose the diagonalization $A = S\Lambda S^{-1}$ is exactly the same as the singular value decomposition $A = U\Sigma V^{\mathrm{T}}$ (so $S = U = V$ and $\Lambda = \Sigma$). What information does this give about $A$? Can it be singular?

(b) What are the eigenvalues of a 3 by 3 Markov projection matrix that has trace 2? Create one matrix that has these properties.

(c) Here is a matrix with orthogonal columns. Find its $SVD$ $A = U\Sigma V^{\mathrm{T}}$.

$$A = \begin{bmatrix} 3 & 0 \\ 4 & 0 \\ 0 & 7 \end{bmatrix}$$

(d) Suppose $A$ is similar to a 3 by 3 matrix $B$ that has eigenvalues 1, 1, 2. What can you say about

1. the eigenvalues of $A$
2. diagonalizability of $A$
3. symmetry of $A$
4. positive definiteness of $A$

In each of (2) (3) and (4) decide if $A$ can't have or might have or must have this property.

**3 (30 pts.)** (a) Find the eigenvalues of the matrix (and fill in the blanks)

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

These eigenvalues are all __________ because this matrix $A$ is __________

(b) If the eigenvectors are $x_1$, $x_2$, $x_3$ (not required to compute them) describe the general solution to the differential equation $\frac{du}{dt} = Au$.

?

(c) At what time $T$ is the solution $u(T)$ guaranteed to equal its initial value $u(0)$?