18.06 Exam III   Professor Strang   May 7, 2014

Your PRINTED Name is: ______________________________

Please circle your section:

R01	T	10	36-144	Qiang Guang
R02	T	10	35-310	Adrian Vladu
R03	T	11	36-144	Qiang Guang
R04	T	11	4-149	Goncalo Tabuada
R05	T	11	E17-136	Oren Mangoubi
R06	T	12	36-144	Benjamin Iriarte Giraldo
R07	T	12	4-149	Goncalo Tabuada
R08	T	12	36-112	Adrian Vladu
R09	T	1	36-144	Jui-En (Ryan) Chang
R10	T	1	36-153	Benjamin Iriarte Giraldo
R11	T	1	36-155	Tanya Khovanova
R12	T	2	36-144	Jui-En (Ryan) Chang
R13	T	2	36-155	Tanya Khovanova
R14	T	3	36-144	Xuwen Zhu
ESG	T	3		G. Stoy

Grading

1:

2:

3:

4:

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1. (28 points) This question is about the differential equation

$$\frac{dy}{dt} = Ay = \begin{bmatrix} 5 & 2 \\ 8 & 5 \end{bmatrix} y \quad \text{with } y(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(a) Find an eigenvalue matrix Λ and an eigenvector matrix S so that $A = S\Lambda S^{-1}$. Compute the matrix exponential e^{tA} by using $e^{t\Lambda}$.

(b) Find $y(t)$ as a **combination of the eigenvectors** of A that has the correct value $y(0)$ at $t = 0$.

2. (a) (24 points) Suppose a symmetric n by n matrix S has eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_n$ and orthonormal eigenvectors $q_1, \ldots, q_n$.

If $x = c_1 q_1 + c_2 q_2 + \cdots + c_n q_n$ show that $x^T x = c_1^2 + \cdots + c_n^2$ and $x^T S x = \lambda_1 c_1^2 + \cdots + \lambda_n c_n^2$.

(b) What is the largest possible value of $R(x) = \frac{x^T S x}{x^T x}$ for nonzero x ?

Describe a vector x that gives this maximum value for this ratio $R(x)$?

3. (24 points)

(a) Show that the matrix $S = A^T A$ is positive semidefinite, for any matrix A. Which test will you use and how will you show it is passed?

(b) If A is 3 by 4, show that $A^T A$ is **not** positive definite.

4. (24 points)

(a) Show that none of the singular values of A are larger than 3.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Why does $B = AQ$ have the same singular values as A? (Q is an orthogonal matrix.)

Scrap Paper