

MIT 18.06 Exam 1, Spring 2022  
Johnson

Your name: \_\_\_\_\_

Recitation: \_\_\_\_\_

problem	score
1	/26
2	/24
3	/25
4	/25
<i>total</i>	/100

**Problem 0 ( $\infty$  points): Honor code**

Copy the following statement with your signature into your solutions:

*I have completed this exam **closed-book/closed-notes** entirely  
on my **own**.*

[your signature]

**Problem 1 (26 points):**

Suppose

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 2 & 5 \\ 1 & 2 & 1 & 1 \end{pmatrix}.$$

(a) Give a basis for  $N(A)$ .

(b) For what value or values (if any) of  $\alpha$  does  $Ax = \begin{pmatrix} 1 \\ 2\alpha \\ \alpha \end{pmatrix}$  have any solution  $x$ ?

*(blank page for your work if you need it)*

**Problem 2 (24 points):**

Give a **basis** for the **nullspace**  $N(A)$  and a basis for the **column space**  $C(A)$  for each of the following matrices:

(a) The one-column matrix  $A = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ .

(b) The one-row matrix  $A = (1 \ 2 \ 3 \ 4)$ .

(c) The 100-row matrix  $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & 4 \end{pmatrix}$  in which every row is  $(1 \ 2 \ 3 \ 4)$ .

*(blank page for your work if you need it)*


**Problem 3 (25 points):**


Suppose that we are solving  $Ax = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ . In each of the parts below, a

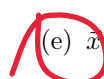
**complete** solution  $x$  is proposed. For each possibility, say **impossible** if that could *not* be a *complete* solution to such an equation, **or** give the the **size**  $m \times n$  and the **rank** of the matrix  $A$  if  $x$  is possible.

(a)  $\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

(b)  $\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \alpha_1 \begin{pmatrix} 1 \\ -1 \\ 5 \\ 17 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$  for all real numbers  $\alpha_1, \alpha_2 \in \mathbb{R}$

 (c)  $\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix}$  for all real numbers  $\alpha \in \mathbb{R}$

 (d)  $\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix}$  for all real numbers  $\alpha \in \mathbb{R}$

 (e)  $\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$  for all real numbers  $\alpha_1, \alpha_2 \in \mathbb{R}$

*important alt method for solve*

**Problem 4 (25 points):**

Let

$$B = \begin{pmatrix} 1 & & \\ 1 & 1 & \\ 1 & 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & -1 & -1 \\ & 2 & -1 \\ & & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ -8 \\ -4 \end{pmatrix}.$$

**Compute:**

$$(CB)^{-1}b.$$

(Hint: Remember what I said in class about inverting matrices!)



*(blank page for your work if you need it)*