18.06 Quiz 2 Professor Strang April 10, 2013

**Please PRINT your name** _____ **1.**

**2.**

**Please Circle your Recitation:** **3.**

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|---|---|---|---|---|---|---|---|---|---|
| r1 | T | 10 | 36-156 | Russell Hewett | r7 | T | 1 | 36-144 | Vinoth Nandakumar |
| r2 | T | 11 | 36-153 | Russell Hewett | r8 | T | 1 | 24-307 | Aaron Potechin |
| r3 | T | 11 | 24-407 | John Lesieutre | r9 | T | 2 | 24-307 | Aaron Potechin |
| r4 | T | 12 | 36-153 | Stephen Curran | r10 | T | 2 | 36-144 | Vinoth Nandakumar |
| r5 | T | 12 | 24-407 | John Lesieutre | r11 | T | 3 | 36-144 | Jennifer Park |
| r6 | T | 1 | 36-153 | Stephen Curran | | | | | |

(1) **(40 pts)**

(a) If $P$ projects every vector $b$ in $\mathbb{R}^5$ to the nearest point in the subspace spanned by $a_1 = (1, 0, 1, 0, 4)$ and $a_2 = (2, 0, 0, 0, 4)$, what is the rank of $P$ and **why?**

(b) If these two vectors are the columns of the 5 by 2 matrix $A$, which of the four fundamental subspaces for $A$ is the nullspace of $P$?

(c) By Gram-Schmidt find an orthonormal basis for the column space of $A$ (spanned by $a_1$ and $a_2$).

(d) If $P$ is any (symmetric) projection matrix, show that $Q = I - 2P$ is an orthogonal matrix.

(2) **(30 pts.)**

(a) Find the determinant of the matrix $A$

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{bmatrix}.$$

(b) The absolute value of $\det A$ tells you the volume of a box in $\mathbb{R}^4$. Describe that box (2 points – describe a different box with the same volume).

(c) Suppose you remove row 3 and column 4 of an invertible 5 by 5 matrix $A$. If that reduced matrix is not invertible, what fact does that tell you about $A^{-1}$?

(3) **(30 pts.)** This 4 by 4 Hadmard matrix is an orthogonal matrix. Its columns are orthogonal unit vectors.

$$Q = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}$$

(a) What projection matrix $P_4$ (give numbers) will project every $b$ in $\mathbb{R}^4$ onto the line through $q_4$?

(b) What projection matrix $P_{123}$ will project every $b$ in $\mathbb{R}^4$ onto the subspace spanned by $q_1$, $q_2$, and $q_3$? Remember that those columns are orthogonal.

(c) Suppose $A$ is the 4 by 3 matrix whose columns are $q_1$, $q_2$, $q_3$. Find the least-squares solution $\widehat{x}$ to the four equations

$$Ax = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = b.$$

What is the error vector $e$?