18.06 Quiz 3 Professor Strang May 10, 2013

**Please PRINT your name** ______________________________ **1.**

**2.**

**Please Circle Your Recitation:** **3.**

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|---|---|---|---|---|---|---|---|---|---|
| r1 | T | 10 | 36-156 | Russell Hewett | r7 | T | 1 | 36-144 | Vinoth Nandakumar |
| r2 | T | 11 | 36-153 | Russell Hewett | r8 | T | 1 | 24-307 | Aaron Potechin |
| r3 | T | 11 | 24-407 | John Lesieutre | r9 | T | 2 | 24-307 | Aaron Potechin |
| r4 | T | 12 | 36-153 | Stephen Curran | r10 | T | 2 | 36-144 | Vinoth Nandakumar |
| r5 | T | 12 | 24-407 | John Lesieutre | r11 | T | 3 | 36-144 | Jennifer Park |
| r6 | T | 1 | 36-153 | Stephen Curran | | | | | |

(1) **(40 pts)**

In all of this problem, the 3 by 3 matrix $A$ has eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with independent eigenvectors $x_1, x_2, x_3$.

(a) What are the trace of $A$ and the determinant of $A$?

(b) Suppose: $\lambda_1 = \lambda_2$. Choose the true statement from 1, 2, 3:

1. $A$ can be diagonalized. Why?
2. $A$ can not be diagonalized. Why?
3. I need more information to decide. Why?

(c) From the eigenvalues and eigenvectors, how could you find the matrix $A$? Give a formula for $A$ and explain each part carefully.

(d) Suppose $\lambda_1 = 2$ and $\lambda_2 = 5$ and $x_1 = (1, 1, 1)$ and $x_2 = (1, -2, 1)$. Choose $\lambda_3$ and $x_3$ so that $A$ is symmetric positive semidefinite but not positive definite.

(2) **(30 pts.)**

Suppose $A$ has eigenvalues $1, \frac{1}{3}, \frac{1}{2}$ and its eigenvectors are the columns of $S$:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{with} \quad S^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

(a) What are the eigenvalues and eigenvectors of $A^{-1}$?

(b) What is the general solution (with 3 arbitrary constants $c_1, c_2, c_3$) to the differential equation $du/dt = Au$? Not enough to write $e^{At}$. Use the $c$'s.

(c) Start with the vector $u = (1, 4, 3)$ from adding up the three eigenvectors: $u = x_1 + x_2 + x_3$. Think about the vector $v = A^k u$ for VERY large powers $k$. What is the limit of $v$ as $k \to \infty$?

(3) **(30 pts.)**

(a) For a really large number $N$, will this matrix be positive definite? Show why or why not.

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 4 & N & 1 \\ 3 & 1 & 4 \end{bmatrix}.$$

(b) Suppose: $A$ is positive definite symmetric
$Q$ is orthogonal (same size as $A$)
$B$ is $Q^T A Q = Q^{-1} A Q$

Show that: 1. $B$ is also symmetric.
2. $B$ is also positive definite.

(c) If the SVD of $A$ is $U \Sigma V^T$, how do you find the orthogonal $V$ and the diagonal $\Sigma$ from the matrix $A$?