Please PRINT your name ______________________________ **1.**

2.

Please Circle your Recitation: **3.**

r1	T	10	36-156	Russell Hewett
r2	T	11	36-153	Russell Hewett
r3	T	11	24-407	John Lesieutre
r4	T	12	36-153	Stephen Curran
r5	T	12	24-407	John Lesieutre
r6	T	1	36-153	Stephen Curran
r7	T	1	36-144	Vinoth Nandakumar
r8	T	1	24-307	Aaron Potechin
r9	T	2	24-307	Aaron Potechin
r10	T	2	36-144	Vinoth Nandakumar
r11	T	3	36-144	Jennifer Park

(1.) **(30 pts.)** For a 3 by 3 matrix A, suppose all three multipliers are $l_{21} = l_{31} = l_{32} = 3$. Each l_{ij} multiplies pivot row j when it is subtracted from row i.

(a) Assuming no row exchanges, what is A, if elimination reaches

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & g \end{bmatrix}?$$

(b) In case $g = 0$, the three columns of A must be dependent. Find the nullspace (a vector space) of A.

(c) In case $g \neq 0$, what is the column space of U? What is the column space of the original matrix A? How do you know?

(2.) **(40 pts.)** A is a 2 by 4 matrix with exactly two special solutions to $Ax = 0$:

$$x = s_1 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad x = s_2 = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

(a) Find the reduced row echelon form R of A.

(b) What is the column space of A?

(c) What is the complete solution to $Rx = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$?

(d) Find a combination of columns 2, 3, 4 that equals the zero vector. (Not OK to use $0\,(\text{col}\,2) + 0\,(\text{col}\,3) + 0\,(\text{col}\,4) = 0$. The problem is to show that these 3 columns are dependent.)

(3.) **(30 pts.)** Suppose A is the 2 by 3 matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}.$$

(a) Find all 3 by 2 matrices X with

$$AX = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

What is a basis for that space of matrices?

(b) Find one 3 by 2 matrix X with

$$AX = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(c) Find the complete solution of this matrix equation: *all* 3 by 2 matrices X with

$$AX = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$