B.Sc. Engg. in CSE / SWE (3rd Semester) 27 August 2020

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2019-2020**
DURATION: 1 Hour **FULL MARKS: 45**

Math 4341: Linear Algebra

General Instructions

- Write your **Name, Student-ID** and **Course Code** on the top of the first page. Maintain a **serial number** on the Top-right corner of each page.
- Answer all the questions. Figures in the right margin indicate marks.
- Sit in proper position and maintain the environment as per the Guidelines.
- No examinee is allowed to scan the file unless 1 hour is finished.
- For any circumstances, follow the instructions of the invigilator.

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1 Suppose, a group of 4 students from IUT CSE'18 went for a tour. They bought a number of souvenirs from there. While packing, they found there were 4 different items. Suppose the items are named A, B, C & D. Everyone bought at least one item or more than that. Now, they tried to put this purchase information into a matrix (Purchase Matrix). So, the columns represent the number of a particular item they bought and the rows represent individual purchase information (number of items) of the students.

a. *It was found out that the items were arranged in the columns in this order- B->C->A->D. How do you put it in the correct order(alphabetical) in the matrix without directly manipulating the columns?* 5

b. The purchase details are as follows- 7

Items	Student 1	Student 2	Student 3	Student 4
A	2	0	1	1
B	0	1	0	2
C	1	0	1	0
D	0	2	0	5

Let's say, V_1=subspace generated by the first column of the purchase matrix
V_2=subspace generated by the third column of the purchase matrix
Find out V_1 & V_2(Draw). What is $V_1 \cap V_2$? Is it a vector subspace as well?

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c. Mention the dimension of all four fundamental subspaces of the purchase matrix. What can be one good basis for the rowspace?

2 Time (in -th seconds) and number of errors done by a machine is given below:

Time (in -th second)	Number of errors
-2	1
0	2
2	4

a. Can you write these data points as a simple linear system? 2
b. Find a linear equation that fits these points by minimizing the error. 10
c. Can you predict the number of errors done by the machine on the 5^{th} second? 3

3 The following system is given- 6

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & h & k \end{array}\right]$$

Find out the values of ***h*** and ***k*** so that the system becomes-

a. Inconsistent. [No solution]
b. Consistent with infinitely many solutions.
c. Consistent with a unique solution.

4 Short Questions-

a. If a matrix has an eigenvalue with $\lambda=0$, what can you decide about the rank and columnspace of the matrix? Is it a matrix with full rank as column? Do the columns span the whole vectorspace? 4
b. If a matrix has two eigenvalues $\lambda_1=3$ and $\lambda_2=4$, 3
   i. Find out the determinant of the matrix.
   ii. Find out the sum of the diagonal elements of the matrix.

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B.Sc. Engg. /HD CSE 3rd Semester **31 May 2018**

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2017-2018**
DURATION: 3 Hours **FULL MARKS: 150**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.
There are 8 (eight) questions. Answer any 6 (six) of them.
Figures in the right margin indicate marks.

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1. a) Show that the projection of a vector $\boldsymbol{b}$ on the column space of given matrix A is obtained from $p = A\hat{x}$. Also derive the value of $\hat{x}$. 7

   b) Prove that $A^T A$ is invertible if and only if A has linear independent columns. 5

   c) Suppose a plane is spanned by the linear combination of two vectors $(1, 2, -1)^T$ and $(1, 0, 1)^T$. Which vector in this plane is closest to $b = (2, 1, 1)^T$? 8

   d) Suppose A is the 4 by 4 identity matrix with its last column removed: A is 4 by 3. Project $b = (1, 2, 3, 4)^T$ onto the column space of A. What shape is projection matrix $\boldsymbol{P}$ and what is P? 5

2. a) Find the equation $y = mx + c$ of the least-squares line that best fits the data points (2, 1), (5, 2), (7, 3), and (8, 3). What is the magnitude of the error vector? 20

   b) Given an $m \times n$ matrix A with linearly independent columns, let A=QR be a QR factorization of A where $\boldsymbol{Q}$ contains orthonormal columns and $\boldsymbol{R}$ is an upper-triangular matrix. Then, prove that for each $\boldsymbol{b}$ in $\mathbf{R}^m$ space, the equation Ax=b has a unique least-squares solution, given by: $\hat{x} = R^{-1}Q^T b$. 5

3. a) If A=QR then $A^T A = R^T R$. *Gram-Schmidt* method on A corresponds to elimination on $A^T A$. Show that the pivots for $A^T A$ must be the squares of diagonal entries of $\boldsymbol{R}$. Find $\boldsymbol{Q}$ and $\boldsymbol{R}$ by Gram-Schmidt for this matrix A: 20

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 2 & 4 \end{bmatrix}$$

   b) Determine whether the following statements are true or false. Give a reason if true or a counter example if false:

      i. The determinant of $I+A$ is $1+\det(A)$. 1
      ii. The determinant of ABC is |A||B||C|. 1
      iii. The determinant of $4A$ is $4\det(A)$. 1
      iv. The determinant of $\boldsymbol{AB-BA}$ is zero. All matrices here are considered square. 2

$$\text{Try with } A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

4. a) The n by n determinant A_n has l's above and below the main diagonal:

$$A_1 = |0| \qquad A_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \qquad A_3 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \qquad A_4 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

      i. What are these determinants A_2, A_3, A_4? 3×5
      ii. By cofactors find the relation between A_n and A_{n-1} and A_{n-2}. Find A_{10}. 5

b) The comers of a triangle are (2, 1) and (3, 4) and (0, 5). Add a corner at (-1, 0) to make a lopsided region (four sides). Find the area of the new region. 5

5. a) Find the eigenvalues for the following matrix A. Calculate the eigenvectors for only nonzero eigenvalues. 20

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

b) Show that if $3I$ is added with a matrix A, then its eigenvalues change but eigenvectors do not. 5

6. a) Factorize the following matrix A in to $S\Lambda S^{-1}$. 7

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

b) The Lucas numbers are like the Fibonacci numbers except they start with $L_1=1$ and $L_2 = 3$. Following the rule $L_{K+2} = L_{K+1} + L_K$ the next Lucas numbers are 4, 7, 11, 18, and so on. Find the Lucas number L_{100}. 18

7. a) List the properties of Markov matrix. Show that the square of a Markov matrix is also a Markov matrix. 2+3

b) Suppose in a city every year 2% of young people become old and 3% of old people become dead. With no births and sudden death of young people, find the steady state for 20

$$\begin{bmatrix} \text{young} \\ \text{old} \\ \text{dead} \end{bmatrix}_{k+1} = \left[3\times3 \ \textit{Markov Matrix}\right] \begin{bmatrix} \text{young} \\ \text{old} \\ \text{dead} \end{bmatrix}_k$$

Show all necessary calculations.

8. a) Compute the product of the following two matrices using the concepts of column picture of a linear system. 10

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

b) Find the inverse of matrix A by the Gauss-Jordan method: 10

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

c) For which values of c and d does the following matrix have a rank of 2? Explain your answer. 5

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix}$$

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B.Sc. Engg./HD CSE 3th Semester

May 28 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION **WINTER SEMESTER, 2018-2019**

DURATION: 3 Hours **FULL MARKS: 150**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.

There are 8 (eight) questions. Answer any 6 (six) of them.

Figures in the right margin indicate marks.

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1. a) Find out all four subspaces of the given matrix A and find out the complete solution for $Ax = b$. 17

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

   b) For the given matrix in 1.(a) find out the dimensions of all four subspaces. 4

   c) With the calculated subspaces, show that nullspace is perpendicular to the row space and left nullspace is perpendicular to the column space. 4

2. a) Define vector space and subspaces with appropriate examples. Mention the types of vector subspaces of $\mathbf{R}^2$ and $\mathbf{R}^3$ space. 5

   b) Prove that K is invertible if $a \neq 0$ and $a \neq b$ for the given matrix K below. 10

$$K = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

   c) Compute L and U for the symmetric matrix A. Also find four conditions on a, b, c, d to get A=LU with all four pivots. 10

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

3. a) Generally when we multiply two matrices by hand (satisfying dimensions), we multiply rows with column. How can we get the same result by multiplying in the other way around i.e. column times row? Mention the special property of the sub-matrix that you get by multiplying a single column with a single row. 7

   b) For which right sides (find a condition on b_1, b_2, and b_3) are these systems solvable? Also find out the rank of the linear systems given. 10

i. $$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

ii. $$\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   c) Reduce the following matrix to its ordinary echelon form and then identify its free variables and pivot variables. 8

$$\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

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4. a) Derive the formula of projection of a vector b onto the column space of a matrix A. If P is the projection matrix, show that $P = P^T$ and $P^2 = P$. 10

b) What linear combination of $(1,2,-1)$ and $(1,0,1)$ is closest to $b = (2,1,1)$? 8

c) With the help of appropriate diagram, for a linear system $Ax = b$, describe how particular solution and nullspace solution maps a matrix A to b. 7

5. a) Prove that $A^T A$ is invertible if and only if A has linearly independent columns. 8

b) Project vector $b = (0,8,8,20)$ onto the line through vector $a = (1,1,1,1)$. Find out the projection of vector b on vector a. Show that the error vector is perpendicular to vector a. 8

c) Find q_1, q_2, q_3 (orthonormal vectors) as combinations of a, b, c (independent columns). Then write A as QR where Q and R have their usual meanings. 9

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{bmatrix}$$

6. a) Define orthonormal vectors. Why it is easier to work with orthonormal vectors while working with projection? Explain with appropriate justification. 6

b) Find the determinants of the following matrices U and U^{-1} and U^3. Where U is given as 9

$$U = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$

c) Using properties of determinants, from the formula $\boldsymbol{AC^T = (\det A)I_n}$ show that $\boldsymbol{\det C = (\det A)^{n-1}}$ 10

7. a) With the help of the properties of the determinant of a matrix, prove that for any matrix A, $\det(A) = \det(A^T)$. 6

b) Use Cramer's Rule with ratios $x_j = {\det B_j}/{\det A}$ to solve the given linear system below. 12

$$\begin{aligned} 2x_1 + 6x_2 + 2x_3 &= 0 \\ 1x_1 + 4x_2 + 2x_3 &= 0 \\ 5x_1 + 9x_2 + 0x_3 &= 1 \end{aligned}$$

c) A box has edges from $(0,0,0)$ to $(3,1,1)$ and $(1,3,1)$ and $(1,1,3)$. Find the volume of the box. 7

8. a) Find the eigenvalues and eigenvectors of the following 3×3 matrix M. 9

$$M = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

b) Start from $u_0 = (1,0)$, Compute u_k when S and Λ contain these eigen vectors and eigen values. 8

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad S = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

c) Find the Fibonacci number F_{98} using eigenvalues and eigenvectors. 8

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B.Sc. Engg. (SWE) 3rd Semester | 28 May 2019

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION — **WINTER SEMESTER, 2018-2019**
DURATION: 3 Hours — **FULL MARKS: 150**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.
There are 8 (eight) questions. **Question no. 3 and 4 is mandatory to answer.**
Answer any 4 (four) from the remaining.
Figures in the right margin indicate marks.

$$B = \begin{bmatrix} 1 & 3 & 1 & 2 & 5 \\ 2 & 6 & 4 & 8 & 10 \\ 0 & 0 & 2 & 4 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

(Use these vectors and matrices to answer the question from 1-3)

1. a) Find out the correct elimination matrix $\boldsymbol{E}$ to convert the matrix $\boldsymbol{B}$ into its upper triangular form $EB = U$. Also factorize B into LU format. 13

   b) For finding the Inverse of a matrix using Gauss-Jordan method, first the augmented matrix is taken as $[A\ I]$. Then the target is to convert A into I which brings A^{-1} in the place of the augmented I simultaneously. 6
   Explain the reason behind the appearing of A^{-1} in the place of the augmented I.

   c) Explain whether the following statements are True or false (with a reason or an example): 1.5×4
      i. Any m by n matrix with $rank = m$ has solution for every $Ax = b$.
      ii. A rank-one matrix might have dependent rows.
      iii. The left Null-space can be empty although the Null-space has special solutions.
      iv. A Matrix-space consisting of all possible m by n matrices has dimension m or n.

2. a) What is the condition on the vector m for the system $Bx = m$ to have a solution? 6

   b) Find out the complete solution of the linear system $Bx = b$. 13

   c) i. Do the vectors lying on the line $2x + y = 7$ form a subspace? Justify your answer. 6
      ii. $S = \{x, y \in R: x > 0, y > 0 \text{ or } x < 0, y < 0\}$ Does this set of vectors form a valid subspace? Justify your answer.

[Mandatory]

3. a) Prove that, if a matrix A has independent columns, A^TA is invertible. 7

   b) Find out the basis and dimension for the four fundamental subspaces associated with the matrix $\boldsymbol{B}$. Also comment about the shapes of the four spaces. 13

   c) Explain why $v = (1,3,1,2,5)$ cannot be a row of $\boldsymbol{B}$ and also in the Null-space at the same time. 5

[Mandatory]

4. a) Suppose a plane in R^n in described by a matrix A. Calculate the *projection* and *error* for any vector 'b' when: 10
   i. $b \in N(A^T)$
   i. $b \perp N(A^T)$

   b) Using the properties of determinant, for any $n \times n$ matrix A, Prove that, $det(A) = det(A^T)$. 8

   c) While taking the projection(p) of a vector(b) on a plane, what are the conditions when the projection will produce an eigenvector? Write the eigenvalues and eigenvectors for those cases. 7

5. a) What linear combination of $a_1 = (1,2,-1)$ and $a_2 = (1,0,1)$ is closest to $b = (2,1,1)$? What will be the error vector*(e)*? 10

   b) Find the orthonormal vectors A, B, C by *Gram-Schmidt* process from a, b, c: 10

$$a = (1,-1,0,0) \quad b = (0,1,-1,0) \quad c = (0,0,1,-1)$$

   c) A box has edges from (0,0,0) to (3,1,1), (1,3,1), (1,1,3). Find the volume of the box. 5

6. a) Derive the formula of the Projection Matrix*(P)* for projecting any vector 'b' onto a plane. Also prove that, *'Projecting the vector 'b' twice onto the same plane will not change the projection.'* 10+5

   b) With $y = 0,8,8,20$ at $x = 0,1,2,3$, find out the equation of the *'best fit'* straight line through these points. What is the minimum value $E = e_1{}^2 + e_2{}^2 + e_3{}^2 + e_4{}^2$ 10

7. a) If a 5 by 5 matrix has $det(A) = \frac{1}{8}$, find $det(-A), det(A^2), det(5A), det(A^{-1})$, $2\,det(A), det(A^{-1}A)$. 1.5×6

   b) Find the determinant of the following matrix in **three** different ways. 12

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

   c) i. Given x an eigenvector, what is the way to find the eigenvalue*(λ)*? 4
   ii. Given λ an eigenvalue, what is the way to find the eigenvector*(x)*?

8. a) Find the eigenvalues and eigenvectors of the matrix 20

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

   Also find the *Trace* and *Determinant* of the matrix using the eigenvalues.

   b) The basic equation of eigenvalue(λ) is $Ax = \lambda x$. What will be the eigenvalue of A^n? Justify with proper mathematical derivation. 5

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B.Sc. Engg. / HD CSE 3rd Semester | 13 March 2018 (Morning)

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION | **WINTER SEMESTER, 2017-2018**
DURATION: 1 Hour 30 Minutes | **FULL MARKS: 75**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.
There are 4 (four) questions. Answer any 3 (three) of them.
Figures in the right margin indicate marks.

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1. a) Determine if the following linear system is consistent (has solution) or not. Calculate the determinant of the coefficient matrix A from its row-echelon form. 10+2

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_2 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1$$

b) Prove that – "If a linear system is consistent, then the solution is unique if and only if every column in the coefficient matrix is a pivot column; otherwise there are infinitely many solutions." 3

c) Find the resultant matrix $C=A\times B$ from matrix multiplication, where 10

$$A = \begin{bmatrix} 2 & 5 & 4 \\ -3 & 0 & -3 \\ 7 & -6 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 & -6 \\ 0 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

All calculations are to be shown from the concepts of column-picture.

2. a) Find the inverse of matrix A using Gauss-Jordan method: 10

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

Show all elimination matrices for performing row eliminations.

b) Factorize the matrix A in Question 2.(a) into its LDU form. 5

c) Find the nullspace of the following matrix A: 10

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$$

Which are the free columns and the pivot columns?

3. a) Find the complete solution to 15

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

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b) Find the dimension and basis for the rowspace, columnspace, nullspace and left-nullspace, respectively, for the coefficient matrix A as given in Question 3.(a) 10

4. a) Define a vector subspace. Prove the followings with examples: 1+3+3
   i. The union of two subspaces is not a subspace.
   ii. The intersection of two subspaces is a subspace

b) Check that the solutions to $Ax=0$ are perpendicular to the rows: 9+ 1+3

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = E^{-1}R$$

How many independent solutions to $A^Ty=0$? Why is the y^T the last row of E?

c) In a $\mathbf{R}^5$vector-space, suppose two sub-spaces, each being a plane, are orthogonal to each other. Can one of them represent a row-space and the other represents a null-space? Explain your answer. 5

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B.Sc.Engg. CSE 3rd Semester

5th December 2022

ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

SEMESTER FINAL EXAMINATION | **WINTER SEMESTER, 2021-2022**

DURATION: 3 HOURS | **FULL MARKS: 150**

Math 4341: Linear Algebra

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer all 6 (Six) questions. Marks of each question and corresponding CO and PO are written in the right margin with brackets.

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1. a) Consider the following matrix A: 10+2 (CO3) (PO1)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

   i. What would be projection matrix (P) of A to project any vector b onto the columnspace of A?

   ii. Find P^3.

   b) Table 1 represents the relationship between the timestamp (t) in hours and total unit of production (u) of an industry on any random day: 11+2 (CO2) (PO1)

Table 1: Table for Question 1.b)

Timestamp (t)	Total unit of production (u)
-2	1
0	2
2	4
4	5

Find a linear equation that fits these data by minimizing the error. Also, predict the unit of production (u) on the 8th hour of that particular day.

2. a) Consider the following matrix A: 13 (CO1) (PO1)

$$A = \begin{bmatrix} 3 & 4 & 6 \\ 0 & 1 & 0 \\ -1 & -2 & -2 \end{bmatrix}$$

Find a basis of R^3 consisting of eigenvectors of A.

   b) Suppose A is a 3 by 3 matrix with eigenvalues 0, 1 and 2. Identify the following: 4×3 (CO3) (PO1)

   i. the rank of A

   ii. the determinant of $A^T A$

   iii. the determinant of $A + I$

   iv. the eigenvalues of $(A + I)^{-1}$

3. a) Consider the following sequence S:

$$0,1,\frac{1}{2},\frac{3}{4},\frac{5}{8},\ldots\ldots\ldots$$

7+7+6 (CO3) (PO1)

i. Find a matrix A that satisfies

$$\begin{bmatrix} F_{k+2} \\ F_{k+1} \end{bmatrix} = A \begin{bmatrix} F_{k+1} \\ F_k \end{bmatrix}$$

where F_k denotes the k^{th} term of the above-mentioned sequence S.

ii. Diagonalize the matrix A so that you can easily produce the A^k for any number k. You do not need to multiply the decomposed elements to get a single matrix.

iii. Find the 100^{th} term of the sequence S.

b) Is it possible to choose all of the eigenvectors of a real symmetric matrix perpendicular to each other? Justify your answer. 5 (CO1) (PO1)

4. a) Consider the following matrix A: 8+7 (CO3) (PO1)

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}$$

i. Find orthonormal vectors q_1, q_2 and q_3 such that q_1 and q_2 form a basis for the columnspace of A and q_3 remains in the left nullspace of A.

ii. Find the closest vector in the columnspace of A to $b = (1, 2, 7)$.

b) Find the determinants of the following matrices: 5+5 (CO3) (PO1)

$$C = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 0 & 5 & 7 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

5. a) Suppose A is a 3 by 3 matrix with eigenvalues 1, 0 and -1. The eigenvector matrix of A is the following matrix S: 10 (CO3) (PO1)

$$S = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

If matrix B is $B = A^9 + I$, give a reason why the matrix B does have or doesn't have each of the following properties:

i. B is invertible

ii. B is symmetric

iii. $trace = B_{11} + B_{22} + B_{33} = 3$

b) Find the Singular Value Decomposition (SVD) of the following matrix A: 15 (CO3) (PO1)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

6. a) Suppose matrix A is the following product: 12 (CO2) (PO1)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 \end{bmatrix}$$

For what values of t (if any) are there solutions to $Ax = (1, 1, t)$?

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b) Find the conditions on a and b that make the matrix A invertible, and find A^{-1} when it exists: 12 (CO1) (PO1)

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$