

**Grading**

**Your PRINTED name is:** \_\_\_\_\_

**1**

**2**

**3**

**Please circle your recitation:**

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r01	T 11	4-159	Ailsa Keating	ailsa
r02	T 11	36-153	Rune Haugseng	haugseng
r03	T 12	4-159	Jennifer Park	jmypark
r04	T 12	36-153	Rune Haugseng	haugseng
r05	T 1	4-153	Dimiter Ostrev	ostrev
r06	T 1	4-159	Uhi Rinn Suh	ursuh
r07	T 1	66-144	Ailsa Keating	ailsa
r08	T 2	66-144	Niels Martin Moller	moller
r09	T 2	4-153	Dimiter Ostrev	ostrev
r10	ESG		Gabrielle Stoy	gstoy

**1 (40 pts.)**

(a) Find the projection  $p$  of the vector  $b$  onto the plane of  $a_1$  and  $a_2$ , when

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad a_1 = \begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -1 \\ 7 \\ 1 \\ -7 \end{bmatrix}.$$

(b) What projection matrix  $P$  will produce the projection  $p = Pb$  for every vector  $b$  in  $\mathbb{R}^4$ ?

~~(c)~~ What is the determinant of  $I - P$ ? Explain your answer.


~~(d)~~ What are all nonzero eigenvectors of  $P$  with eigenvalue  $\lambda = 1$ ? 

How is the number of independent eigenvectors with  $\lambda = 0$  of an  $n \times n$  square matrix  $A$  connected to the rank of  $A$ ? 

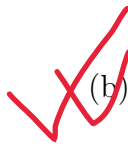

(You could answer (c) and (d) even if you don't answer (b).)

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**2 (30 pts.)**

-  (a) Suppose the matrix  $A$  factors into  $A = PLU$  with a permutation matrix  $P$ , and 1's on the diagonal of  $L$  (lower triangular) and pivots  $d_1, \dots, d_n$  on the diagonal of  $U$  (upper triangular).

What is the determinant of  $A$ ?      EXPLAIN WHAT RULES YOU ARE USING.

-  (b) Suppose the first row of a new matrix  $A$  consists of the numbers 1, 2, 3, 4. Suppose the cofactors  $C_{ij}$  of that first row are the numbers 2, 2, 2, 2. 
- (Cofactors already include the  $\pm$  signs.)

Which entries of  $A^{-1}$  does this tell you and what are those entries?

- (c) What is the determinant of the matrix  $M(x)$ ? For which values of  $x$  is the determinant equal to zero?

$$M(x) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & x \\ 1 & 1 & 4 & x^2 \\ 1 & -1 & 8 & x^3 \end{bmatrix}.$$

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**3 (30 pts.)**

- ~~(a)~~ Starting from independent vectors  $a_1$  and  $a_2$ , use Gram-Schmidt to find formulas for two orthonormal vectors  $q_1$  and  $q_2$  (combinations of  $a_1$  and  $a_2$ ):

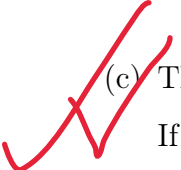
$$q_1 =$$

$$q_2 =$$

•

- (b) The connection between the matrices  $A = [a_1 \ a_2]$  and  $Q = [q_1 \ q_2]$  is often written  $A = QR$ . From your answer to Part (a), what are the entries in this matrix  $R$ ?



 (c) The least squares solution  $\hat{x}$  to the equation  $Ax = b$  comes from solving what equation?  
If  $A = QR$  as above, show that  $R\hat{x} = Q^T b$ .

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