18.06 Professor Strang Quiz 3 May 7th, 2012

Grading

Your PRINTED name is: ______________________

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Please circle your recitation:

r01	T 11	4-159	Ailsa Keating	`ailsa`
r02	T 11	36-153	Rune Haugseng	`haugseng`
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r04	T 12	36-153	Rune Haugseng	`haugseng`
r05	T 1	4-153	Dimiter Ostrev	`ostrev`
r06	T 1	4-159	Uhi Rinn Suh	`ursuh`
r07	T 1	66-144	Ailsa Keating	`ailsa`
r08	T 2	66-144	Niels Martin Moller	`moller`
r09	T 2	4-153	Dimiter Ostrev	`ostrev`
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1 (33 pts.)

Suppose an $n \times n$ matrix A has n independent eigenvectors $x_1, \ldots, x_n$. Then you could write the solution to $\frac{du}{dt} = Au$ in three ways:

$$u(t) = e^{At}u(0), \quad \text{or}$$
$$u(t) = Se^{\Lambda t}S^{-1}u(0), \quad \text{or}$$
$$u(t) = c_1 e^{\lambda_1 t}x_1 + \ldots + c_n e^{\lambda_n t}x_n.$$

Here, $S = [x_1 \mid x_2 \mid \ldots \mid x_n]$.

(a) From the definition of the exponential of a matrix, show why e^{At} is the same as $Se^{\Lambda t}S^{-1}$.

(b) How do you find $c_1, \ldots, c_n$ from $u(0)$ and S?

(c) For this specific equation, write $u(t)$ in any one of the three forms, using *numbers* not symbols: You can choose which form.

$$\frac{du}{dt} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} u, \quad \text{starting from} \quad u(0) = \begin{bmatrix} 4 \\ 3 \end{bmatrix}.$$

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2 (30 pts.)

This question is about the real matrix

$$A = \begin{bmatrix} 1 & c \\ 1 & -1 \end{bmatrix}, \quad \text{for} \quad c \in \mathbb{R}.$$

(a) - Find the eigenvalues of A, depending on c.

- For which values of c does A have real eigenvalues?

(b) - For one particular value of c, convince me that A is similar to both the matrix

$$B = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix},$$

and to the matrix

$$C = \begin{bmatrix} 2 & 2 \\ 0 & -2 \end{bmatrix}.$$

- Don't forget to say which value c this happens for.

(c) For one particular value of c, convince me that A cannot be diagonalized. It is not similar to a diagonal matrix Λ, when c has that value.

- Which value c?

- Why not?

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3 (37 pts.)

(a) Suppose A is an $n \times n$ symmetric matrix with eigenvalues $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$.

- What is the largest number real number c that can be subtracted from the diagonal entries of A, so that $A - cI$ is positive semidefinite?

- Why?

(b) Suppose B is a matrix with independent columns.

- What is the nullspace $N(B)$?
- Show that $A = B^T B$ is positive definite. Start by saying what that means about $x^T A x$.

(c) This matrix A has rank $r = 1$:

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}.$$

- Find its largest singular value σ from $A^T A$.
- From its column space and row space, respectively, find unit vectors u and v so that

$$Av = \sigma u, \quad \text{and} \quad A = u\sigma v^T.$$

- From the nullspaces of A and A^T put numbers into the full SVD (Singular Value Decomposition) of A:

$$A = \begin{bmatrix} | & | \\ u & \cdots \\ | & | \end{bmatrix} \begin{bmatrix} \sigma & 0 \\ & \\ 0 & \cdots \end{bmatrix} \begin{bmatrix} | & | \\ v & \cdots \\ | & | \end{bmatrix}^T.$$

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