

Your PRINTED Name is: _____

Please circle your section:

R01	T	10	36-144	Qiang Guang
R02	T	10	35-310	Adrian Vladu
R03	T	11	36-144	Qiang Guang
R04	T	11	4-149	Goncalo Tabuada
R05	T	11	E17-136	Oren Mangoubi
R06	T	12	36-144	Benjamin Iriarte Giraldo
R07	T	12	4-149	Goncalo Tabuada
R08	T	12	36-112	Adrian Vladu
R09	T	1	36-144	Jui-En (Ryan) Chang
R10	T	1	36-153	Benjamin Iriarte Giraldo
R11	T	1	36-155	Tanya Khovanova
R12	T	2	36-144	Jui-En (Ryan) Chang
R13	T	2	36-155	Tanya Khovanova
R14	T	3	36-144	Xuwen Zhu
ESG	T	3		Gabrielle Stoy

Grading 1:

2:

3:

4:

1. (24 points total)

(a) (6 points) What matrix P projects every vector in \mathbf{R}^3 onto the line that passes through origin and $a = (3, 4, 5)$?

(b) (6 points) What is the nullspace of that matrix P ?

(c) (6 points) What is the row space of P^2 ?

(d) (6 points) What is the determinant of P ?

2. (25 points total)

(a) (11 points) Suppose \hat{x} is the best least squares solution to $Ax = b$ and \hat{y} is the best least squares solution to $Ay = c$.

Does this tell you the best least squares solution \hat{z} to $Az = b + c$? If so, what is the best \hat{z} and *why*?

(b) (7 points) If Q is an m by n matrix with orthonormal columns, find the best least squares solution \hat{x} to $Qx = b$.

(c) (7 points) If $A = QR$, where R is square invertible and Q is the same as in (b), find the least squares solution to $Ax = b$.

3. (25 points total)

(a) (17 points) Find the determinant of this matrix A (with an unknown x in 4 entries).

$$A = \begin{bmatrix} x & 1 & 0 & 0 \\ 2 & x & 2 & 0 \\ 0 & 3 & x & 3 \\ 0 & 0 & 4 & x \end{bmatrix} \quad B = \begin{bmatrix} x & 1 & 0 & 1 \\ 2 & x & 2 & 0 \\ 0 & 3 & x & 3 \\ 0 & 0 & 4 & x \end{bmatrix}$$

You could use the big formula or the cofactor formula or possibly the pivot formula.

(b) (5 points) Find the determinant for matrix B which has an additional 1 in the corner. What new contribution to the determinant does this 1 make?

(c) (3 points) If M is any 3 by 3 matrix, let $f(x) = \det(xM)$. Find the derivative of f at $x = 1$.

4. (26 points total)

(a) (6 points) Find the projection p of the vector b onto the column space of A .

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 2 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

(b) (7 points) Use Gram-Schmidt to find an orthogonal basis q_1, q_2 for the column space of A .

(c) (6 points) Find the projection p of the same vector b onto the column space of the new matrix Q with columns q_1 and q_2 .

(d) (7 points) True or False: The best least squares solution \hat{x} to $Ax = b$ is the same as the best least squares solution \hat{y} to $Qy = b$. Explain why.

Scrap Paper