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1 (36 pts.) Suppose Q is a 4 by 3 matrix with orthonormal columns q_1, q_2, q_3 .

- (a) Starting from a vector v (not in the column space of Q), give a formula for the fourth orthonormal vector q_4 that is produced by Gram-Schmidt from q_1, q_2, q_3, v .
- (b) Describe the nullspace of Q (the same 4 by 3 matrix) and the nullspace of Q^T . (You can answer even if you didn't find the particular formula for q_4 in part a.) Describe also the nullspaces of $Q^T Q$ and $Q Q^T$.
- (c) Suppose $b = q_1 + 2q_2 + 3q_3 + 4q_4$. Find the least squares solution \bar{x} to $Qx = b$. What is the projection p of this b onto the column space of Q ?

- 2 (24 pts.)**
- (a) Fitting the best (least squares) straight line through the points $(t, b) = (2, 3), (3, 5),$ and $(4, K)$ is the same as solving what system of equations $Ax = b$ by least squares? Is there any value of K for which this system $Ax = b$ has an exact solution?
- (b) For general A and B , under what condition does the equation $Ax = b$ have $\bar{x} = 0$ as its least squares solution? In the example of part (a), prove that there is or there isn't a value of K so that $\bar{x} = 0$ is the least squares solution.

3 (40 pts.) (a) Suppose A is a 4 by 4 matrix. If you add 1 to the entry a_{14} in the northeast corner, how much will the determinant change? 

(b) Explain why the determinant of every projection matrix is either 0 or 1.

(c) Find the determinant of the “circulant matrix”

$$A = \begin{bmatrix} 0 & b & 0 & a \\ a & 0 & b & 0 \\ 0 & a & 0 & b \\ b & 0 & a & 0 \end{bmatrix}.$$