Some basic concepts underlying computer architecture

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Some basic concepts underlying computer architecture

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Decimal and binary number systems

- The binary system is used in computers because 1 and 0 represent the two voltage levels of on and off.
  - Whereas in base 10 there are 10 distinct symbols, 0, 1, 2, ... , 9, in base 2 there are only two, 0 and 1, with which to generate numbers.
  - Base 10 contains digits 0 through 9; binary contains digits 0 and 1 only.
    - These two binary digits, 0 and 1, are commonly referred to as *bits.*

Converting from decimal to binary

- One method of converting from decimal to binary is to divide the decimal number by 2 repeatedly, keeping track of the remainders.
- This process continues until the quotient becomes zero.
- The remainders are then written in reverse order to obtain the binary number.

Convert 25_{10} to binary.

Solution:

	Quotient	Remainder	
25/2 =	12	1	LSB (least significant bit)
12/2 =	6	0	
6/2 =	3	0	
3/2 =	1	1	
1/2 =	0	1	MSB (most significant bit)

Therefore, $25_{10} = 11001_2$.

Converting from binary to decimal

- To convert from binary to decimal, it is important to understand the concept of weight associated with each digit position.

$110101_2 =$

					Decimal	***Binary***
1×2^0	=	1×1	=	1	1
0×2^1	=	0×2	=	0	00
1×2^2	=	1×4	=	4	100
0×2^3	=	0×8	=	0	0000
1×2^4	=	1×16	=	16	10000
1×2^5	=	1×32	=	<u>32</u>	<u>100000</u>
				53	110101

Convert 11001_2 to decimal.

Solution:

Weight:	16	8	4	2	1
Digits:	1	1	0	0	1
Sum:	16 +	8 +	0 +	0 +	$1 = 25_{10}$

Hexadecimal system

- Base 16, or the *hexadecimal* system as it is called in computer literature, is used as a convenient representation of binary numbers.
- The binary system has 2 digits, 0 and 1.
- The base 10 system has 10 digits, 0 through 9.
- The hexadecimal (base 16) system has 16 digits.
  - In base 16, the first 10 digits, 0 to 9, are the same as in decimal, and for the remaining six digits, the letters A, B, C, D, E, and F are used.

Hexadecimal system

Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Converting between binary and hex

- To represent a binary number as its equivalent hexadecimal number, start from the right and group 4 bits at a time, replacing each 4-bit binary number with its hex equivalent.

Represent binary 100111110101 in hex.

Solution:

First the number is grouped into sets of 4 bits: 1001 1111 0101.
Then each group of 4 bits is replaced with its hex equivalent:

1001	1111	0101
9	F	5

Therefore, 100111110101_2 = 9F5 hexadecimal.

Convert hex 29B to binary.

Solution:

		2	9	B
29B	=	0010	1001	1011

Dropping the leading zeros gives 1010011011.

Converting from decimal to hex

- Converting from decimal to hex could be approached in two ways:
  - Convert to binary first and then convert to hex.
  - Convert directly from decimal to hex by repeated division, keeping track of the remainders.

(a) Convert 45_{10} to hex.

<u>32</u>	<u>16</u>	<u>8</u>	<u>4</u>	<u>2</u>	<u>1</u>	First, convert to binary.
1	0	1	1	0	1	$32 + 8 + 4 + 1 = 45$

$45_{10} = 0010\ 1101_2 = 2D$ hex

Converting from hex to decimal

- Conversion from hex to decimal can also be approached in two ways:
  - Convert from hex to binary and then to decimal. Example 0-7 demonstrates this method of converting from hex to decimal.
  - Convert directly from hex to decimal by summing the weight of all digits

Converting from hex to decimal

Convert the following hexadecimal numbers to decimal.

(a) $6B2_{16} = 0110\ 1011\ 0010_2$

1024	512	256	128	64	32	16	8	4	2	1
1	1	0	1	0	1	1	0	0	1	0

$1024 + 512 + 128 + 32 + 16 + 2 = 1714_{10}$

(b) $9F2D_{16} = 1001\ 1111\ 0010\ 1101_2$

32768	16384	8192	4096	2048	1024	512	256	128	64	32	16	8	4	2	1
1	0	0	1	1	1	1	1	0	0	1	0	1	1	0	1

$32768 + 4096 + 2048 + 1024 + 512 + 256 + 32 + 8 + 4 + 1 = 40{,}749_{10}$

Addition of binary and hex numbers

Add the following binary numbers. Check against their decimal equivalents.

Solution:

	Binary	*Decimal*
	1101	13
+	1001	9
	10110	22

Perform hex addition: 23D9 + 94BE.

Solution:

	23D9	LSD:	9 + 14 = 23	23 – 16 = 7 with a carry
+	94BE		1 + 13 + 11 = 25	25 – 16 = 9 with a carry
	B897		1 + 3 + 4 = 8	
		MSD:	2 + 9 = B	

2's complement

- To get the 2 's complement of a binary number, invert all the bits and then add 1 to the result.
- Inverting the bits is simply a matter of changing all 0s to 1s and 1s to 0s. This is called the *1 s complement.*

Subtraction of hex numbers

Perform hex subtraction: 59F – 2B8.

Solution:

	59F	LSD:	8 from 15 = 7
–	2B8		11 from 25 (9 + 16) = 14 (E)
	2E7		2 from 4 (5 – 1) = 2

ASCII code

- Because all information in the computer must be represented by 0s and 1s, binary patterns must be assigned to letters and other characters.
- In the 1960s a standard representation called *ASCII* (American Standard Code for Information Interchange) was established.
- The ASCII code assigns binary patterns for numbers 0 to 9, all the letters of the English alphabet, both uppercase (capital) and lower-case, and many control codes and punctuation marks.
- The great advantage of this system is that it is used by most computers, so that information can be shared among computers.
- The ASCII system uses a total of 7 bits to represent each code.