Boolean Algebra

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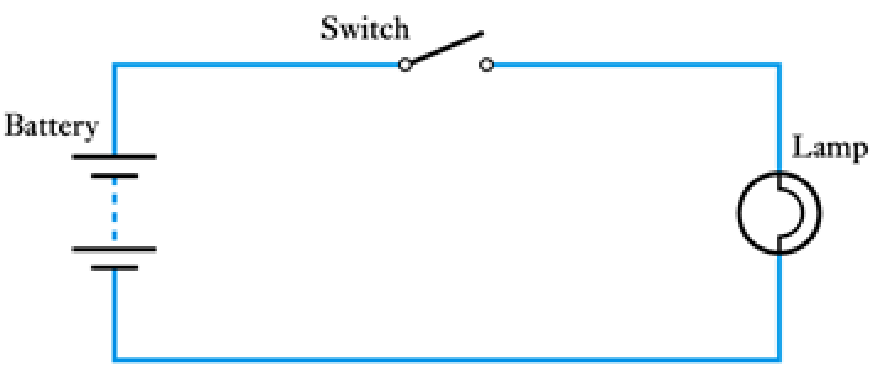
Introduction

- Digital systems are concerned with digital signals.
- Here we will concentrate on **binary signals** since these are the most common form of digital signals.

Binary Quantities and Variables

- A **binary quantity** is one that can take only 2 states.



A simple binary arrangement

S	L
OPEN	OFF
CLOSED	ON

S	L
0	0
1	1

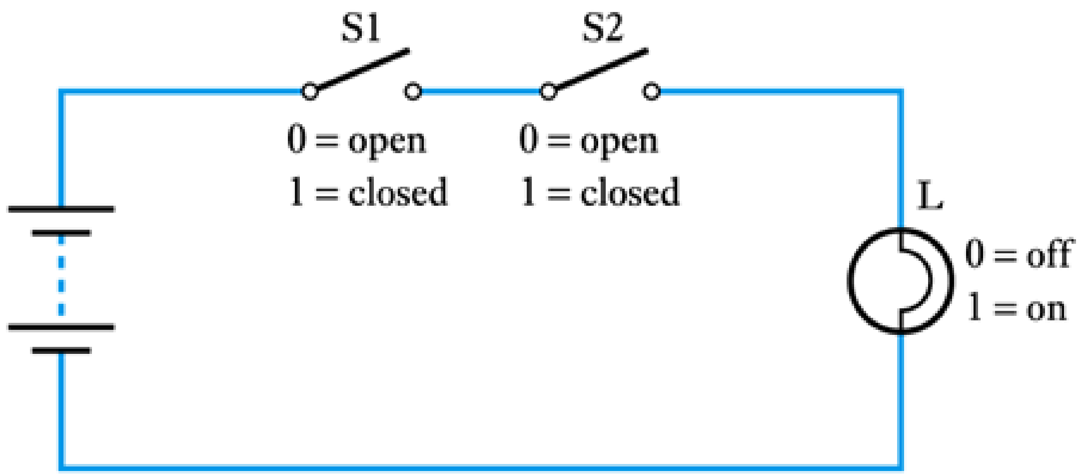
A truth table

Logic Gates

- The building blocks used to create digital circuits are **logic gates.**
- There are three elementary logic gates and a range of other simple gates.
  - AND
  - OR
  - NOT
- Each gate has its own **logic symbol** which allows complex functions to be represented by a logic diagram.
- The function of each gate can be represented by a **truth table** or using **Boolean notation.**

The AND gate

- A binary arrangement with two switches in series



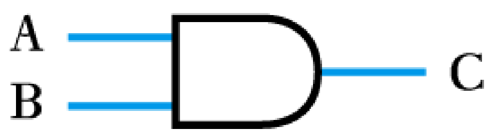
(a) Circuit

S1	S2	L
0	0	0
0	1	0
1	0	0
1	1	1

(b) Truth table

L = $S1$ AND $S2$

The AND gate



(a) Circuit symbol

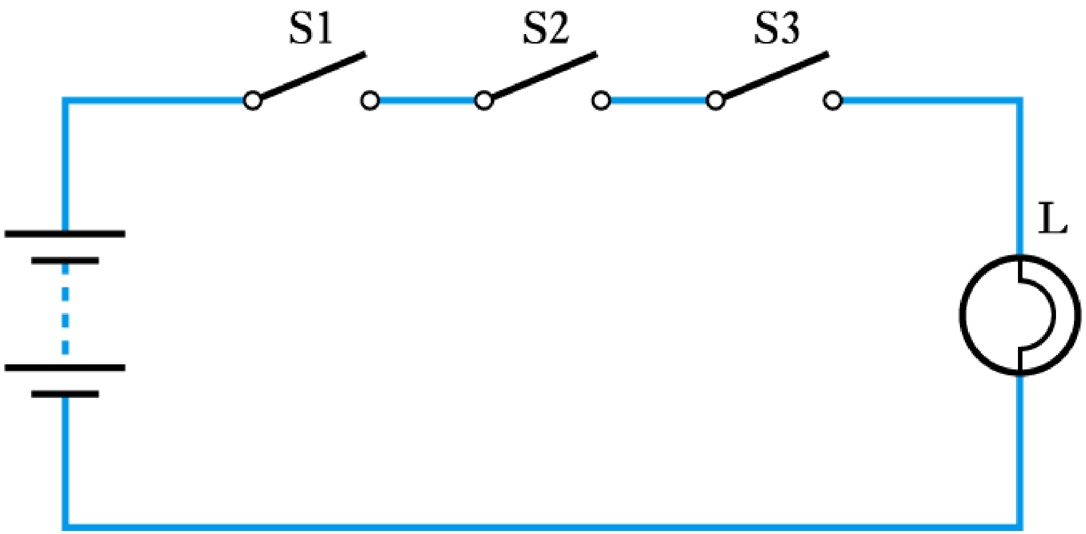
A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

(b) Truth table

$$C = A \cdot B$$

(c) Boolean expression

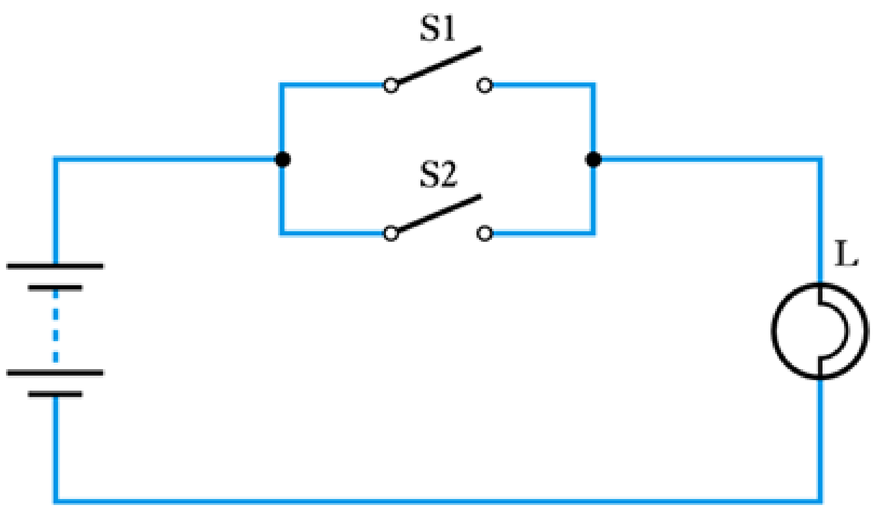
The AND gate



S1	S2	S3	L
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

The OR gate

- A binary arrangement with two switches in parallel



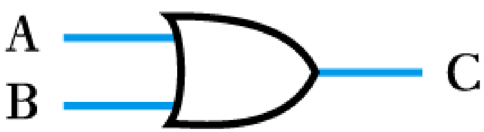
(a) Circuit

S1	S2	L
0	0	0
0	1	1
1	0	1
1	1	1

(b) Truth table

$L = S1 \text{ OR } S2$

The OR gate



(a) Circuit symbol

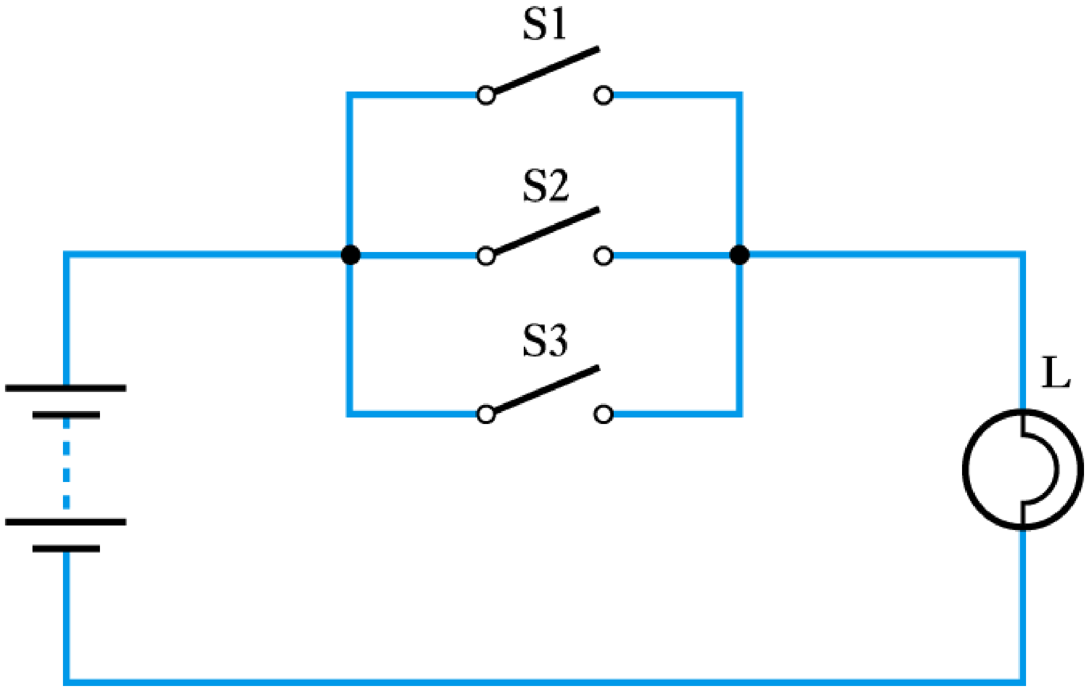
A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

(b) Truth table

$$C = A + B$$

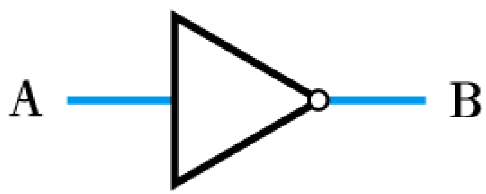
(c) Boolean expression

The OR gate



S1	S2	S3	L
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

The NOT gate (or inverter)



(a) Circuit symbol

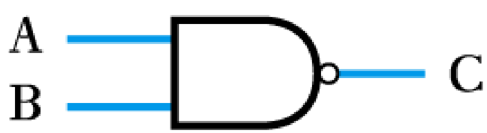
A	B
0	1
1	0

(b) Truth table

$$B = \overline{A}$$

(c) Boolean expression

The NAND gate



(a) Circuit symbol

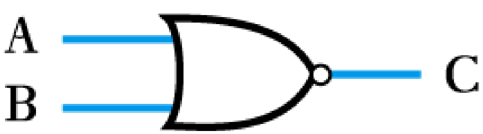
A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

(b) Truth table

$$C = \overline{A \cdot B}$$

(c) Boolean expression

The NOR gate



(a) Circuit symbol

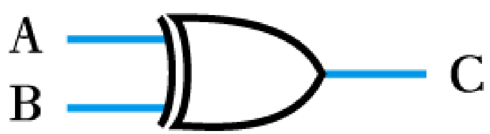
A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

(b) Truth table

$$C = \overline{A + B}$$

(c) Boolean expression

The Exclusive OR gate



(a) Circuit symbol

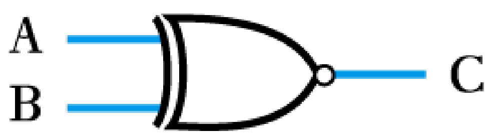
A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

(b) Truth table

$$C = A \oplus B$$

(c) Boolean expression

The Exclusive NOR gate



(a) Circuit symbol

A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

(b) Truth table

$$C = \overline{A \oplus B}$$

(c) Boolean expression

Boolean Algebra

- **Boolean Constants**
  - these are ‘0’ (false) and ‘1’ (true)
- **Boolean Variables**
  - variables that can only take the vales ‘0’ or ‘1’
- **Boolean Functions**
  - each of the logic functions (such as AND, OR and NOT) are represented by symbols as described above
- **Boolean Theorems**
  - a set of **identities** and **laws**

Boolean Algebra

- **Boolean identities (Basic Operation)**

AND Function	**OR Function**	**NOT function**
$0 \bullet 0=0$	$0+0=0$	$\overline{0}=1$
$0 \bullet 1=0$	$0+1=1$	$\overline{1}=0$
$1 \bullet 0=0$	$1+0=1$	$\overline{\overline{A}}=A$
$1 \bullet 1=1$	$1+1=1$	
$A \bullet 0=0$	$A+0=A$	
$0 \bullet A=0$	$0+A=A$	
$A \bullet 1=A$	$A+1=1$	
$1 \bullet A=A$	$1+A=1$	
$A \bullet A=A$	$A+A=A$	

Boolean Algebra laws

Commutative law $AB = BA$ $A + B = B + A$	**Absorption law** $A + AB = A$ $A(A + B) = A$
Distributive law $A(B + C) = AB + AC$ $A + BC = (A + B)(A + C)$	**De Morgan's law** $\overline{A + B} = \overline{A} \bullet \overline{B}$ $\overline{A \bullet B} = \overline{A} + \overline{B}$
Associative law $A(BC) = (AB)C$ $A + (B + C) = (A + B) + C$	$A + \overline{A}B = A + B$ $A(\overline{A} + B) = AB$

Boolean Algebra laws proof

(Distributive law)

A + BC = A.1 + BC [Since, A.1 = A]

= A(1 + B) + BC [Since, B+1 = 1]

= A.1 + AB + BC

= A.(1 + C) + AB + BC

= A (A + C) + B (A+C) [Since, A.A = A.1 = A]

A + BC = (A+B)(A+C)

Boolean Algebra laws proof

(Absorption Law):

$$A \,.\, (A+B) = A$$

Proof.

$A.(A+B) = A.A + A.B$

$= A+AB$ [Since, A.A = A]

$= A(1+B)$

$= A.1$

$$A + (A \cdot B) = A$$

Proof.

$A+(A.B) = A.1 + AB$ [Since A.1 = A]

$= A(1+B)$ [Since, 1 + B = 1]

$= A.1 = A$

Boolean Algebra laws proof

(De Morgan's Law)

For every pair a, b in set B:

$(A+B)' = A'B'$, and $(AB)' = A'+B'$.

Proof: We should show that a+b and a'b' are complementary.

In other words, we show that both of the following are true

$$(A+B) + (A'B') = 1$$

$$(A+B)(A'B') = 0$$

Identity element

- An **identity element** (or **neutral element**) is a special type of element of a set with respect to a binary operation on that set.
  - It leaves other elements unchanged when combined with them.
  - An identity element with respect to + and . is 0 and 1, respectively.

Complement element

- b is a complement of a if $a+b=1$, $a.b=0$.
  - So a unique complement must be a unique solution to *both* equations (involving *both* operations), not just a single operation.