Multi-Modality Image Reconstruction with Parallel Level Sets

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Introduction: Recent advantages in technology have enabled us to combine imaging systems. Among others positron emission tomography (PET) and magnetic resonance imaging (MRI) scanners can be combined to simultaneously image function and anatomy of the human body [1]. As function follows anatomy the images to be reconstructed are expected to share a lot of information. By coupling these two inverse problems we aim to exploit the shared information in the data [2]. Similarly shared structures have been exploited in a joint framework in colour image processing [3,4] and geophysics [5].

Reconstruction with Parallel Level Sets: The joint reconstruction of two imaging modalities can be posed as a joint minimization problem

$$(u^\sharp, v^\sharp) \in \operatorname{argmin}_{(u,v)} \Phi(u,v)$$

with the objective function having the form

$$\Phi(u,v) = d_{1,f}(\mathcal{P}u) + d_{2,g}(\mathcal{B}v) + \alpha\Psi(u,v)$$

where the data fidelities depend on the noise models considered and $\Psi(u,v)$ expresses our prior knowledge about the state of u and v [2]. One way of modelling the shared structure in the two modalities is to say that the spatial gradients in both images align or equivalently the images having parallel level sets [4] which can be achieved by setting the prior to be

$$\Psi(u,v) = \int \|\nabla u\|\|\nabla v\| - |\langle \nabla u, \nabla v\rangle|.$$

As Ψ vanishes when one of the images is flat this prior is not guaranteed to have a regularizing effect. This can be introduced for instance by combining joint structural information and local smoothness as

$$\Psi_\beta(u,v) = \frac{1}{\beta}\int \|\nabla u\|_\beta \|\nabla v\|_\beta - \|\langle \nabla u, \nabla v\rangle\|_{\beta^2}$$

where $\|x\|_\beta = \sqrt{\|x\|^2 + \beta^2}$ [4].

Results: Here we restrict ourselves to estimate only one image given a second. We will apply this prior to reconstruct from highly undersampled MRI data [6] assuming our image is similar to another image either obtained from PET or from another MRI contrast [7]. It can be shown numerically that by exploiting this similarity far less data is needed. Even in the case of noise far less radial samplings are needed to reconstruct the Shepp Logan phantom than by exploiting sparsity in the gradients as in [8]. In addition the results indicate that the side information does not help only locally but globally to improve the image quality.

References

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