L1-penalized linear mixed-effects models for high dimensional data with application to BCI

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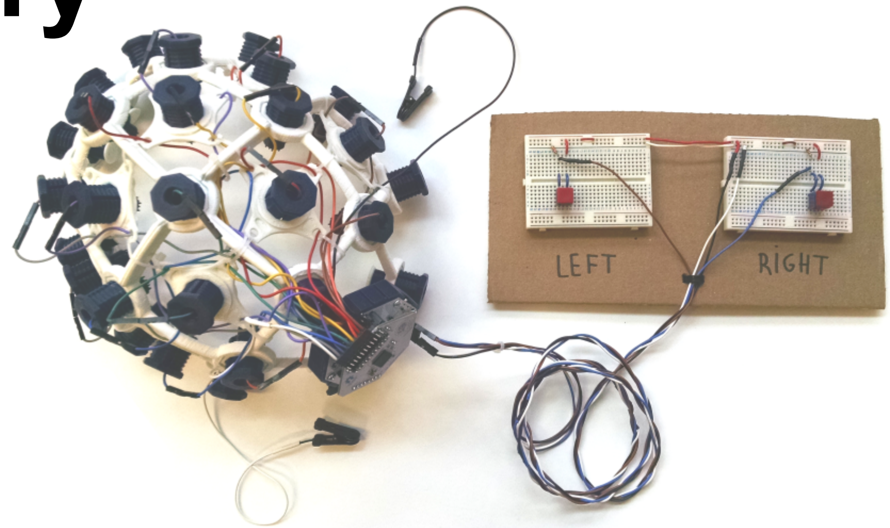
1. **Introduction**
2. **Statistical Model:** model setup + L1-penalized maximum likelihood estimator + prediction of the random-effects + model selection + computational implementation
3. **Available data and experiments:** BCI setup and data basis: Generation of the ensemble + Temporal filters + Spatial filters and classifiers + Validation
4. **Results:** experimental results: Subject-to-subject transfer + Session-to-session transfer + Relation of baseline misclassification to σ2 and τ2 + Effective spatial filters and distances thereof
5. **Discussion and conclusions**

Motor Imagery

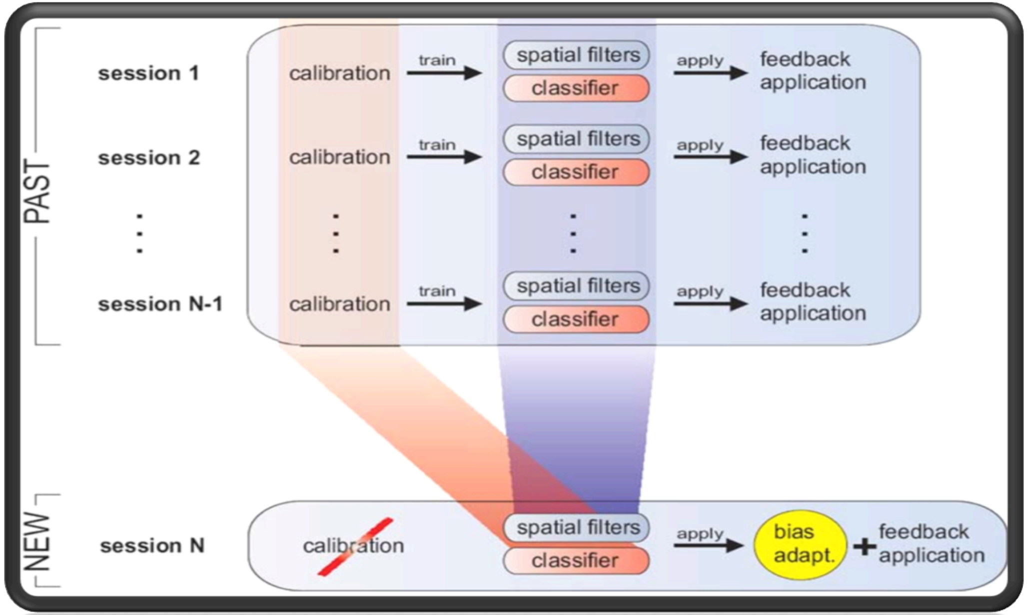
- Training Set



a mental process by which an individual rehearses or simulates a given action.

Zero Training!

- Depending on the number of past sessions, this procedure creates a larger set of filters.
- Once this set of historic filters is created, 6 so-called prototype filters are chosen



* Image from 'Towards Zero Training for Brain-Computer Interfacing'
Matthias Krauledat*, Michael Tangermann, Benjamin Blankertz, Klaus-Robert Mu¨ller

L1-penalized linear mixed-effects model

- Less calibration time (practically zero) (just bias)
- more accuracy
- within subject and across subject variation
- sparse
- better features

mixed effect: within subject and across subject variation
L1: Few data + high dimension (No need for calibration)

- mixed effect
- L1

Linear mixed-effects model

- inbuilt heterogeneity of data, distinct sources
  - Variance: Individual experiment and a fixed effect
- Grouping Structure
- Mixed Model: fixed and random effects

Problems:

- High Dimensional Data
- Computationally intractable for large covariates

Lasso method

- least absolute shrinkage and selection operator
- performs both variable selection and regularization in order to enhance the prediction accuracy and interpretability of the statistical model it produces.
- Min Σe^2 s.t. $\Sigma|\beta j| \leq s$
- regression with an l1-norm penalty
- convex problem

L1 penalized

- assuming that the number of potential fixed effects is large and that the underlying true fixed-effects vector is sparse.
- splits high **variance** of BCI
- quantify both sources of variance + improved ensemble model
- Practically zero training (no calibration)
- BCI community goals: **reduce setup cost, increase Information Transfer Rates (ITR)**

How to implement Statistical Model?

Mixed effect model setup

- extensions of linear regression models for grouped data
- with coefficients that can vary with respect to one or more grouping variables
- Fixed and random effects
- individual experiments

Mixed effect model setup

- $y_i = X_i\beta + Z_i b_i + \varepsilon_i \qquad i = 1, \ldots, N$
- $i)\ b_i \sim \mathcal{N}_q(0, \tau^2 I_q),\ ii)\ \varepsilon_i \sim \mathcal{N}_{n_i}(0, \sigma^2 I_{n_i})$
- $y_i \sim \mathcal{N}_{n_i}(X_i\beta, \Lambda_i(\sigma^2, \tau^2)) \quad \text{with} \quad \Lambda_i(\sigma^2, \tau^2) = \sigma^2 I_{n_i} + \tau^2 Z_i Z_i^{\top}.$

N subjects
ni observations per subject
NT = Σni : total number of observations
yi: ni dimensional observation of each subject

Xi: ni*p fixed-effects covariates
Zi: ni*q random-effects covariates
β: p-dimensional fixed-effects vector
**bi: (i=1:N) q-dimensional random-effects vectors.
Assume: errors εi are mutually independent of the random effects bi.

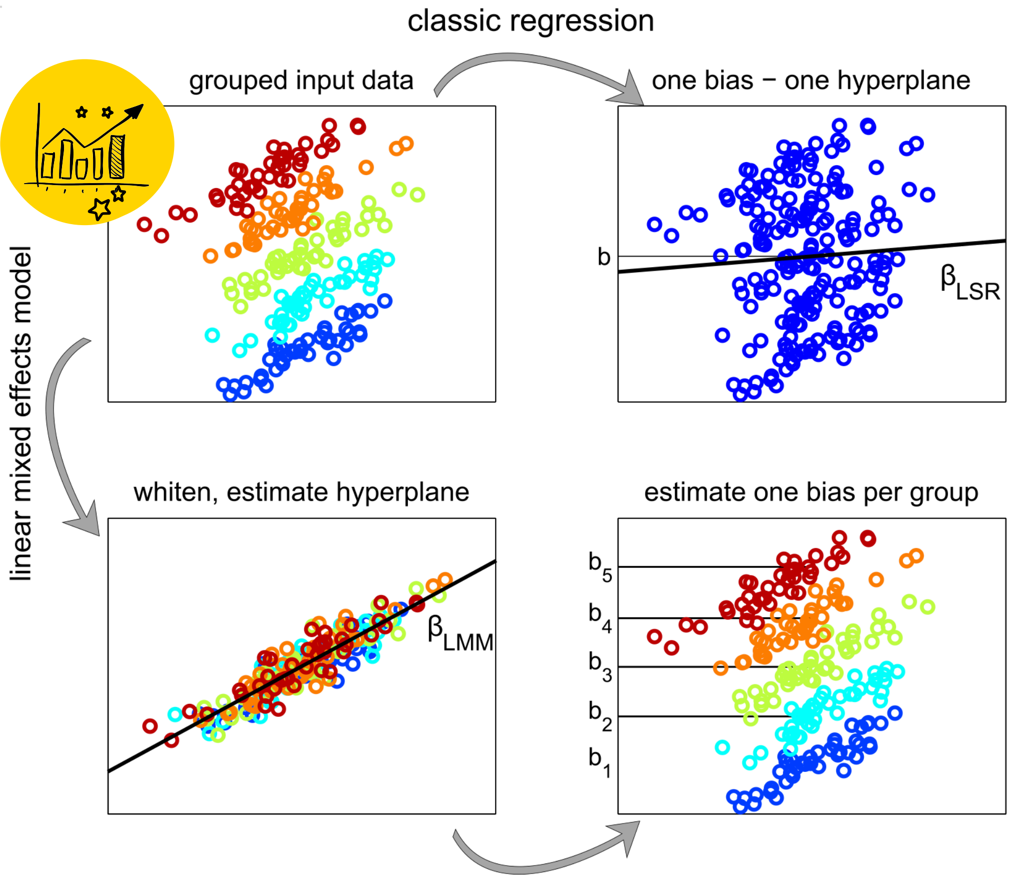


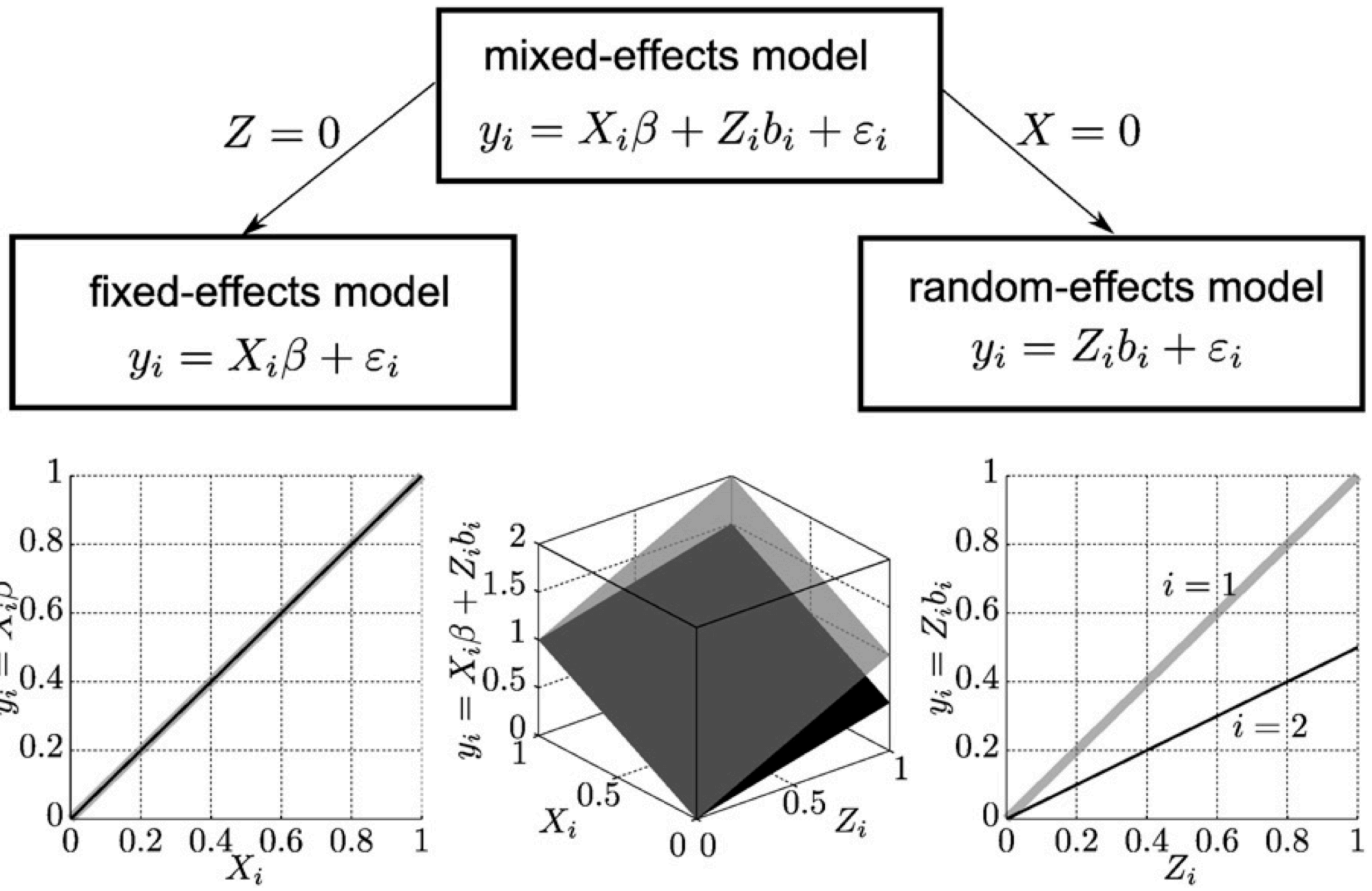
Figure 1

INTUITION

- $Z_i = I_{ni}$
  $\beta_{ORIG} = 0.5$
  $b_{ORIG} = [-2; -1; 0; 1; 2]$
  $\varepsilon_{ORIG} \sim N(0, 0.2)$

- $\beta_{LSR} = 0.048$
  $b_{LSR} = 0.075$

- $\beta_{LMM} = 0.504$
  $b_{LMM} = [-1.96; -1.015; -0.014; 0.973; 2.013]$



mixed-effects model
$y_i = X_i\beta + Z_i b_i + \varepsilon_i$
$Z = 0$
$X = 0$
fixed-effects model
$y_i = X_i\beta + \varepsilon_i$
random-effects model
$y_i = Z_i b_i + \varepsilon_i$
$y_i = X_i\beta$
X_i
$y_i = X_i\beta + Z_i b_i$
X_i
Z_i
$y_i = Z_i b_i$
Z_i
$i = 1$
$i = 2$

foreach $(\sigma^2, \tau^2, \lambda)$ **do**

  foreach i **do**

    (Whiten data and labels)

$$\Lambda_i = \sigma^2 I_{n_i} + \tau^2 Z_i Z_i^\top$$

$$\bar{X}_i = \Lambda_i^{-1/2} X_i, \quad \bar{y}_i = \Lambda_i^{-1/2} y_i$$

  end

  (Fit ℓ_1-penalized least-squares to concatenated data)

$$\hat{\beta} = \operatorname{argmin} \|\bar{X}\beta - \bar{y}\|_2^2 + 2\lambda \textstyle\sum_{k=2}^{p} |\beta_k|$$

  foreach i **do**

    (Find random effects)

$$\hat{b}_i = [Z_i^\top Z_i + \sigma^2/\tau^2 I_q]^{-1} Z_i^\top (y_i - X_i \hat{\beta})$$

  end

end

Algorithm 1. Algorithm for fitting the mixed effects model.

L1-penalized maximum likelihood estimator

- $S_\lambda(\beta;\sigma^2;\tau^2) := (-1/2) \sum \{\log|\Lambda_i| + (y_i - X_i\beta)^T \Lambda_i^{-1} (y_i - X_i\beta)\} - \lambda \sum |\beta_k|$

λ: nonnegative regularization parameter.

- $\hat{\beta};\hat{\sigma}^2;\hat{\tau}^2 = \arg\max S_\lambda(\beta;\sigma^2;\tau^2)$

- *Prediction of the random-effects (b_i)(using MAP β, σ^2, τ^2)*

$$b_i = [\, Z^T{}_i \, Z_i + \sigma^2 / \tau^2 I_q]^{-1} \; Z^T{}_i \, (y_i - X_i \beta)$$

- *Model selection*

$$-2L(\beta\hat{}\,;\, \sigma\hat{}\,^2 ; \tau\hat{}^2) + \log N_T \cdot d\hat{}f_\lambda$$

- *Computational implementation*

$$\beta\hat{} = \arg\min \sum ||\Lambda_i^{-12} X_i \beta - y_i||_2{}^2 + 2\lambda \sum |\beta_k|$$

Data & Experiment

Data & Experiment

83 BCI experiments (sessions) * 150 trials

83 individual subjects

90 sessions*60-600 trials

44 subjects.

Balanced Data Set

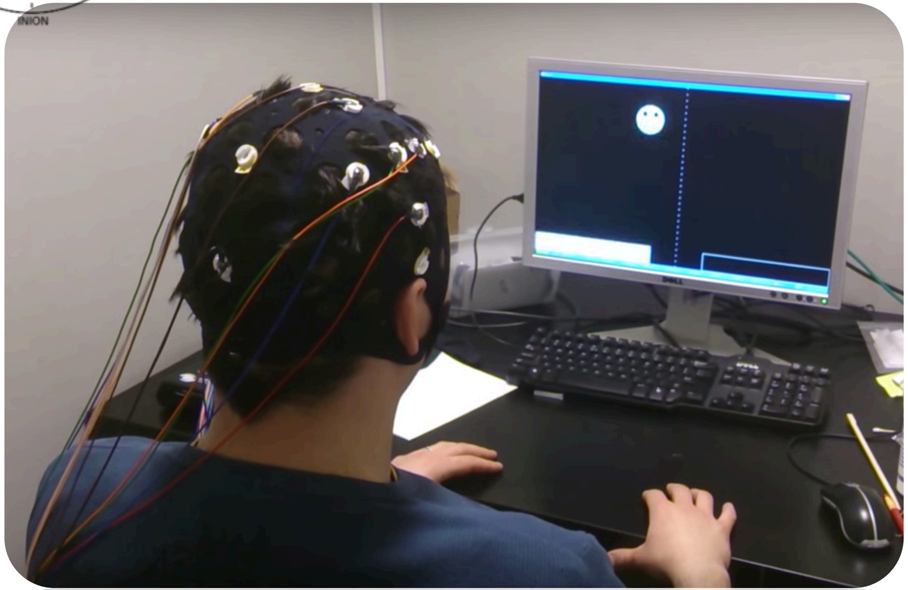
Unbalanced Data Set

'clean' model

individual sessions, underlying processes.

Experiment

I.



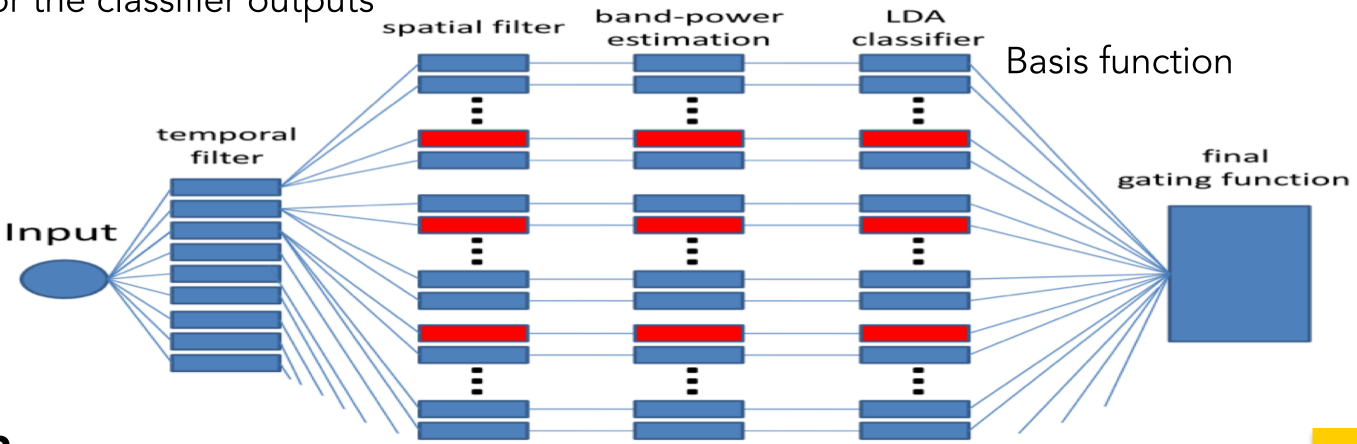
II.

III. L R

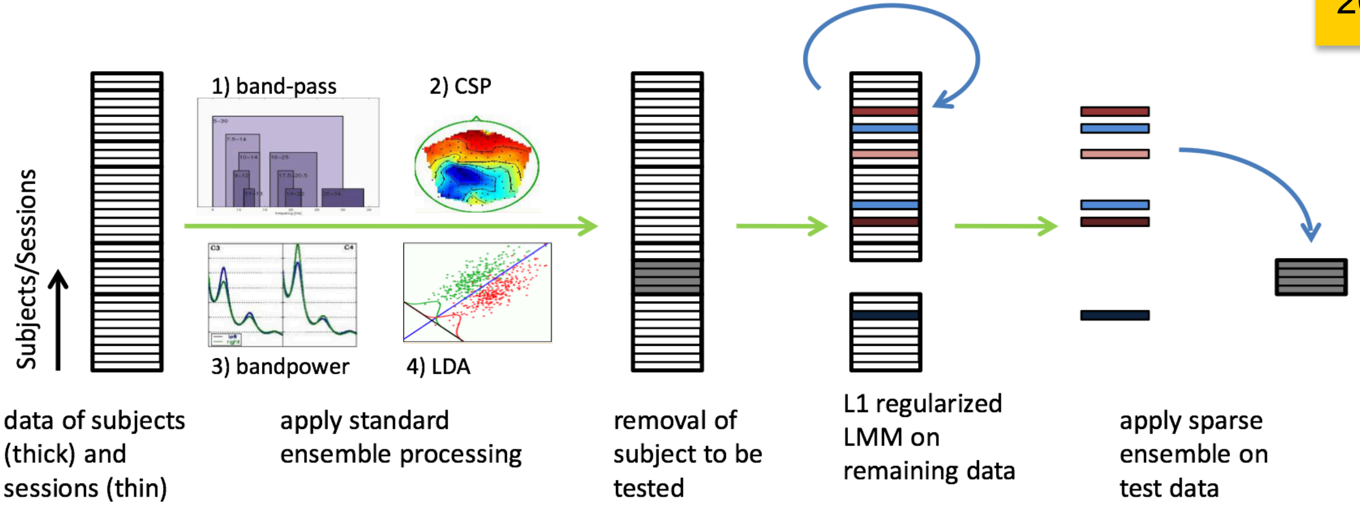
EMG, EOG

- *Generation of the ensemble*
- *Temporal filters*
- *Spatial filters and classifiers*
- *Validation*

appropriate weighting for the classifier outputs



- ***Generation of the ensemble***

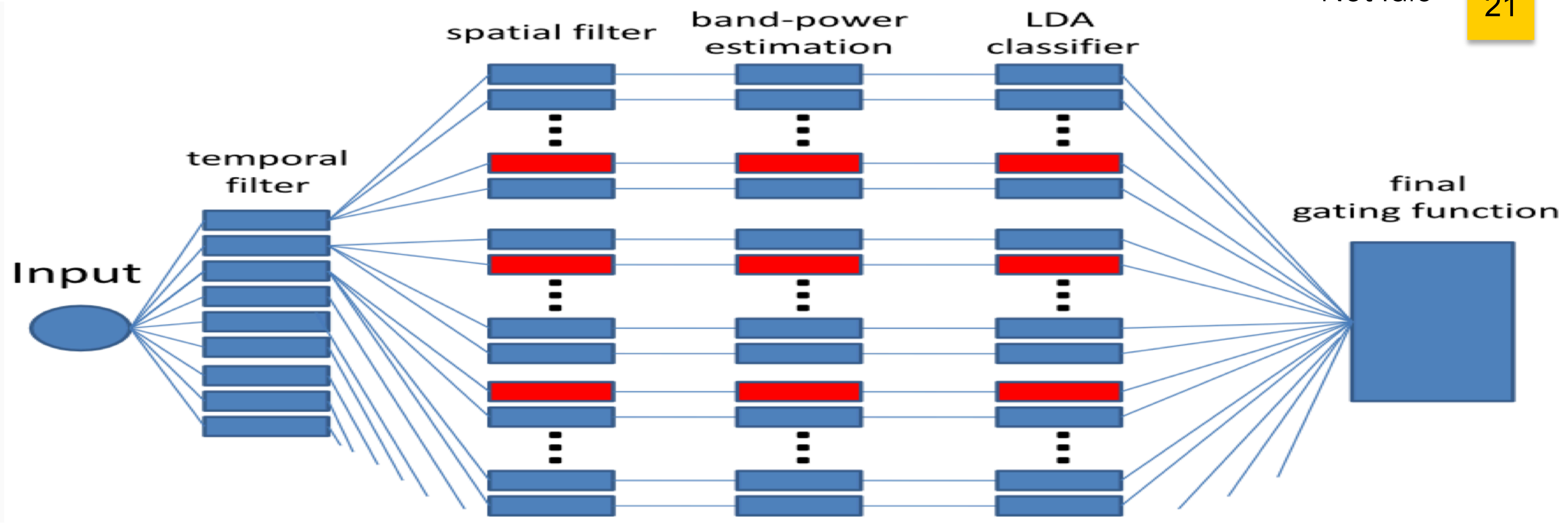


Remove frequencies within the raw signal, that are not of interest.

Temporal filters

Not Idle

21



Spatial filters and classifiers:

Σ_1 , Σ_2 → **CSP** → W , D → **LDA**

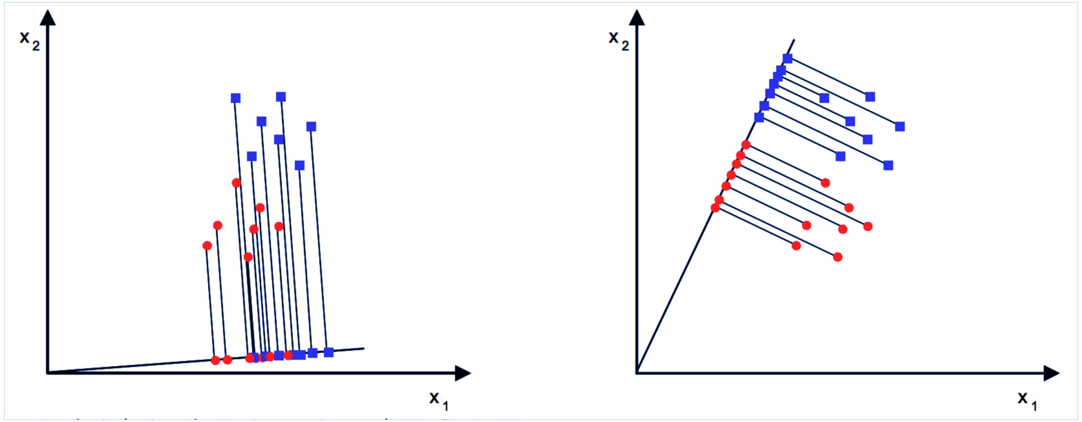
a matrix of projections
Diagonal

$W \Sigma_1 W^T = D$ and $W \Sigma_2 W^T = I - D$

separating a multivariate signal into additive subcomponents which have maximum differences in variance

Spatial filters and classifiers:

LDA



Linear Discriminate analysis

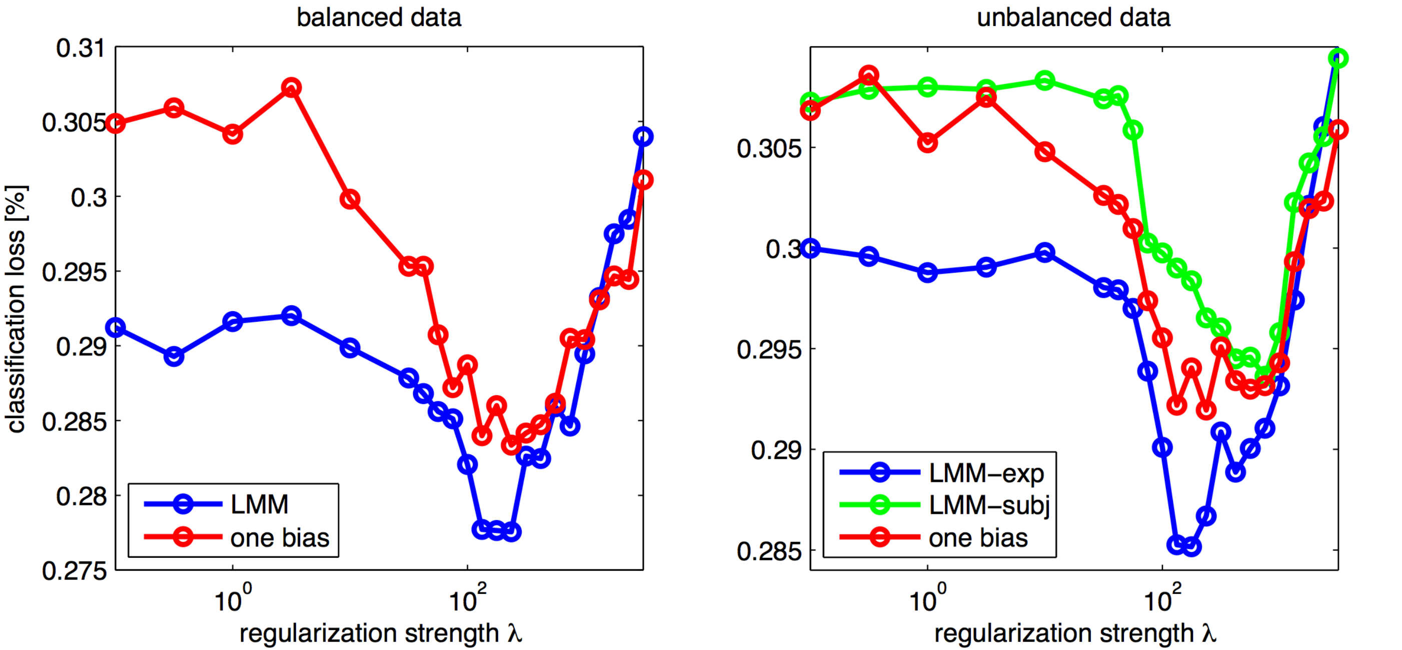
*Figure from Introduction to Common Spatial Pattern Filters for EEG Motor Imagery Classification
Tatsuya Yokota
Tokyo Institute of Technology
July 17, 2012

Validation

8-fold cross-validation,
splitting the data chronologically.

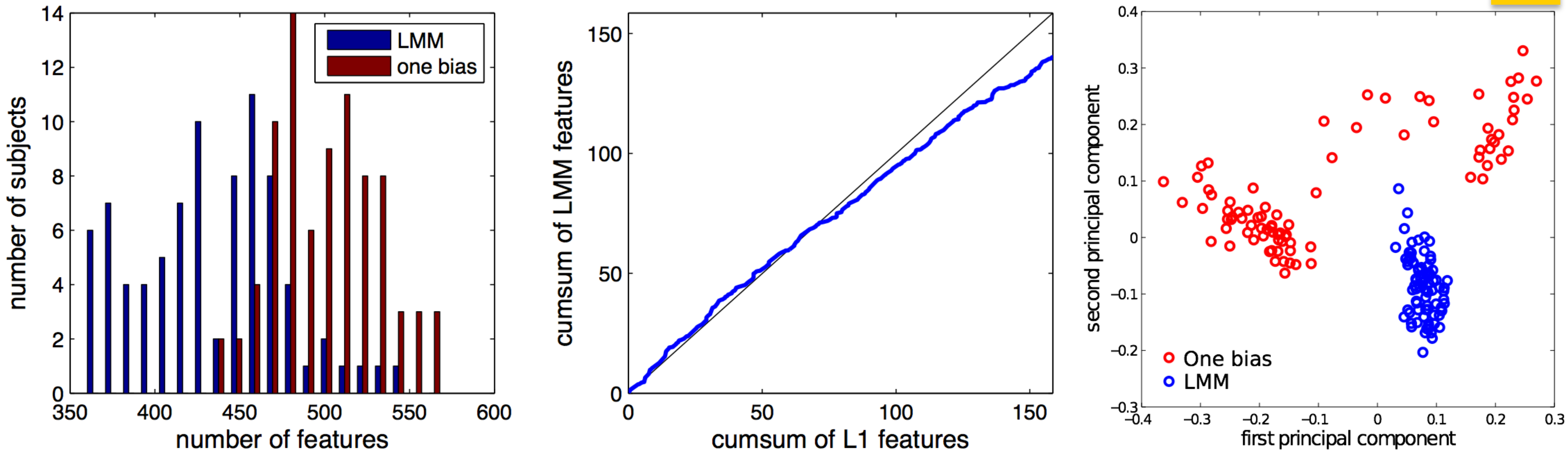
chronological | since the non- stationarity of the data is thus preserved

Results

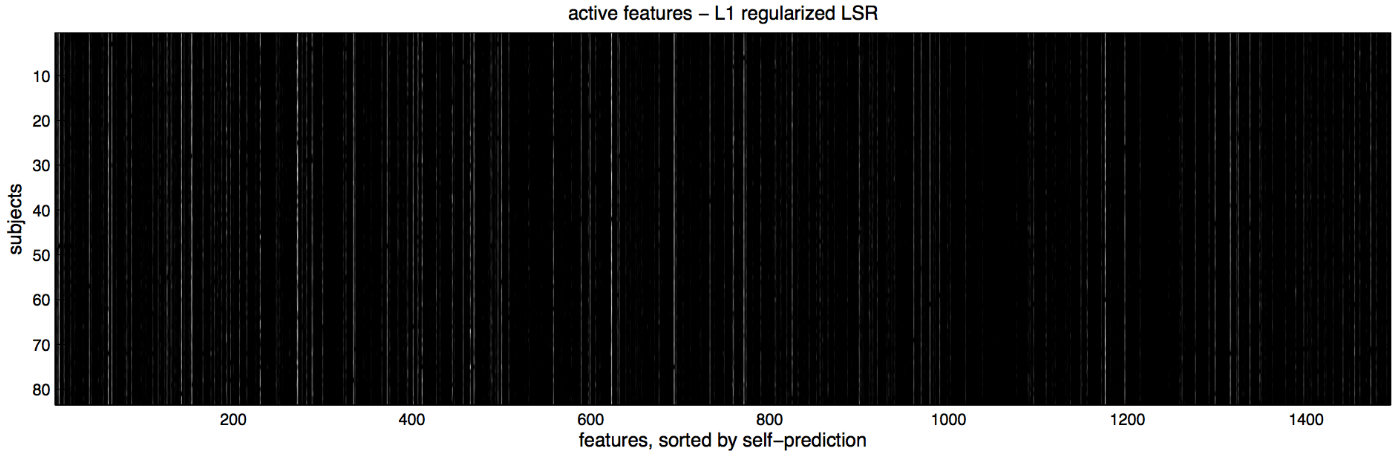


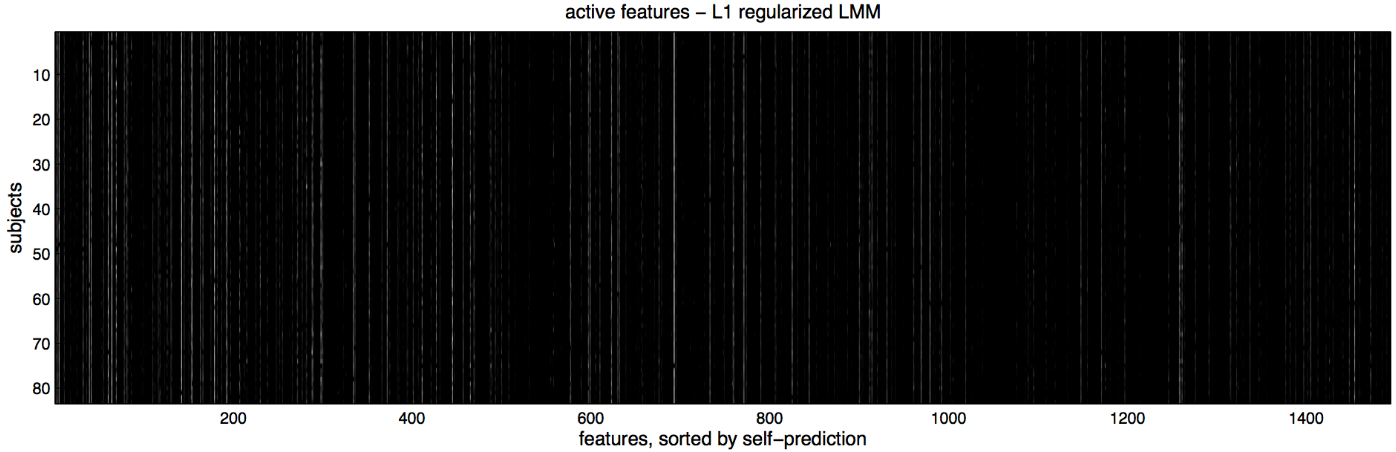
LMM-exp one bias per experiment

LMM-subj estimates one bias per subject



LMM: higher fraction of features with low self-prediction errors
sorted by increasing self-prediction accuracy.


active features – L1 regularized LSR
subjects
10
20
30
40
50
60
70
80
200
400
600
800
1000
1200
1400
features, sorted by self–prediction


active features – L1 regularized LMM
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10
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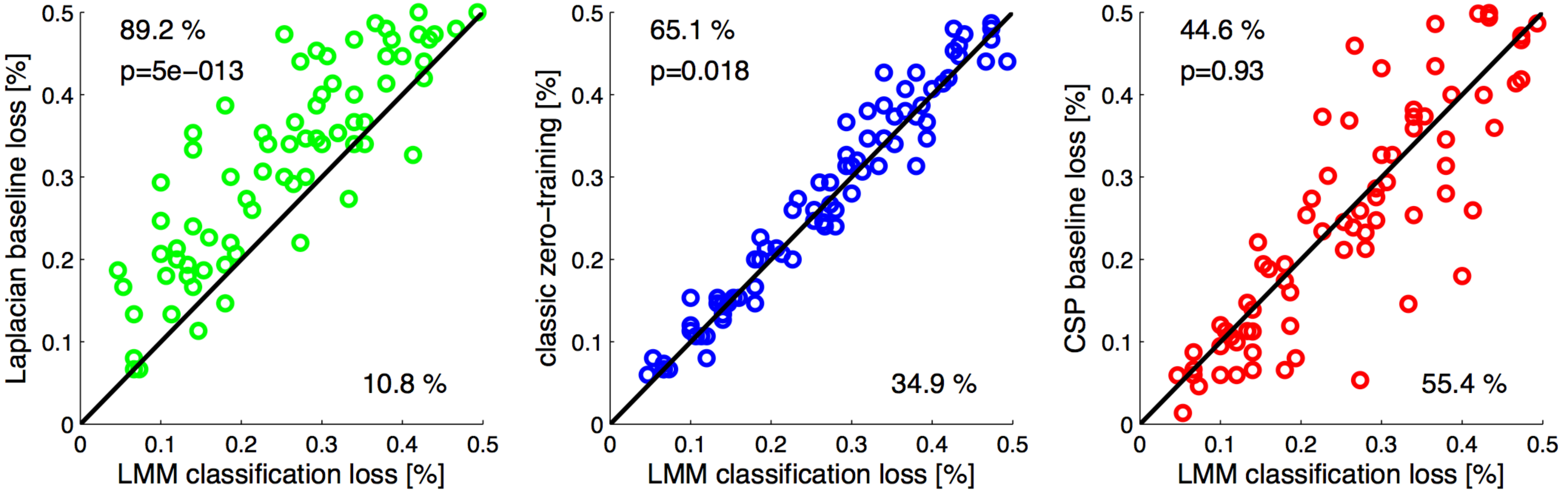
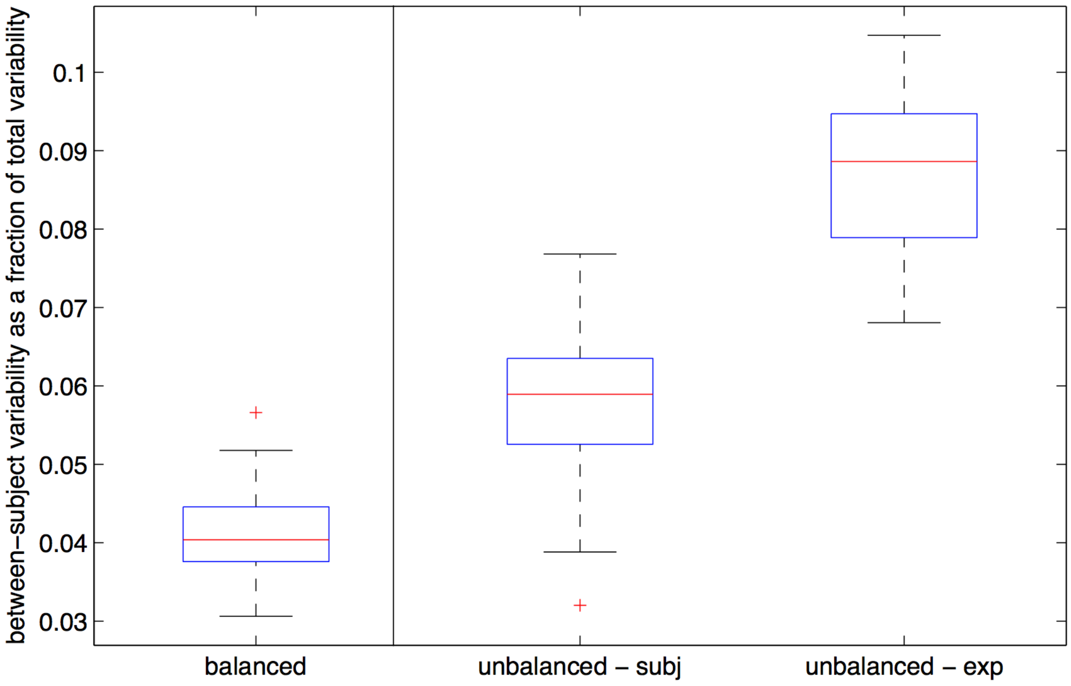

89.2 %
p=5e−013
10.8 %
Laplacian baseline loss [%]
LMM classification loss [%]
65.1 %
p=0.018
34.9 %
classic zero−training [%]
LMM classification loss [%]
44.6 %
p=0.93
55.4 %
CSP baseline loss [%]
LMM classification loss [%]

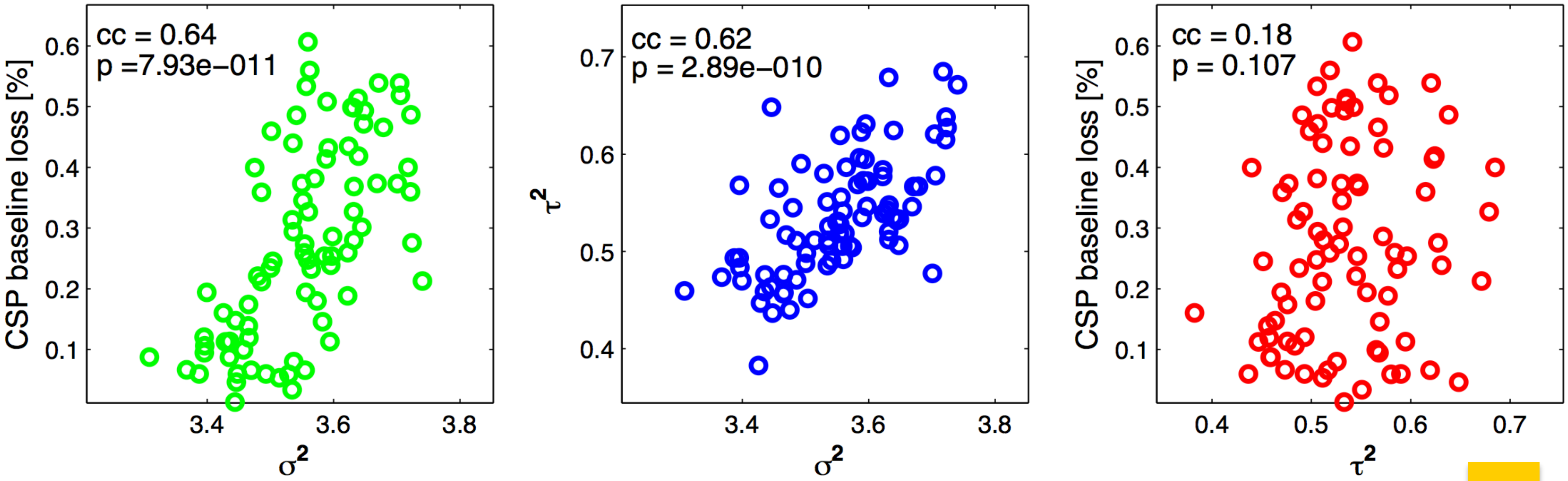
30

the **transfer of classifiers from sessions to sessions** required a **bias correction**

Valid: we are able to **capture a meaningful part of the variability** which would otherwise **be ignored as noise**.



It is not possible to draw conclusions about the quality of a subject's data by the variance of its assigned biases.



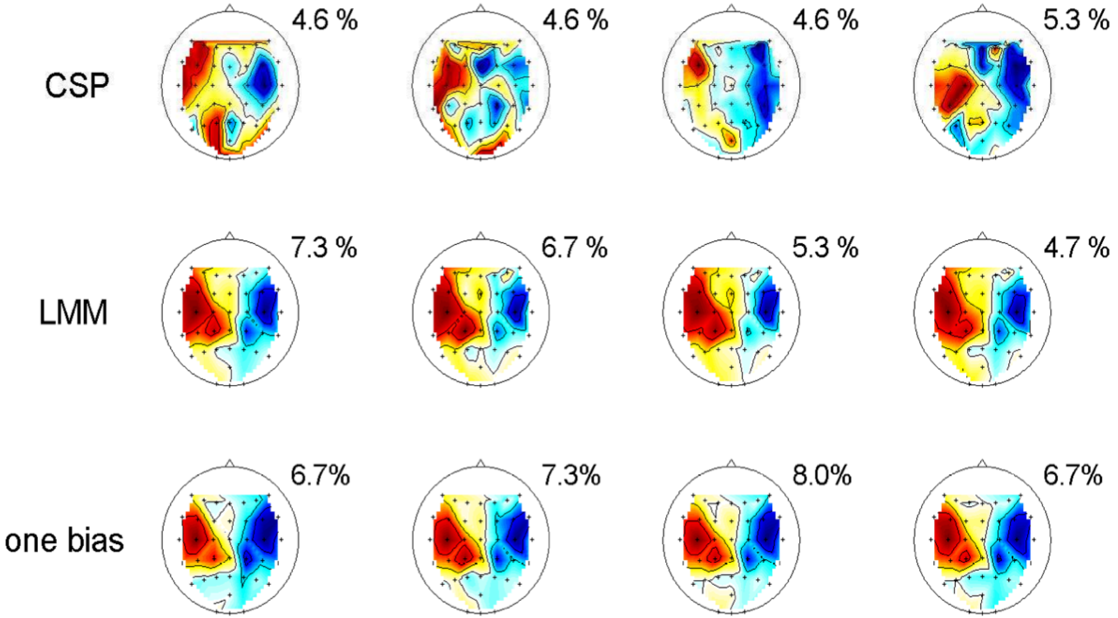
σ2 have **a strong positive correlation**

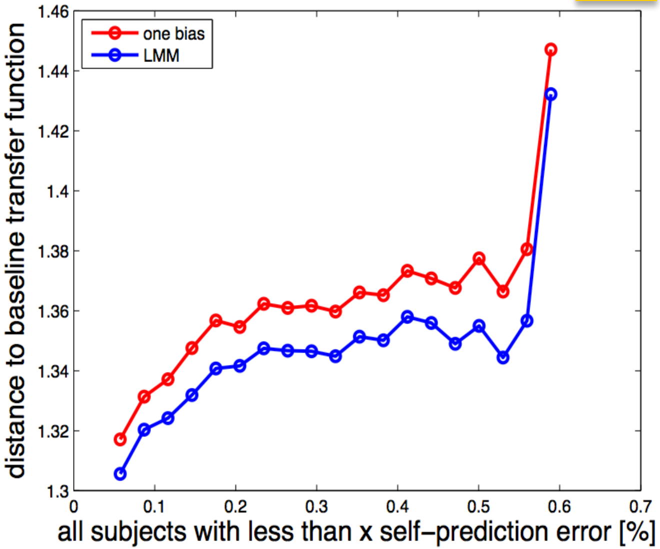
of **τ2** and **σ2** and find a **strong positive relation**.

within-subject variability σ^2, between-subject variability τ^2

sinusoid

LEFT: resulting response matrices





Discussion and conclusions

- Sparse modeling approach based on L1-penalized linear mixed-effects model and novel BCI zero- training model

- Better overall prediction (improved zero-training model)
- Attributing variability to differences between subjects, not noise.
- Same subject on two subsequent days, on the second day the classifier can often be reused without much retraining, only the bias needs to be adjusted
- More robust feature selection
- **Future study:** study online adaptation of penalized linear mixed-effects models in the context of medical diagnosis.

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Thanks!

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