

Understanding classification failure in Machine Learning using CNNs and GANs

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ABSTRACT

The question of fooling neural networks has been around for a long time. Ever since deep neural networks were shown to perform better than competing conventional methods for image classification in 2012 by AlexNet[1] in the ImageNet competition, researchers have pursued the scenario that such powerful classifiers are not perfect and may be fooled. Though this may be applied to neural nets across various applications, such as text or even audio trained networks, it is explicitly visible in the field of image and video classification. The call to place such networks in ubiquitous technology such as phones (face recognition) and autonomous vehicles pushes the need of the hour toward trustworthy deep learning models that may not be fooled by any factors, obvious to humans or otherwise. We explore both manual and generative adversarial methods to explore the effect of noise on trained classifiers. We prepare several different experiments varying image size, resolution and noise type and magnitude of noise and note the drop in classification accuracy i.e. the effect of noise induced misclassification on CNN, Pre-trained and GAN models trained to high accuracy.

CCS CONCEPTS

• Image Classification • Machine Learning Failure • Neural Networks • Convolutional Neural Networks • Generative Adversarial Networks

KEYWORDS

Image Misclassification, Adversarial Attacks, CNNs, Conditional GANs

1. Introduction

Understanding how and why neural networks can be fooled into misclassifying images is an interesting topic that has been widely explored. This topic has gained much attention of researchers as we are rapidly entering the world of self-driving cars and these kinds of failures can cause major hurdles in bringing technologies to the mainstream society. Some papers [2] show that one-pixel change is enough to force a misclassification. This can be done using CNNs or GANs [2], and even uses evolutionary

algorithms in some cases [3]. We feel like though a one-pixel attack may be easy, a more common phenomenon in our experience is dealing with more general noisy images when training or testing image classification networks.

Deep neural networks have been widely used for classifying images with the high accuracy. This project is our quest to explore the tolerance and identify weaknesses of image classifying networks to different types and degrees of noise. The addition of noise is almost indistinguishable to the human eyes but surprisingly it can completely fool neural networks. Moreover, another important point which drives this research is its application in real-time scenarios e.g. If a self-driving car just ignores or misclassifies a stop-sign or a pedestrian because its neural networks have mis-classified its sensors' input images. With this we hypothesize that:

1. It should be possible to identify certain noise thresholds beyond which a network starts regularly failing,
2. Similarly, certain noise patterns may be more disruptive to the classification process of trained networks and even pre-trained networks.
3. Training GANs on structured noise augmented datasets may help us understand the failure of the machine learning models or the solution to overcome this problem.

2. Background

2.1 Supervised learning

Supervised learning can be defined as the process of building a function that maps an input to an output based on previous examples. The neural network learns in a supervised manner by tuning its parameters to adapt to known examples so that it may then generalize to other samples of data previously unknown to it.

2.2 Deep Neural Networks

Deep Neural Networks, or DNNs, can be defined as non-linear approximation functions. Given an input, they can combine several non-linear transformations onto weighted sums to provide an output mapping in a higher or lower dimensional space.

2.3 Convolutional Neural Network (CNN)

A class of a deep neural network mostly used to analyze visual imagery based on the shared weight architecture of filters and convolutional kernels which extract the features instead of crafting hand-designed feature set from the dataset[9].

2.4 Pre-trained Networks

As advanced CNNs get larger and deeper, the resources and compute time taken to train these models becomes larger than most regular machines can handle. One way to be able to use these powerful models in our experiments is to re-use the model weights from models already trained on large image classification datasets like ImageNet[4-5]. E.g. ResNet50, InceptionV3

2.5 Generative Adversarial Networks

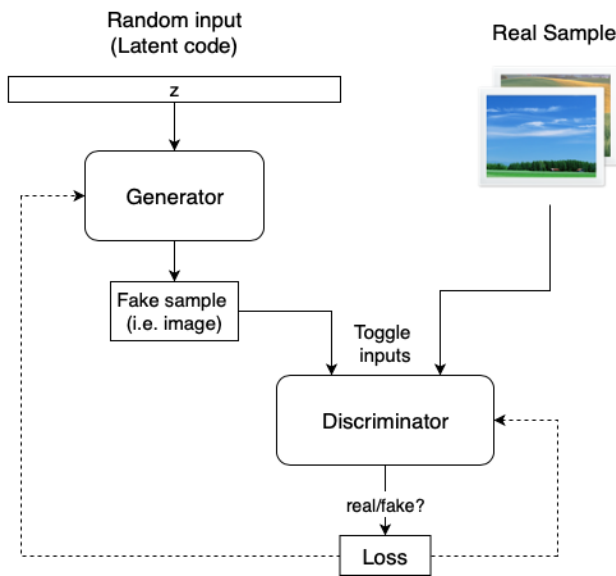


Figure 1: GAN Description: Ref [6]

A Generative Adversarial Network (seen in Figure 1) is a compound neural network architecture designed of two parts. The working of each part is based on a two-player min-max game where the generator G tries to model an input data distribution to generate realistic data to fool the discriminator network while the discriminator D tries to distinguish between real and generated, synthetic data.[7] The value function to be optimized is shown in the below equation (Fig 2.)

$$\min_{G,E} \max_D V(G, E, D) = \mathbb{E}_{\mathbf{x} \sim p_{data}(\mathbf{x})} \log D(\mathbf{x}, E(\mathbf{x})) + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} \log(1 - D(G(\mathbf{z}), \mathbf{z}))$$

Figure 2: MinMax Objective function for a GAN

2.6 Conditional GAN(cGANs)

One drawback of the original GAN[12], we had no control over what output was generated; since the network's input was pure noise. Mahdi and Simon presented the first paper on Conditional GANs in 2014. In the described implementation of this cGAN we can add a conditional input \mathbf{c} to the random noise \mathbf{z} so that the generated image is defined by $G(\mathbf{c}; \mathbf{z})$ [8]. Typically, the conditional input vector \mathbf{c} is concatenated with the noise vector \mathbf{z} , and the resulting vector is put into the generator as it is in the original GAN.[7]

2.7 Adversarial Examples

Adversarial examples are examples designed by adding artificial perturbations and noise to a regular image from a dataset with the specific aim to throw off a DNNs classification of that image. Such examples can be made in a simple additive manner by adding random or structured values to pixels in an image. Such modifications can cause the classifier to label the modified image as a completely different class.[2]

2.8 Data Augmentation

Data augmentation is the technique to artificially create the new training data. It helps to deal with the problem of imbalanced classes in a dataset and the performance of deep learning models rely heavily on the amount of data in each class i.e. the class representation. More data in each class will improve both the intra-class variation and inter-class variation, both of which are beneficial to the performance of the deep learning model [10].

2.9 Image Noise

Image noise is the variation in the color/pixel information of in the images. It changes the brightness of the images depending on the type of the noise. Overall, it is an unwanted signal in the image. Depending on the level of the noise added to the images vary the deep learning model performance which is almost indistinguishable to the human eyes but it can fool the neural networks [11].

2.10 Transfer Learning

Transfer learning is a technique of machine learning in which model that is trained on one task can be applied to the related task. So, it is a technique which helps in increase performance of the second task by just fine tuning few layers of the first deep model. This way the training time and compute resources requirements reduced a lot for the related task [13].

3. Motivation

When we started this project, we wanted to understand misclassification of supervised learning algorithms and neural nets. Over time we started work and focused on a subset of this problem, namely the effect of noise and its types on trained and pre-trained networks. Later, we stumbled upon the field of Adversarial attacks and generative adversarial networks. This allowed us to start experimenting with GANs and Conditional GANs in code.

4. Experiments

In this section we describe our dataset and methodology. After a short introduction covering our implementation details, we write about our DNN models, approach to adding noise and our experiments. We also show the results of using both GANs and Conditional GANs to synthesize traffic sign images. Tables 2, 3 and 4 briefly go over our experiments and their results.

4.1 Dataset

The German Traffic Sign Recognition Benchmark Dataset [2] was published by researchers at the Ruhr-Universität Bochum, Germany in 2011 for the International Joint Conference on Neural Networks (IJCNN). The dataset has the following properties:

- Single-image, multi-class classification problem.
- Dataset contains 43 classes of different traffic size.
- Training and test sets of 39209 and 12630 respectively.

The images in this dataset are of uneven size and range between 25x25 to 225x225 images of road signs. The class dataset distribution graph is shown in the Fig. 3. Additionally, to mitigate this problem of imbalanced classes we have performed different data augmentation techniques like Random rotation of 40 degrees, Random contrast of 0.6 and Horizontal and vertical flip. we have Fig 2 shows a few samples of the original dataset without augmentation is being applied.[16]



Figure 3: Some samples from the GTSRB dataset



Figure 4: Types and levels of noise introduced to the dataset. From the top, Gaussian Noise, Periodic Noise and Salt and Pepper Noise. Left to right: Min to Max noise.

Table 1: Sample Class Labels

ClassId	SignName	ClassId	SignName
0	Speed limit (20km/h)	22	Bumpy road
1	Speed limit (30km/h)	23	Slippery road
2	Speed limit (50km/h)	24	Road narrows on the right
3	Speed limit (60km/h)	25	Road work
4	Speed limit (70km/h)	26	Traffic signals
5	Speed limit (80km/h)	27	Pedestrians
6	End of speed limit (80km/h)	28	Children crossing
7	Speed limit (100km/h)	29	Bicycles crossing
8	Speed limit (120km/h)	30	Beware of ice/snow
19	Dangerous curve to the left	41	End of no passing

4.2 Preprocessing

Using these training and testing images to train our CNNs, pre-trained networks and GANs required some modification. Since all the images were not of uniform shape/size (in terms of width and height), we generated copies of the dataset, each in different sizes to support the specific neural network. The images were regularized by cropping or padding them to the expected size, while making sure that most of the traffic sign still remained in the center of the image. e.g, The InceptionV3 was built to accept inputs of 75*75 while the ResNet50 was designed to allow a minimum image size of 50*50; The GANs we built took an input of 28*28 so that processing time would be quicker and we also trained the CNNs on the 40*40 dataset. All the images were RGB i.e. had three channels of color. This made their final shape $\sim x \times x \times 3$, where x was in [40,50,75]

The noisy image datasets were prepared by adding perturbations to the original images. The main three categories of the noise we have taken care of are Salt & Pepper Noise, Periodic Noise and Gaussian Noise. We wrote python scripts to re-generate the datasets according to the type and level of the noise. In total we have generated 9 different datasets. Fig. 4 shows different types of noise based on the level.

Across all experiments, three gaussian datasets were generated, where the perturbation introduced was changing the standard deviation of pixels in the image by +50(Max) standard deviations, +25(Avg) or +5(Min) standard deviations. Similarly, for the periodic datasets, we generated noise using a combination of sine and cosine wave patterns with fixed amplitude. The perturbations added to in these datasets were $(\sin(x) + \cos(x))$ (Max), $0.5(\sin(x) + \cos(x))$ (Avg) and $0.25(\sin(x) + \cos(x))$ (Min), where x was a constant. Finally for Salt and pepper noise, we generated noise as a random increase of values to random pixels in the image. Here, (Max) corresponds to 0.4% of pixels being activated, (Avg) is 0.2% and (Min) is 0.1%.

Salt and pepper noise: In this type of noise certain amount of the pixels are black or white. This noise can be used to make images which machine learning model cannot classify. Given the probability p which is $(0 \leq p \leq 1)$ which says the chances of the pixel is being corrupted. Depending on the probability value i.e $p = 0.4$ then that many pixels are randomly set to the black color and rest of the pixels are randomly set to white color [15].

Periodic noise: Periodic noise is another type of noise in which it has different patterns. While adding that noise we have added combinations of sine and cosine signals into spatial domain which would add either vertical or horizontal patterns to the image. We could have added for the higher degree of the cosine and sine function but then image quality is degraded a lot. So we were restricted to use the lower value of the degree range between $(0.5 < d < 1)$ [17].

Gaussian noise: This noise falls into the category of the statistical noise where noise level is equivalent to the normal distribution of the Gaussian Distribution type. A random function is added to the original image and which distorts the image depending on the level of the Gaussian function. Gaussian noise adds noise to an image with the variance of the image with mean value and standard deviation [18].

Initially due to memory constraints, we made several copies of the noisy datasets, each with its own version of training and testing images. Later in the experiments this proved useful to prevent load on the RAM which was already being strained under the pressure of training a GAN.

When memory was free, we developed pipelines to transform the images in memory so that we would not have to continuously read and save datasets from the filesystem.

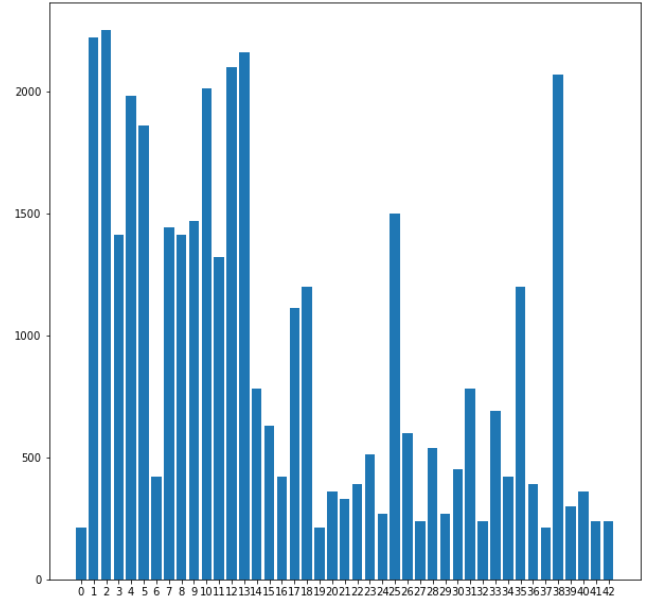


Figure 5: Class Distribution of Training Samples (Class vs Number of Training Samples)

4.3 Implementation

Our implementations of a simple image classification model used as baseline were built with TensorFlow and Keras. After failing to build working networks in Tensorflow, we found some helpful resources that guided us to use PyTorch to build and test our GAN and cGAN[14]. The structures of our networks can be seen in the below figures. The baseline CNN was trained with an Adam optimizer with a learning rate of 0.01 and minimized categorical cross entropy loss while being trained for 10 epochs each. The generative network and discriminative network are trained with Adam optimizer with $\beta_1 = 0.5$, $\beta_2 = 0.999$, and learning rate of 0.0002. The batch size is 30, and the iterations for training were iteratively set as 20, 60, 100 and 200.

```

Discriminator(
  (model): Sequential(
    (0): Linear(in_features=2352, out_features=512, bias=True)
    (1): LeakyReLU(negative_slope=0.2, inplace=True)
    (2): Linear(in_features=512, out_features=256, bias=True)
    (3): LeakyReLU(negative_slope=0.2, inplace=True)
    (4): Linear(in_features=256, out_features=1, bias=True)
    (5): Sigmoid()
  )
)

```

Figure 6: Discriminator Architecture

```

Generator(
  (model): Sequential(
    (0): Linear(in_features=100, out_features=128, bias=True)
    (1): LeakyReLU(negative_slope=0.2, inplace=True)
    (2): Linear(in_features=128, out_features=256, bias=True)
    (3): BatchNorm1d(256, eps=0.8, momentum=0.1, affine=True, track_running_stats=True)
    (4): LeakyReLU(negative_slope=0.2, inplace=True)
    (5): Linear(in_features=256, out_features=512, bias=True)
    (6): BatchNorm1d(512, eps=0.8, momentum=0.1, affine=True, track_running_stats=True)
    (7): LeakyReLU(negative_slope=0.2, inplace=True)
    (8): Linear(in_features=512, out_features=1024, bias=True)
    (9): BatchNorm1d(1024, eps=0.8, momentum=0.1, affine=True, track_running_stats=True)
    (10): LeakyReLU(negative_slope=0.2, inplace=True)
    (11): Linear(in_features=1024, out_features=2352, bias=True)
    (12): Tanh()
  )
)

```

Figure 7: Generator Architecture

Model: "sequential_5"

Layer (type)	Output Shape	Param #
rescaling_2 (Rescaling)	(None, 40, 40, 3)	0
conv2d_10 (Conv2D)	(None, 40, 40, 16)	448
max_pooling2d_10 (MaxPooling)	(None, 20, 20, 16)	0
conv2d_11 (Conv2D)	(None, 20, 20, 32)	4640
max_pooling2d_11 (MaxPooling)	(None, 10, 10, 32)	0
conv2d_12 (Conv2D)	(None, 10, 10, 64)	18496
max_pooling2d_12 (MaxPooling)	(None, 5, 5, 64)	0
flatten_5 (Flatten)	(None, 1600)	0
dense_10 (Dense)	(None, 128)	204928
dense_11 (Dense)	(None, 43)	5547
Total params: 234,059		
Trainable params: 234,059		
Non-trainable params: 0		

Figure 8: Baseline CNN Architecture

4.3.1 CNNs

Our experiments in CNNs were the first step in exploring the effect of noise on misclassification. We first designed, built and trained a simple baseline classification network for evaluation on the Test set of the GTSRB dataset. Then, using its performance as baseline, we evaluated the trained module on 9 other test sets, each with one of three variants of noise: Salt and Pepper noise, Gaussian noise, and Periodic noise.

Table 2: CNN Experiment Table

Model	Image Size	Noise Type	Noise Level	Accuracy(%)
CNN	40*40*3	None	None	89.09

		Gaussian	Max	39.97
		Gaussian	Avg	41.83
		Gaussian	Min	60.49
		Periodic	Max	10.54
		Periodic	Avg	57.91
		Periodic	Min	74.44
		Salt & Pepper	Max	68.11
		Salt & Pepper	Avg	75.33
		Salt & Pepper	Min	80.57

4.3.2 Pre-trained Models

We chose to use largely successful and widely studied networks for our pretrained focused experiments, namely Google's InceptionV3 and the ResNet50. Both these networks are huge in size compared to our baseline CNN model and their architectures are many times deeper. In these experiments, the non-uniformity of the datasets image size caused us to pad smaller images with zeroes and crop larger images as mentioned in section 4.2. This cropping/padding transformation had an impact on the performance of the networks, particularly the InceptionV3.

Table 3: Pre-trained network Experiment Table

Model	Image Size	Noise Type	Noise Level	Accuracy(%)
InceptionV3	75*75*3	None	None	68.17
		Gaussian	Max	60.55
		Gaussian	Avg	61.23
		Gaussian	Min	63.89
		Periodic	Max	15.09
		Periodic	Avg	35.04
		Periodic	Min	50.42
		Salt & Pepper	Max	58.32
		Salt & Pepper	Avg	62.50
		Salt & Pepper	Min	65.19
ResNet50	50*50*3	None	None	81.58

		Gaussian	Max	80.76
		Gaussian	Avg	80.82
		Gaussian	Min	80.99
		Periodic	Max	20.24
		Periodic	Avg	63.12
		Periodic	Min	71.07
		Salt & Pepper	Max	23.39
		Salt & Pepper	Avg	54.12
		Salt & Pepper	Min	68.12

4.3.3 GANs and cGANs

The GAN and cGAN experiments were of a different nature compared to the ones above, in these experiments, we trained two GANs: one Vanilla and one Conditional GAN and evaluated the kinds of images that were synthesized by the trained Generators. GANs were a new concept for us and understanding how they worked, their architectures, losses, forward and backward passes was a lengthy and confusing road. Our initial GANs written in TensorFlow and Keras worked fine for simpler datasets such as Fashion MNIST, but when faced with multichannel (RGB) complex objects like traffic signs, did not perform as well. Even training them for up to 9 hours of training, around 100 epochs only led them to learn a bunch of white noise speckled with patches of color. Increasing the learning rate to 0.01 from the initial 0.0002 while keeping the momentum constant may have led us to face the regularly discussed “exploding gradients” issue faced by GANs[ref].

Our next tries with reference code in PyTorch were more successful, but even the simplest GAN architectures took a whole night to train (15 epochs in an hour). Some of the PyTorch

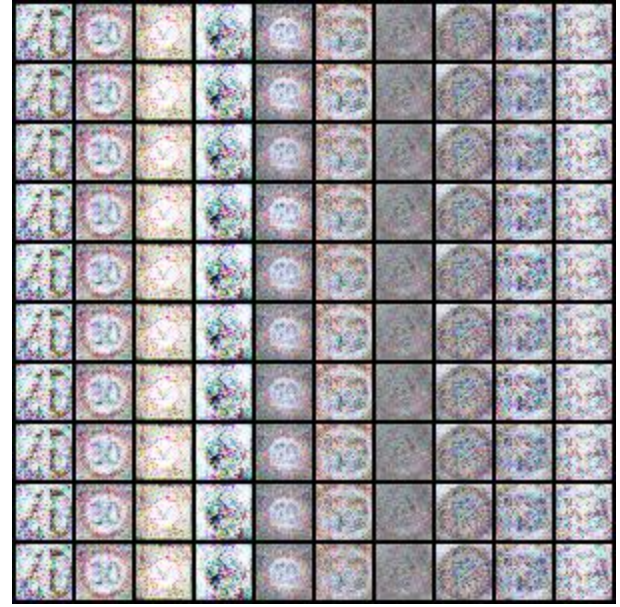


Figure 9: Training Conditional GAN Outputs

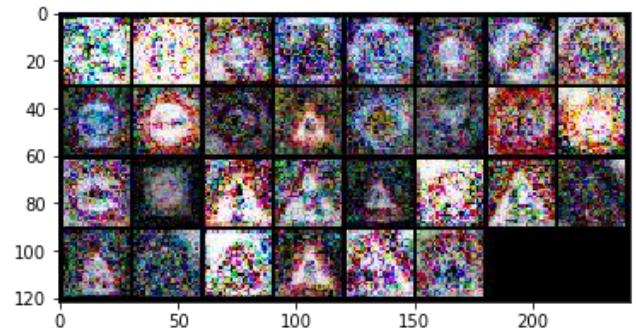


Figure 10: Conditional GAN Evaluation after 200 epochs.

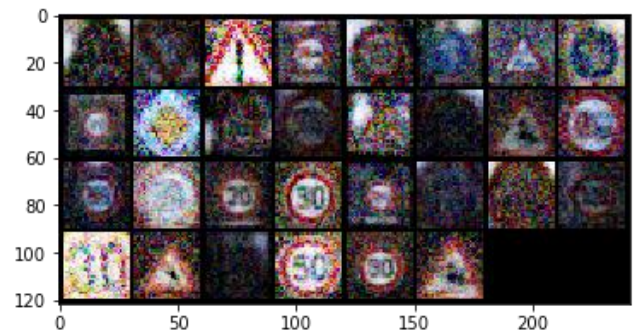


Figure 11: Vanilla GAN Evaluation after 200 epochs

5. Results

From our initial CNN based experiments, we were expecting to find a “worst” kind of noise that uniformly affects all Image classifier networks in the same detrimental manner. What we see instead from the pre-trained and GAN/cGAN experiments is that

different networks respond to different types of noise differently. E.g. the InceptionV3 pre-trained network evaluated poorly on periodic noise whereas the ResNet50 suffered on the Salt and Pepper noise test set. The vanilla CNNs performance on Gaussians was worse off compared to the other networks. We see from our experiments that **different networks respond differently to different sorts of noise**.

Our second result may sound obvious, **but more noise is inhibitive to the performance of image classifiers**. We can see a clear pattern of decreasing performance as noise increases. For higher levels of noise, we can induce misclassification in a 89% accurate baseline CNN as seen in Figures 12 and 13.

While training the vanilla GANs we experimented with learning rate, batch size and other parameters, but the long training time was prohibitive to our experiments. We plan to 1) Train the GANs further and 2) Use the trained Vanilla Generators to generate batches of noise and smooth images (as seen in Figs 9-11) and pass them to our highly trained CNNs and Pre-trained networks in order to “fool” them.

The conditional GAN, while more flexible and controllable in its output, does not offer any significant improvements over the vanilla GAN.

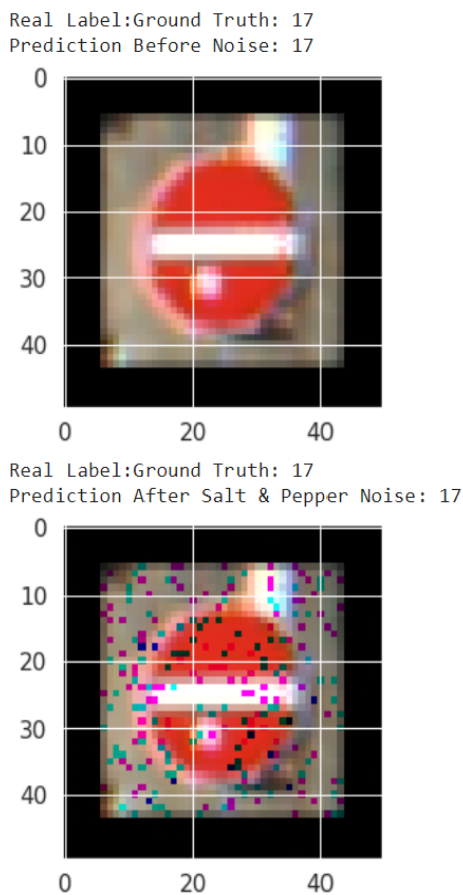


Figure 12: With less noise, the baseline CNN is robust enough to label the image to the right class.

We see plenty of experiments in exploring different generator and discriminator architectures and input types for both the GAN and cGAN and how the images generated will interact with the trained networks.

5.1 Heat map of the confusion matrix

The confusion matrix is a performance measurement for Machine Learning classification problem where output can be two or more classes. In our case it is 43 x 43 confusion matrix which has 43 different classification. Diagonal Entry represents the truly predicted values out of all the values in particular row whereas other values in the row showcase the false predictions made by models. The values which are very large and does not fit into the width of the cell in the confusion matrix are represented with exponential value. But the value is visible from the adjacent bar graph where the lowest value is 0 and the highest value is just above 700 the categories with the appropriate color scheme. i.e. $1.1e+02$ means 110 when expanded into Decimal notation from Scientific notation. Fig 14. Represent the confusion matrix in the next page for the pretrained ResNet50 model. Majority of the classes are correctly classified whereas classes 12 and 38 have majority misclassified prediction for the images without any noise.

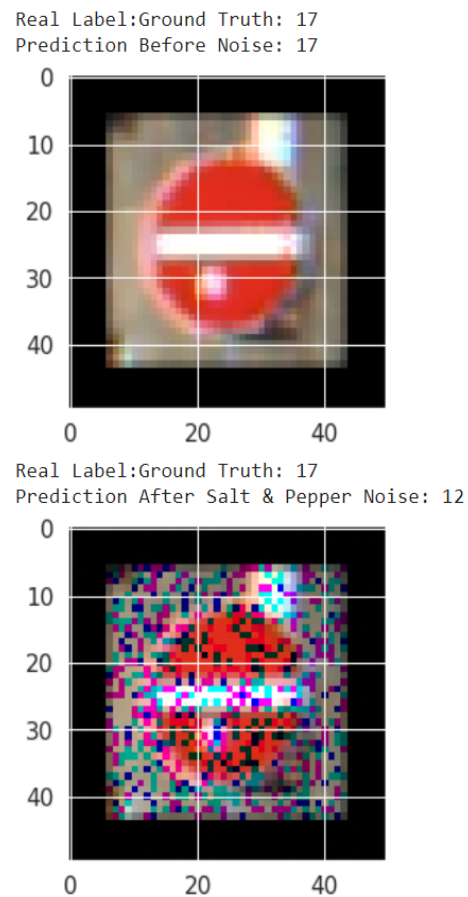


Figure 13: Results of noise induced misclassification on our baseline CNN

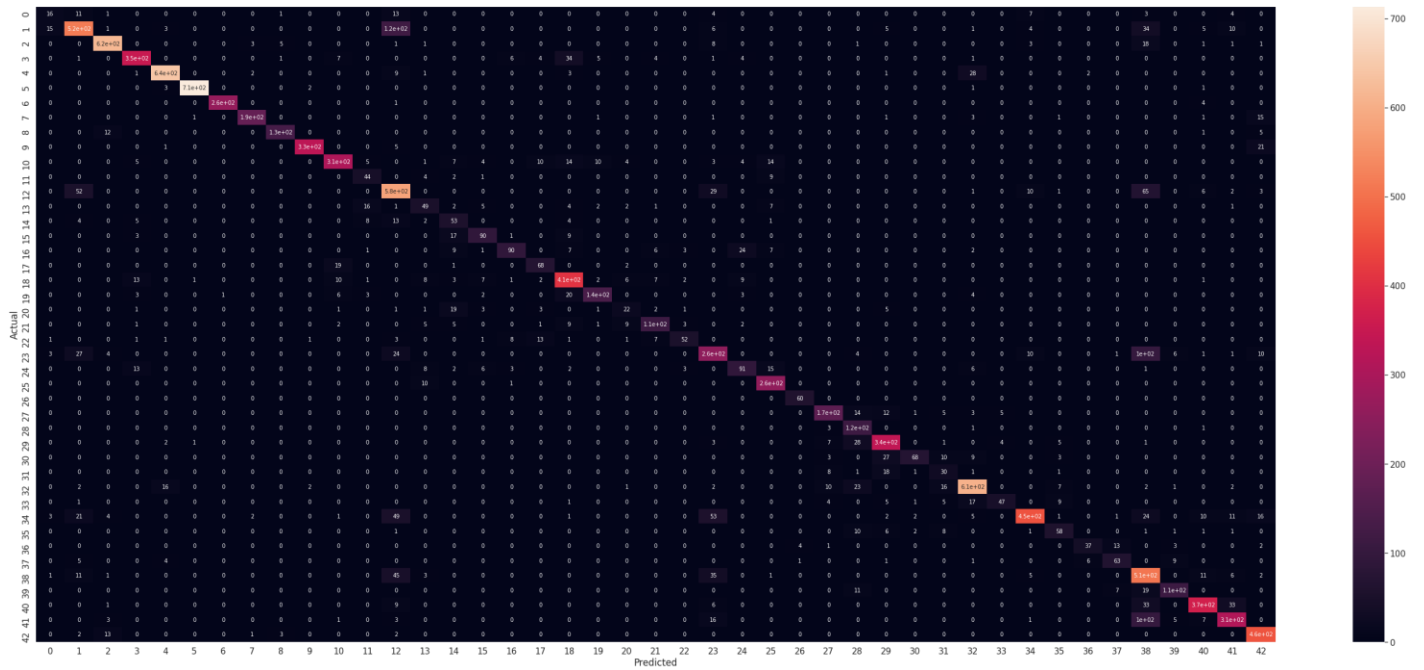


Figure 14: A Heatmap Confusion Matrix of Images Classified by the ResNet50

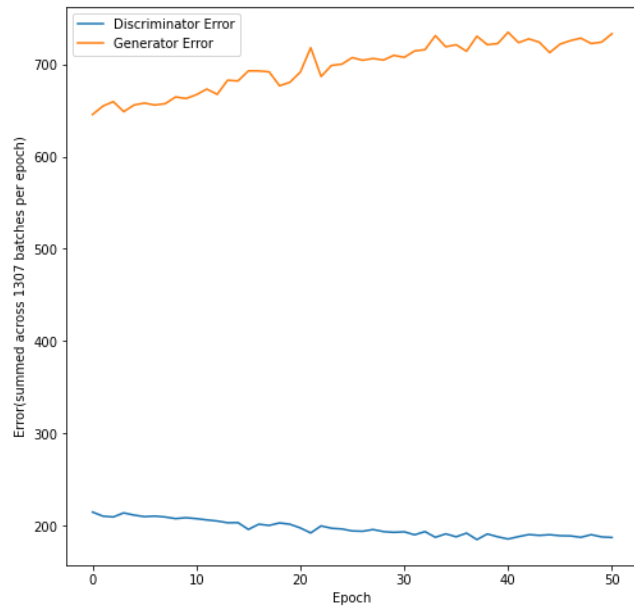


Figure 15: Diverging error curves in conditional GAN Training

6. Conclusion

Over the course of our implementation and experiments, we managed to

1. Observe the robustness of image classifying networks to different types of noise. In our results, we note reactions of networks to different types and levels of noise.
2. CNNs and Pre-trained networks react as expected to increasing levels of noise.
3. Hyper parameter tuning has certainly improved the performance of the deep neural networks.
4. Training GANs for over 200 epochs on a complex dataset like the GTRSB is not enough to generate clear and smooth images to test networks. Instead we note from the training error curves that the generator and discriminator are diverging instead of converging and from literature surveys[16] we expect that 2000-2500 epochs is a better training time to get recognizable images.
5. GANs are very sensitive to the learning rate and image size. The higher learning rate result into exploding gradients and hig
6. Evaluate performance of the network specific to noise type and range

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