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Stability of Image-Reconstruction Algorithms

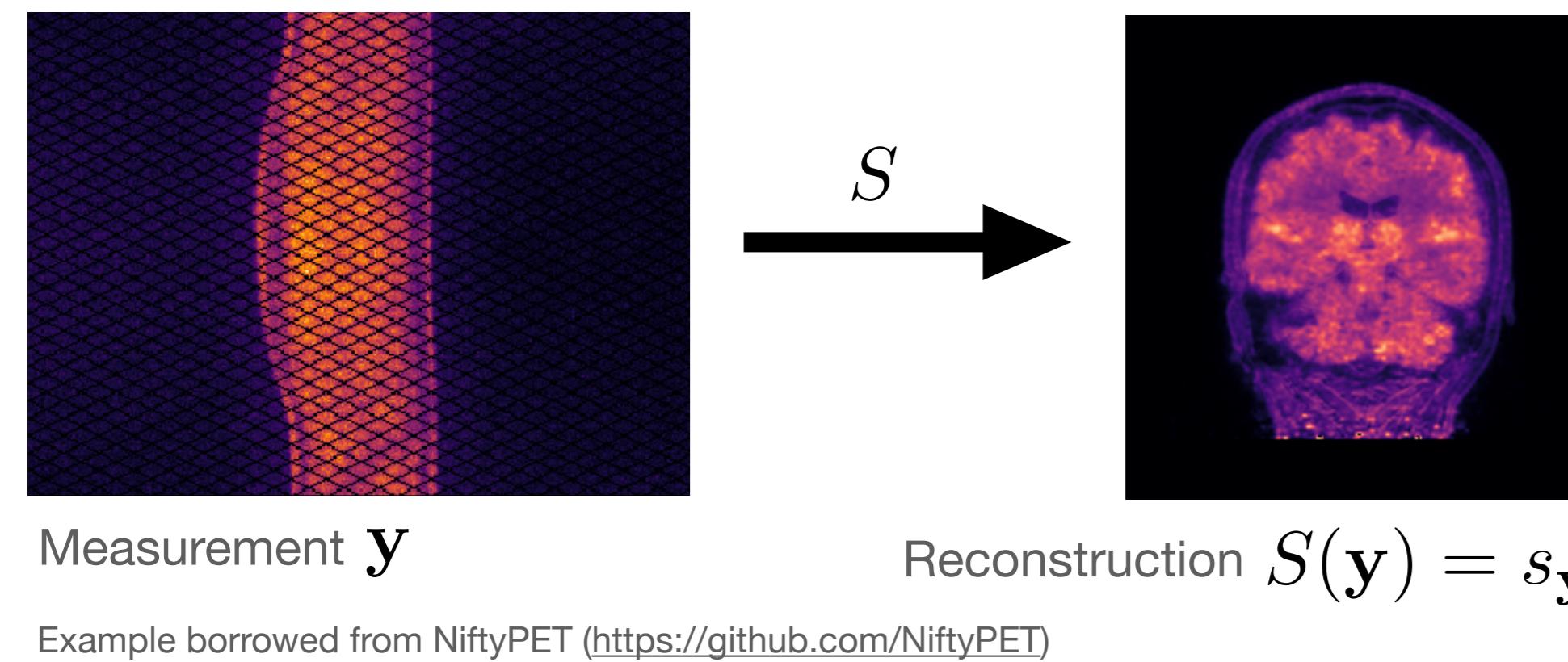
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Motivation

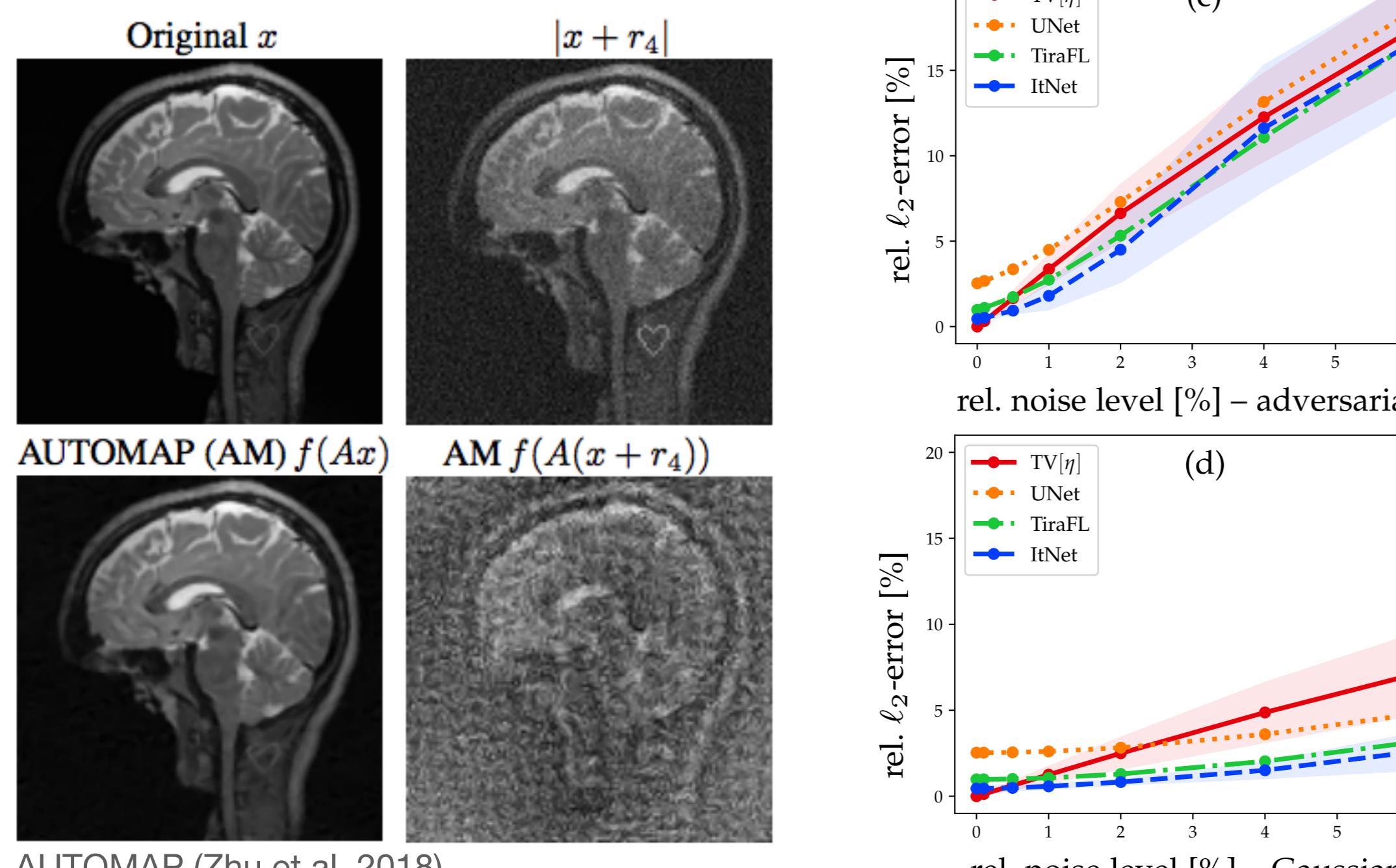
- Image reconstruction



- Reliability critical to applications

- State-of-the-art approaches are not stable

Learning-based image rec.
(Antun et al. 2020)



- Going back to fundamentals

Which methods guarantee

$$\|s_{\mathbf{y}_1} - s_{\mathbf{y}_2}\|_{\ell_p} \leq K \|\mathbf{y}_1 - \mathbf{y}_2\|_2^\beta \quad ?$$

Discussion: How should K be?

Stability through the history of image rec.

- A reference variational problem

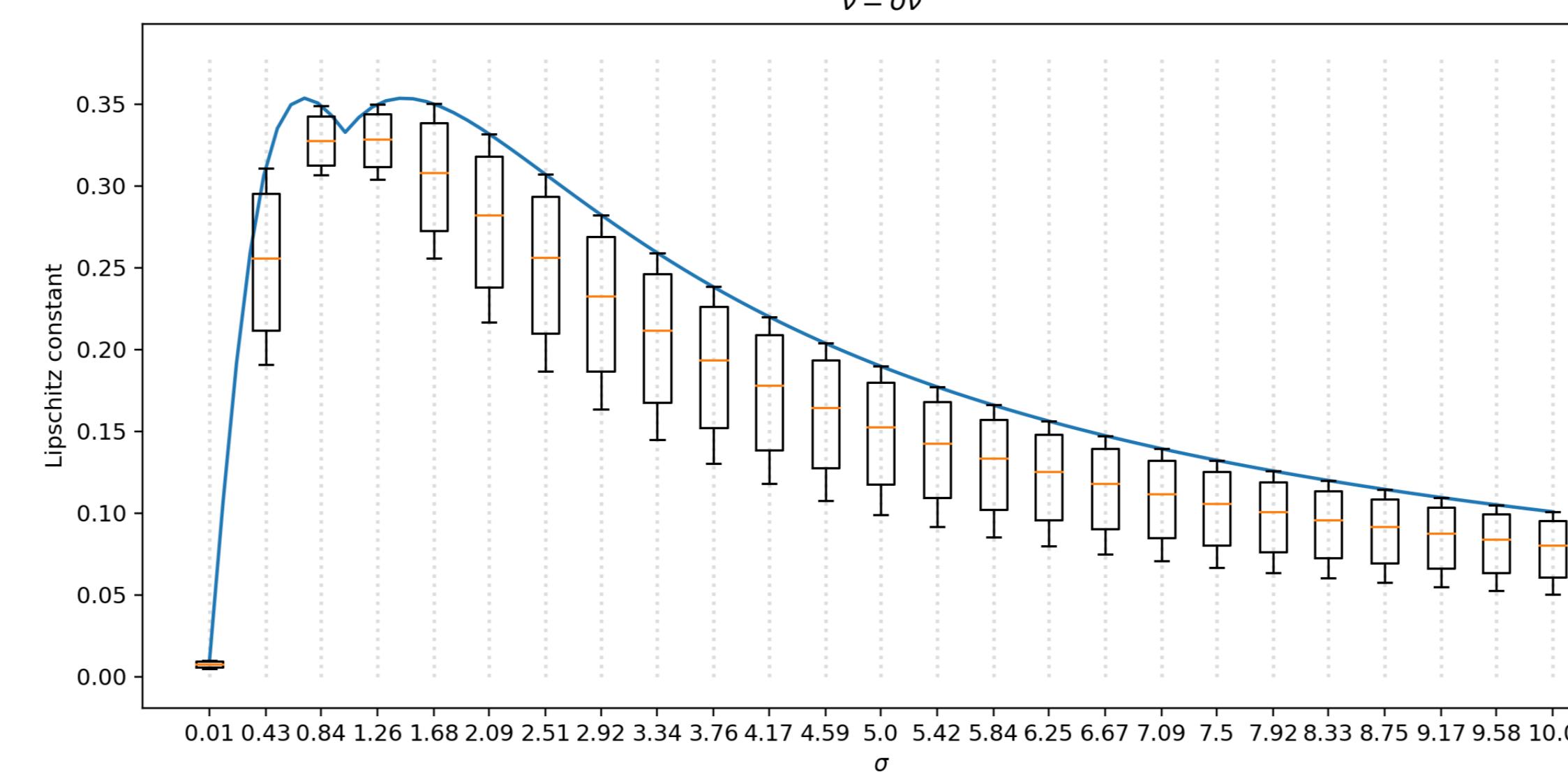
$$\min_{f \in \mathcal{X}} \{E(\mathbf{y}, \nu(f)) + \psi(\|f\|_{\mathcal{X}})\}$$

measurement operator $\nu(f) = (\langle \nu_1, f \rangle_{\mathcal{X}' \times \mathcal{X}}, \dots, \langle \nu_M, f \rangle_{\mathcal{X}' \times \mathcal{X}})$.

- Tikhonov regularization

$$E(\mathbf{y}_1, \mathbf{y}_2) = \frac{1}{2} \|\mathbf{y}_1 - \mathbf{y}_2\|_2^2, \mathcal{X} = \ell_2, \text{ and } \psi(x) = x$$

$$\Rightarrow f_{\mathbf{y}} = \sum_{m=1}^M a_{\mathbf{y},m} \varphi_m \text{ with } \varphi_m = \mathcal{R}(\nu_m) \Rightarrow \mathbf{a}_{\mathbf{y}} = (\mathbf{H} + 2\lambda \mathbf{Id})^{-1} \mathbf{y}$$

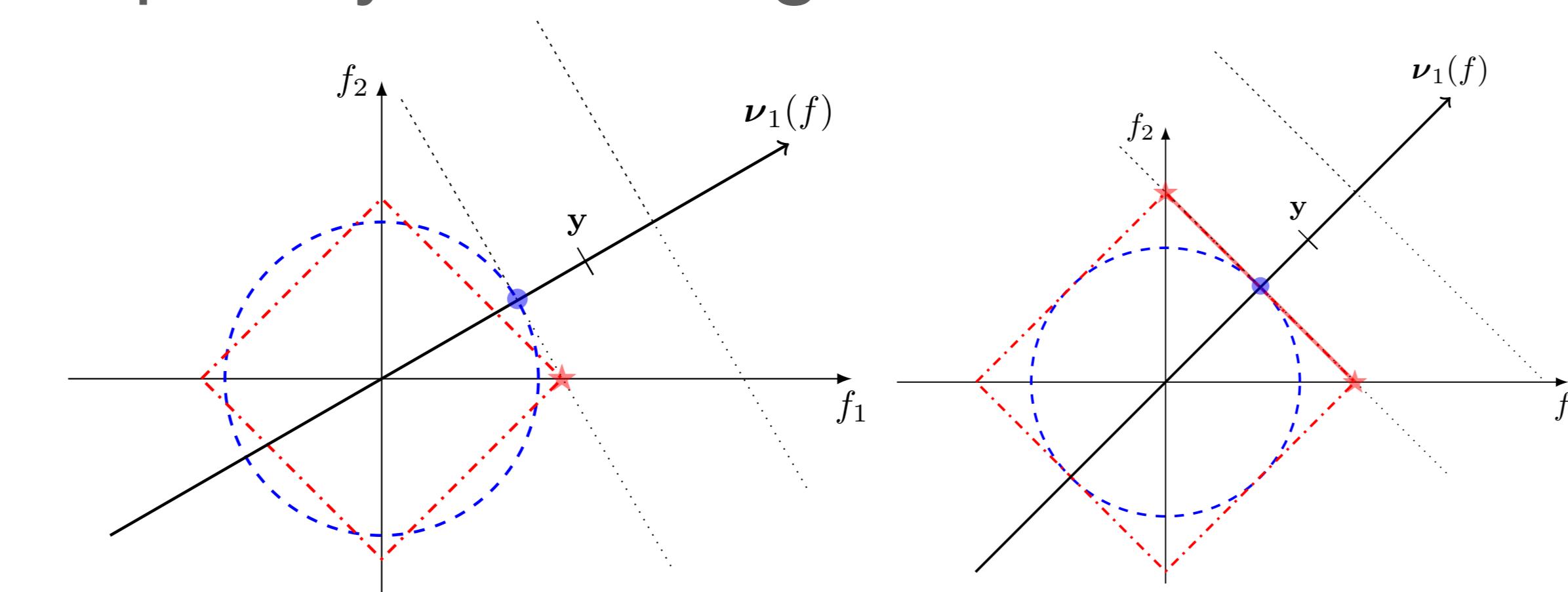


$$\|f_{\mathbf{y}_1} - f_{\mathbf{y}_2}\|_{\mathcal{H}}^2 \leq \max_{m \in \{1, 2, \dots, M\}} \left\{ \frac{\sigma_m}{(\sigma_m + 2\lambda)^2} \right\} \|\mathbf{y}_1 - \mathbf{y}_2\|_2^2$$

where $\{\sigma_m\}$ are the eigenvalues of \mathbf{H} with $\mathbf{H}_{m,n} = \langle \varphi_m, \varphi_n \rangle_{\ell_2}$.

Discussion: The role of the operator norm

- Sparsity-based regularization

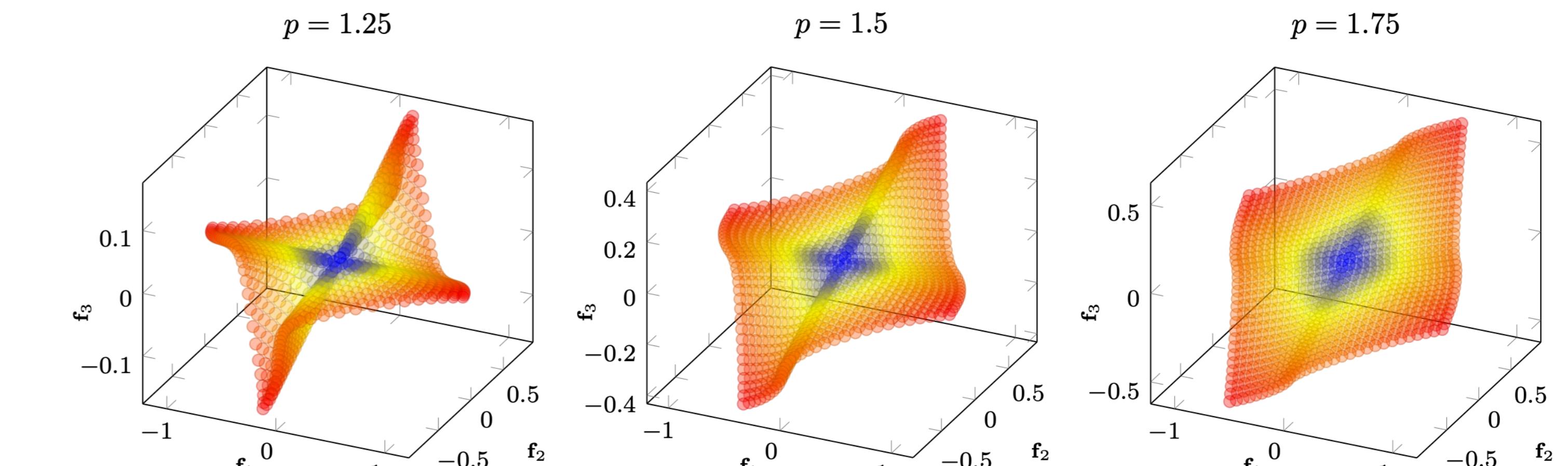


K hard to compute or bound

Our study

- ℓ_p regularization for $p \in (1, \infty)$

$\mathcal{X} = \ell_p$ or $\mathcal{X} = L_p(\Omega)$ and $\psi(x) = x^p$



Under technical conditions on E and ψ ,

$$\Rightarrow f_{\mathbf{y}} = \mathcal{J}_{\mathcal{X}} \left(\sum_{m=1}^M a_{\mathbf{y},m} \nu_m \right) \quad (\text{Unser 2021})$$

$$\text{with } \mathcal{J}_{\mathcal{X}}(\nu) = \frac{|\nu|^{p-2}}{\|\nu\|_{L_p}^{p-2}} \nu.$$

Results

- For $p \in (1, 2)$

$$\|f_{\mathbf{y}_1} - f_{\mathbf{y}_2}\|_{L_p} \leq \frac{(2r_p(Y))^{2-p}K_p}{\lambda p(p-1)} \|\mathbf{y}_1 - \mathbf{y}_2\|_2$$

Locally Lipschitz-stable

K depends on the locality

K depends on the operator norm

Discussion: The role of the operator norm

- For $p \in (2, \infty)$

$$\|f_{\mathbf{y}_1} - f_{\mathbf{y}_2}\|_{L_p} \leq \left(\frac{2^{p-2}K_p}{\lambda p} \right)^{\frac{1}{p-1}} \|\mathbf{y}_1 - \mathbf{y}_2\|_2^{\frac{1}{p-1}}$$

Hölder stable

K depends on the operator norm