PRML - exercise 1.1

(1) $$y(x, \vec{w}) = \sum_{j=0}^{M} w_j x^j$$

(2) $$\mathcal{L} = \frac{1}{2} \sum_{n=1}^{N} \left(y(x_n - \vec{w}) - t_n \right)^2$$

(3) $$\vec{\nabla} \mathcal{L} = 0 \qquad w^*$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \frac{1}{2} \sum_{n=1}^{N} 2 \left(\frac{\partial y(x_n, \vec{w})}{\partial w_i} \right) \left(y(x_n, \vec{w}) - t_n \right)$$

$$\frac{\partial y(x_n, \vec{w})}{\partial w_i} = \frac{\partial}{\partial w_i} \left(\sum_{j=0}^{M} w_j x_n^j \right) = x_n^i$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \frac{1}{2} \sum_{n=1}^{N} 2 (x_n)^i \left(\sum_{j=0}^{M} w_j x_n^j - t_n \right)$$

$$= \sum_{n=1}^{N} \sum_{j=0}^{M} w_j x_n^{i+j} - x_n^i t_n$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = 0 \Rightarrow \sum_{n=1}^{N} \sum_{j=0}^{M} w_j x_n^{i+j} = \sum_{n=1}^{N} x_n^i t_n$$

$$\sum_{j=0}^{M} w_j \sum_{n=1}^{N} x_n^{i+j} = \sum_{n=1}^{N} x_n^i t_n$$

$$\sum_{j=0}^{M} w_j \underbrace{\sum_{n=1}^{N} x_n^{i+j}}_{A_{ij}} = \underbrace{\sum_{n=1}^{N} x_n^i t_n}_{T_i}$$

$$\sum_{j=0}^{M} w_j A_{ij} = T_i$$

To get w^* we need to solve the following M equations:

for $i \in [0, M]$: $$W_0 \times N + W_1 \times \sum_{n=1}^{N} x_n^{i+1} + W_2 \sum_{n=1}^{N} x_n^{i+2} + \dots + W_M \sum_{n=1}^{N} x_n^{i+M} = \sum_{n=1}^{N} x_n^i t_n$$

(M+1 unknowns & M+1 equations)