

PRML - exercise 1.1

$$(1) \quad y(x, \vec{w}) = \sum_{j=0}^M w_j x^j$$

$$(2) \quad \mathcal{L} = \frac{1}{2} \sum_{n=1}^N (y(x_n, \vec{w}) - t_n)^2$$

$$(3) \quad \vec{\nabla} \mathcal{L} = 0 \quad w^*$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \frac{1}{2} \sum_{n=1}^N 2 \left(\frac{\partial y(x_n, \vec{w})}{\partial w_i} \right) (y(x_n, \vec{w}) - t_n)$$

$$\frac{\partial y(x_n, \vec{w})}{\partial w_i} = \frac{\partial}{\partial w_i} \left(\sum_{j=0}^M w_j x_n^j \right) = x_n^i$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = \frac{1}{2} \sum_{n=1}^N 2 (x_n)^i \left(\sum_{j=0}^M w_j x_n^j - t_n \right)$$

$$= \sum_{n=1}^N \sum_{j=0}^M w_j x_n^{i+j} - x_n^i t_n$$

$$\frac{\partial \mathcal{L}}{\partial w_i} = 0 \Rightarrow \sum_{n=1}^N \sum_{j=0}^M w_j x_n^{i+j} = \sum_{n=1}^N x_n^i t_n$$

$$\sum_{j=0}^M w_j \sum_{n=1}^N x_n^{i+j} = \sum_{n=1}^N x_n^i t_n$$

$$\sum_{j=0}^M w_j \underbrace{\sum_{n=1}^N x_n^{i+j}}_{A_{ij}} = \underbrace{\sum_{n=1}^N x_n^i t_n}_{T_i}$$

$$\sum_{j=0}^M w_j A_{ij} = T_i$$