

# Données haute fréquence

## Analyse et modélisation statistique multi-échelle de séries chronologiques financières

Cours de Master - Paris 6

Transparents de la Partie VI :  
Faits stylisés et modèles “classiques”

Emmanuel Bacry

Centre de Mathématiques Appliquées

Ecole Polytechnique

emmanuel.bacry@polytechnique

	Moyenne	Variance	Skewness	Kurtosis	Test de JB
CAD/USD	0.0013	0.1054	-0.0065	5.7243	0.0000
JPY/USD	-0.0115	0.4624	-0.4802	6.9016	0.0000
CHF/USD	-0.0090	0.5680	-0.0192	5.6205	0.0000
GBP/USD	-0.0003	0.3856	0.1011	6.2693	0.0000

TAB. 1 – Forex. 1977-2006 : 7208 points. Moyenne, variance, skewness, kurtosis et  $p$ -valeur du test statistique de Jarque et Bera des rendements logarithmiques journaliers.

Thèse Alexei Kozhemyak

CMAP - Ecole Polytechnique 2007

	Moyenne	Variance	Skewness	Kurtosis	Test de JB
Accor	0.0164	4.1302	-0.1946	6.5537	0.0000
Air Liquide	0.0351	2.9136	0.0965	5.3988	0.0000
Alcatel	-0.0140	9.2631	-0.8909	28.2650	0.0000
Axa	0.0311	5.0735	0.0287	7.5109	0.0000
Bouygues	0.0426	5.2142	0.1934	7.2622	0.0000
Capgemini	-0.0173	8.7527	-0.0704	8.3784	0.0000
Carrefour	0.0504	3.4422	-0.0804	6.1774	0.0000
Casino Guichard	0.0356	3.5825	0.2289	5.4663	0.0000
Danone	0.0314	2.3257	-0.0291	6.6562	0.0000
Essilor International	0.0580	3.9690	0.0948	7.9286	0.0000
L'Oréal	0.0585	3.8911	-0.0182	5.0729	0.0000

TAB. 2 – Extrait CAC 40. 1990-2005 : 3700 points.

**Table 6.6.** Value of the parameters corresponding to a TLD or Student (S) fit to the daily and monthly returns of a pool of U.S. stocks. The variance of each stock is normalized to one using the method explained in Section 4.4.

Time lag	$a^{2/3}\alpha$	$\mu$	Kurtosis $\kappa_1$			Log likelihood per point	
			Measured	TLD	S	TLD	S
Daily	0.174	4.16	38.3	9.7	37.5	-1.3383	-1.3369
Monthly	0.332	5.68	5.38	3.7	3.6	-1.3818	-1.3818

Bouchaud J.-Ph. and Potters M.  
 Theory of Financial Risk and Derivative Pricing : From Statistical Physics to  
 Risk Management Cambridge University Press, (2nd edition) 2003.

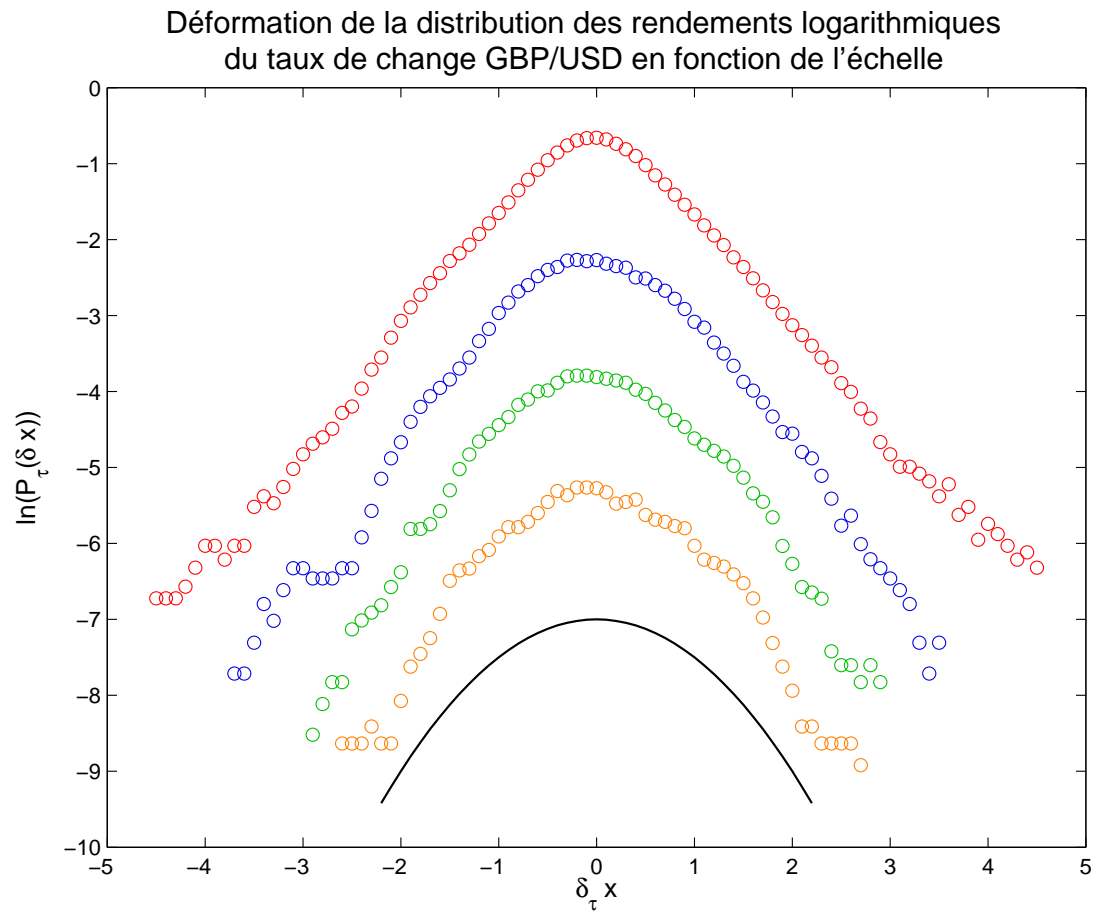
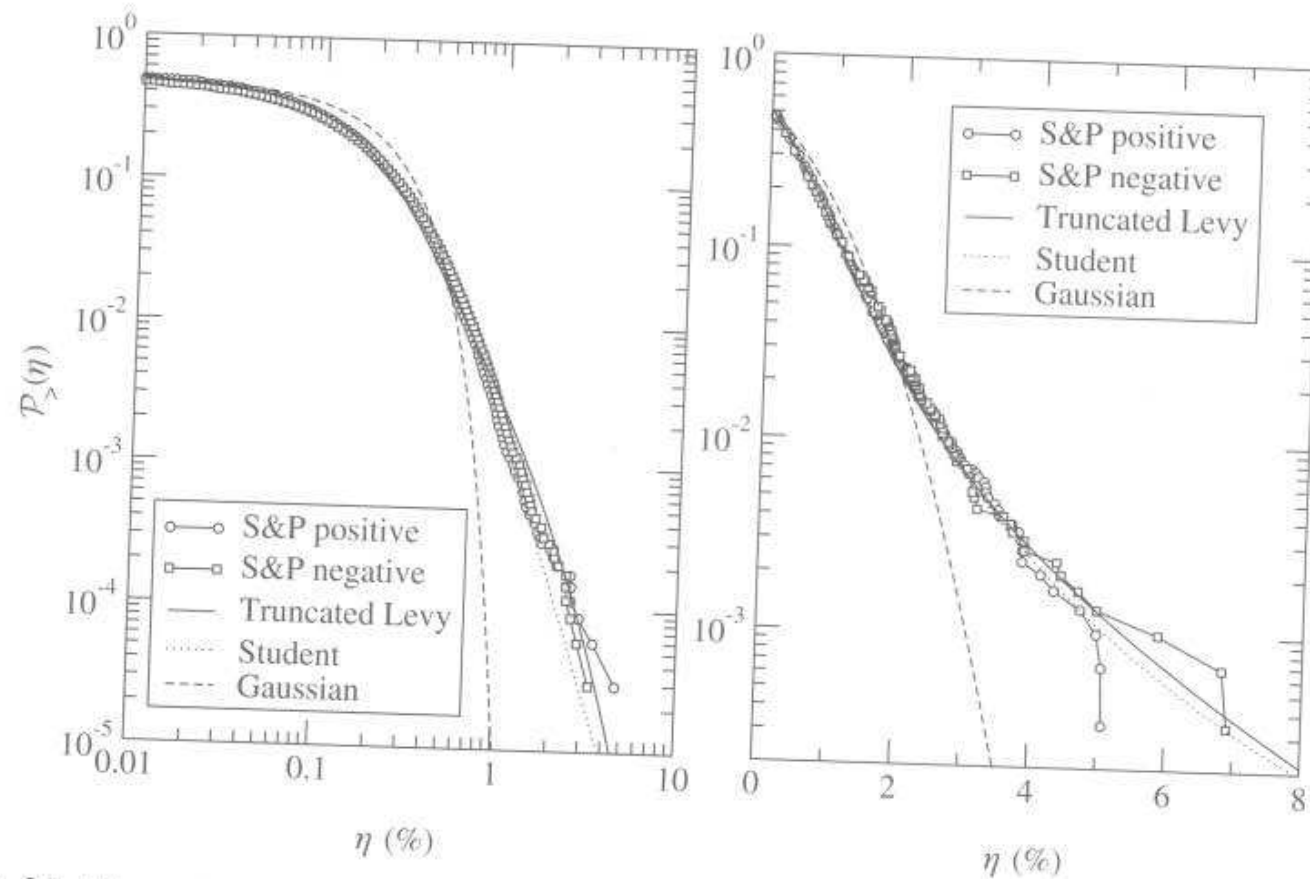


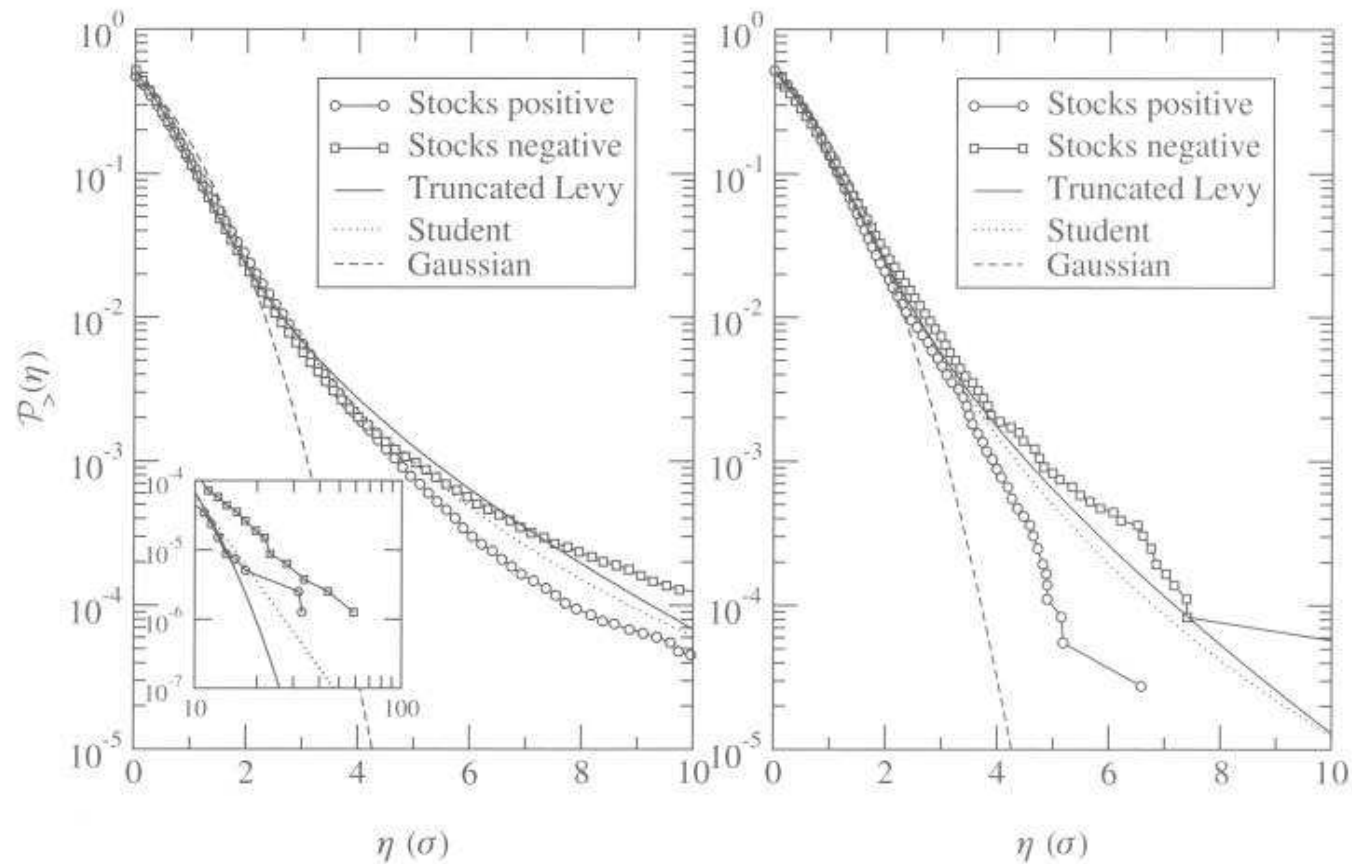
FIG. 1 – Lois des rendements logarithmiques GBP/USD de haut en bas :  $\tau = 1$  jour, 5 jours, 10 jours et 20 jours.



**Fig. 6.5.** Elementary cumulative distribution  $\mathcal{P}_{1>}(\eta)$  (for  $\eta > 0$ ) and  $\mathcal{P}_{1<}(\eta)$  (for  $\eta < 0$ ), for the S&P 500 returns, with  $\tau = 30$  min (left, log-log scale) and 1 day (right, semi-log scale). The thick line corresponds to the best fit using a symmetric TLD  $L_{\mu}^{(\eta)}$ , of index  $\mu = \frac{3}{2}$ . We have also shown the best Student distribution and the Gaussian of same RMS.

Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.



**Fig. 6.8.** Daily (left) and monthly (right) distribution returns of a pool of U.S. stocks. The RMS of the log-returns of each stock was normalized to one using the method explained in Section 4.4. Daily extreme events are shown in left inset. Note power law with  $\mu = 3$  for the negative tails (open squares).

Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.

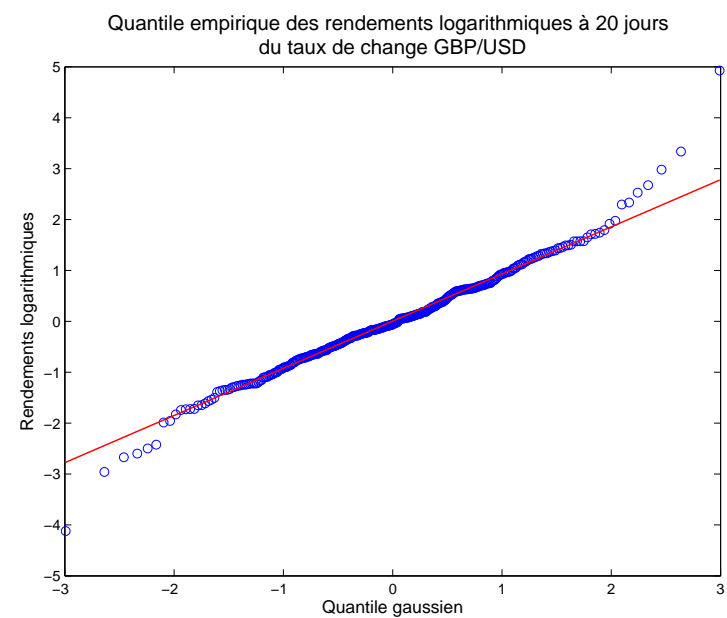
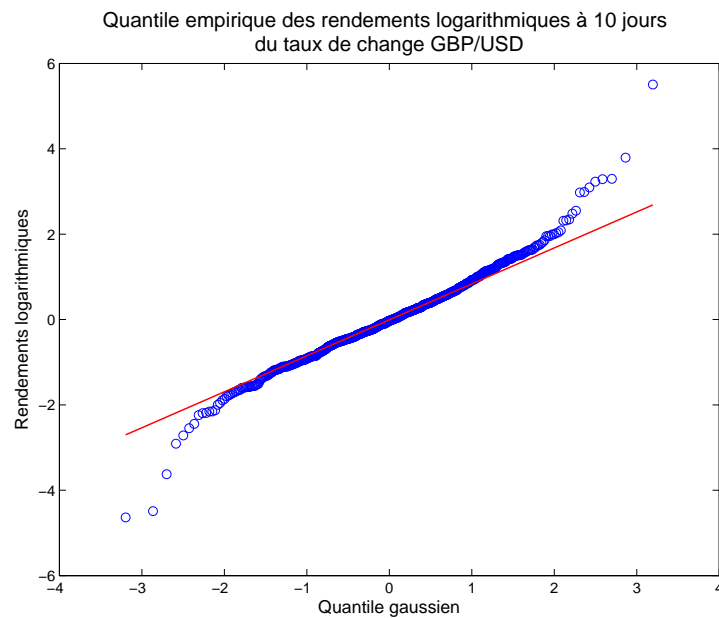
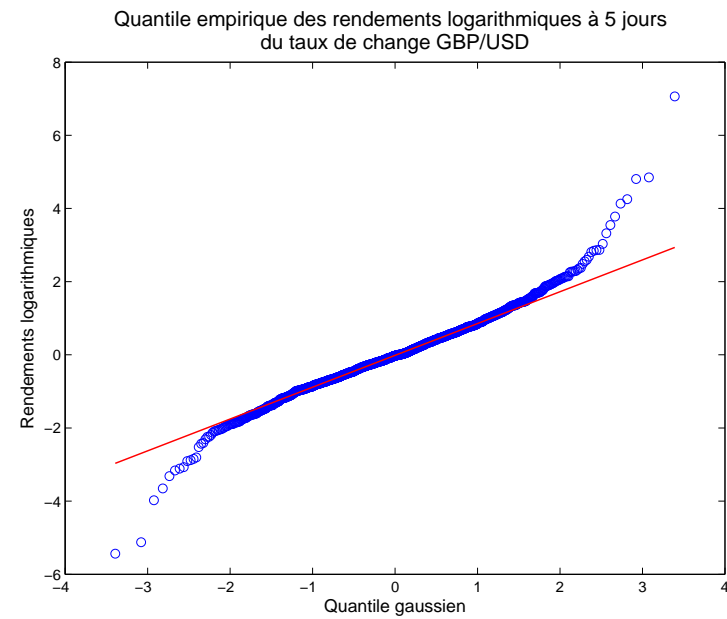
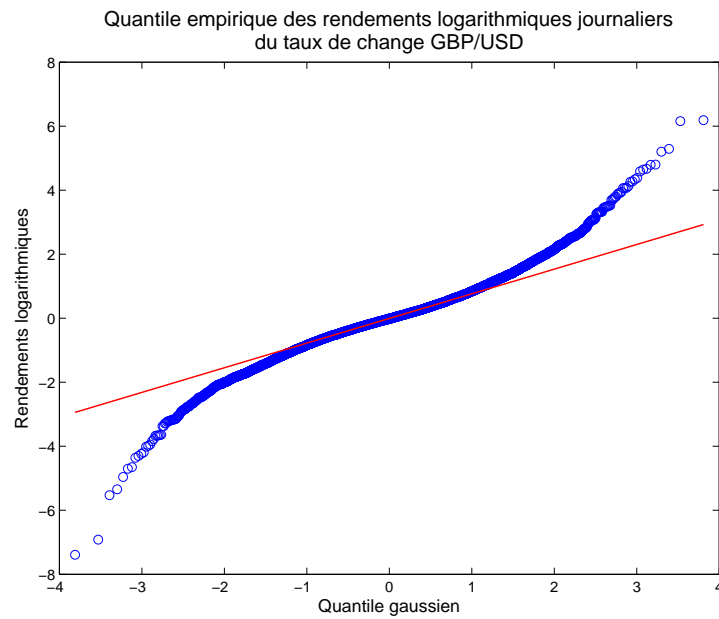
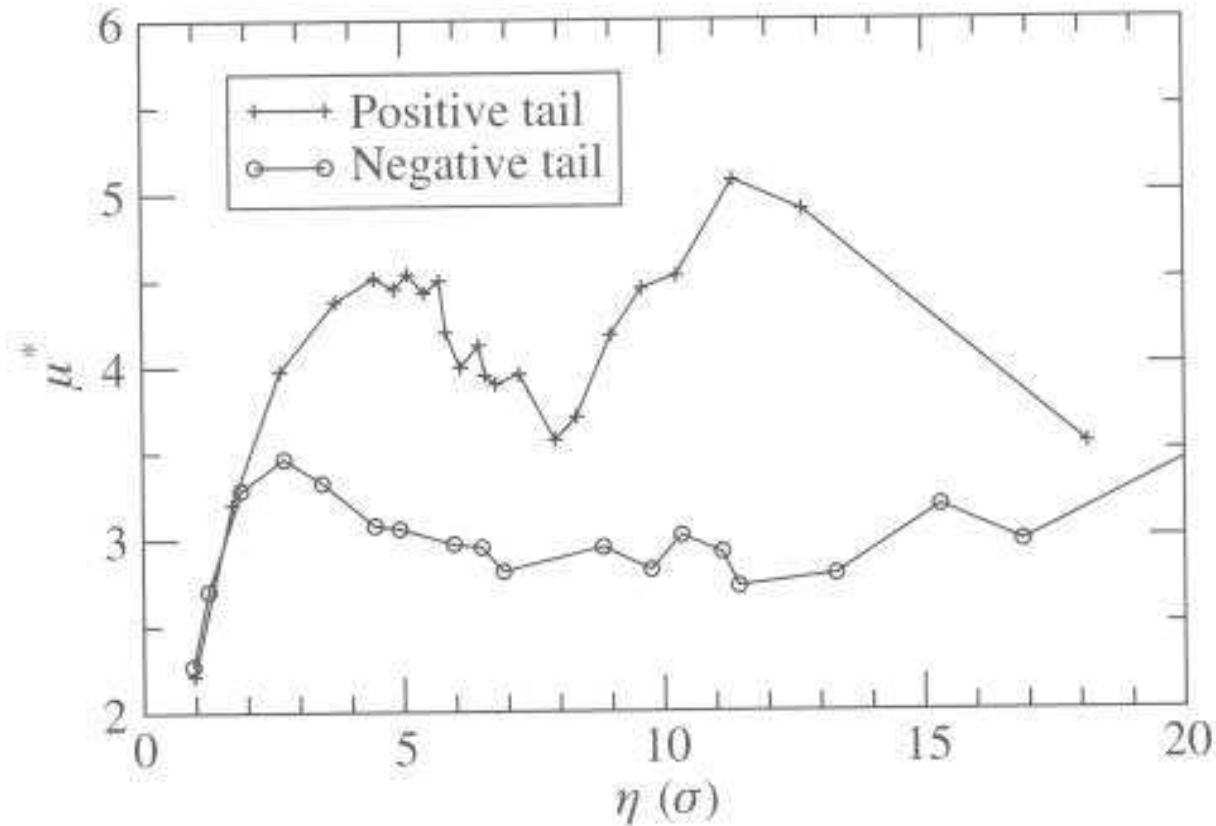


FIG. 2 — Thèse Alexei Kozhemyak. CMAP - Ecole Polytechnique 2007

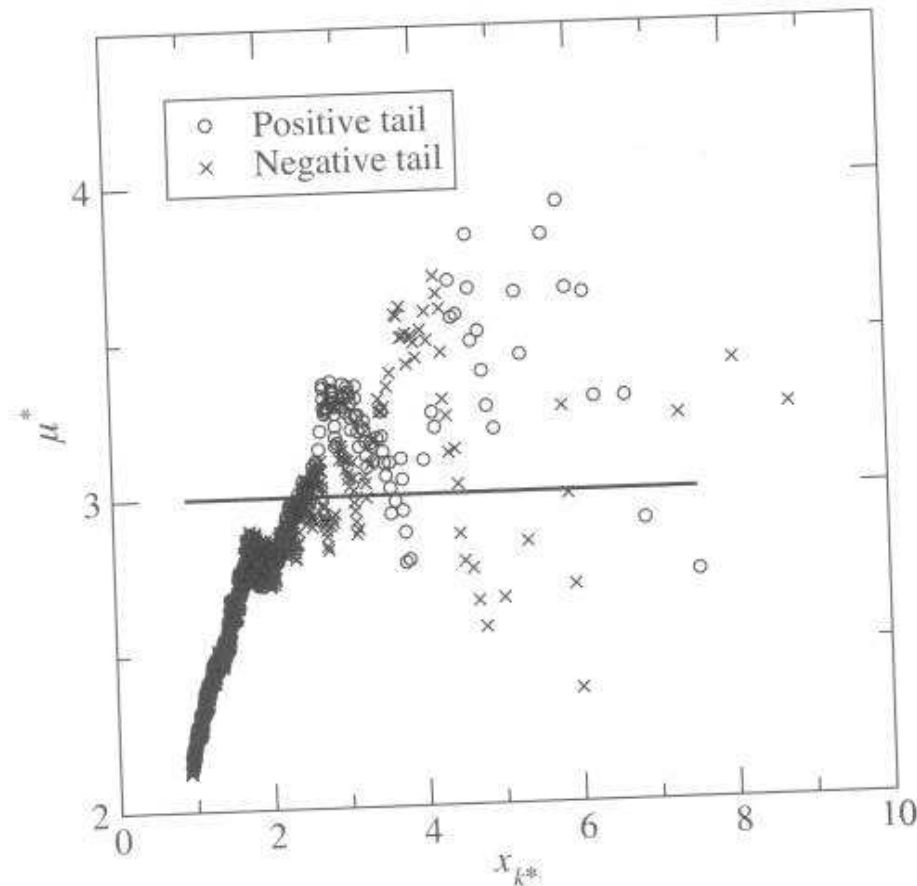




**Fig. 6.10.** Hill estimator of the tail exponent of the individual U.S. stock returns, normalized by their volatility using the trick explained in Eq. (4.37). The negative tail exponent is  $\mu \approx 3$ , whereas the positive tail exponent is  $\mu \approx 4$ .

Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.



**Fig. 4.1.** Hill estimator  $\mu^*(k^*)$  as a function of  $x_{k^*}$  for a 1000 point sample of a  $\mu = 3$  Student distribution. The two curves corresponds to the left and right tail. Note that for small  $x$  one leaves the tail region. The plain horizontal line is the expected value  $\mu = 3$ .

Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.

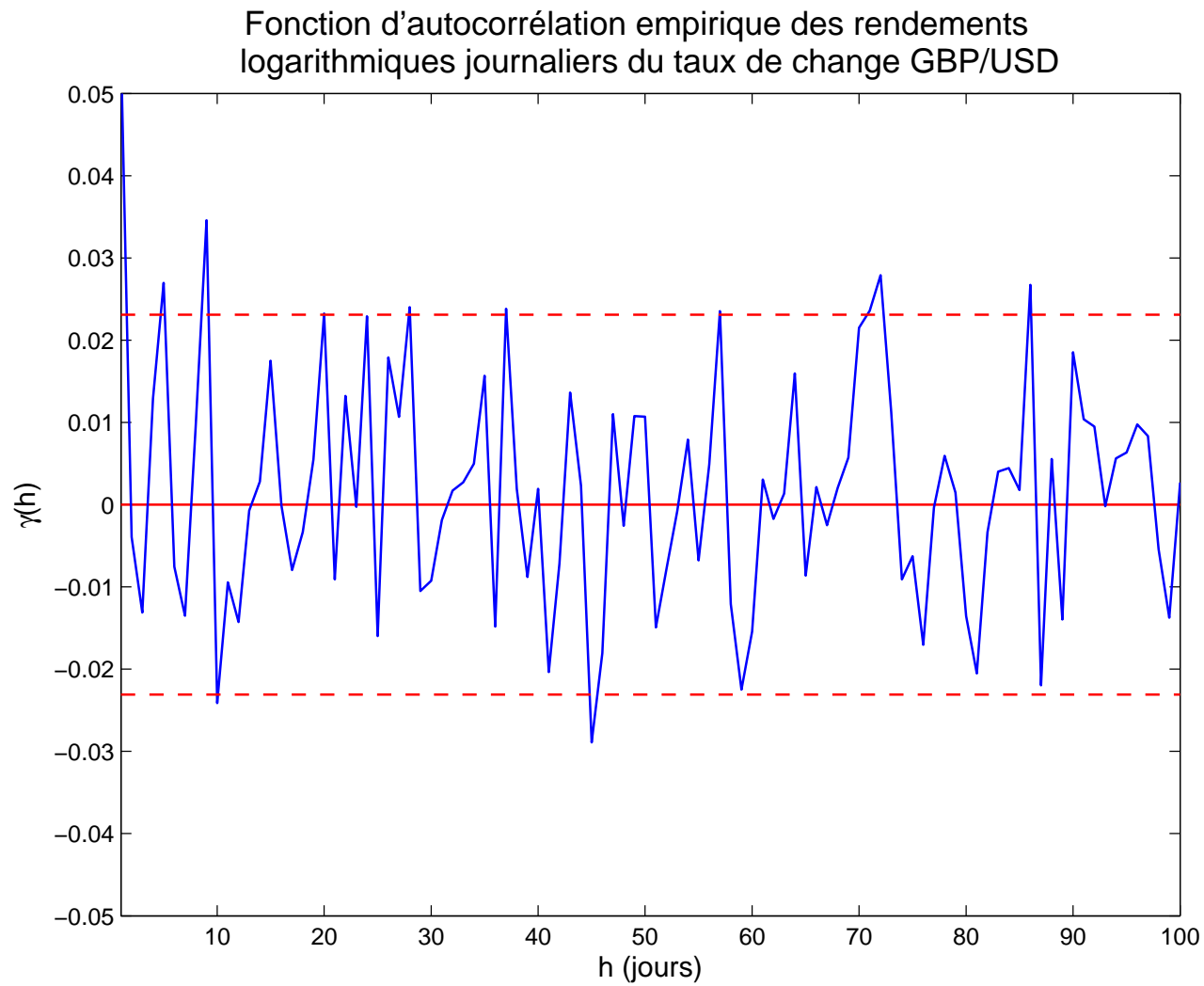


FIG. 3 – Thèse Alexei Kozhemyak - CMAP - Ecole Polytechnique 2007.

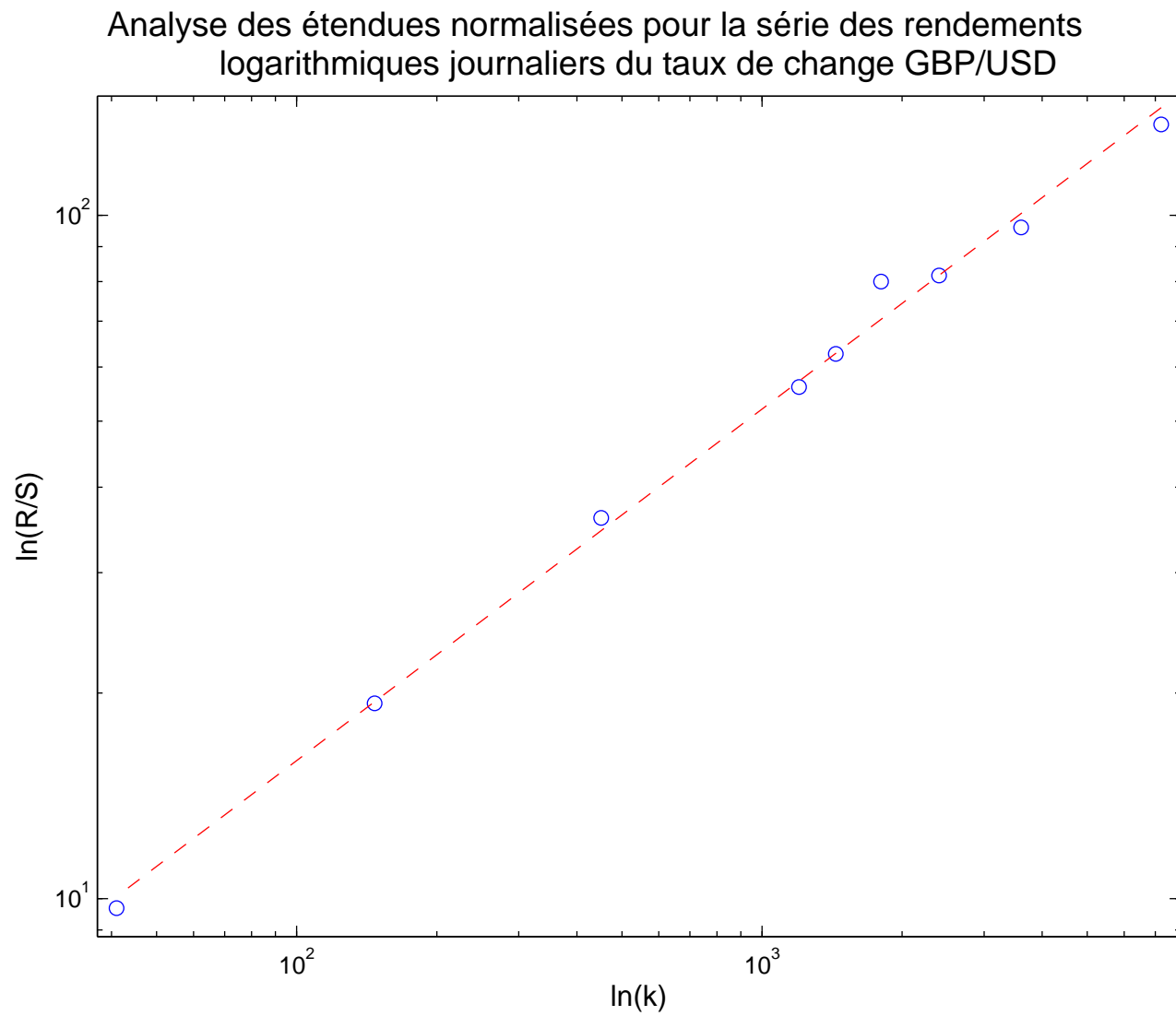
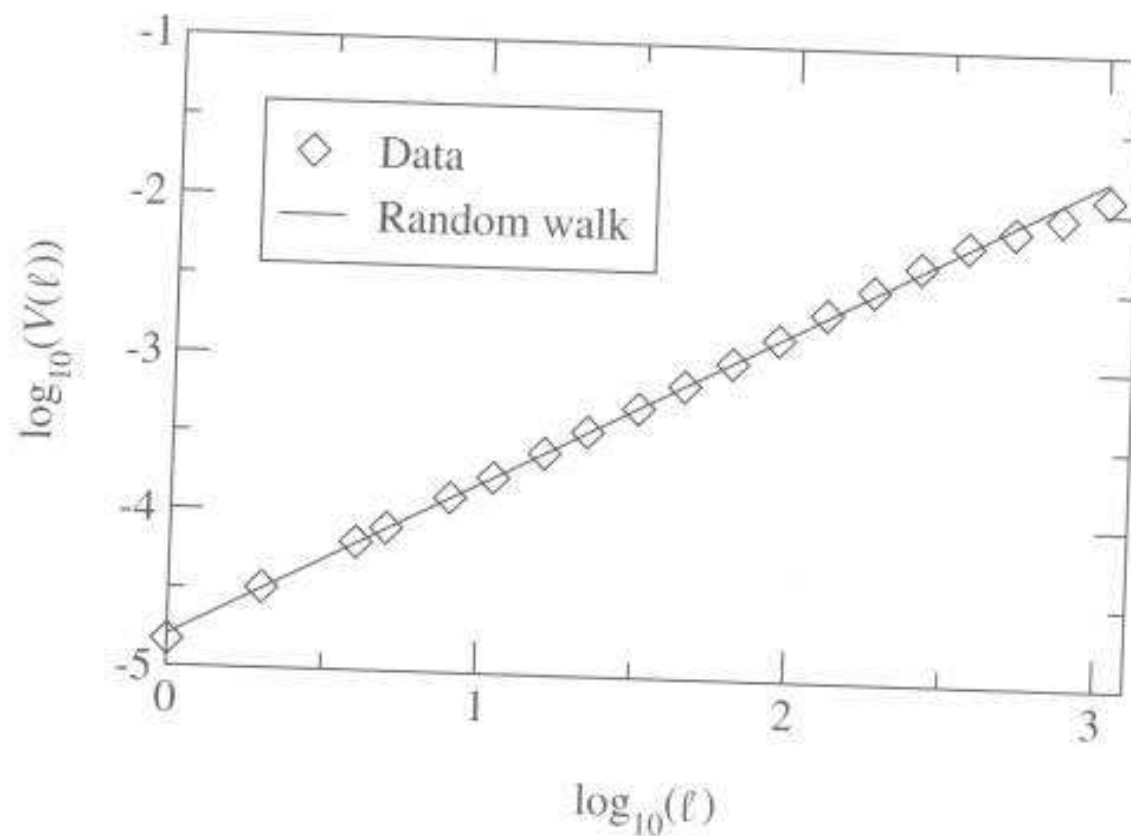


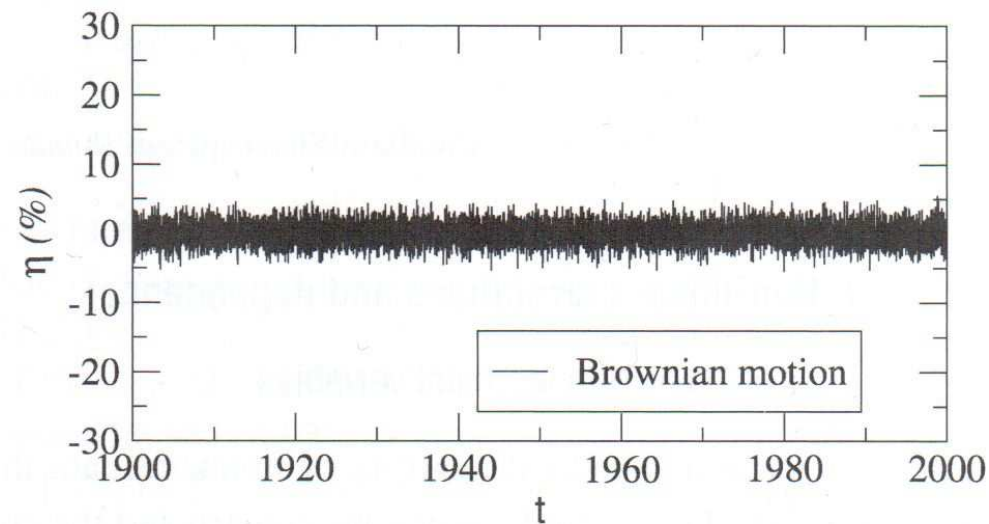
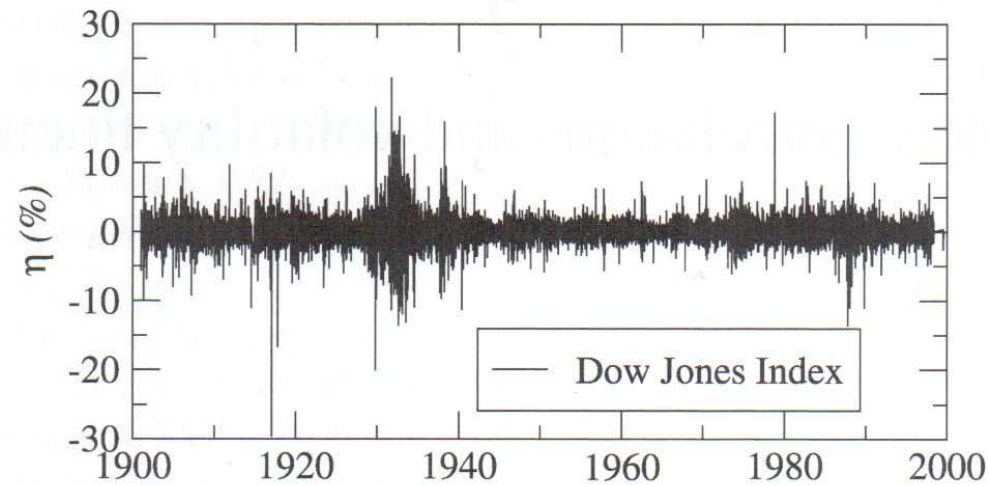
FIG. 4 – Thèse Alexei Kozhemyak - CMAP - Ecole Polytechnique 2007.



**Fig. 6.3.** Variogram of the detrended logarithm of the Dow Jones index, during the period 1950–2000.  $\ell$  is in days. Except perhaps for the last few points, the variogram shows no sign of saturation.

Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.



Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.

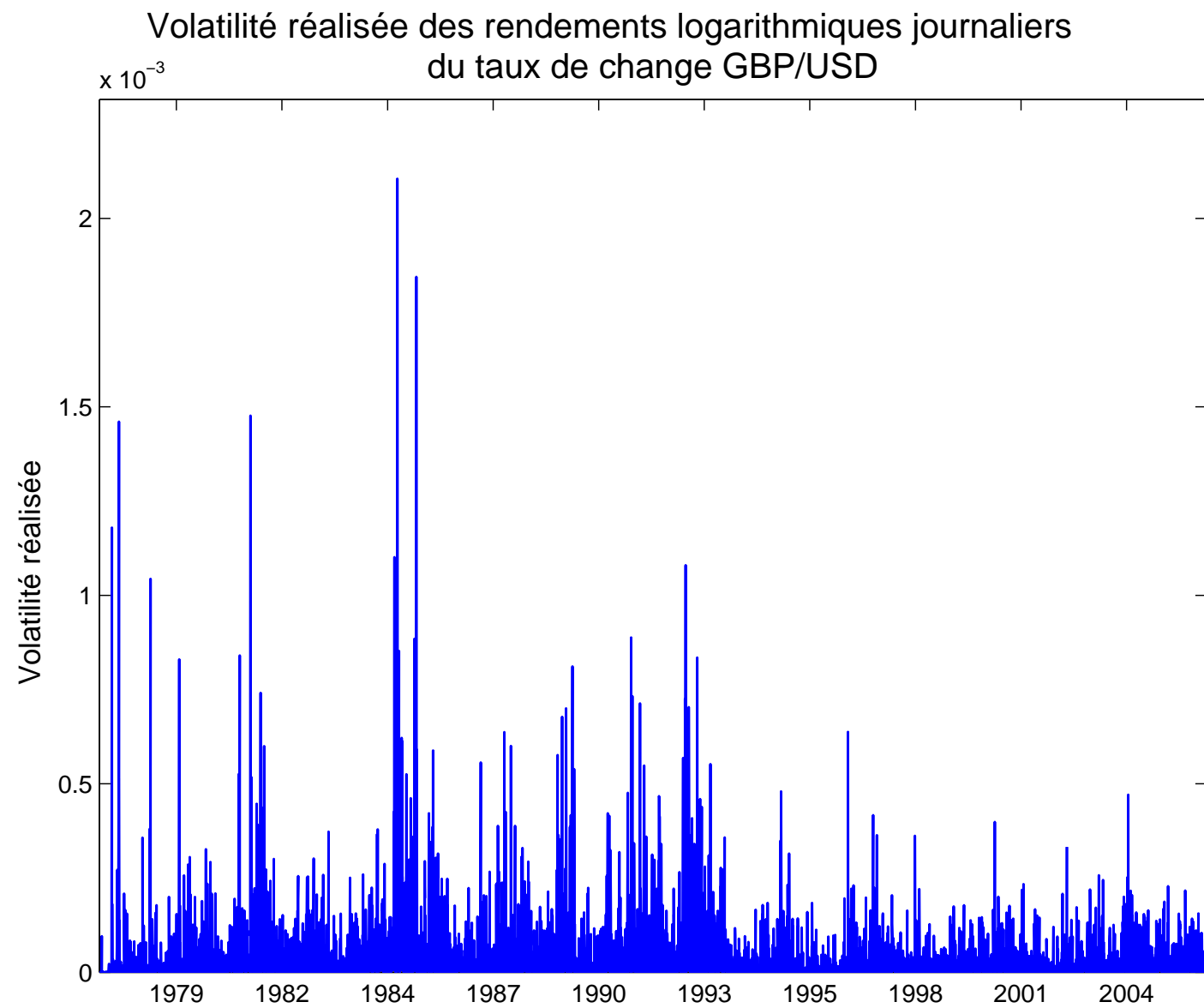


FIG. 5 – Thèse Alexei Kozhemyak - CMAP - Ecole Polytechnique 2007.

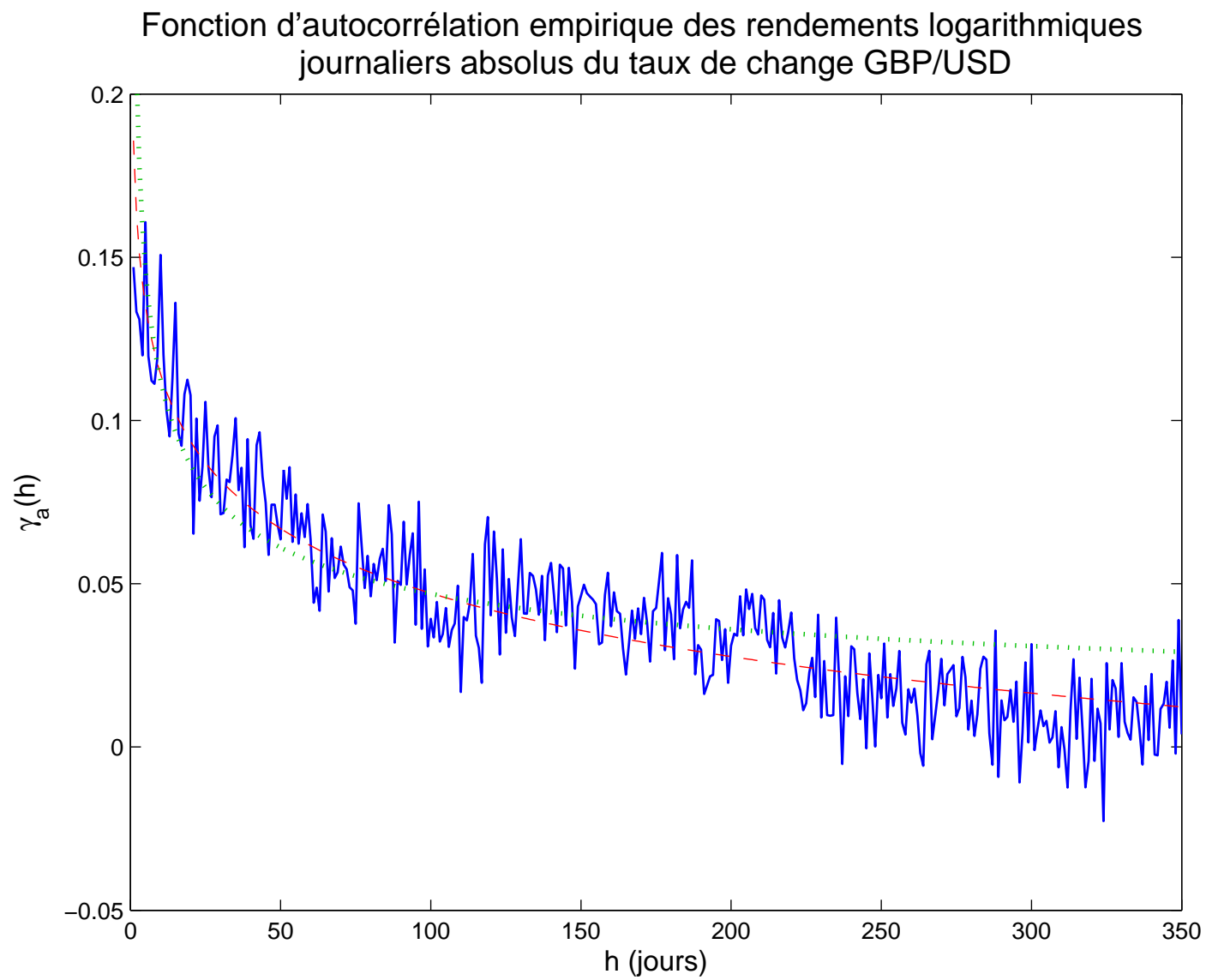


FIG. 6 – Thèse Alexei Kozhemyak - CMAP - Ecole Polytechnique 2007.

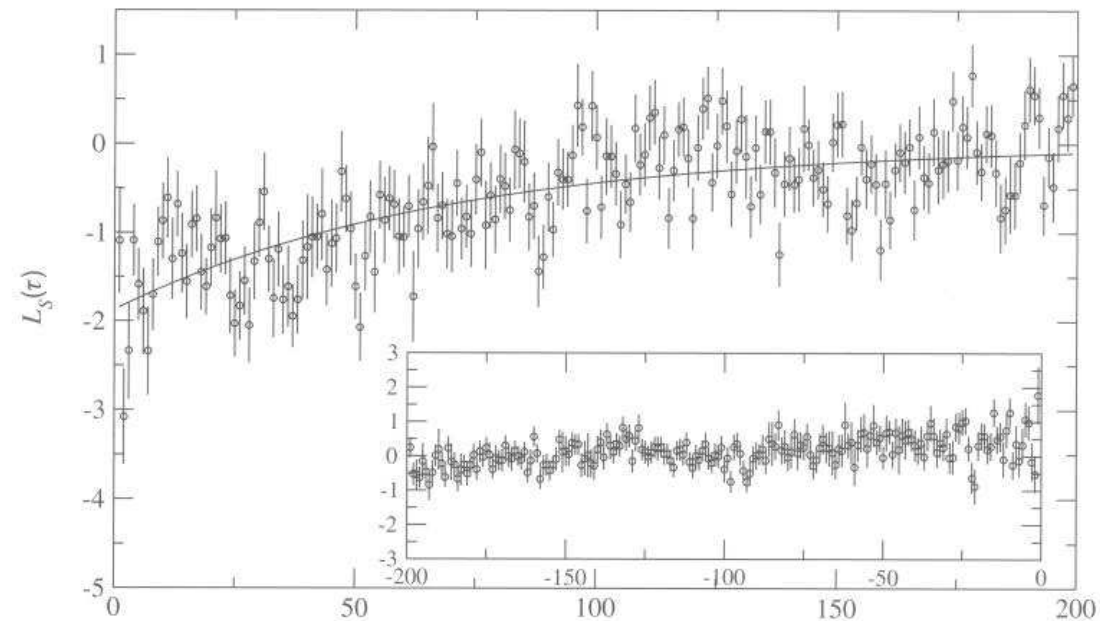


	GARCH(1,1) gaussien				GARCH(1,1) Student				
	$\omega$	$\alpha_1$	$\beta_1$	$\alpha_1 + \beta_1$	$\nu$	$\omega$	$\alpha_1$	$\beta_1$	$\alpha_1 + \beta_1$
CAD/USD	0.0020	0.135	0.845	0.980	6.686	0.0015	0.116	0.868	0.984
JPY/USD	0.0215	0.091	0.863	0.954	3.877	0.0114	0.104	0.888	0.992
CHF/USD	0.0160	0.090	0.888	0.978	5.805	0.0116	0.077	0.909	0.986
GBP/USD	0.0134	0.066	0.905	0.971	4.915	0.0021	0.072	0.928	1.000

TAB. 3 – Thèse Alexei Kozhemyak - CMAP - Ecole Polytechnique 2007.

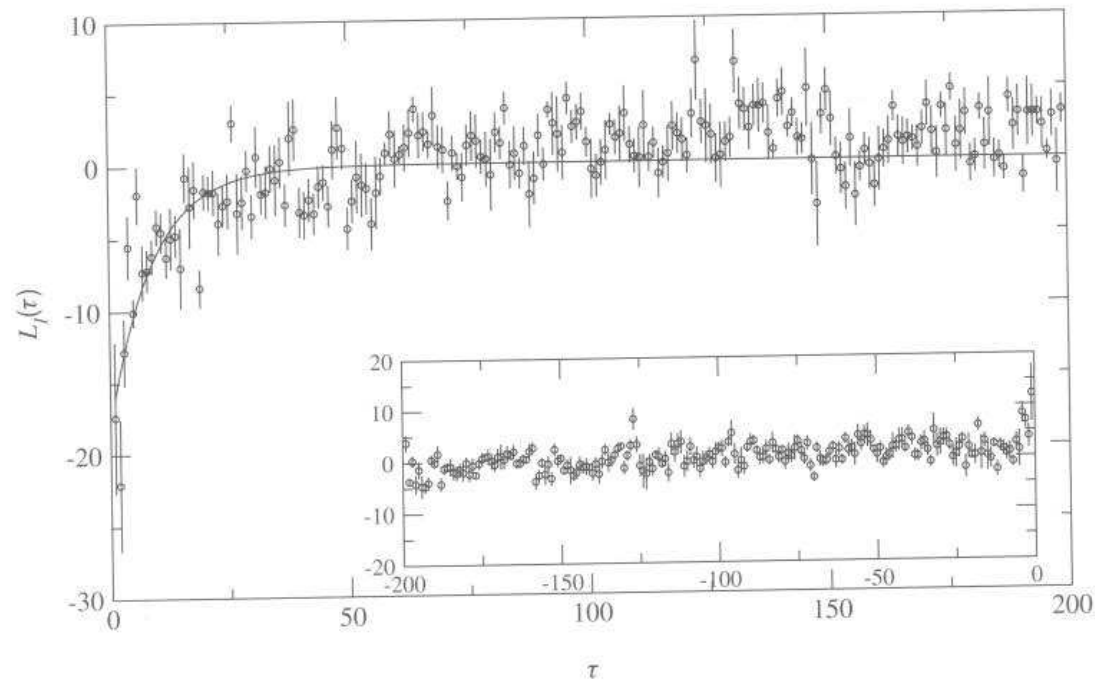
	GARCH(1,1) gaussien				GARCH(1,1) Student				
	$\omega$	$\alpha_1$	$\beta_1$	$\alpha_1 + \beta_1$	$\nu$	$\omega$	$\alpha_1$	$\beta_1$	$\alpha_1 + \beta_1$
Accor	0.127	0.085	0.883	0.967	6.527	0.066	0.086	0.901	0.987
Air Liquide	0.036	0.068	0.920	0.988	8.721	0.030	0.070	0.921	0.991
Alcatel	0.128	0.150	0.850	1.000	5.161	0.067	0.114	0.886	1.000
Axa	0.080	0.097	0.886	0.984	7.795	0.057	0.095	0.895	0.990
Bouygues	0.053	0.072	0.919	0.991	5.491	0.048	0.080	0.915	0.995
Capgemini	0.153	0.086	0.899	0.985	5.235	0.173	0.121	0.870	0.991
Carrefour	0.043	0.061	0.926	0.987	7.372	0.037	0.061	0.929	0.990
Casino	0.070	0.050	0.930	0.981	5.159	0.093	0.079	0.900	0.979
Danone	0.050	0.072	0.907	0.979	6.344	0.037	0.074	0.912	0.986
Essilor	0.311	0.110	0.812	0.922	4.122	0.152	0.098	0.873	0.971
L'Oréal	0.072	0.092	0.891	0.983	13.689	0.061	0.086	0.900	0.986

TAB. 4 – Thèse Alexei Kozhemyak - CMAP - Ecole Polytechnique 2007.



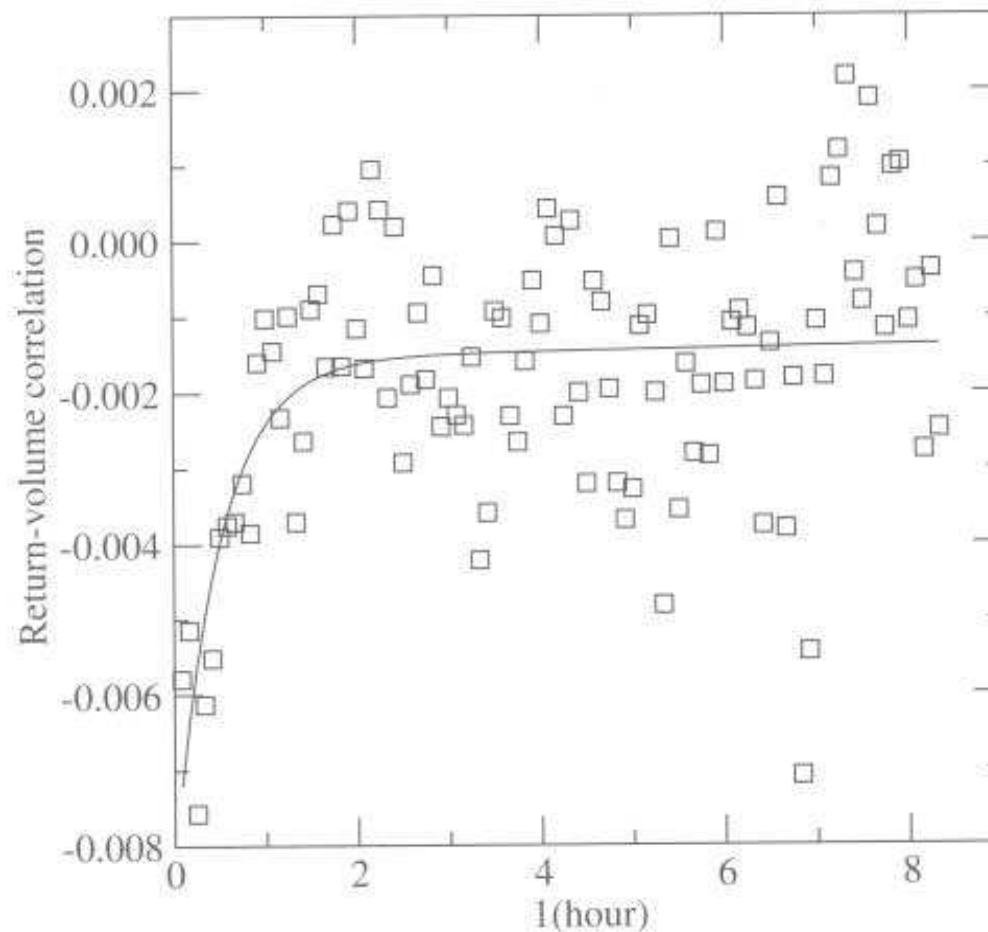
Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.



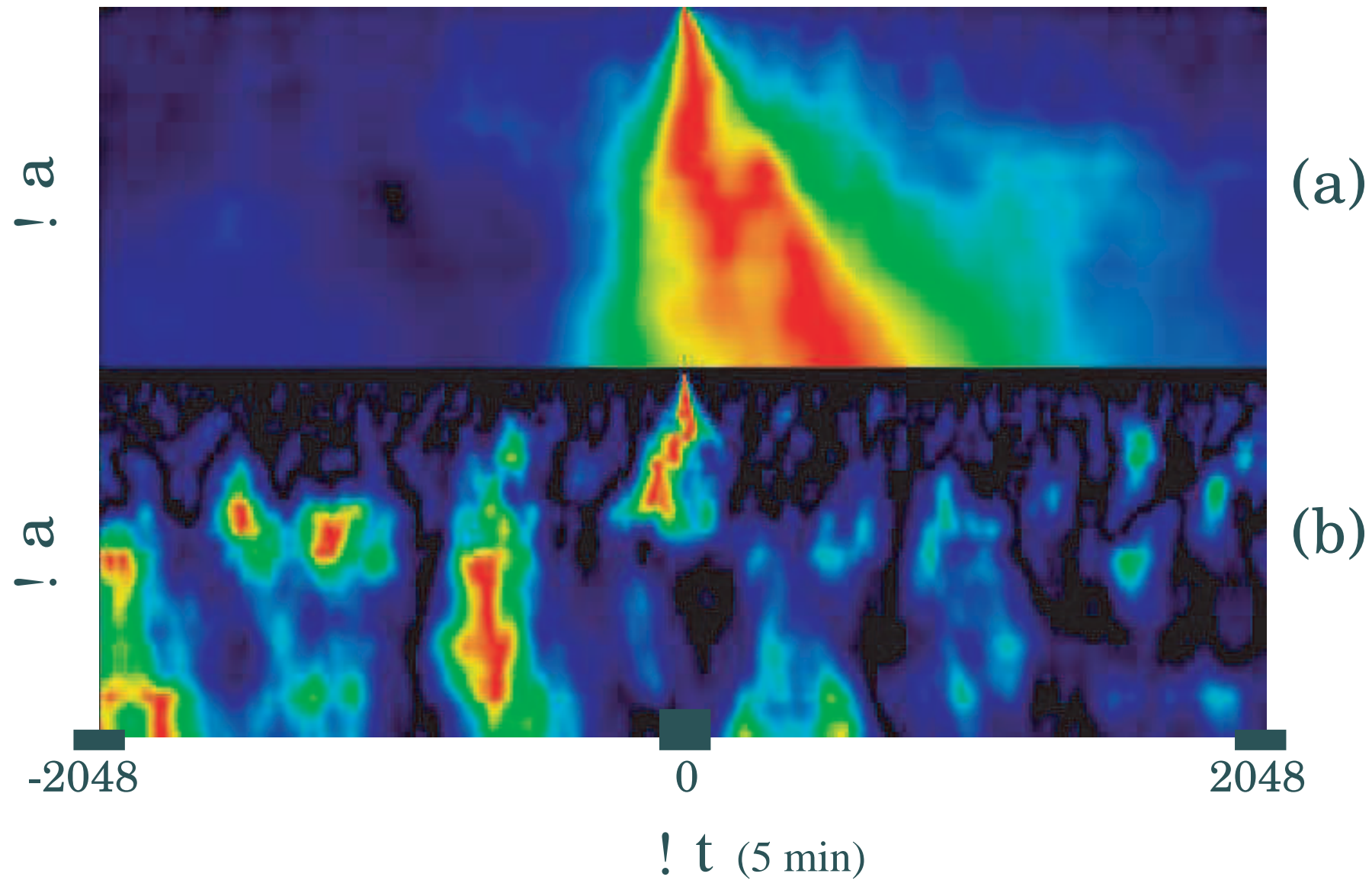
Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.



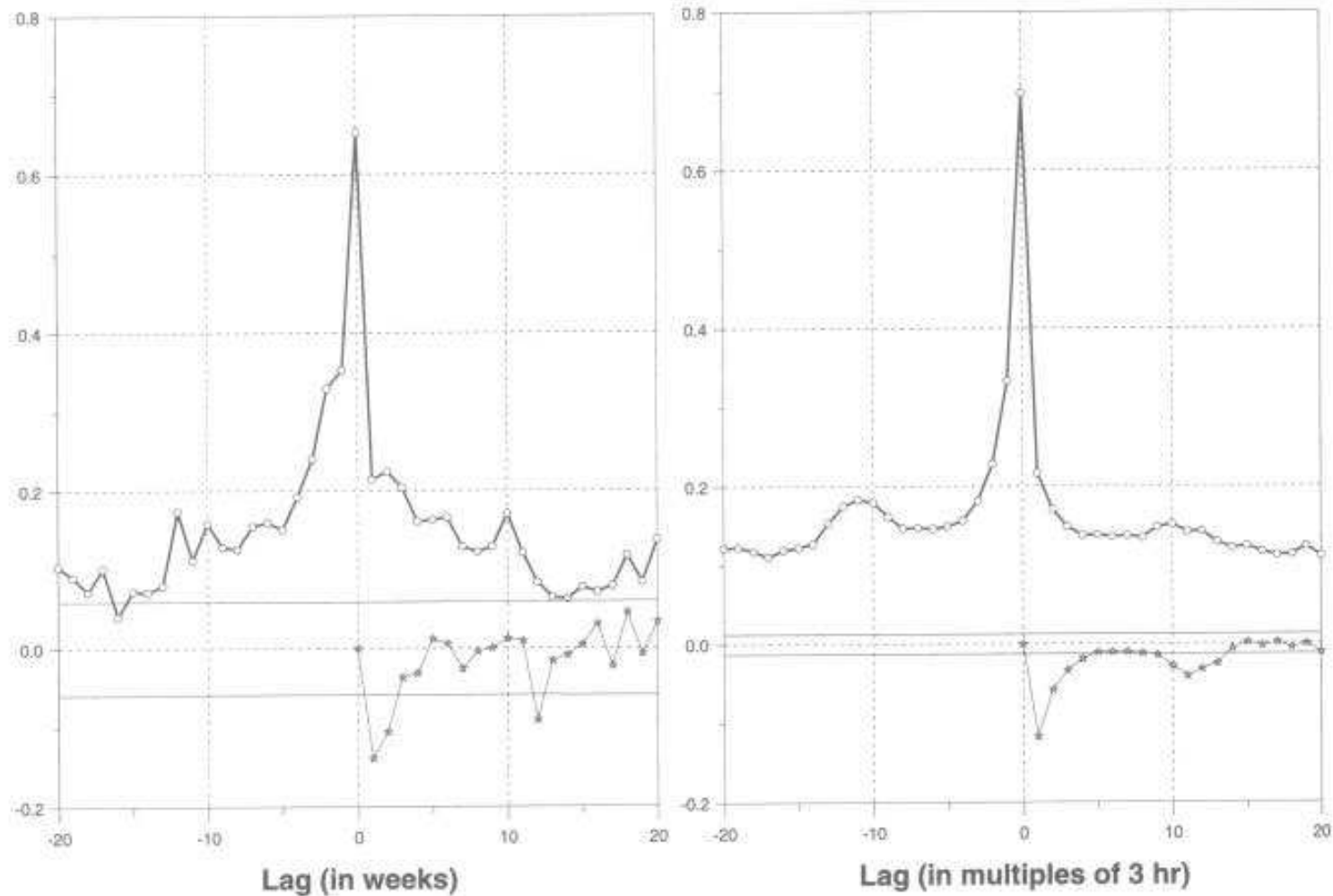
Bouchaud J.-Ph. and Potters M.

Theory of Financial Risk and Derivative Pricing : From Statistical Physics to Risk Management Cambridge University Press, (2nd edition) 2003.



Arneodo, Muzy, Sornette

"Direct" causal cascade in the stock market, Eur. Phys. J. B **2**, **2** (1998).



Michael Dacorogna, Ramazan Gencay and Ulrich A Muller  
An Introduction to High-Frequency Finance Academic Press (May 2001)