MSB Priority Reduction Rule: A Recursive Discrete Model for Sequential State Reduction

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November 2025

Abstract

This paper introduces the MSB Priority Reduction Rule, a discrete recursive method for sequentially reducing an initial state vector a_0 to the zero vector. The process follows a predefined priority sequence that determines which component of the state vector is decremented at each step. We formally define the rule, demonstrate it with an example, and discuss its properties, including determinism and termination. Applications of this rule span discrete systems, scheduling, resource allocation, and sequential game analysis.

1 Introduction

Modeling discrete processes where the system state is reduced according to a predefined priority sequence has applications ranging from computer science (e.g., scheduling) to discrete systems theory. In this note, we introduce a simple but rigorous priority-based reduction rule called the **MSB Priority Reduction Rule**.

2 Basic Definitions

Definition 1 (State Space). *Let the initial state vector a_0 be defined in $\mathbb{Z}^4_{\geq 0}$ as:*

$$a_0 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

Definition 2 (Priority Sequence). *Define a* ***priority sequence*** *P of length $L = 7$:*

$$P = (p_0, p_1, p_2, p_3, p_4, p_5, p_6) = (1, 2, 3, 4, 1, 2, 1)$$

This sequence determines which component is reduced at each step.

3 Recursive Relation

3.1 Mathematical Definition

The reduction sequence a_n for $n = 0, \dots, L-1$ is defined recursively:

$$a_{n+1} = a_n - e_{p_n}, \quad n = 0, \dots, 6$$

where e_{p_n} is a standard unit vector in $\mathbb{R}^4$ with 1 at component p_n and 0 elsewhere.

3.2 Sequence Example

Applying the recursive rule produces:

$$\begin{pmatrix}3\\2\\1\\1\end{pmatrix} \xrightarrow{p_0=1} \begin{pmatrix}2\\2\\1\\1\end{pmatrix} \xrightarrow{p_1=2} \begin{pmatrix}2\\1\\1\\1\end{pmatrix} \xrightarrow{p_2=3} \begin{pmatrix}2\\1\\0\\1\end{pmatrix} \xrightarrow{p_3=4} \begin{pmatrix}2\\1\\0\\0\end{pmatrix} \xrightarrow{p_4=1} \begin{pmatrix}1\\1\\0\\0\end{pmatrix} \xrightarrow{p_5=2} \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \xrightarrow{p_6=1} \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

4 Properties

Theorem 1 (Termination)**.** *The process always reaches the zero vector after L steps provided:*

$$\sum_{i=1}^{4} a_{0,i} = \sum_{n=0}^{L-1} \delta_{i,p_n} \quad \forall i \in \{1,2,3,4\}$$

which holds in this example.

Theorem 2 (Determinism)**.** *Given the fixed priority sequence P, the process is fully deterministic.*

5 Applications

- **Queueing systems:** Modeling service processes with rotating priorities.
- **Resource management:** Allocating resources in computer systems with predefined access patterns.
- **Game theory:** Analyzing sequential moves in turn-based games.

6 Conclusion

We defined the MSB Priority Reduction Rule as a discrete recursive relation. This rule models the state transition of a vector system following a fixed priority sequence. The example with

$$a_0 = \begin{pmatrix}3\\2\\1\\1\end{pmatrix} \text{ and priority sequence } (1,2,3,4,1,2,1)$$

provides a formal mathematical template for this reduction pattern.

References

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