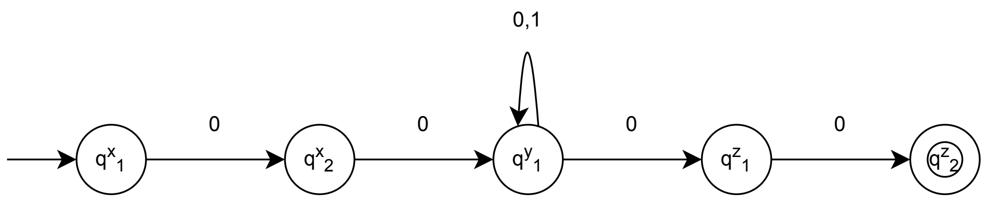

Problem 1.71. Let $\Sigma = 0, 1$.

Part a. Let $A = \{0^k u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that A is regular.

Proof Idea. State diagram of an NFA that recognizes $A_{k=2}$.



Proof. The proof is by construction. Construct the NFA $N = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$ that recognizes A:

1. $Q = Q^x \cup Q^y \cup Q^z$, where:
$$Q^x = \{q^x_1,\ q^x_2,\ q^x_3,\ \cdots,\ q^x_k\}$$
$$Q^y = \{q^y_1\}$$
$$Q^z = \{q^z_1,\ q^z_2,\ q^z_3,\ \cdots,\ q^z_k\}$$
2. $q_0 = q^a_1$
3. $F = \{q^c_k\}$
4. Define δ so that for any $q^u_i \in Q$ and any $a \in \Sigma$:
$$\delta(q^u_i,\ a) = \begin{cases} \{q^x_{i+1}\} & u = x\ ,\ i < k \text{ and } a = 0 \\ \{q^y_1\} & u = x\ ,\ i = k \text{ and } a = 0 \\ \{q^y_1,\ q^z_1\} & u = y\ ,\ i = 1 \text{ and } a = 0 \\ \{q^y_1\} & u = y\ ,\ i = 1 \text{ and } a = 1 \\ \{q^z_{i+1}\} & u = z\ ,\ i < k \text{ and } a = 0 \\ \phi & u = z\ ,\ i = k \end{cases}$$

□

Part b. Let $B = \{0^k 1 u 0^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$. Show that B is not regular.

Proof. The proof is by contradiction. Assume to the contrary that B is regular. Let p be the pumping length given by the pumping lemma. Choose s to be the string $0^p 1 0^p$ ($u = \epsilon$). Because $s \in B$ and $|s| \geq p$, the pumping lemma guarantees that s can be split into three pieces, $s = xyz$, where for any $i \geq 0$ the string $xy^iz \in B$. According to the condition 3 of the pumping lemma, y can only be $0s$. The string $xyyz$ has more $0s$ before the 1 then after it, so $xyyz \notin B$, which is a contradiction. □