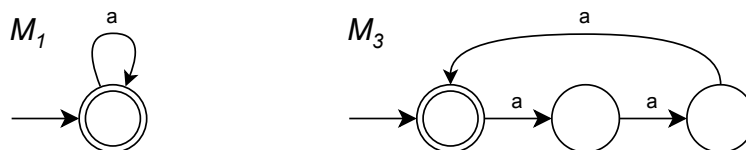**Problem 1.31.** Let $B_n = \{a^k \mid k \text{ is a multiple of } n\}$. Show that for each $n \geq 1$, the language B_n is regular.

Proof Idea. $B_1 = \{\epsilon, a, aa, aaa, aaaa, \cdots\}$ and $B_3 = \{\epsilon, aaa, aaaaaa, aaaaaaaaa, \cdots\}$.

State diagrams of DFAs that recognize B_1 and B_3.

Proof. The proof is by construction. Construct the DFA $M_n = (Q, \Sigma, \delta, q_0, F)$ to recognize B_n:

1. $Q = \{q_1, q_2, q_3, \cdots, q_n\}$, where $n \geq 1$.

2. $\Sigma = \{a\}$

3. $q_0 = q_1$

4. $F = \{q_1\}$

5. Define $\delta(q, a)$ so that for any $q \in Q$ and any $a \in \Sigma$:

$$\delta(q,\ a) = \begin{cases} q_{i+1} & 1 \leq i < n \\ q_1 & i = n \end{cases}$$

$\square$