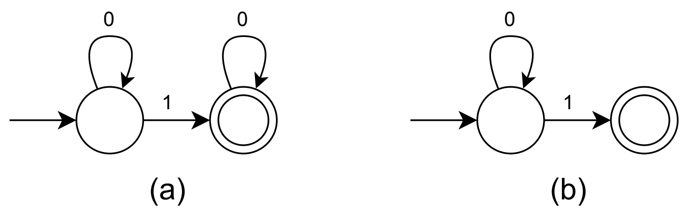
Problem 1.40. Recall that string x is a **prefix** of string y if a string z exists where $xz = y$, and that x is a proper prefix of y if in addition $x \neq y$. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

Part a. $NOPREFIX(A) = \{w \in A \mid no\ proper\ prefix\ of\ w\ is\ a\ member\ of\ A\}$.

Proof Idea. Let the regular expression $R = 0^*10^*$ describe the language A.

$$A = \{1, 01, 10, 010, 001, \cdots\}$$

$$NOPREFIX(A) = \{1, 01, 001, \cdots\}$$



State diagrams of finite automata that recognize A (a) and $NOPREFIX(A)$ (b). Take the finite automaton that recognizes A, and remove all transitions from accept states to any other state to construct the finite automaton to recognize $NOPREFIX(A)$.

Proof. The proof is by construction. Let $N = (Q, \Sigma, \delta, q_0, F)$ be the DFA that recognizes A. Construct the NFA $N' = (Q', \Sigma, \delta', q_0', F')$ to recognize $NOPREFIX(A)$:

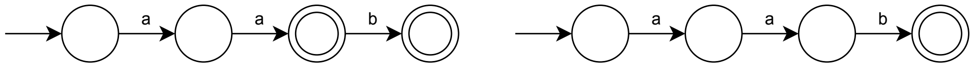
1. $Q' = Q$
2. $q_0' = q_0$
3. $F' = F$
4. Define δ'(q, a) so that for any $q \in Q'$ and any $a \in \Sigma$:

$$\delta'(q,\ a)\ = \begin{cases} \{\delta(q,\ a)\} & q \notin F \\ \phi & q \in F \end{cases}$$

□

Part b. $NOEXTEND(A) = \{w \in A \mid w \; is \; not \; the \; proper \; prefix \; of \; any \; string \; in \; A\}$.

Proof Idea. Let $A = \{aa, aab\}$, then $NOEXTEND(A) = \{aab\}$



(a)

(b)

State diagrams of finite automata that recognize A (a) and $NOEXTEND(A)$ (b). Take the finite automaton that recognizes A, and change any accept state q into non-accept state, if there exists a sequence of transitions from q to any accept state (including q itself).

Proof. The proof is by construction. Let $N = (Q, \Sigma, \delta, q_0, F)$ be the DFA that recognizes A. Construct the DFA $N' = (Q, \Sigma, \delta, q_0, F')$ to recognize $NOEXTEND(A)$:

1. N' has the same states Q, start state q_0 and δ as N.
2. $F' = \{q \mid q \in F \; and \; there \; is \; no \; sequence \; of \; transitions \; from \; q \; to \; any \; accept \; state \; \}$

$\square$