
Problem 5.26. Use Rice's theorem, which appears in Problem 5.28, to prove the undecidability of each of the following languages.

Part b. $\{\langle M \rangle \mid M \text{ is a } TM \text{ and } 1011 \in L(M)\}$.

Proof. Let $P = \{\langle M \rangle \mid M \text{ is a } TM \text{ and } 1011 \in L(M)\}$. P is a language of ***TM*** descriptions. It satisfies the two conditions of Rice's theorem. First, it is nontrivial because languages some ***TM***s contain 1011 and others do not. Second, it depends only on the language. If two ***TM***s recognize the same language, either both have descriptions in P or neither do. Consequently, Rice's theorem implies that P is undecidable. □

Part c. $ALL_{TM} = \{\langle M \rangle \mid M \text{ is a } TM \text{ and } L(M) = \Sigma^*\}$.

Proof. ALL_{TM} is a language of ***TM*** descriptions. It satisfies the two conditions of Rice's theorem. First, it is nontrivial because languages some ***TM***s contain all possible strings and others do not. Second, it depends only on the language. If two ***TM***s recognize the same language, either both have descriptions in ALL_{TM} or neither do. Consequently, Rice's theorem implies that ALL_{TM} is undecidable. □