

Problem 4.24. A *useless state* in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

Proof. Let $U_{PDA} = \{\langle M \rangle \mid M \text{ is a PDA and } M \text{ contains a useless state}\}$. To show that U_{PDA} is decidable, we present a **TM** I that decides it.

$I =$ “On input $\langle M \rangle$, where M is a PDA:

1. Check all states that are reachable in M by performing *Depth-First-Search* on M 's start state.
2. Each time a state is visited by *Depth-First-Search* procedure, mark it.
3. If some state is unmarked after *Depth-First-Search* finishes, *accept*. Otherwise *reject*.”

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