
Problem 1.66. A homomorphism is a function $f : \Sigma \longrightarrow \Gamma^*$ from one alphabet to strings over another alphabet. We can extend f to operate on strings by defining $f(w) = f(w_1)f(w_2)\cdots f(w_n)$, where $w = w_1w_2\cdots w_n$ and each $w_i \in \Sigma$. We further extend f to operate on languages by defining $f(A) = \{f(w) \mid w \in A\}$, for any language A.

Part a. Show, by giving a formal construction, that the class of regular languages is closed under homomorphism. In other words, given a DFA M that recognizes B and a homomorphism f, construct a finite automaton M' that recognizes $f(B)$. Consider the machine M' that you constructed. Is it a DFA in every case?

Proof. Solution Replace this text with the details of your proof or solution. □

Part b. Show, by giving an example, that the class of non-regular languages is not closed under homomorphism.

Proof. Solution Replace this text with the details of your proof or solution. □