
Problem 1.38. An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if every posible state that M could be in after reading input x is a state from F. Note, in contrast, that an ordinary NFA accepts a string if some state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

Proof. The proof is in two parts. In the first part, show that for any regular language A and an NFA N that recognizes it, there exists an all-NFA M that reocognizes the complement of A. In the second part, use the result of the first part to show that for every regular language there exists an all-NFA that recognizes it.

Part 1. Let A be any regular language, and let N be an NFA that recognizes A. Use the construction give in the Theorem 1.39 to construct an equivalent DFA D for N that also recognizes A. Swap the accept and non-accept states of D to get a new DFA D' that accepts the complement of A[1].

Let $D' = (Q', \Sigma, \delta', q'_0, F')$. Construct the all-NFA $M = (Q, \Sigma, \delta, q_0, F)$ by reversing the construction of the Theorem 1.39.

1. $Q = Q'$
2. $q_0 = \{q'_0\}$

Part 2. □

[1]Exercise 1.14 a.