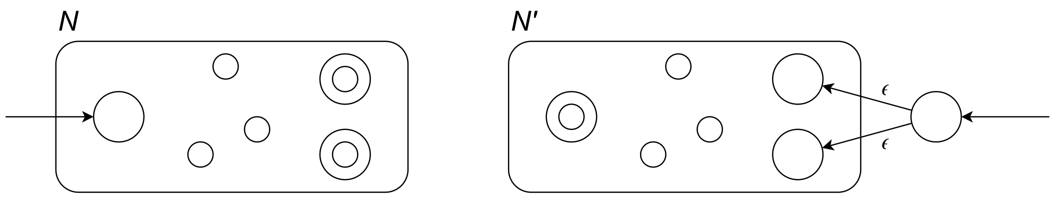
Problem 1.31. For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written w^R , is the string w in reverse order, $w_n \cdots w_1 w_2$. For any language A, let $A^R = \{w^R \,|\, w \,\epsilon\, A\}$. Show that if A is regular, so is A^R.

Proof Idea. If A is regular, then there exists a DFA, say N that recognizes it. Take N and construct a new NFA N' to recognize A^R. N' has all the states of N, but the transitions are reversed. Start state of N is the accept state of N' has a new start state, say q_s with ϵ transitions to every accept state in N.



When N computes on some $w \,\epsilon\, A$, it starts at the accept state, tranisitions through some intermediate states and finally stops at some accept state, whereas N' starts simultaneously from all the accept states of N and transitions backwards to reach the start state of N, which is the only accept state in N'.

Proof. The proof is by construction. Let $N = (Q, \Sigma, \delta, q_0, F)$ be the DFA that recognizes A. Construct $N' = (Q', \Sigma, \delta', q_0', F')$ to recognize A^R:

1. $Q' = Q \cup \{q_s\}$
2. q_s is the start state
3. $F' = \{q_0\}$
4. Define δ'(q, a) so that for any $q \,\epsilon\, Q'$ and any $a \,\epsilon\, \Sigma_\epsilon$:

$$\delta'(q,\ a) \ = \begin{cases} F & q = q_s \ and \ a \ = \ \epsilon \\ \phi & q = q_s \ and \ a \ \neq \ \epsilon \\ \{q' \mid q' \ \epsilon \ Q \ and \ \delta(q',\ a) \ = \ q\} & q \ \epsilon \ Q \end{cases}$$

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