

**Problem 4.17.** Prove that  $EQ_{DFA}$  is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.

*Proof.* We present a **TM**  $I$  that decides  $EQ_{DFA}$  by testing the two DFAs on all strings up to a certain size.

$I =$  “On input  $\langle A, B \rangle$ , where  $A$  and  $B$  are DFAs:

1. Let  $k = \max(m, n)$ , where  $m$  and  $n$  are the number of states in DFAs  $A$  and  $B$  respectively.
2. Generate list of all possible strings  $w_1, w_2, \dots$ , where each  $w_i = w_1 w_2 \dots w_n$ , and  $n \leq k$ .
3. Repeat the following for each string  $w_i$ :
4.     Run the  $A_{DFA}$  decider  $M$  from Theorem 4.1 on two inputs  $\langle A, w_i \rangle$  and  $\langle B, w_i \rangle$  separately.
5.     If  $M$  accepts one input and rejects the other, *reject*.
6. *Accept*.”

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