

Problem 4.11. Let $INFINITE_{PDA} = \{\langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$. Show that $INFINITE_{PDA}$ is decidable.

Proof. We present a **TM** T that decides $INFINITE_{PDA}$ using the result of the Problem 2.35¹.

$T =$ “On input $\langle M \rangle$, where M is a PDA:

1. Convert M to an equivalent CFG G using the construction given in Lemma 2.27.
2. Convert G to an equivalent grammar G_c in Chomsky normal form.
3. Let b be the number of variables in G_c . Generate all strings s_1, s_2, \dots , where each s_i has a derivation of 2^b steps in G_c .
4. Run the decider S for A_{CFG} given in Theorem 4.7 on each s_i .
5. If S accepts some s_i , then *accept*, otherwise *reject*.”

□

¹Let G be a CFG in Chomsky normal form that contains b variables. If G generates some string with a derivation having at least 2^b steps, then $L(G)$ is infinite.