
Problem 1.47. Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1\#x_2\#\cdots\#x_k \ for\ k \geq 0,\ each\ x_i \in 1^*, and\ x_i \neq x_j\ for\ i \neq j\}.$$

Prove that Y is not regular.

Proof Idea. For $k = 1$ and $k = 2$, some of the members of Y are:

$$Y = \{\epsilon,\ 1,\ 11,\ 111,\ \cdots\ 1\#,\ \#1,\ 11\#,\ \#11,\ 1\#11,\ 11\#1\ \cdots\}.$$

The idea is to choose a string s, for which $k = 2$, and show that x_1 and x_2 can be equal.

Proof. The proof is by contradiction. Assume Y is a ragular language. Let p be the pumping lenght given by the pumping lemma. Choose s to be the string:

$$s = 1^p\#1^{p+p!}$$

The string s is a member of Y, and $|s| \geq p$. Therefore, the pumping lemma guarantees that the string s can be split into three parts, $s = xyz$, and for $i \geq 0$, $xy^iz \in Y$. According to the condition 3 ($|xy| \leq p$) of the pumping lemma, y can only be 1s.

Let $m + n = p$, $x = 1^m$ and $y = 1^n$. Then

$$1^m\left[1^n\right]^i\#1^{p+p!} \in Y.$$

But for $i = 1+\frac{p!}{n}$, the number of 1s on both sides of the # are the same, which is a contradiction. $\square$