
Problem 1.46. Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

Part c. $\{w \mid w \in \{0,1\}^* \textit{ is not a palindrome}\}$

Proof. The proof is by contadiction. Assume A is a regular language. Let p be the pumping lenght given by the pumping lemma. Let s be the string:

$$s = 0^p 1 0^{p+p!}$$

The string s is a member of A, and $|s| \geq p$, therefore the pumping lemma guarantees that s can be split into three pieces, $s = xyz$, where for any $i \geq 0$ the string $xy^iz \in A$. According to the condition 3 ($|xy| \leq p$) of the pumping lemma, y consists of 0s. Let $m + n = p$, where $0 \leq m < p$ and $1 \leq n \leq p$. Then

$$0^m \left[0^n\right]^i 1 0^{p+p!} \in A.$$

But for $i = 1 + \frac{p!}{n}$, the number of 0s on both sides of the 1 become the same and the resulting string is a palindrome, which is a contradiction. $\square$

Part d. $\{wtw \mid w,\ t \in \{0,1\}^+\}$

Proof. $\square$