
Problem 4.16. Let $A = \{\langle R \rangle \mid R \textit{ is a regular expression describing a language containing at least one string } w \textit{ that has } 111 \textit{ as a substring } (\textit{i.e., } w = x111y \textit{ for some } x \textit{ and } y)\}$. Show that A is decidable.

Proof. We present a **TM** M that decides A.

$M =$ "On input $\langle R \rangle$, where R is a regular expression:

1. Construct a DFA D, such that $L(D) = L(R) \cap \Sigma^*111\Sigma^*$.
2. Test $L(D) = \phi$ using the E_{DFA} decider T from Theorem 4.4.
3. If T accepts, *reject*; if T rejects, *accept*."

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