
Problem 7.17. Let $UNARY\text{-}SSUM$ be the subset sum problem in which all numbers are represented in unary. Why does the NP-completeness proof for $SUBSET\text{-}SUM$ fail to show $UNARY\text{-}SSUM$ is NP-complete? Show that $UNARY\text{-}SSUM \in P$.

Part a. Why does the NP-completeness proof for $SUBSET\text{-}SUM$ fail to show $UNARY\text{-}SSUM$ is NP-complete?

Part b. Show that $UNARY\text{-}SSUM \in P$.

Proof Idea. The $UNARY\text{-}SSUM$ problem involving $S = \{x_1, \cdots, x_k\}$, and t reduces to testing if $t \in S^*$. For example, let $S_1 = \{\varepsilon, 11, 1111\}$, and $t_1 = 111111$. Clearly, $t_1 \in S_1^*$, therefore $\langle S_1, t_1 \rangle \in UNARY\text{-}SSUM$. S is finite, therefore membership in S, $MEMBER_S = \{t \mid t \in S\}$ can be decided in polynomial time. $MEMBER_S \in P$, and P is closed under the star operation [1], therefore $UNARY\text{-}SSUM \in P$.

Proof. Let

$$\begin{aligned} UNARY\text{-}SSUM = \{\langle S,t\rangle \mid S = \{x_1, \cdots, x_k\},\ &\text{and for some} \\ \{y_1, \cdots, y_l\} \subseteq \{x_1, \cdots, x_k\},\ &\text{we have } \Sigma y_i = t, \\ &\text{and } x_i,\ y_j \text{ and } t \text{ are represented in unary}\}. \end{aligned}$$

To show $UNARY\text{-}SSUM \in P$, we give a polynomial time reduction from $UNARY\text{-}SSUM$ to $MEMBER_S$ as described above.

$F =$ "On input $\langle S, t\rangle$, where $S = \{x_1, \cdots, x_k\}$, each x_i and t is a non-negative number represented in unary:

1. If $t = \varepsilon$ and $\varepsilon \notin S$, then output $\langle \{11\}, 1\rangle$.
2. Output $\langle S, t\rangle$."

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[1] See solution to Problem 7.15.