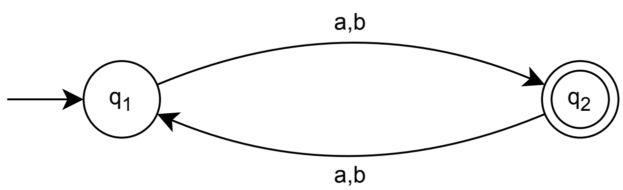
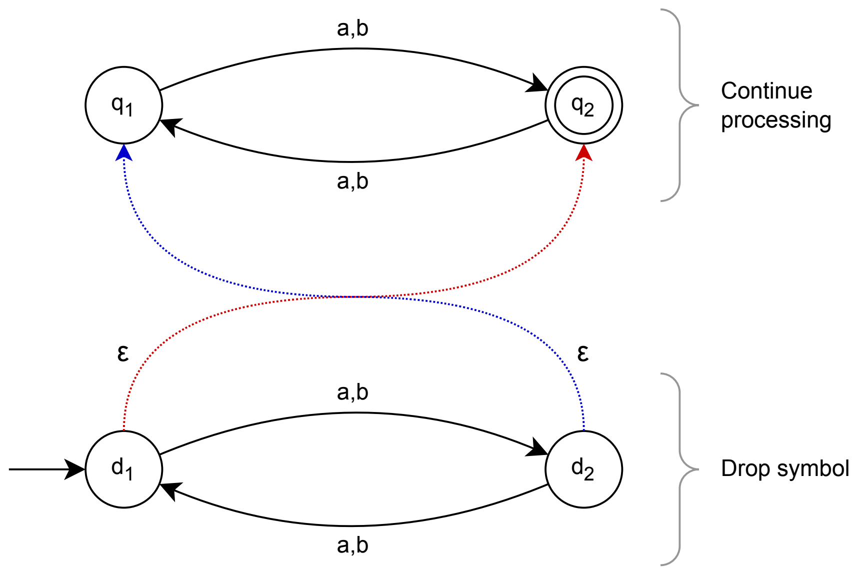
Problem 1.43. Let A be any language. Define $DROP-OUT(A)$ to be the language containing all strings that can be obtained by removing one symbol from a string in A. Thus, $DROP-OUT(A) = \{xz \mid xyz \in A \; where \; x, \; z \in \Sigma^*, \; y \in \Sigma\}$. Show that the class of regular languages is closed under the $DROP-OUT$ operation. Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

Part 1. Proof by picture. Let $\Sigma = \{a, b\}$ and $A = \{w \mid w \; has \; odd \; length\}$. A contains the strings a, b, aaa, aab, and bbb. $DROP - OUT(A)$ contains all strings of even length, such as ϵ, aa, ab, and bb.



The above DFA recognizes A.

The idea is to construct a finite automaton using the DFA of A, which drops one symbol at a time in strings of A. Take the DFA that recognizes A, for each state, keep all of its transitions, but also skip all transitions to the next states by adding ϵ-transitions, because each symbol is represented by a tranistion in automata. After skipping a symbol, we need to make sure that no subsequent symbols are skipped. This can be achieved by using two copies of A's DFA in the new finite automaton. One DFA selects a position to drop a symbol, while the other continues normally after that. The two DFAs are connected using ϵ-transitions that simulate dropping of a symbol.



NFA that recognizes $DROP - OUT(A)$.