
Problem 1.41. Let B and C be languages over $\Sigma = \{0,1\}$. Define

$$B \overset{1}{\leftarrow} C = \{w \in B \mid for\ some\ y \in C,\ strings\ w\ and\ y\ contain\ equal\ numbers\ of\ 1s\}.$$

Show that the class of regular languages is closed under the $\overset{1}{\leftarrow}$ operation.

Proof. To show that the operation $B \overset{1}{\leftarrow} C$ is closed under the class of regular languages we use the closure properties of operations intersection, $Zero^-$ and $Zero^+$, and give a regular expression that uses these operations to describes operation $B \overset{1}{\leftarrow} C$.

Let A be any language over $\Sigma = \{0,1\}$. Define

$$Zero^-(A) = \{w \mid w\ is\ constructed\ by\ removing\ all\ 0s\ from\ some\ string\ in\ A\}.$$

$$Zero^+(A) = \{w \mid w\ is\ constructed\ by\ inserting\ any\ number\ of\ 0s\ anywhere\ in\ some\ string\ of\ A\}.$$

The closure properties of $Zero^-$ and $Zero^+$ are proved in Corollaries 1 and 2. Let B and C be any regular languages over $\Sigma = \{0,1\}$. The regular expression that describes the operation $B \overset{1}{\leftarrow} C$ is:

$$B \overset{1}{\leftarrow} C = Zero^+\Big(Zero^-(B) \cap Zero^-(C)\Big) \cap B$$

□

Corollary 1. Show that the class of regular languages is closed under the $Zero^-$ operation.

Proof. □

Corollary 2. Show that the class of regular languages is closed under the $Zero^+$ operation.

Proof. □