Problem 1.41. For languages A and B, let the perfect shuffle of A and B be the language

$$\{w \mid w = a_1b_1\cdots a_kb_k, where \ a_1\cdots a_k \in A \ and \ b_1\cdots b_k \in B, \ each \ a_i,b_i \in \Sigma\}.$$

Show that the class of regular languages is closed under perfect shuffle.

 $Proof\ Idea.$

Proof.