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**Problem 3.9.** Let a $k$-PDA be a pushdown automaton that has $k$ stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.

**Part a.** Show that 2-PDAs are more powerful than 1-PDAs.

*Proof.* A 2-PDA can simulate any 1-PDA by using only one of it's stacks. Therefore, 2-PDAs can recognize all the languages that are recognized by 1-PDAs. Additionally, we show that 2-PDAs can also recognize more languages, which are not recognized by 1-PDAs. For example, the language $A = \{a^n b^n c^n \mid n \geq 0\}$. The language $A$ is not a context free language[1], but we can construct a 2-PDA to recognize $A$ as follows:

1. Read and push $a$'s on both stacks until a $b$ is read.
2. Once a $b$ is read, match it with an $a$ by popping an $a$ from the first stack. Keep reading and matching any subsequent $b$'s. Reject if the number of $a$'s and $b$'s are not equal.
3. Next, match $c$'s with $a$'s on the second stack. Accept if the number of $a$'s and $c$'s are equal, reject otherwise.
4. In above steps, also make sure the input string is in $a^*b^*c^*$.

□

**Part b.** Show that 3-PDAs are not more powerful than 2-PDAs.

*Proof.* □

[1] Example 2.36, Chapter 2.