
Problem 5.23. Show that A is decidable iff $A \leq_m 0^*1^*$

The proof is in two parts.

Part a. If A is decidable, then $A \leq_m 0^*1^*$

Proof. Let A be any decidable language and let R be a decider for A. To show $A \leq_m 0^*1^*$, we give a reduction f from A to 0^*1^*.

The following machine F computes reduction f.

$F =$ "On input w:

1. Run R on w.
2. If R accepts, output 01. Otherwise, output 10."

□

Part a. If $A \leq_m 0^*1^*$, then A is decidable.

Proof. If $A \leq_m 0^*1^*$, then there exists a computable function f from A to 0^*1^*. Let F be the **TM** that computes f. We can construct the decider S for A as follows.

$S =$ "On input w:

1. Run F on w to compute $f(w)$.
2. Construct a DFA D, such that $L(D) = 0^*1^*$.
3. Run the decider M for A_{DFA} from Theorem 4.1 on input $\langle D, f(w) \rangle$
4. If M accepts, *accept*. Otherwise, *reject*."

□