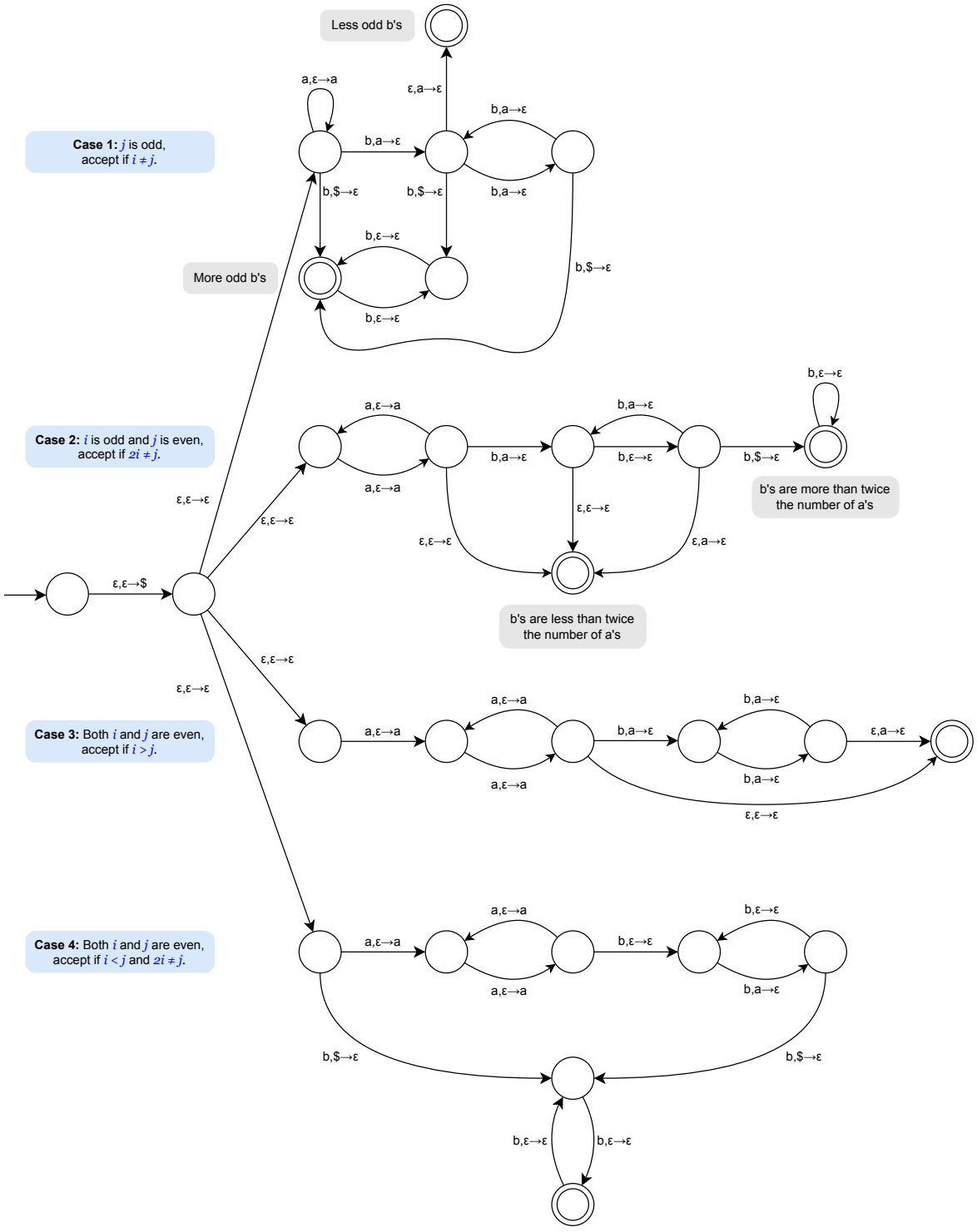

Problem 2.24. Let $E = \{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$. Show that E is a context-free language.

Proof Idea. There are four cases when we take into account the parity of i and j:

1. If j is odd, then accept if $i \neq j$.
2. If i is odd and j is even, then accept if $2i \neq j$.
3. If both i and j are even, then accept if $i > j$.
4. If both i and j are even, then accept if $i < j$.

Proof. Proof is by construction. Construct a PDA that non-deterministically tests the four cases mentioned above to recognize the language E. □



A PDA that recognizes E. It non-deterministically tests the four cases based on the parity of i and j.