Problem 5.30. Use Rice's theorem, which appears in Problem 5.28, to prove the undecidability of each of the following languages.

Part b. $\{\langle M \rangle \mid M$ is a TM and $1011 \in L(M)\}$.

Proof. Let $P = \{\langle M \rangle \mid M$ is a TM and $1011 \in L(M)\}$. P is a language of **TM** descriptions. It satisfies the two conditions of Rice's theorem. First, it is nontrivial because languages some **TM**s contain 1011 and others do not. Second, it depends only on the language. If two **TM**s recognize the same language, either both have descriptions in P or neither do. Consequently, Rice's theorem implies that P is undecidable. $\qquad\square$

Part c. $ALL_{TM} = \{\langle M \rangle \mid M$ is a TM and $L(M) = \Sigma^*\}$.

Proof. ALL_{TM} is a language of **TM** descriptions. It satisfies the two conditions of Rice's theorem. First, it is nontrivial because languages some **TM**s contain all possible strings and others do not. Second, it depends only on the language. If two **TM**s recognize the same language, either both have descriptions in ALL_{TM} or neither do. Consequently, Rice's theorem implies that ALL_{TM} is undecidable. $\qquad\square$