

**Problem 4.13.** Let  $A = \{\langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$ . Show that  $A$  is decidable.

*Proof.* We present a **TM**  $M$  that decides  $A$ .

$M =$  “On input  $\langle R, S \rangle$ , where  $R$  and  $S$  are regular expressions:

1. Construct a DFAs  $B$  and  $C$ , such that  $L(B) = L(R)$ , and  $L(C) = L(S)$ .
2. Test  $L(B) = \phi$  and  $L(C) = \phi$  using the  $E_{DFA}$  decider  $T$  from Theorem 4.4.
3. If  $T$  accepts in both cases, *accept*; if  $T$  accepts in one case and rejects in the other, *reject*.
4. Construct a DFA  $D$  such that  $L(D) = L(R) \cap \overline{L(S)}$ .
5. Test  $L(D) = \phi$  using  $T$ .
6. If  $T$  accepts, *accept*; if  $T$  rejects, *reject*.”

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