
Problem 5.17. Show that the Post Correspondence Problem (PCP) is decidable over the unary alphabet $\Sigma = \{1\}$.

Proof. Let $P_1 = \{\langle P \rangle \mid P \text{ is an instance of PCP over the unary alphabet } \{1\}, \text{ and } P \text{ has a match.}$ We construct a **TM** S that decides P_1 as follows.

$S =$ "On input $\langle P \rangle$, where P is an instance of PCP over the unary alphabet $\{1\}$:

1. Let n be the number of dominoes in P.
2. If $|t_i| > |b_i|$ for all $1 \leq i \leq n$, then *reject*.
3. If $|t_i| < |b_i|$ for all $1 \leq i \leq n$, then *reject*.
4. *Accept*."

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