
Problem 5.26. Define a ***two-headed finite automaton*** (2DFA) to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a 2DFA is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-hand end and one on the right-hand end, that serve as delimiters. A 2DFA accepts its input by entering a special accept state. For example, a 2DFA can recognize the language $a^n b^n c^n \mid n \geq 0$.

Part a. Let $A_{2DFA} = \{\langle M, x\rangle \mid M$ is a 2DFA and M accepts $x\}$. Show that A_{2DFA} is decidable.

Proof. First, we define a notation for representing different configurations of a 2DFA, and then we calculate the number of distinct configurations of a 2DFA for an input of length n.

For a state q and two strings u and v over the input alphabet Σ, we write $q \sqcup \circ\ u \bullet v \sqcup$ for the configuration where the current state is q, the current input is uv, the current first and second head locations are the first symbols of u and v respectively. For a 2DFA with q states, there are exactly $q(n+2)^2$ distinct configurations for an input of length n. Construct decider S for A_{2DFA} as follows.

$S =$ "On input $\langle M, x\rangle$, where M is a 2DFA and x is a string:

1. Let n be the length of string x.
2. Simulate M on x for $q(n+2)^2$ steps or until it halts.
3. If M has halted, *accept* if it has accepted and *reject* if it has rejected. If it has not halted, *reject*."

□

Part a. Let $E_{2DFA} = \{\langle M\rangle \mid M$ is a 2DFA and $L(M) = \emptyset\}$. Show that E_{2DFA} is not decidable.

Proof. □