
Problem 1.55. The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or more. If p is a pumping length for language A, so is any length $p' \geq p$. The minimum pumping length for A is the smallest p that is a pumping length for A. For example, if $A = 01^*$, the minimum pumping length is 2. The reason is that the string $s = 0$ is in A and has length 1 yet s cannot be pumped; but any string in A of length 2 or more contains a 1 and hence can be pumped by dividing it so that $x = 0$, $y = 1$, and z is the rest. For each of the following languages, give the minimum pumping length and justify your answer.

Part a. 0001^*

The minimum pumping length is 4. The strings 0, 00 and 000 cannot be divided and pumped. The string $s = 0001$ can be pumped by dividing it so that $x = 000$, $y = 1$, and z is the rest.

Part d. $0^*1^+0^+1^* \cup 10^*1$

The minimum pumping length for $0^*1^+0^+1^*$ is 2, because all strings in it are at least of length 2, contain at least one 1 and 0, and can be pumped by dividing according to following two cases.

1. If the string s starts with 0, then it can be pumped by dividing it so that $x = \epsilon$, $y = 0$, and z is the rest.

2. If the string s starts with 1, then it can be pumped by dividing it so that $x = \epsilon$, $y = 1$, and z is the rest.

The minimum pumping length for 10^*1 is 3. The strings 1 and 11 can not be pumped. A string of length at least 3 contains at least one 0, so it can be pumped by dividing it so that $x = 1$, $y = 0$, and z is the rest.

The minimum pumping length for $0^*1^+0^+1^* \cup 10^*1$ is 3, because 11 cannot be pumped.