

Problem 1.70. We define the **avoids** operation for languages A and B to be

$$A \text{ avoids } B = \{w \mid w \in A \text{ and } w \text{ doesn't contain any string in } B \text{ as a substring}\}.$$

Prove that the class of regular languages is closed under the *avoids* operation.

Proof Idea. If both A and B are regular languages, then we can build a regular expression that describes $A \text{ avoids } B$ as follows:

1. Construct a language C that consists of all strings that contain strings of B as substring.
2. Take complement of C to construct a new language \overline{C} that consists of all strings that contains all strings that do not contain strings of B as substring.
3. Take intersection of A and \overline{C} to construct $A \text{ avoids } B$.

Proof. To prove that the class of regular languages is closed under the *avoids* operation we use the closure of the class of regular languages under intersection and complement. The regular expression $A \cap (\Sigma^* \cdot B \cdot \Sigma^*)$ describes the language $A \text{ avoids } B$. \square