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**Problem 5.18.** Show that the Post Correspondence Problem is undecidable over the binary alphabet $\Sigma = 0, 1$.

*Proof.* Let $PCP_b = \{\langle P \rangle \mid P \text{ is an instance of } PCP \text{ over binary alphabet } \Sigma = \{0, 1\}\}$. To show that $PCP_b$ is undecidable, we give a reduction from $PCP$ to $PCP_b$. Let $\Sigma_{PCP}$ be a finite set of symbols $a_1, a_2, \ldots, a_n$, where $n \geq 1$. Let $g$ be a function that maps each symbol $a_i$ to the binary representation of its index $i$ using $\lfloor log_2 n \rfloor + 1$ characters. A reduction $f$ from $PCP$ to $PCP_b$ can be constructed using $g$ that converts an instance of $PCP$ to $PCP_b$. As $PCP$ is undecidable and $PCP \leq_m PCP_b$, so according to the Corollary 5.23, $PCP_b$ is also undecidable. $\square$