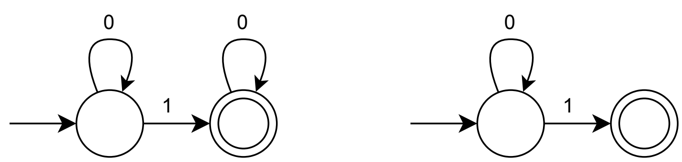
**Problem 1.40.** Recall that string $x$ is a **prefix** of string $y$ if a string $z$ exists where $xz = y$, and that $x$ is a proper prefix of $y$ if in addition $x \neq y$. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

**Part a.** $NOPREFIX(A) = \{w \in A \mid no\ proper\ prefix\ of\ w\ is\ a\ member\ of\ A\}$.

*Proof Idea.* Let the regular expression $R = 0^*10^*$ describe a language $A$.

$$A = \{1, 01, 10, 010, 001, \cdots\}$$

$$NOPREFIX(A) = \{1, 01, 001, \cdots\}$$




(a) (b)

State diagrams of finite automata that recognize $A$ (a) and $NOPREFIX(A)$ (b). Take the finite automaton that recognizes $A$, and remove all transitions from accept states to any other state to construct the finite automaton to recognize $NOPREFIX(A)$.

*Proof.* The proof is by construction. Let $N = (Q, \Sigma, \delta, q_0, F)$ be the DFA that recognizes $A$. Construct the NFA $N' = (Q', \Sigma, \delta', q_0', F')$ to recognize $NOPREFIX(A)$:

1. $Q' = Q$
2. $q_0' = q_0$
3. $F' = F$
4. Define $\delta'$(q, a) so that for any $q \in Q'$ and any $a \in \Sigma$:

$$\delta'(q,\ a) = \begin{cases} \{\delta(q,\ a)\} & q \notin F \\ \phi & q \in F \end{cases}$$

□

**Part b.** $NOEXTEND(A) = \{w \in A \mid w\ is\ not\ the\ proper\ prefix\ of\ any\ string\ in\ A\}$.