Problem 4.26. Let $PAL_{DFA} = \{\langle M \rangle \mid M \text{ is a DFA that accepts some palindrome}\}$. Show that PAL_{DFA} is decidable.

Proof. Let $\Sigma = \{a_1, a_2, \cdots, a_n\}$. The language of all palindromes over Σ is a context-free language, generated by the grammar:

$$
\begin{aligned}
&S \to \varepsilon \\
&S \to a_1 \mid a_2 \mid \cdots \mid a_n \\
&S \to a_1 S a_1 \mid a_2 S a_2 \mid \cdots \mid a_n S a_n
\end{aligned}
$$

Let P be the PDA that recognizes this language. Build a TM I for E, which operates as follows. On input $\langle M \rangle$, where M is a DFA, use M and P to construct a new PDA R that recognizes the intersection of the languages of M and P. Then test whether R's language is empty. If its language is empty, *reject*; otherwise, *accept*. □

Problem 4.27. Let $E = \{\langle M \rangle \mid M \text{ is a DFA that accepts some string with more 1s than 0s}\}$. Show that E is decidable.

Proof. The language of all strings with more 1s than 0s is a context-free language, generated by the grammar $S \longrightarrow T1T,\ T \longrightarrow TT \mid 1T0 \mid 0T1 \mid 1 \mid \varepsilon$. Let P be the PDA that recognizes this language. Build a TM I for E, which operates as follows. On input $\langle M \rangle$, where M is a DFA, use M and P to construct a new PDA R that recognizes the intersection of the languages of M and P. Then test whether R's language is empty. If its language is empty, *reject*; otherwise, *accept*. □