
Problem 5.33. Consider the problem of determining whether a PDA accepts some string of the form $\{ww \mid w \in \{0,1\}^*\}$. Use the computation history method to show that this problem is undecidable.

Proof. Let $B = \{\langle P \rangle \mid P \text{ is a PDA that accepts some string of the form } \{ww \mid w \in \{0,1\}^*\}$. This proof is by contradiction. To get the contradiction, we assume that B is decidable and use this assumption to show that A_{TM} is decidable.

Follow the construction given in Theorem 5.13 to construct a CFG G that generates all strings that are not accepting computation histories for M on w.
We use the construction given in Theorem 5.13 to construct a PDA D
This proof is similar to that of Theorem 5.10 but with a small extra twist: It is a reduction from ATM via computation histories, but we modify the representation of the computation histories slightly for a technical reason that we will explain later. □