
Problem 7.18. Show that if $P = NP$, then every language $A \in P$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.

Proof. Let A be any language in P, except $\emptyset$ and Σ^*. To show that if $P = NP$, then A is NP-complete, we show that $SAT \leq_p A$. Let S be the TM that decides SAT in polynomial time. There exist two strings w_a and w_b, such that $w_a \in A$ and $w_b \notin A$. Construct polynomial time reduction F from SAT to A as follows:

$F =$ "On input $\langle \phi \rangle$, where ϕ is a Boolean formula:

1. Run S on $\langle \phi \rangle$.
2. If S accepts, then output w_a, otherwise output w_b."

□