
Problem 1.67. Let the rotational closure of language A be $RC(A) = \{yx \mid xy \in A\}$.

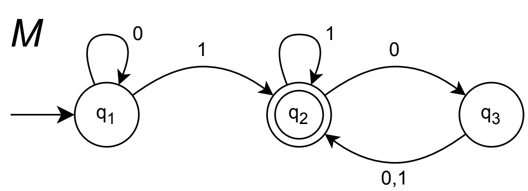
Part a. Show that for any language A, we have $RC(A) = RC(RC(A))$.

Proof. abc
abc y's = a[bc], ab[c], abc[] bca y's = b[ca], bc[a], bca[] cab y's = c[ab], ca[b], cab[]
there are n possible values for y... n1, n1.n2, ... n1.n2.....nn n is length of a string in the language.... w=w1w2....wn
because there are n possible values for y... therefore there are n rotations for each w, n = —w—...... w2w3.....wnw1 .. .. .. wnw1w2....w(n-1) w=w1w2....wn
these rotations ar cyclical.. have same length as the original string w .... and have the same.. applying the rotation operation on any of the rotated string does not generate any new string.
any rotated string is rotate n times... show that: R(w) = R(wr) ... where wr is in R(w)

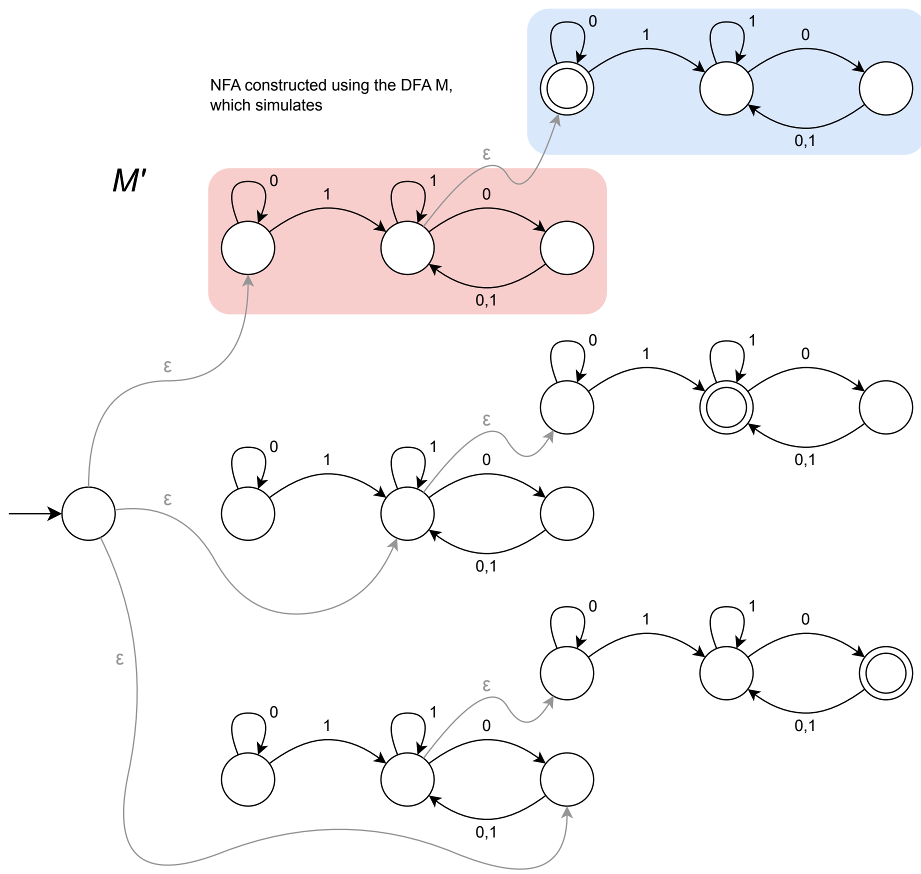
□

Part b. Show that the class of regular languages is closed under rotational closure.

Proof Idea.



State diagram of DFA M that recognizes A.
$A = \{w \mid w \; contains \; at \; least \; one \; 1 \; and \; an \; even \; number \; of \; 0s \; follow \; the \; last \; 1\}$.



State diagram of DFA M' that recognizes $RC(A)$.

Proof. Let $M = (Q, \Sigma, \delta, q_0, F)$ be the DFA that recognizes A. Construct the NFA $M' = (Q', \Sigma, \delta', q_0', F')$ to recognize $RC(A)$.

1. $Q' = Q_s \cup Q_x \cup Q_y$, where

$$Q_s = \{q_s\}\ ,\ Q_x = \bigcup_{i=1}^{|Q|} X_{q_i}\ ,\ Q_y = \bigcup_{i=1}^{|Q|} Y_{q_i}$$

   For each $q \in Q$

$$X_{q_i} = \{(q_i, x, q_1), (q_i, x, q_2), \cdots, (q_i, x, q_n)\}$$
$$Y_{q_i} = \{(q_i, y, q_1), (q_i, y, q_2), \cdots, (q_i, y, q_n)\}$$

   The state q_s is the new start state. For every state in the DFA M, there are two copies of M....

2. $q_0' = q_s$

3. $F' = \{(q_1, x, q_1), (q_2, x, q_2), \cdots, (q_n, x, q_n)\}$, where each $q_i \in Q$.

4. Define δ'(q, a) so that for any $q \in Q'$ and any $a \in \Sigma_\epsilon$: $q = (q_i, x, q_j)$

$$\delta'(q,\ a) \ = \begin{cases} \{(q_1, y, q_1), (q_2, y, q_2), \cdots, (q_n, y, q_n)\} & q = q_s \ and \ a \ = \ \epsilon \\ \{(q_i, x, \delta(q_j, a))\} & q \in Q_x \\ \{(q_i, y, \delta(q_j, a))\} & q \in Q_y \ and \ q_j \notin F \\ \{(q_i, y, \delta(q_j, a))\} & q \in Q_y,\ q_j \in F \ and \ a \neq \epsilon \\ \{(q_i, x, q_0)\} & q \in Q_y,\ q_j \in F \ and \ a = \epsilon \end{cases}$$

□