
Problem 4.21. Let $S = \{\langle M \rangle \mid M \text{ is a } DFA \text{ that accepts } w^R \text{ whenever it accepts } w\}$. Show that S is decidable.

Proof. We present a **TM** I that decides S.

I = "On input $\langle M \rangle$, where M is a DFA:

1. Construct an NFA N, such that $L(N) = S^R$ by following the construction given in solution to Problem 1.31[1].
2. Convert the NFA N to equivalent DFA D.
3. Test $L(M) = L(D)$ using the EQ_{DFA} decider F from Theorem 4.5.
4. If F accepts, *accept*; if F rejects, *reject*."

□

[1]Problem 1.31. For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n \cdots w_1 w_2$. For any language A, let $A^R = \{w^R \mid w \,\epsilon\, A\}$. Show that if A is regular, so is A^R.