

**Problem 6.6.** Describe two different Turing machines,  $M$  and  $N$ , where  $M$  outputs  $\langle N \rangle$  and  $N$  outputs  $\langle M \rangle$ , when started on any input.

*Solution.* Construct **TMs**  $M$  and  $N$ , so that  $L(M) = \Sigma^*$  and  $L(N) = \emptyset$ . Also, both  $M$  and  $N$  obtain intersection of their own description and print it.

$M$  = “On any input:

1. Obtain, via the recursion theorem, own description  $\langle SELF \rangle$ .
2. Construct a new **TM**  $S$ , such that  $L(S) = \overline{\langle SELF \rangle}$ .
3. Print  $\langle S \rangle$ .
4. *Accept.*”

$N$  = “On any input:

1. Obtain, via the recursion theorem, own description  $\langle SELF \rangle$ .
2. Construct a new **TM**  $S$ , such that  $L(S) = \overline{\langle SELF \rangle}$ .
3. Print  $\langle S \rangle$ .
4. *Reject.*”

□