
Problem 7.20. We generally believe that $PATH$ is not NP-complete. Explain the reason behind this belief. Show that proving $PATH$ is not NP-complete would prove $P \neq NP$.

Proof. In solution of Problem 7.18, we showed that

$$P = NP \Rightarrow A \in P, \text{ except } A = \emptyset \text{ and } A = \Sigma^* \text{ is NP-complete.}$$

Clearly, $PATH \neq \emptyset$ and $PATH \neq \Sigma^*$. Therefore, proving $PATH$ is not NP-complete would prove $P \neq NP$. □