
Problem 4.29. Let $C_{CFG} = \{\langle G, k\rangle \mid G \text{ is a } CFG \text{ and } L(G) \text{ contains exactly } k \text{ strings where } k \geq 0 \text{ or } k = \infty\}$. Show that C_{CFG} is decidable.

Proof. We present a **TM** I that decides C_{CFG}.

$I =$ "On input $\langle M\rangle$, where M is a PDA:

1. If $k = 0$:
2. Test $L(G) = \phi$ using the E_{CFG} decider R from Theorem 4.8.
3. If R accepts, *accept*; otherwise, *reject*.
4. If $k = \infty$:
5. Convert G to equivalent PDA P.
6. Test $L(P)$ is infinite using the $INFINITE_{PDA}$ decider T from solution to Problem 4.11.
7. If T accepts, *accept*; otherwise, *reject*.
8. Generate all possible strings $s_1, s_2, \cdots, s_n$ in G.
9. If $n = k$, *accept*; otherwise, *reject*."

□