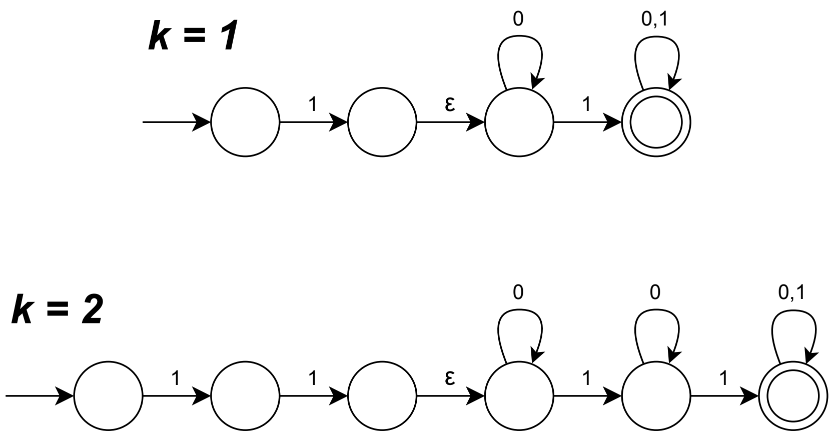
Problem 1.49 a. Let $B = \{1^k y \mid y \in \{0,1\}^* \; and \; y \; contains \; at \; least \; k \; 1s, \; for \; k \geq 1\}$. Show that B is a regular language.

Proof Idea. The regular expression, $1^k(0^*1)^k\{0,1\}^*$ describes the language B_k for some fixed k.



State diagrams of finite automata that recognize $B_{k=1}$ and $B_{k=2}$.

Proof. □

Part b. Let $C = \{1^k y \mid y \in \{0,1\}^* \; and \; y \; contains \; at \; most \; k \; 1s, \; for \; k \geq 1\}$. Show that C isn't a regular language.

Proof. The proof is by contradiction. Assume C is a regular language. Let p be the pumping lenght given by the puming lemma. Choose s to be the string:

$$s = 1^p 0 1^p$$

The string s is a member of C, and $|s| \geq p$. Therefore, the pumping lemma guarantees that the string s can be split into three pieces, $s = xyz$, and for each $i \geq 0$ the string $xy^iz \in C$. According to the condition 3 ($|xy| \leq p$), y can only be 1s. But, pumping down the y with $i = 0$ results in fewer than p 1s at the start of the string. Thus $xy^0z \notin C$, which is a contradiction. □