
Problem 4.22. Let $PREFIX{-}FREE_{REX} = \{\langle R\rangle \mid R$ is a regular expression and $L(R)$ is prefix-free$\}$. Show that $PREFIX - FREEREX$ is decidable. Why does a similar approach fail to show that $PREFIX - FREE_{CFG}$ is decidable?

Proof. We present a **TM** I that decides $PREFIX - FREE_{REX}$.

$I =$ "On input $\langle R\rangle$, where R is a regular expression:

1. Convert R to equivalent NFA N.
2. Construct an NFA M, such that $L(M) = NOPREFIX(N)$ by following the construction given in solution to Problem 1.40 a.
3. Convert M to equivalent DFA D.
4. Test $L(D) = \phi$ using the E_{DFA} decider T from Theorem 4.4.
5. If T accepts, *accept*; if F rejects, *reject*."

□

The class of context-free languages is not closed under $NOPREFIX$[1]. Therefore, the approach used to construct a decider for $PREFIX - FREE_{REX}$ cannot be used to show that $PREFIX - FREE_{CFG}$ is decidable.

[1] Refer to the proof given in solution to Problem 2.41 a.