
Problem 4.28. Let $C = \{\langle G, x\rangle \mid G \text{ is a CFG and } x \text{ is a substring of some } y \in L(G)\}$. Show that C is decidable.

Proof. We present a **TM** I that decides C.

I = "On input $\langle G, x\rangle$, where G is a CGA and x is a string:

1. Construct PDA P, such that $L(P) = L(G) \cap \Sigma^* x \Sigma^*$.
2. Convert PDA P to equivalent CFG G'.
3. Test $L(G') = \phi$ using the E_{CFG} decider R from Theorem 4.8.
4. If R accepts, *reject*; if R rejects, *accept*."

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