
Problem 2.21. Let $\Sigma = \{a, b\}$. Give a CFG generating the language of strings with twice as many a's as b's. Prove that your grammar is correct.

Solution. Let $G = (V, \Sigma, R, S)$ be the following grammar.

$$
\begin{aligned}
S &\to Saab \mid Sbaa \mid Saba \\
S &\to bSaa \mid baaS \mid baSa \\
S &\to aaSb \mid aabS \\
S &\to aSab \mid aSba \\
S &\to abSa \mid abaS \\
S &\to \varepsilon
\end{aligned}
$$

G generates the language of strings with twice as many a's as b's. □

Proof. The proof is in two parts.

Part 1. All strings in the language can be generated by G.

Part 2. All strings generated by G are in the language.

□