

**Problem 3.15.** Show that the collection of decidable languages is closed under the operation of.

**Part b.** concatenation.

*Proof.*

□

**Part c.** star.

*Proof.*

□

**Part d.** complementation.

For any decidable language  $L$ , let  $M$  be the **TM** that decides it. Construct a **TM**  $M'$  that decides the complement of  $L$ :

“On input  $w$ :

1. Run  $M$  on  $w$ . If it accepts, *reject*. Otherwise, *accept*.”

**Part e.** intersection.

For any two decidable languages  $L_1$  and  $L_2$ , let  $M_1$  and  $M_2$  be the **TMs** that decide them. We construct a **TM**  $M'$  that decides the intersection of  $L_1$  and  $L_2$ :

“On input  $w$ :

1. Run  $M_1$  on  $w$ . If it rejects, *reject*.
2. Run  $M_2$  on  $w$ . If it accepts, *accept*. Otherwise, *reject*.”