

**Problem 4.11.** Let  $INFINITE_{PDA} = \{\langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$ . Show that  $INFINITE_{PDA}$  is decidable.

*Proof.* We present a **TM**  $T$  that decides  $INFINITE_{PDA}$  using the result of the Problem 2.35<sup>1</sup>.

$T =$  “On input  $\langle M \rangle$ , where  $M$  is a PDA:

1. Convert  $M$  to an equivalent CFG  $G$  using the construction given in Lemma 2.27.
2. Convert  $G$  to an equivalent grammar  $G_c$  in Chomsky normal form.
3. Let  $b$  be the number of variables in  $G_c$ . Generate all strings  $s_1, s_2, \dots$ , where each  $s_i$  has a derivation of  $2^b$  steps in  $G_c$ .
4. Run the decider  $S$  for  $A_{CFG}$  given in Theorem 4.7 on each  $s_i$ .
5. If  $S$  accepts some  $s_i$ , then *accept*, otherwise *reject*.”

□

<sup>1</sup>Let  $G$  be a CFG in Chomsky normal form that contains  $b$  variables. If  $G$  generates some string with a derivation having at least  $2^b$  steps, then  $L(G)$  is infinite.