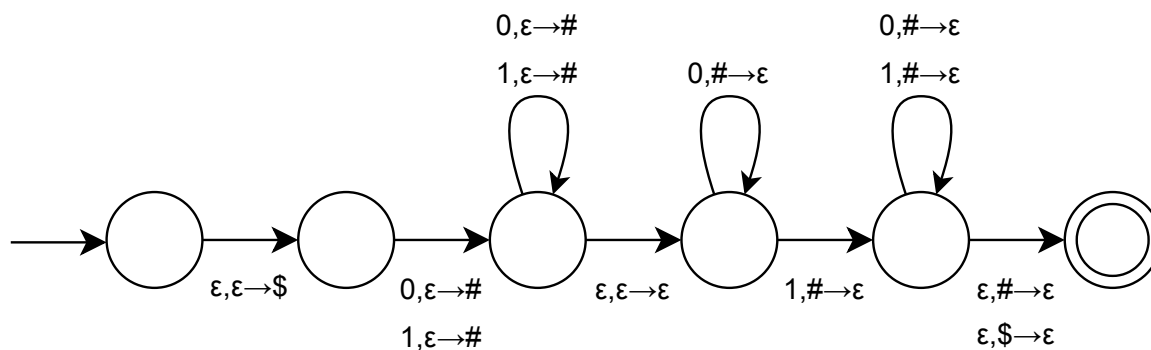


**Problem 2.47.** Let  $\Sigma = \{0, 1\}$  and let  $B$  be the collection of strings that contain at least one 1 in their second half. In other words,  $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^*1\Sigma^* \text{ and } |u| \geq |v|\}$ .

**Part a.** Give a PDA that recognizes  $B$ .



A PDA that recognizes  $B$ .

**Part b.** Give a CFG that generates  $B$ .

$$\begin{aligned}
 S &\rightarrow 0S0 \mid 1S0 \mid 0T1 \mid 1T1 \\
 T &\rightarrow 0T0 \mid 0T1 \mid 1T0 \mid 1T1 \mid 0 \mid 1 \mid \varepsilon
 \end{aligned}$$