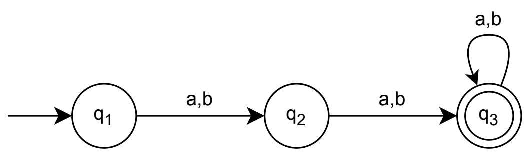

Problem 1.65. Prove that for each $n > 0$, a language B_n exists where

Part a. B_n is recognizable by an NFA that has n states.

Proof. Let Σ be any nonempty set of symbols. Define B_n to be the language:

$$B_n = \{w \mid w \textit{ has length at least } n - 1\}$$



Example of an NFA that recognizes B_3, where $\Sigma = \{a, b\}$.

Construct the NFA $N = (Q, \Sigma, \delta, q_0, F)$ to recognize B_n:

1. $Q = \{q_1,\ q_2,\ \cdots,\ q_n\}$
2. $q_0 = q_1$
3. $F = \{q_n\}$
4. Define $\delta(q_i, a)$ so that for any $q_i \in Q$ and any $a \in \Sigma$:

$$\delta(q_i,\ a)\ = \begin{cases} \{q_{i+1}\} & i < n \\ \{q_n\} & i = n \end{cases}$$

□

Part b. If $B_n = A_1 \cup \cdots \cup A_k$, for regular languages A_i, then at least one of the A_i requires a DFA with exponentially many states.

Proof. Solution Replace this text with the details of your proof or solution. □