
Problem 3.15. Show that the collection of decidable languages is closed under the operation of.

Part b. concatenation.

For any two decidable languages L_1 and L_2, let M_1 and M_2 be the **TM**s that decide them. For a string $w = w_1 w_2 w_3 \cdots w_n$, define split $W = (P,\ S)$, where $P, S \in \Sigma^*$, and $w = PS$. In other words, the strings P and S are prefix and suffix of w, such that their concatenation is equivalent to w. We construct a **TM** $M^{'}$ that decides the concatenation of L_1 and L_2:

"On input w:

1. Calculate the list of all possible splits $W_1 = (P_1, S_1), W_2 = (P_2, S_2), \cdots W_n = (P_n, S_n)$ of w.
2. Repeat the following for each $i = 1, 2, 3, \cdots$.
3. Run M_1 on P_i, and M_2 on S_i.
4. If in any computation, both M_1 and M_2 accept, *accept.* "

Part c. star.

Part d. complementation.

For any decidable language L, let M be the **TM** that decides it. Construct a **TM** $M^{'}$ that decides the complement of L:

"On input w:

1. Run M on w. If it accepts, *reject.* Otherwise, *accept.*"

Part e. intersection.

For any two decidable languages L_1 and L_2, let M_1 and M_2 be the **TM**s that decide them. We construct a **TM** $M^{'}$ that decides the intersection of L_1 and L_2:

"On input w:

1. Run M_1 on w. If it rejects, *reject.*
2. Run M_2 on w. If it accepts, *accept.* Otherwise, *reject.*"