Problem 7.46. Say that two Boolean formulas are ***equivalent*** if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is ***minimal*** if no shorter Boolean formula is equivalent to it. Let $MIN - FORMULA$ be the collection of minimal Boolean formulas. Show that if P = NP, then $MIN - FORMULA \in P$.

Proof. The P = NP assumption implies that SAT is in P, so testing satisfiability is solvable in polynomial time. To show that if P = NP, then $MIN - FORMULA \in P$, we give a polynomial time algorithm M.

$M = $ "On input $\langle \phi \rangle$, where ϕ is a Boolean formula:

1. Repeat for every literal a in ϕ:

2.   Construct a new Boolean formula ϕ_t by replacing a with *true* in ϕ.

3.   Construct a new Boolean formula ϕ_f by replacing a with *false* in ϕ.

4.   Test satisfiablity of ϕ_t and ϕ_f.

5.   If satisfiability of ϕ_t is same as ϕ_f, then *reject*.

6. *Accept.*"

$\square$