
Problem 7.25. Let $CNF_H = \{\langle\phi\rangle \mid \phi$ is a satisfiable cnf-formula where each clause contains any number of literals, but at most one negated literal$\}$. Show that $CNF_H \in P$.

Proof. Consider a cnf-formula ϕ where each clause contains any number of literals, but at most one negated literal. ϕ can only contains follow four types of clauses.

$$\phi = (\overline{a_1}) \ \wedge \ (a_2) \ \wedge \ (\overline{a_3} \ \vee b_3 \ \vee \cdots) \ \wedge \ (a_4 \ \vee b_4 \ \vee \cdots) \ \wedge \ \cdots$$

If ϕ is satisfiable, then at least one literal in every clause must be true. We can test satisfiability of ϕ as follows:

1. Assign *false* value to any variable that appears negated in a single variable clause.
2. Assign *true* value to all other variables.
3. Test assignment for satisfiability.

Stages 1 to 3 are easily implemented in polynomial time on any reasonable deterministic model. □