
Problem 7.26. Let ϕ be a 3cnf-formula. An ***$\neq$-assignment*** to the variables of ϕ is one where each clause contains two literals with unequal truth values. In other words, an *$\neq$-assignment* satisfies ϕ without assigning three true literals in any clause.

Part a. Show that the negation of any *$\neq$-assignment* to ϕ is also an *$\neq$-assignment.*

Proof. Let ϕ be a 3cnf-formula with n variables. Let $A = (v_1, v_2, \cdots, v_n)$ be an *$\neq$-assignment* to the variables $x_1, x_2, \cdots, x_n$ of ϕ. The negation of A is $A' = (\overline{v_1}, \overline{v_2}, \cdots, \overline{v_n})$. Every clause in ϕ contains at least one literal that is assigned $false$ value by A, and at least one literal that is assigned *true*. Therefore, A' also assigns at lease one literal $false$ and at least one literal *true* in each clause. Thus, A' is also an *$\neq$-assignment.* □

Part b. Let $\neq SAT$ be the collection of 3cnf-formulas that have an *$\neq$-assignment.* Show that we obtain a polynomial time reduction from $3SAT$ to $\neq SAT$ by replacing each clause c_i

$$(y_1 \vee y_2 \vee y_3)$$

with the two clauses

$$(y_1 \vee y_2 \vee z_i) \textbf{ and } (\overline{z_i} \vee y_3 \vee b),$$

where z_i is a new variable for each clause c_i, and b is a single additional new variable.

Part c. Conclude that $\neq SAT$ is NP-complete.