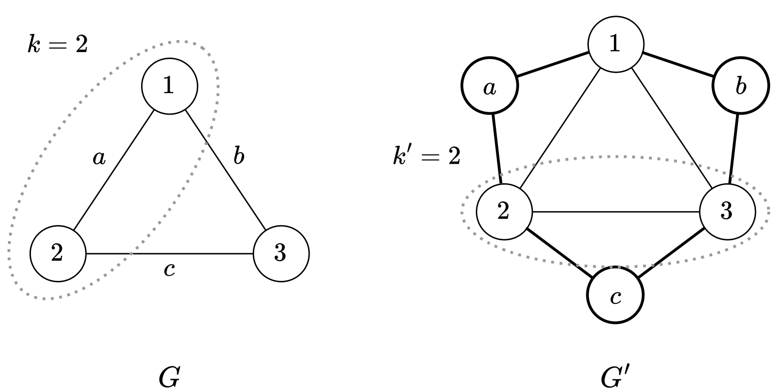
Problem 7.35. A subset of the nodes of a graph G is a ***dominating*** set if every other node of G is adjacent to some node in the subset. Let

$$DOMINATING-SET = \{\langle G, k\rangle \mid G \text{ has a dominating set with } k \text{ nodes}\}.$$

Show that it is NP-complete by giving a reduction from $VERTEX-COVER$.

Proof. To show that $DOMINATING-SET$ is NP-complete, we must show that it is in NP and that all NP-problems are polynomial time reducible to it. The first part is easy; a certificate is simply the subset of nodes. To prove the second part, we show that $VERTEX-COVER$ is polynomial time reducible to $DOMINATING-SET$. The reduction converts an undirected graph G and integer k into an undirected graph G' and integer k', so that G has a k-node vertex cover, iff G' has a k'-node dominating set.

The graph G' contains a node for every node and edge in G. The nodes in G' that correspond to nodes in G are all connected to each other. Every node in G' that corresponds to an edge u in G is connected to two nodes x and y in G', where u connects the corresponding nodes of x and y in G. The integer k' equals k.



Graph G has a 2-node vertex cover, and G' has 2-node dominating set.

Suppose G has a k-node vertex cover. We show that the subset D of k-nodes in G' that correspond to the k-node vertex cover forms a k-node dominating set. First, all nodes in G' that corresponds to nodes in G are adjacent to every node in D. Second, all nodes in G' that correspond to edges in G are also adjacent to one of the nodes in D.

Suppose G' has a k'-node dominating set. Then, the dominating set gives a k'-node vertex cover in G. $\square$