

Signed Values

Explanation: In order to better understand the proof given in the notes I have tried reworking it for myself. I wanted to also test fitch.sty a package for writing logic proofs for latex. The end result is similar to a two column proof. The premises/assumptions are the statements at the start, new statements are on the left with their respective justifications and referenced lines on the right.

What I am trying to prove:

$$((x_{m-1})?(-(2^m - \sum_{i=0}^{m-1} x_i 2^i)) : (\sum_{i=0}^{m-1} x_i 2^i)) \Rightarrow ((\sum_{i=0}^{m-2} x_i 2^i) - x_{m-1} 2^{m-1})$$

How: A ternary expression effectively has the form "if a then b follows and if not a then c follows." I have chosen to treat this as a two case conditional proof starting first with $x_{m-1} = 1$ as case 1.

Case 1: $x_{m-1} = 1$

1	$x_{m-1} = 1$	Assumption
2	$(x_{m-1})?(-(2^m - \sum_{i=0}^{m-1} x_i 2^i)) : (\sum_{i=0}^{m-1} x_i 2^i)$	Assumption
3	$-(2^m - \sum_{i=0}^{m-1} x_i 2^i)$	From 1 and 2, defn of ternary stmt
4	$(\sum_{i=0}^{m-1} x_i 2^i) - 2^m$	From 3, Multiplication
5	$(\sum_{i=0}^{m-2} x_i 2^i) + x_{m-1} 2^{m-1} - 2^m$	From 4, Expansion of a Series
6	$(\sum_{i=0}^{m-2} x_i 2^i) - (2^m - x_{m-1} 2^{m-1})$	5, Factoring
7	$(\sum_{i=0}^{m-2} x_i 2^i) - (2^m - (1)2^{m-1})$	6 and 1, Multiplicative identity
8	$(\sum_{i=0}^{m-2} x_i 2^i) - (2^m - 2^{m-1})$	7, Simplification
9	$(\sum_{i=0}^{m-2} x_i 2^i) - (2^m - 2^m 2^{-1})$	8, Rules of Exponents
10	$(\sum_{i=0}^{m-2} x_i 2^i) - 2^m (1 - \frac{1}{2})$	9, Factoring
11	$(\sum_{i=0}^{m-2} x_i 2^i) - 2^m (\frac{1}{2})$	10, Simplification
12	$(\sum_{i=0}^{m-2} x_i 2^i) - 2^{m-1}$	11, Rules of Exponents
13	$(\sum_{i=0}^{m-2} x_i 2^i) - (1)2^{m-1}$	12, Multiplicative identity
14	$(\sum_{i=0}^{m-2} x_i 2^i) - x_{m-1} 2^{m-1}$	1 and 13, Substitution

Which completes the proof for Case 1.

Case 2: $x_{m-1} = 0$

1	$x_{m-1} = 0$	Assumption
2	$(x_{m-1})?(-(2^m - \sum_{i=0}^{m-1} x_i 2^i)) : (\sum_{i=0}^{m-1} x_i 2^i)$	Assumption
3	$\sum_{i=0}^{m-1} x_i 2^i$	From 1 and 2, defn of ternary stmt
4	$(\sum_{i=0}^{m-2} x_i 2^i) + x_{m-1} 2^{m-1}$	From 3, Expansion of a Series
5	$(\sum_{i=0}^{m-2} x_i 2^i) + (0) 2^{m-1}$	1 and 4, Substitution
6	$(\sum_{i=0}^{m-2} x_i 2^i) + 0$	5, Multiplying by 0 gives Additive Identity
7	$\sum_{i=0}^{m-2} x_i 2^i$	6, Additive Identity
8	$(\sum_{i=0}^{m-2} x_i 2^i) - (0) 2^{m-1}$	7, Additive Identity
9	$(\sum_{i=0}^{m-2} x_i 2^i) - x_{m-1} 2^{m-1}$	8 and 1, Substitution

Which completes the proof for Case 2 and ends the proof.