Fits to Brody distribution, and calculations for density of states

In[∘]:=
```
PBrody[x_, w_] := (1 + w) (Gamma[(2 + w)/(1 + w)])^(1+w) x^w Exp[- (Gamma[(2 + w)/(1 + w)] x)^(1+w)];
bins = Table[x, {x, 0, 5, 0.2}];
bFit = Table[0.5 * (bins[[i]] + bins[[i + 1]]), {i, 1, Length[bins] - 1}];
SampleRange = # < Ecutoff1 &
```

Out[∘]= `#1 < Ecutoff1 &`

- **Run 0**

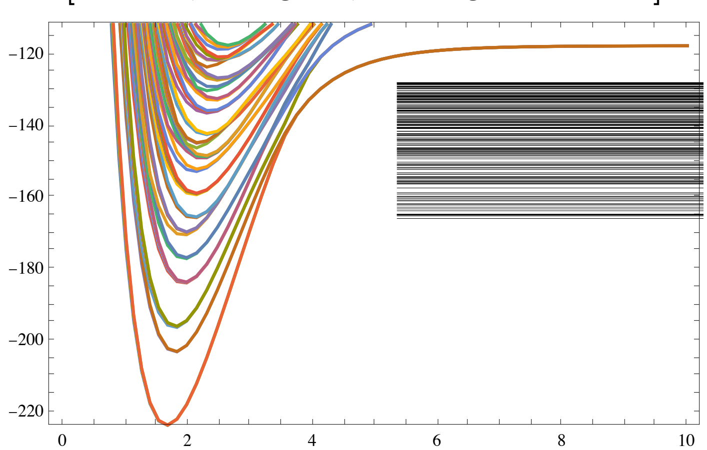
In[∘]:=
```
SampleRange = # < Ecutoff0 &
curvestemp0 =
  Transpose[Import["/Users/niravmehta/Documents/GitHub/4BodySVD/AdiabaticCurves.dat"]];
Curves0 = Table[Table[{curvestemp0[[1, i]], curvestemp0[[j, i]]},
    {i, 1, Length[curvestemp0[[1]]]}], {j, 2, Length[curvestemp0]}];
Evals0 = Flatten[Import["/Users/niravmehta/Documents/GitHub/4BodySVD/Eigenvals.dat"]];
Ecutoff0 = Evals0[[4]]
Evals0 = Sort[Drop[Evals0, 1 ;; 4]];
eValsBound0 = Sort[Select[Evals0, SampleRange]];
```

Out[∘]= `#1 < Ecutoff0 &`

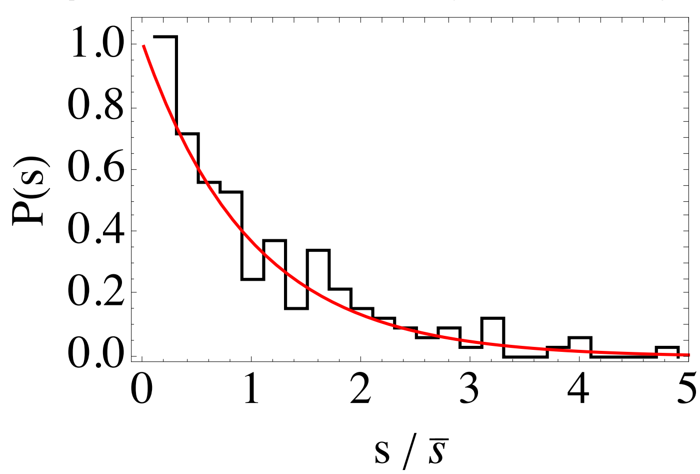
Out[∘]= `-117.562`

In[∘]:=
```
pcurves0 = ListPlot[Curves0, Frame → True, PlotMarkers → None,
   Joined → True, PlotRange → {Min[curvestemp0[[2]]], Ecutoff0 + 6}];
penergies0 = ListPlot[Table[{8, eValsBound0[[i]]}, {i, 1, Length[eValsBound0]}],
   Frame → True, PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{0, 0}, {5, 0}}]}]];
Show[pcurves0, penergies0, PlotRange → Automatic]
```

Out[∘]=



```
In[•]:= Es0 = Table[eValsBound0[[i + 1]] - eValsBound0[[i]], {i, 1, Length[eValsBound0] - 1}];
        eTrim0 = Select[Es0, # > 0.0000001 &];
        avg0 = Mean[eTrim0];
        eTrim0 = eTrim0 / avg0;
        (*Sort[eTrim0];*)
        Length[Es0];
        Length[eTrim0];
        ρ0 = 1 / avg0;
        EspaceBin0 = BinCounts[eTrim0, {bins}];
        NormBrodyBinCs0 = Table[EspaceBin0[[i]] / Total[EspaceBin0] / (bins[[i + 1]] - bins[[i]]),
           {i, 1, Length[bins] - 1}];
        brodyPdist0 = Table[{bFit[[i]], NormBrodyBinCs0[[i]]}, {i, 1, Length[bFit]}]
        phist0 = ListPlot[brodyPdist0, InterpolationOrder → 0,
           Joined → True, PlotRange → All, PlotStyle → Black];
        pars0 = FindFit[brodyPdist0, {PBrody[s, w], w > 0, w < 1}, {w}, s]
        pfit0 = Plot[PBrody[s, w /. pars0], {s, 0, 5}, PlotRange → All, PlotStyle → Red];

Out[•]= {{0.1, 1.03125}, {0.3, 0.71875}, {0.5, 0.5625}, {0.7, 0.53125},
         {0.9, 0.25}, {1.1, 0.375}, {1.3, 0.15625}, {1.5, 0.34375}, {1.7, 0.21875},
         {1.9, 0.15625}, {2.1, 0.125}, {2.3, 0.09375}, {2.5, 0.0625}, {2.7, 0.09375},
         {2.9, 0.03125}, {3.1, 0.125}, {3.3, 0.}, {3.5, 0.}, {3.7, 0.03125},
         {3.9, 0.0625}, {4.1, 0.}, {4.3, 0.}, {4.5, 0.}, {4.7, 0.03125}, {4.9, 0.}}

Out[•]= {w → 4.32273 × 10^-7}

In[•]:= Show[phist0, pfit0, FrameLabel → {"s / s̅", "P(s)"}, Frame → True, LabelStyle → Large]

Out[•]=
```



Run1

In[∘]:=
```
rundata1 = Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run1/4BodySVD.par", "text"]
```

Out[∘]=
```
xPoints (phi)        yPoints (theta)
30            60

LobattoPoints        NumChannels    Order
64            40        5

PotentialDepth        Rmin        Rmax        alpha (turns V on and off)
40.d0            0d0        10.d0        1.d0

m1        m2        m3        m4
1.d0        1.d0        1.d0        1.d0

xMin        xMax        yMin        yMax   (enter in units of pi)
0d0        0.25d0        0d0        0.5d0

Left        Right        Bot        Top
0        0        2        1



!  At top -- Phi(theta=pi/2,phi)=0 for odd parity so choose Top = 0
               Phi'(theta=pi/2,phi)=0 for even parity so choose Top = 1
```

```
SampleRange = Ecutoff1 - 30.0 < # < Ecutoff1 - 1.0 &
curvestemp1 = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run1/AdiabaticCurves.dat"]];
Curves1 = Table[Table[{curvestemp1[[1, i]], curvestemp1[[j, i]]},
    {i, 1, Length[curvestemp1[[1]]]}], {j, 2, Length[curvestemp1]}];
Evals1 = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run1/Eigenvals.dat"]];
Ecutoff1 = Evals1[[4]]
Evals1 = Sort[Drop[Evals1, 1 ;; 4]];
eValsBound1  =  Sort[Select[Evals1, SampleRange]]
```

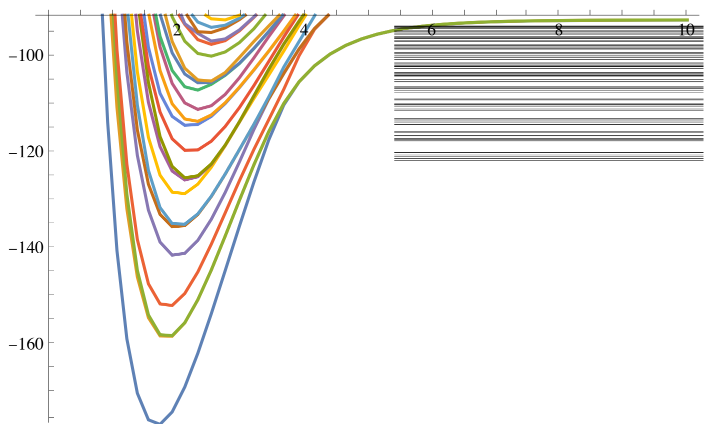
Out[∘]= Ecutoff1 - 30. < #1 < Ecutoff1 - 1. &

Out[∘]= -92.4717

Out[∘]= {-121.61, -121.216, -120.765, -120.5, -119.99, -117.553, -117.246, -117.118, -116.484, -115.777, -115.681, -114.184, -113.676, -113.449, -113.215, -112.903, -111.217, -110.945, -110.367, -110.101, -110.014, -109.749, -109.055, -108.859, -107.449, -107.042, -107.022, -106.686, -106.434, -106.232, -105.278, -104.819, -104.126, -104.017, -103.859, -103.542, -103.394, -102.559, -102.159, -102.087, -101.971, -101.602, -101.375, -100.701, -100.207, -100.181, -99.9092, -99.4754, -99.2255, -98.5014, -98.3682, -98.0902, -97.8463, -97.7517, -97.4842, -96.9466, -96.666, -96.3003, -96.2263, -96.0905, -95.8742, -95.3924, -95.1579, -94.8176, -94.574, -94.2601, -94., -93.9054, -93.8451, -93.7054}

In[∘]:=
```
pcurves1 = ListPlot[Curves1, PlotMarkers → None,
   Joined → True, PlotRange → {Min[curvestemp1[[2]]], Ecutoff1 + 1}];
penergies1 = ListPlot[Table[{8, eValsBound1[[i]]}, {i, 1, Length[eValsBound1]}],
   PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{15, 0}, {20, 0}}]}]];
Show[pcurves1, penergies1]
```

Out[∘]=



In[◦]:=
```
Es1 = Table[eValsBound1[[i + 1]] - eValsBound1[[i]], {i, 1, Length[eValsBound1] - 1}];
eTrim1 = Select[Es1, # > 0.0000001 &];
avg1 = Mean[eTrim1];
eTrim1 = eTrim1 / avg1;
(*Sort[eTrim1];*)
Length[Es1];
Length[eTrim1];
ρ1 = 1 / avg1;
EspaceBin1 = BinCounts[eTrim1, {bins}];
NormBrodyBinCs1 = Table[EspaceBin1[[i]] / Total[EspaceBin1] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist1 = Table[{bFit[[i]], NormBrodyBinCs1[[i]]}, {i, 1, Length[bFit]}]
phist1 = ListPlot[brodyPdist1, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars1 = FindFit[brodyPdist1, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit1 = Plot[PBrody[s, w /. pars1], {s, 0, 5}, PlotRange → All, PlotStyle → Red];
```

Out[◦]=
```
{{0.1, 0.367647}, {0.3, 0.882353}, {0.5, 0.514706}, {0.7, 1.25},
 {0.9, 0.441176}, {1.1, 0.367647}, {1.3, 0.294118}, {1.5, 0.147059},
 {1.7, 0.367647}, {1.9, 0.}, {2.1, 0.0735294}, {2.3, 0.0735294}, {2.5, 0.},
 {2.7, 0.}, {2.9, 0.}, {3.1, 0.}, {3.3, 0.}, {3.5, 0.0735294}, {3.7, 0.0735294},
 {3.9, 0.}, {4.1, 0.0735294}, {4.3, 0.}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.}}
```

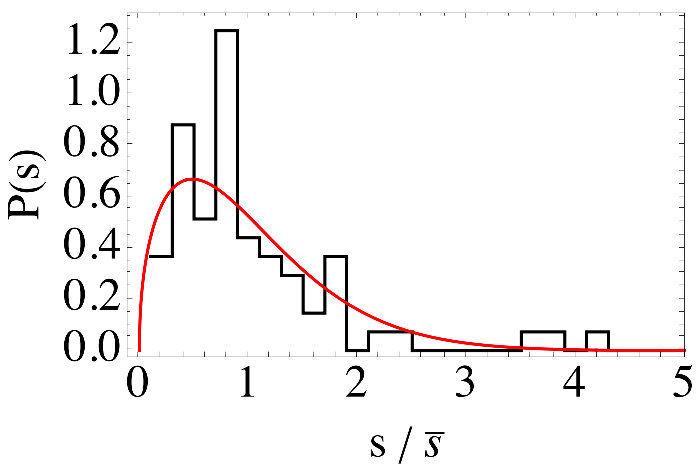
Out[◦]=
```
{w → 0.440578}
```

In[◦]:=
```
ρ1
```

Out[◦]=
```
2.47269
```

In[◦]:=
```
Show[phist1, pfit1, FrameLabel → {"s / s̅", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[◦]=



Run2

In[∘]:=
```
rundata2 = Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run2/4BodySVD.par", "text"]
```

Out[∘]=
```
xPoints (phi)          yPoints (theta)
30            60

LobattoPoints          NumChannels    Order
64            80         5

PotentialDepth         Rmin         Rmax         alpha (turns V on and off)
40.d0             0d0         10.d0         1.d0

m1         m2         m3         m4
1.d0         1.d0         1.d0         1.d0

xMin         xMax         yMin         yMax    (enter in units of pi)
0d0         0.25d0         0d0          0.5d0

Left         Right         Bot         Top
0         1         2         1



!  At top -- Phi(theta=pi/2,phi)=0 for odd parity so choose Top = 0
               Phi'(theta=pi/2,phi)=0 for even parity so choose Top = 1
```

In[∘]:=
```
SampleRange = Ecutoff2 - 30.0 < # < Ecutoff2 - 1 &
curvestemp2 = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run2/AdiabaticCurves.dat"]];
Curves2 = Table[Table[{curvestemp2[[1, i]], curvestemp2[[j, i]]},
    {i, 1, Length[curvestemp2[[1]]]}], {j, 2, Length[curvestemp2]}];
Evals2 = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run2/Eigenvals.dat"]];
Ecutoff2 = Evals2[[4]]
Evals2 = Sort[Drop[Evals2, 1 ;; 4]];
eValsBound2  = Sort[Select[Evals2, SampleRange]]
```

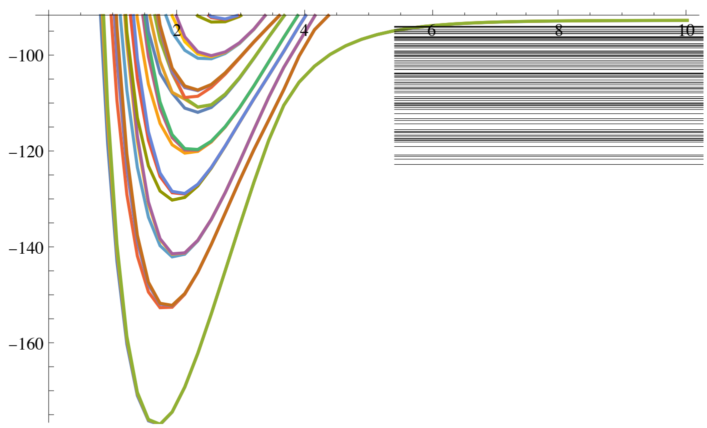
Out[∘]= `Ecutoff2 - 30. < #1 < Ecutoff2 - 1 &`

Out[∘]= `-92.4717`

Out[∘]=
```
{-122.413, -121.312, -120.73, -120.439, -118.683, -117.761, -117.45,
 -117.288, -117.048, -116.962, -116.542, -116.306, -115.805, -115.616,
 -115.541, -115.17, -113.967, -113.265, -112.881, -111.855, -110.973, -110.58,
 -110.215, -110.1, -109.867, -109.704, -109.611, -109.134, -108.734, -108.663,
 -108.309, -107.528, -107.188, -106.715, -106.577, -106.345, -105.85, -105.486,
 -105.296, -104.848, -104.334, -104.104, -103.972, -103.621, -103.533, -103.509,
 -103.394, -102.646, -102.632, -102.13, -101.855, -101.504, -101.13, -100.824,
 -100.347, -99.9505, -99.8741, -99.7308, -99.6589, -99.3399, -99.163, -98.9669,
 -98.786, -98.1867, -97.881, -97.7348, -97.67, -97.3511, -97.2895, -96.6686,
 -96.4676, -96.287, -96.0921, -96.0597, -95.988, -95.9237, -95.8149, -95.2412,
 -95.0797, -94.7664, -94.6698, -94.3024, -93.898, -93.8351, -93.795, -93.6754}
```

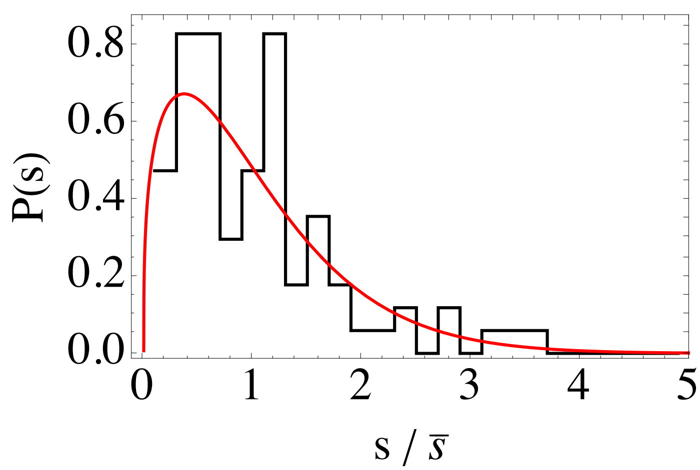
In[∘]:=
```
pcurves2 = ListPlot[Curves2, PlotMarkers → None,
   Joined → True, PlotRange → {Min[curvestemp2[[2]]], Ecutoff2 + 1}];
penergies2 = ListPlot[Table[{8, eValsBound2[[i]]}, {i, 1, Length[eValsBound2]}],
   PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{15, 0}, {20, 0}}]}]];
Show[pcurves2, penergies2]
```

Out[∘]=



```
In[•]:= Es2 = Table[eValsBound2[[i + 1]] - eValsBound2[[i]], {i, 1, Length[eValsBound2] - 1}];
eTrim2 = Select[Es2, # > 0.0000001 &];
avg2 = Mean[eTrim2];
eTrim2 = eTrim2 / avg2;
(*Sort[eTrim2];*)
Length[Es2];
Length[eTrim2];
ρ2 = 1 / avg2;
EspaceBin2 = BinCounts[eTrim2, {bins}];
NormBrodyBinCs2 = Table[EspaceBin2[[i]] / Total[EspaceBin2] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist2 = Table[{bFit[[i]], NormBrodyBinCs2[[i]]}, {i, 1, Length[bFit]}]
phist2 = ListPlot[brodyPdist2, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars2 = FindFit[brodyPdist2, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit2 = Plot[PBrody[s, w /. pars2], {s, 0, 5}, PlotRange → All, PlotStyle → Red];

Out[•]= {{0.1, 0.47619}, {0.3, 0.833333}, {0.5, 0.833333}, {0.7, 0.297619},
 {0.9, 0.47619}, {1.1, 0.833333}, {1.3, 0.178571}, {1.5, 0.357143},
 {1.7, 0.178571}, {1.9, 0.0595238}, {2.1, 0.0595238}, {2.3, 0.119048}, {2.5, 0.},
 {2.7, 0.119048}, {2.9, 0.}, {3.1, 0.0595238}, {3.3, 0.0595238}, {3.5, 0.0595238},
 {3.7, 0.}, {3.9, 0.}, {4.1, 0.}, {4.3, 0.}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.}}

Out[•]= {w → 0.319464}

In[•]:= Show[phist2, pfit2, FrameLabel → {"s / s̅", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[•]=



Run3

In[•]:=
```
rundata3 = Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run3/4BodySVD.par", "text"]
```

Out[•]=
```
xPoints (phi)        yPoints (theta)
60              60

LobattoPoints         NumChannels    Order
64              80        5

PotentialDepth         Rmin         Rmax         alpha (turns V on and off)
40.d0            0d0         10.d0        1.d0

m1         m2         m3         m4
1.d0         1.d0         1.d0         1.d0

xMin         xMax         yMin         yMax    (enter in units of pi)
0d0         0.5d0         0d0          0.5d0

Left         Right         Bot         Top
0         0         2         1



!  At top -- Phi(theta=pi/2,phi)=0 for odd parity so choose Top = 0
               Phi'(theta=pi/2,phi)=0 for even parity so choose Top = 1
```

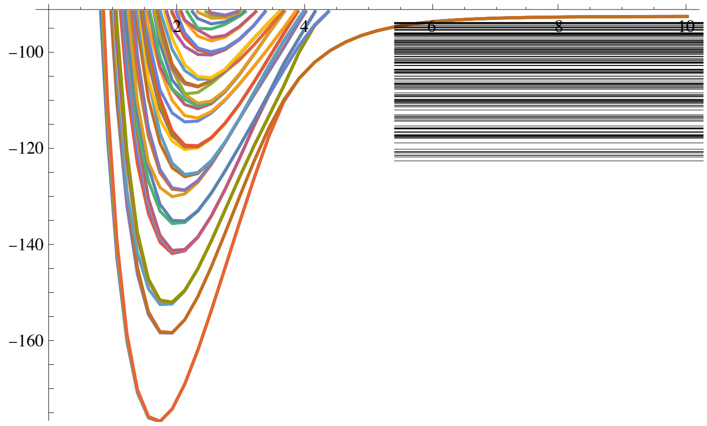
In[∘]:=
```
SampleRange = Ecutoff3 - 30.0 < # < Ecutoff3 - 1 &
curvestemp3 = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run3/AdiabaticCurves.dat"]];
Curves3 = Table[Table[{curvestemp3[[1, i]], curvestemp3[[j, i]]},
    {i, 1, Length[curvestemp3[[1]]]}], {j, 2, Length[curvestemp3]}];
Evals3 = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run3/Eigenvals.dat"]];
Ecutoff3 = Evals3[[4]]
Evals3 = Sort[Drop[Evals3, 1 ;; 4]];
eValsBound3 = Sort[Select[Evals3, SampleRange]]
```

Out[∘]=
```
Ecutoff3 - 30. < #1 < Ecutoff3 - 1 &
```

Out[∘]=
```
-92.4718
```

Out[∘]=
```
{-122.413, -121.61, -121.312, -121.216, -120.765, -120.73, -120.5, -120.439, -119.99,
 -118.683, -117.761, -117.553, -117.45, -117.288, -117.246, -117.118, -117.048,
 -116.962, -116.542, -116.483, -116.306, -115.805, -115.776, -115.681, -115.616,
 -115.541, -115.169, -114.183, -113.966, -113.675, -113.449, -113.265, -113.215,
 -112.903, -112.881, -111.854, -111.217, -110.972, -110.945, -110.58, -110.367,
 -110.214, -110.101, -110.1, -110.013, -109.866, -109.749, -109.704, -109.611, -109.134,
 -109.054, -108.859, -108.734, -108.663, -108.308, -107.526, -107.449, -107.187,
 -107.042, -107.021, -106.714, -106.686, -106.576, -106.434, -106.345, -106.232,
 -105.85, -105.485, -105.295, -105.277, -104.847, -104.819, -104.332, -104.125,
 -104.103, -104.017, -103.972, -103.858, -103.62, -103.542, -103.532, -103.507,
 -103.394, -103.393, -102.646, -102.631, -102.558, -102.159, -102.129, -102.087,
 -101.971, -101.855, -101.601, -101.503, -101.374, -101.13, -100.818, -100.701,
 -100.346, -100.205, -100.18, -99.947, -99.9089, -99.873, -99.729, -99.6572, -99.4747,
 -99.3371, -99.2252, -99.1617, -98.9652, -98.785, -98.5003, -98.368, -98.185, -98.089,
 -97.8786, -97.8458, -97.7511, -97.7328, -97.6679, -97.484, -97.3496, -97.2859,
 -96.9458, -96.6676, -96.6653, -96.4657, -96.2998, -96.2844, -96.2252, -96.0917,
 -96.0903, -96.0572, -95.9872, -95.9228, -95.8734, -95.814, -95.3921, -95.2395,
 -95.1576, -95.0783, -94.8173, -94.7656, -94.6652, -94.5738, -94.3002, -94.2596,
 -93.9996, -93.9048, -93.8965, -93.8431, -93.8305, -93.7943, -93.7049, -93.6707}
```

In[∘]:=
```
pcurves3 =
  ListPlot[Curves3, Joined → True, PlotRange → {Min[curvestemp3[[2]]], Ecutoff3 + 1}];
penergies3 = ListPlot[Table[{8, eValsBound3[[i]]}, {i, 1, Length[eValsBound3]}],
   PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{15, 0}, {20, 0}}]}]];
Show[pcurves3, penergies3]
```

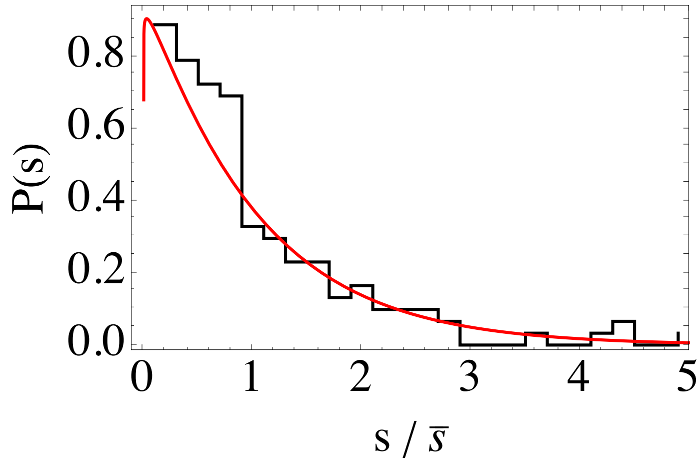
Out[∘]=


In[∘]:=
```
Es3 = Table[eValsBound3[[i + 1]] - eValsBound3[[i]], {i, 1, Length[eValsBound3] - 1}];
eTrim3 = Select[Es3, # > 0.0000001 &];
avg3 = Mean[eTrim3];
eTrim3 = eTrim3 / avg3;
Sort[eTrim3];
Length[Es3];
Length[eTrim3];
ρ3 = 1 / avg3;
EspaceBin3 = BinCounts[eTrim3, {bins}];
NormBrodyBinCs3 = Table[EspaceBin3[[i]] / Total[EspaceBin3] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist3 = Table[{bFit[[i]], NormBrodyBinCs3[[i]]}, {i, 1, Length[bFit]}]
phist3 = ListPlot[brodyPdist3, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars3 = FindFit[brodyPdist3, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit3 = Plot[PBrody[s, w /. pars3], {s, 0, 5}, PlotRange → All, PlotStyle → Red];
```

Out[∘]=
```
{{0.1, 0.888158}, {0.3, 0.789474}, {0.5, 0.723684}, {0.7, 0.690789}, {0.9, 0.328947},
 {1.1, 0.296053}, {1.3, 0.230263}, {1.5, 0.230263}, {1.7, 0.131579}, {1.9, 0.164474},
 {2.1, 0.0986842}, {2.3, 0.0986842}, {2.5, 0.0986842}, {2.7, 0.0657895},
 {2.9, 0.}, {3.1, 0.}, {3.3, 0.}, {3.5, 0.0328947}, {3.7, 0.}, {3.9, 0.},
 {4.1, 0.0328947}, {4.3, 0.0657895}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.0328947}}
```

Out[∘]=
```
{w → 0.0248492}
```

In[∘]:=
```
pall3 = Show[phist3, pfit3, FrameLabel → {"s / s̅", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[∘]=


- ***Analysis so far***

Runs 1 and 2 have taken advantage of a possible symmetry by further reducing the coordinate space. If my guess is right, then the density of states for run3 should be roughly twice that of run 1 and 2 since I expect both "even and odd" states about phi = $\pi/4$ to be present in run 3, but only even states in run1 and odd states in run2

In[∘]:=
```
ρ1
ρ2
ρ3
```

Out[∘]= 2.47269

Out[∘]= 2.95779

Out[∘]= 5.39274

In[∘]:=
```
ρ1 + ρ2
```

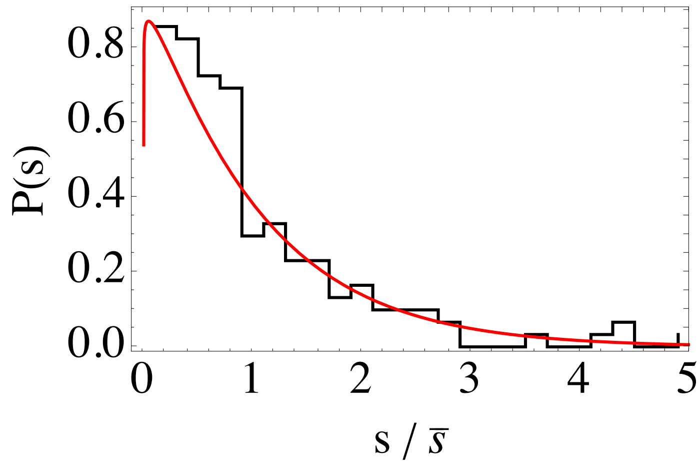
Out[∘]= 5.68733

Further, I expect that if these states (even and odd) do not couple since the Hamiltonian is symmetric about $\phi=\pi/4$, then overlaying these two distributions will result in a distribution similar to that of run3

```
In[•]:= eValsBoundCombined = Sort[ Flatten[{eValsBound1, eValsBound2}]]
EsCombined = Table[eValsBoundCombined[[i + 1]] - eValsBoundCombined[[i]],
    {i, 1, Length[eValsBoundCombined] - 1}];
eTrimCombined = Select[EsCombined, # > 0.0000001 &];
avgCombined = Mean[eTrimCombined];
eTrimCombined = eTrimCombined / avgCombined;
(*Sort[eTrimCombined];*)
Length[eTrimCombined];
ρCombined = 1 / avgCombined;
EspaceBinCombined = BinCounts[eTrimCombined, {bins}];
NormBrodyBinCsCombined =
  Table[EspaceBinCombined[[i]] / Total[EspaceBinCombined] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdistCombined = Table[{bFit[[i]], NormBrodyBinCsCombined[[i]]}, {i, 1, Length[bFit]}]
phistCombined = ListPlot[brodyPdistCombined,
    InterpolationOrder → 0, Joined → True, PlotRange → All, PlotStyle → Black];
parsCombined = FindFit[brodyPdistCombined, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfitCombined =
  Plot[PBrody[s, w /. parsCombined], {s, 0, 5}, PlotRange → All, PlotStyle → Red];

Out[•]= {-122.413, -121.61, -121.312, -121.216, -120.765, -120.73, -120.5, -120.439, -119.99,
 -118.683, -117.761, -117.553, -117.45, -117.288, -117.246, -117.118, -117.048,
 -116.962, -116.542, -116.484, -116.306, -115.805, -115.777, -115.681, -115.616,
 -115.541, -115.17, -114.184, -113.967, -113.676, -113.449, -113.265, -113.215,
 -112.903, -112.881, -111.855, -111.217, -110.973, -110.945, -110.58, -110.367,
 -110.215, -110.101, -110.1, -110.014, -109.867, -109.749, -109.704, -109.611, -109.134,
 -109.055, -108.859, -108.734, -108.663, -108.309, -107.528, -107.449, -107.188,
 -107.042, -107.022, -106.715, -106.686, -106.577, -106.434, -106.345, -106.232,
 -105.85, -105.486, -105.296, -105.278, -104.848, -104.819, -104.334, -104.126,
 -104.104, -104.017, -103.972, -103.859, -103.621, -103.542, -103.533, -103.509,
 -103.394, -103.394, -102.646, -102.632, -102.559, -102.159, -102.13, -102.087,
 -101.971, -101.855, -101.602, -101.504, -101.375, -101.13, -100.824, -100.701,
 -100.347, -100.207, -100.181, -99.9505, -99.9092, -99.8741, -99.7308, -99.6589,
 -99.4754, -99.3399, -99.2255, -99.163, -98.9669, -98.786, -98.5014, -98.3682,
 -98.1867, -98.0902, -97.881, -97.8463, -97.7517, -97.7348, -97.67, -97.4842, -97.3511,
 -97.2895, -96.9466, -96.6686, -96.666, -96.4676, -96.3003, -96.287, -96.2263,
 -96.0921, -96.0905, -96.0597, -95.988, -95.9237, -95.8742, -95.8149, -95.3924,
 -95.2412, -95.1579, -95.0797, -94.8176, -94.7664, -94.6698, -94.574, -94.3024,
 -94.2601, -94., -93.9054, -93.898, -93.8451, -93.8351, -93.795, -93.7054, -93.6754}

Out[•]= {{0.1, 0.855263}, {0.3, 0.822368}, {0.5, 0.723684}, {0.7, 0.690789}, {0.9, 0.296053},
 {1.1, 0.328947}, {1.3, 0.230263}, {1.5, 0.230263}, {1.7, 0.131579}, {1.9, 0.164474},
 {2.1, 0.0986842}, {2.3, 0.0986842}, {2.5, 0.0986842}, {2.7, 0.0657895},
 {2.9, 0.}, {3.1, 0.}, {3.3, 0.}, {3.5, 0.0328947}, {3.7, 0.}, {3.9, 0.},
 {4.1, 0.0328947}, {4.3, 0.0657895}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.0328947}}

Out[•]= {w → 0.039776}
```

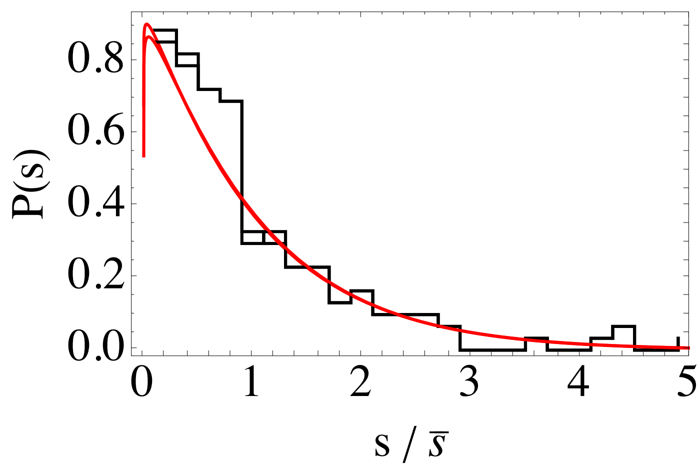
In[∘]:=
```
pcomb = Show[phistCombined, pfitCombined,
  FrameLabel → {"s / s̄", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[∘]=



In[∘]:=
```
Show[pall3, pcomb]
```

Out[∘]=



Run4

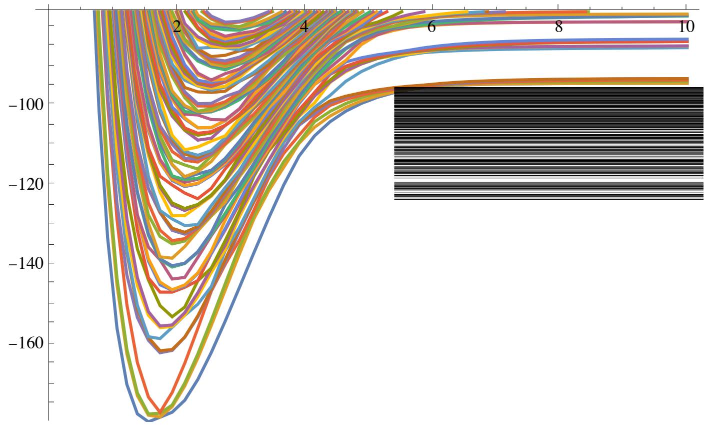
In[∘]:=
```
rundata4 = Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run4/4BodySVD.par", "text"]
```

Out[∘]=
```
xPoints (phi)          yPoints (theta)
60             60

LobattoPoints          NumChannels    Order
64             80        5

PotentialDepth          Rmin         Rmax         alpha (turns V on and off)
40.d0            0d0         10.d0         1.d0

m1         m2         m3         m4
1.d0         1.d0         1.5d0         1.5d0

xMin         xMax         yMin         yMax    (enter in units of pi)
0d0         0.5d0         0d0         0.5d0

Left         Right         Bot         Top
0        0        2        1



!  At top -- Phi(theta=pi/2,phi)=0 for odd parity so choose Top = 0
               Phi'(theta=pi/2,phi)=0 for even parity so choose Top = 1
```

```
In[•]:= SampleRange = Ecutoff4 - 30.0 < # < Ecutoff4 - 1 &
curvestemp4 = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run4/AdiabaticCurves.dat"]];
Curves4 = Table[Table[{curvestemp4[[1, i]], curvestemp4[[j, i]]},
    {i, 1, Length[curvestemp4[[1]]]}], {j, 2, Length[curvestemp4]}];
Evals4 = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run4/Eigenvals.dat"]];
Ecutoff4 = Evals4[[4]]
Evals4 = Sort[Drop[Evals4, 1 ;; 4]];
eValsBound4 = Sort[Select[Evals4, SampleRange]]

Out[•]= Ecutoff4 - 30. < #1 < Ecutoff4 - 1 &

Out[•]= -94.392

Out[•]= {-123.638, -123.483, -123.229, -122.899, -122.578, -122.548, -122.369, -122.266, -121.789,
  -121.28, -121.041, -121.006, -120.872, -120.686, -120.514, -120.346, -120.194,
  -120.06, -119.801, -119.732, -119.543, -119.303, -118.449, -118.206, -117.73,
  -117.598, -117.484, -117.101, -116.851, -116.809, -116.646, -116.491, -116.453,
  -116.387, -116.148, -115.903, -115.779, -115.535, -115.275, -114.847, -114.795,
  -114.545, -114.482, -114.251, -114.072, -114.057, -113.935, -113.838, -113.81,
  -113.543, -113.44, -113.159, -112.727, -112.706, -112.671, -112.259, -112.005,
  -111.753, -111.478, -111.279, -111.056, -111., -110.958, -110.878, -110.663, -110.622,
  -110.547, -110.31, -110.188, -110.142, -109.665, -109.456, -109.341, -109.232,
  -109.01, -108.852, -108.581, -108.395, -108.277, -108.189, -108.146, -107.992,
  -107.959, -107.9, -107.84, -107.621, -107.502, -107.408, -107.259, -106.811,
  -106.721, -106.659, -106.603, -106.403, -106.131, -106.062, -105.983, -105.846,
  -105.592, -105.506, -105.431, -105.251, -105.068, -104.943, -104.797, -104.688,
  -104.631, -104.484, -104.206, -104.039, -103.96, -103.822, -103.704, -103.504,
  -103.435, -103.359, -103.241, -103.013, -102.835, -102.775, -102.722, -102.522,
  -102.48, -102.283, -102.178, -102.125, -102.061, -102.018, -101.85, -101.737,
  -101.602, -101.529, -101.337, -101.241, -101.048, -100.977, -100.895, -100.874,
  -100.826, -100.55, -100.432, -100.329, -100.202, -100.192, -100.079, -100.04,
  -99.9209, -99.8663, -99.617, -99.5607, -99.4032, -99.3792, -99.1974, -99.169,
  -99.0855, -98.9663, -98.8387, -98.7409, -98.683, -98.5592, -98.5348, -98.3665,
  -98.3266, -98.1581, -97.9124, -97.8677, -97.6901, -97.6398, -97.552, -97.4548,
  -97.4388, -97.2449, -97.1531, -97.0118, -96.9606, -96.8284, -96.7692, -96.7321,
  -96.7241, -96.5818, -96.5663, -96.5223, -96.3512, -96.3376, -96.2726, -96.2309,
  -96.0708, -96.0121, -95.8815, -95.7311, -95.6541, -95.6229, -95.5481, -95.4112}
```

In[•]:=
```
pcurves4 =
  ListPlot[Curves4, Joined → True, PlotRange → {Min[curvestemp4[[2]]], Ecutoff4 + 18}];
penergies4 = ListPlot[Table[{8, eValsBound4[[i]]}, {i, 1, Length[eValsBound4]}],
   PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{15, 0}, {20, 0}}]}]];
Show[pcurves4, penergies4]
```

Out[•]=



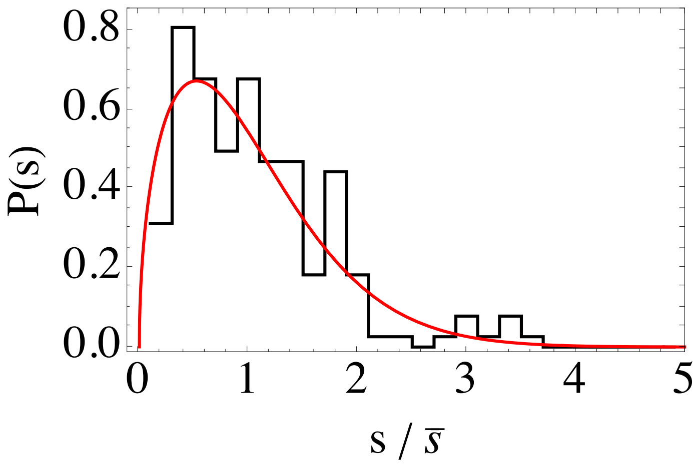
In[•]:=
```
Es4 = Table[eValsBound4[[i + 1]] - eValsBound4[[i]], {i, 1, Length[eValsBound4] - 1}];
eTrim4 = Select[Es4, # > 0.0000001 &];
avg4 = Mean[eTrim4];
eTrim4 = eTrim4 / avg4;
Sort[eTrim4];
Length[Es4];
Length[eTrim4];
ρ4 = 1 / avg4;
EspaceBin4 = BinCounts[eTrim4, {bins}];
NormBrodyBinCs4 = Table[EspaceBin4[[i]] / Total[EspaceBin4] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist4 = Table[{bFit[[i]], NormBrodyBinCs4[[i]]}, {i, 1, Length[bFit]}]
phist4 = ListPlot[brodyPdist4, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars4 = FindFit[brodyPdist4, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit4 = Plot[PBrody[s, w /. pars4], {s, 0, 5}, PlotRange → All, PlotStyle → Red];
```

Out[•]=
```
{{0.1, 0.3125}, {0.3, 0.807292}, {0.5, 0.677083}, {0.7, 0.494792},
 {0.9, 0.677083}, {1.1, 0.46875}, {1.3, 0.46875}, {1.5, 0.182292}, {1.7, 0.442708},
 {1.9, 0.182292}, {2.1, 0.0260417}, {2.3, 0.0260417}, {2.5, 0.}, {2.7, 0.0260417},
 {2.9, 0.078125}, {3.1, 0.0260417}, {3.3, 0.078125}, {3.5, 0.0260417},
 {3.7, 0.}, {3.9, 0.}, {4.1, 0.}, {4.3, 0.}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.}}
```

Out[•]=
```
{w → 0.488697}
```

In[∘]:=
```
pall4 = Show[phist4, pfit4, FrameLabel → {"s / s̄", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[∘]=



In[∘]:=
```
ρ4
```

Out[∘]=
```
6.83743
```

We need to more closely examine the identical particle symmetries and any other symmetries in the Hamiltonian in order to understand these spectra.

We defined the Jacobi coordinates in the H-tree above so that:

$$\vec{y}^{(12)} = S_{12}\,\vec{x} \tag{1}$$

where:

$$\frac{1}{\sqrt{\mu}}\begin{pmatrix} \sqrt{\mu_{12}} & -\sqrt{\mu_{12}} & 0 & 0 \\ 0 & 0 & \sqrt{\mu_{34}} & -\sqrt{\mu_{34}} \\ \frac{\sqrt{\mu_{12,34}}\ m_1}{m_1+m_2} & \frac{\sqrt{\mu_{12,34}}\ m_2}{m_1+m_2} & -\frac{\sqrt{\mu_{12,34}}\ m_3}{m_3+m_4} & -\frac{\sqrt{\mu_{12,34}}\ m_4}{m_3+m_4} \\ \frac{m_1}{\sqrt{M}} & \frac{m_2}{\sqrt{M}} & \frac{m_3}{\sqrt{M}} & \frac{m_4}{\sqrt{M}} \end{pmatrix} \tag{2}$$

To keep things as general as possible I'll assume for now that only particle 1 is of different mass and let $m_2 = m_3 = m_4 = m$, and $m_1 = \gamma\, m$ Then

$$\mu_{12} = \frac{\gamma\, m\, m}{\gamma\, m + m} = m\,\frac{\gamma}{1+\gamma} \longrightarrow \frac{m}{2} \tag{3}$$

$$\mu_{34} = \frac{m}{2} \tag{4}$$

$$\mu_{12,34} = \frac{(\gamma\, m + m)\,(2\, m)}{3\, m + \gamma\, m} = 2\, m\,\frac{(1+\gamma)}{(3+\gamma)} \longrightarrow m \tag{5}$$

$$\mu = \left(\frac{\gamma\, m\, m\, m\, m}{\gamma\, m + 3\, m}\right)^{1/3} = m\left(\frac{\gamma}{3+\gamma}\right)^{1/3} \longrightarrow \left(\frac{m}{2^{2/3}}\right) \tag{6}$$

So for the equal-mass case:

$$\frac{2^{1/3}}{\sqrt{m}}\begin{pmatrix} \sqrt{\frac{m}{2}} & -\sqrt{\frac{m}{2}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{m}{2}} & -\sqrt{\frac{m}{2}} \\ \frac{\sqrt{m}}{2} & \frac{\sqrt{m}}{2} & -\frac{\sqrt{m}}{2} & -\frac{\sqrt{m}}{2} \\ \frac{m}{\sqrt{4m}} & \frac{m}{\sqrt{4m}} & \frac{m}{\sqrt{4m}} & \frac{m}{\sqrt{4m}} \end{pmatrix} = 2^{1/3}\begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

and

$$\vec{x} = S_{12}{}^{-1}\,\vec{y}^{(12)} \tag{8}$$

Then

$$\vec{y}^{(13)} = S_{13}\,\vec{x} = S_{13}\,S_{12}{}^{-1}\,\vec{y}^{(12)} \tag{9}$$

Now note that we could use the same matrix S_{12} to construct $\vec{y}^{(13)}$ if the matrix were to act on a column vector $\begin{pmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{pmatrix}$ instead of the usual vector $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$. We can then say:

$$\begin{pmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = P_{23}\,\vec{x} \tag{10}$$

Hence:

$$S_{13} = S_{12}\,P_{23} \tag{11}$$

and

$$\vec{y}^{(13)} = S_{13}\,\vec{x} = S_{13}\,S_{12}{}^{-1}\,\vec{y}^{(12)} = S_{12}\,P_{23}\,S_{12}{}^{-1}\,\vec{y}^{(12)} \tag{12}$$

Similarly,

$$\vec{y}^{(14)} = S_{14}\,\vec{x} = S_{14}\,S_{12}{}^{-1}\,\vec{y}^{(12)} = S_{12}\,P_{24}\,S_{12}{}^{-1}\,\vec{y}^{(12)} \tag{13}$$

where

$$P_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{14}$$

In[∘]:= $\mathtt{S12} = 2^{1/3}\begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix};$

In[∘]:=
```
P24 = {{1, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}};

P23 = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}};

P34 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}};

P14 = {{0, 0, 0, 1}, {0, 1, 0, 0}, {0, 0, 1, 0}, {1, 0, 0, 0}};

P13 = {{0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 1}};

P12 = {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

$$P24 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \quad P23 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad P34 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix};$$

$$P14 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; \quad P13 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad P12 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

In[∘]:=
```
𝒫12 = Simplify[S12.P12.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫12]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
𝒫34 = Simplify[S12.P34.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫34]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
𝒫14 = Simplify[S12.P14.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫14]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

In[∘]:=
```
𝒫13 = Simplify[S12.P13.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫13]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

In[∘]:=
```
𝒫23 = Simplify[S12.P23.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫23]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

In[∘]:=
```
𝒫24 = Simplify[S12.P24.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫24]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

In[∘]:=
```
TraditionalForm[FullSimplify[𝒫23.𝒫14]]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
TraditionalForm[FullSimplify[𝒫24.𝒫13]]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

In[∘]:=
```
TraditionalForm[FullSimplify[𝒫23.𝒫14.𝒫24.𝒫13]]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4} \tag{15}$$

$$\rho_1^{(12)} = x_1 - x_2 \tag{16}$$

$$\rho_2^{(12)} = x_3 - x_4 \tag{17}$$

$$\rho_3^{(12)} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - \frac{m_3 x_3 + m_4 x_4}{m_3 + m_4} \tag{18}$$

and *mass-scaled* Jacobi coordinates as:

$$y_1^{(12)} = \sqrt{\frac{\mu_{12}}{\mu}}\,(x_1 - x_2) \tag{19}$$

$$y_2^{(12)} = \sqrt{\frac{\mu_{34}}{\mu}}\,(x_3 - x_4) \tag{20}$$

$$y_3^{(12)} = \sqrt{\frac{\mu_{12,34}}{\mu}}\left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - \frac{m_3 x_3 + m_4 x_4}{m_3 + m_4}\right) \tag{21}$$

$$y_4^{(12)} = \sqrt{\frac{M}{\mu}}\; X_{\text{cm}} \tag{22}$$

The convention here is that the superscript (12) indicates this coordinate system is convenient for treating the interaction between particles 1 and 2. In matrix notation:

$$\begin{pmatrix} y_1^{(12)} \\ y_2^{(12)} \\ y_3^{(12)} \\ y_4^{(12)} \end{pmatrix} = \frac{1}{\sqrt{\mu}} \begin{pmatrix} \sqrt{\mu_{12}} & -\sqrt{\mu_{12}} & 0 & 0 \\ 0 & 0 & \sqrt{\mu_{34}} & -\sqrt{\mu_{34}} \\ \frac{\sqrt{\mu_{12,34}}\; m_1}{m_1+m_2} & \frac{\sqrt{\mu_{12,34}}\; m_2}{m_1+m_2} & -\frac{\sqrt{\mu_{12,34}}\; m_3}{m_3+m_4} & -\frac{\sqrt{\mu_{12,34}}\; m_4}{m_3+m_4} \\ \frac{m_1}{\sqrt{M}} & \frac{m_2}{\sqrt{M}} & \frac{m_3}{\sqrt{M}} & \frac{m_4}{\sqrt{M}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{23}$$

Now we define the hyperspherical coordinates as:

$$y_1^{(12)} = \cos\theta_{12} \tag{24}$$

$$y_2^{(12)} = \sin\theta_{12}\cos\phi_{12} \tag{25}$$

$$y_3^{(12)} = \sin\theta_{12}\sin\phi_{12} \tag{26}$$

In[∘]:=
```
Clear[μ4]
```

In[∘]:=
```
μ4 = (μ12 μ34 μ1234)^(1/3) /.
  {μ12 → (m1 m2/(m1 + m2)), μ34 → (m3 m4/(m3 + m4)), μ1234 -> ((m1 + m2) (m3 + m4)/(m1 + m2 + m3 + m4)), M → m1 + m2 + m3 + m4}
```

Out[∘]=

$$\left(\frac{\text{m1 m2 m3 m4}}{\text{m1} + \text{m2} + \text{m3} + \text{m4}}\right)^{1/3}$$

In[∘]:=
$$A = \frac{1}{\sqrt{\mu 4}} \begin{pmatrix} \sqrt{\mu 12} & -\sqrt{\mu 12} & 0 & 0 \\ 0 & 0 & \sqrt{\mu 34} & -\sqrt{\mu 34} \\ \frac{\sqrt{\mu 1234}\ m1}{m1+m2} & \frac{\sqrt{\mu 1234}\ m2}{m1+m2} & -\frac{\sqrt{\mu 1234}\ m3}{m3+m4} & -\frac{\sqrt{\mu 1234}\ m4}{m3+m4} \\ \frac{m1}{\sqrt{M}} & \frac{m2}{\sqrt{M}} & \frac{m3}{\sqrt{M}} & \frac{m4}{\sqrt{M}} \end{pmatrix} \text{/. } \left\{\mu 12 \to \left(\frac{m1\ m2}{m1+m2}\right),\right.$$
$$\left.\mu 34 \to \left(\frac{m3\ m4}{m3+m4}\right),\ \mu 1234 \text{ -> } \left(\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}\right),\ M \to m1+m2+m3+m4\right\} \text{ // FullSimplify}$$

Out[∘]=
$$\left\{\left\{\frac{\sqrt{\frac{m1\ m2}{m1+m2}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}, -\frac{\sqrt{\frac{m1\ m2}{m1+m2}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}, 0, 0\right\},\right.$$
$$\left\{0, 0, \frac{\sqrt{\frac{m3\ m4}{m3+m4}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}, -\frac{\sqrt{\frac{m3\ m4}{m3+m4}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}\right\}, \left\{\frac{m1\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m1+m2)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}},\right.$$
$$\left.\frac{m2\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m1+m2)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}, -\frac{m3\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m3+m4)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}, -\frac{m4\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m3+m4)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}\right\},$$
$$\left\{\frac{m1}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}}, \frac{m2}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}},\right.$$
$$\left.\left.\frac{m3}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}}, \frac{m4}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}}\right\}\right\}$$

Let's just treat the equal mass case here...

In[∘]:=
$$\begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \text{FullSimplify}\left[\text{Inverse[A]}.\begin{pmatrix} x \\ y \\ z \\ XCM \end{pmatrix} \text{/. } \{m1 \to m,\ m2 \to m,\ m3 \to \beta\, m,\ m4 \to \beta\, m\}\right];$$

In[∘]:= Clear[R]

In[∘]:= x12 = Simplify[x1 - x2]; FullSimplify[x12, {m > 0, β > 0}]
x12hyp = Simplify[x12, m > 0] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x12hyp, m > 0]

Out[∘]=
$$\frac{2^{1/3}\ x\ \beta^{1/3}}{(1+\beta)^{1/6}}$$

Out[∘]=
$$2^{1/3}\ R\left(\frac{\beta^2}{1+\beta}\right)^{1/6} \text{Cos}[\phi]\ \text{Sin}[\theta]$$

In[∘]:=
```
x13 = FullSimplify[x1 - x3, {m > 0, β > 0}]
x13hyp = Simplify[x13] /. {x → R Sin[θ] Cos[ϕ], y →  R Sin[θ] Sin[ϕ], z →  R Cos[θ]};
FullSimplify[x13hyp, {m > 0, β > 0}]
```

Out[∘]= $$\frac{\sqrt{m}\ x\,\beta - y\sqrt{m\,\beta} + z\sqrt{m\,\beta\,(1+\beta)}}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

Out[∘]= $$\frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]\left(\sqrt{m}\ \beta\,\mathrm{Cos}[\phi] - \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

In[∘]:=
```
x14 = FullSimplify[x1 - x4, {m > 0, β > 0}]
x14hyp = Simplify[x14] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x14hyp, {m > 0, β > 0}]
```

Out[∘]= $$\frac{\sqrt{m}\ x\,\beta + y\sqrt{m\,\beta} + z\sqrt{m\,\beta\,(1+\beta)}}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

Out[∘]= $$\frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]\left(\sqrt{m}\ \beta\,\mathrm{Cos}[\phi] + \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

In[∘]:=
```
x23 = Simplify[x2 - x3, {m > 0, β > 0}]
x23hyp = Simplify[x23] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x23hyp, {m > 0, β > 0}]
```

Out[∘]= $$-\frac{\sqrt{m}\ x\,\beta + y\sqrt{m\,\beta} - z\sqrt{m\,\beta\,(1+\beta)}}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

Out[∘]= $$\frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] - \mathrm{Sin}[\theta]\left(\sqrt{m}\ \beta\,\mathrm{Cos}[\phi] + \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

In[∘]:=
```
x24 = Simplify[x2 - x4, {m > 0, β > 0}]
x24hyp = Simplify[x24] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x24hyp, {m > 0, β > 0}]
```

Out[∘]= $$\frac{-\sqrt{m}\ x\,\beta + y\sqrt{m\,\beta} + z\sqrt{m\,\beta\,(1+\beta)}}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

Out[∘]= $$\frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]\left(-\sqrt{m}\ \beta\,\mathrm{Cos}[\phi] + \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\ (1+\beta)^{1/6}}$$

In[∘]:=
```
x34 = Simplify[x3 - x4, {m > 0, β > 0}]
x34hyp = Simplify[x3 - x4, {m > 0, β > 0}] /.
   {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x34hyp, {m > 0, β > 0}]
```

Out[∘]=
$$\frac{2^{1/3}\, y}{(\beta\,(1+\beta))^{1/6}}$$

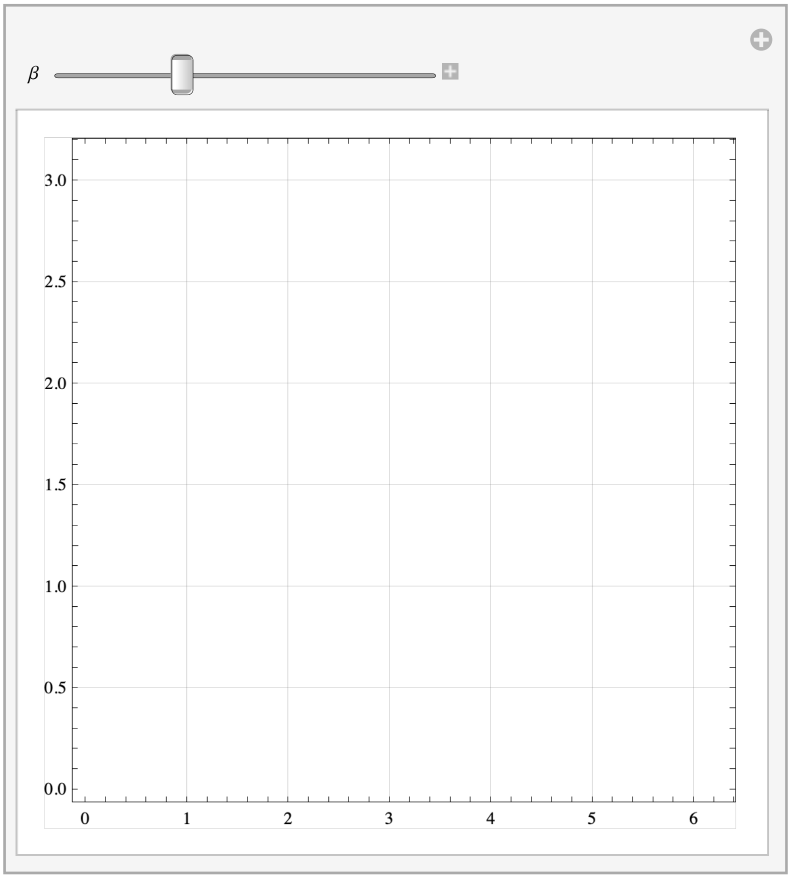
Out[∘]=
$$\frac{2^{1/3}\, R\, \mathrm{Sin}[\theta]\, \mathrm{Sin}[\phi]}{(\beta\,(1+\beta))^{1/6}}$$

In[∘]:=
```
unequalmassxij[β_] = FullSimplify[{x12hyp, x13hyp, x14hyp, x23hyp, x24hyp, x34hyp}]
```

Out[∘]=
$$\left\{2^{1/3}\, R \left(\frac{\beta^2}{1+\beta}\right)^{1/6} \mathrm{Cos}[\phi]\, \mathrm{Sin}[\theta],\ \frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]\left(\sqrt{m}\ \beta\, \mathrm{Cos}[\phi] - \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\,(1+\beta)^{1/6}},\right.$$
$$\frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]\left(\sqrt{m}\ \beta\, \mathrm{Cos}[\phi] + \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\,(1+\beta)^{1/6}},$$
$$\frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] - \mathrm{Sin}[\theta]\left(\sqrt{m}\ \beta\, \mathrm{Cos}[\phi] + \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\,(1+\beta)^{1/6}},$$
$$\left.\frac{R\left(\sqrt{m\,\beta\,(1+\beta)}\ \mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]\left(-\sqrt{m}\ \beta\, \mathrm{Cos}[\phi] + \sqrt{m\,\beta}\ \mathrm{Sin}[\phi]\right)\right)}{2^{2/3}\sqrt{m}\ \beta^{2/3}\,(1+\beta)^{1/6}},\ \frac{2^{1/3}\, R\, \mathrm{Sin}[\theta]\, \mathrm{Sin}[\phi]}{\left(\beta+\beta^2\right)^{1/6}}\right\}$$

In[∘]:=
```
Manipulate[ContourPlot[Evaluate[unequalmassxij[β] == 0 /. {R → 1, m → 1}],
  {ϕ, 0, 2 π}, {θ, 0, π}, GridLines → Automatic], {β, 1, 10}]
```

Out[∘]=



In[∘]:=
```
unequalmassxij = FullSimplify[
  {x12hyp, x13hyp, x14hyp, x23hyp, x24hyp, x34hyp} /. {R → 1, m → 1, θ → t π, ϕ → f π}]
```

Out[∘]=

$$\left\{2^{1/3}\left(\frac{\beta^2}{1+\beta}\right)^{1/6}\mathrm{Cos}[f\,\pi]\,\mathrm{Sin}[\pi\,t],\ \frac{\sqrt{\beta\,(1+\beta)}\ \mathrm{Cos}[\pi\,t]+\beta\,\mathrm{Cos}[f\,\pi]\,\mathrm{Sin}[\pi\,t]-\sqrt{\beta}\ \mathrm{Sin}[f\,\pi]\,\mathrm{Sin}[\pi\,t]}{2^{2/3}\,\beta^{2/3}\,(1+\beta)^{1/6}},\right.$$

$$\frac{\sqrt{\beta\,(1+\beta)}\ \mathrm{Cos}[\pi\,t]+\beta\,\mathrm{Cos}[f\,\pi]\,\mathrm{Sin}[\pi\,t]+\sqrt{\beta}\ \mathrm{Sin}[f\,\pi]\,\mathrm{Sin}[\pi\,t]}{2^{2/3}\,\beta^{2/3}\,(1+\beta)^{1/6}},$$

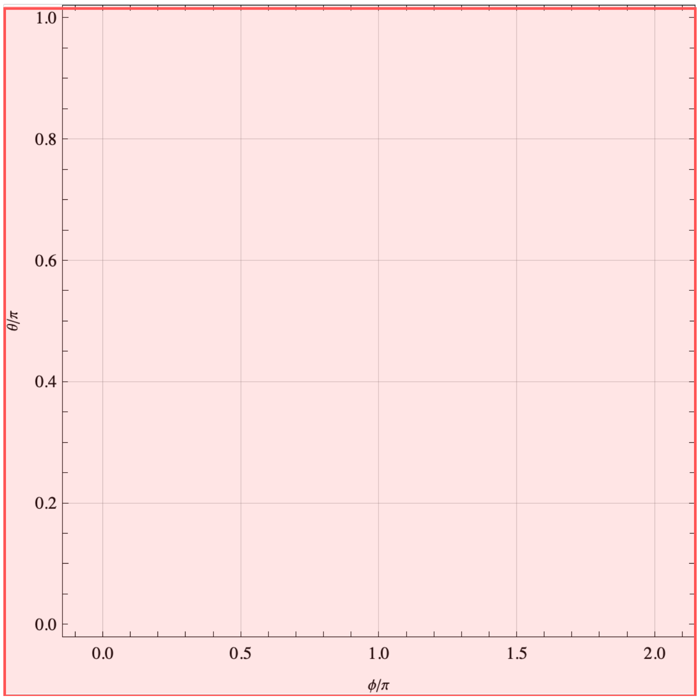
$$\frac{\sqrt{\beta\,(1+\beta)}\ \mathrm{Cos}[\pi\,t]-\sqrt{\beta}\ \left(\sqrt{\beta}\ \mathrm{Cos}[f\,\pi]+\mathrm{Sin}[f\,\pi]\right)\mathrm{Sin}[\pi\,t]}{2^{2/3}\,\beta^{2/3}\,(1+\beta)^{1/6}},$$

$$\left.\frac{\sqrt{\beta\,(1+\beta)}\ \mathrm{Cos}[\pi\,t]-\beta\,\mathrm{Cos}[f\,\pi]\,\mathrm{Sin}[\pi\,t]+\sqrt{\beta}\ \mathrm{Sin}[f\,\pi]\,\mathrm{Sin}[\pi\,t]}{2^{2/3}\,\beta^{2/3}\,(1+\beta)^{1/6}},\ \frac{2^{1/3}\,\mathrm{Sin}[f\,\pi]\,\mathrm{Sin}[\pi\,t]}{\left(\beta+\beta^2\right)^{1/6}}\right\}$$

In[∘]:=
```
Manipulate[DensityPlot[Product[unequalmassxij[[i]], {i, 1, 6}] == 0 /. β → beta,
  {f, -0.1, 2.1}, {t, 0, 1}, GridLines → Automatic,
  ColorFunction → "TemperatureMap", FrameLabel → {"ϕ/π", "θ/π"}], {beta, 1, 4}]
```

Out[ ]=

beta



Part: Part specification unequalmassxij⟦1⟧ is longer than depth of object.

Part: Part specification unequalmassxij⟦2⟧ is longer than depth of object.

Part: Part specification unequalmassxij⟦3⟧ is longer than depth of object.

General: Further output of Part::partd will be suppressed during this calculation.

Part: Part specification unequalmassxij⟦1⟧ is longer than depth of object.

Part: Part specification unequalmassxij⟦2⟧ is longer than depth of object.

Part: Part specification unequalmassxij⟦3⟧ is longer than depth of object.

General: Further output of Part::partd will be suppressed during this calculation.

Part: Part specification unequalmassxij⟦1⟧ is longer than depth of object.

Part: Part specification unequalmassxij⟦2⟧ is longer than depth of object.

Part: Part specification unequalmassxij⟦3⟧ is longer than depth of object.

General: Further output of Part::partd will be suppressed during this calculation.

Part: Part specification unequalmassxij⟦1⟧ is longer than depth of object.

Part: Part specification unequalmassxij⟦2⟧ is longer than depth of object.

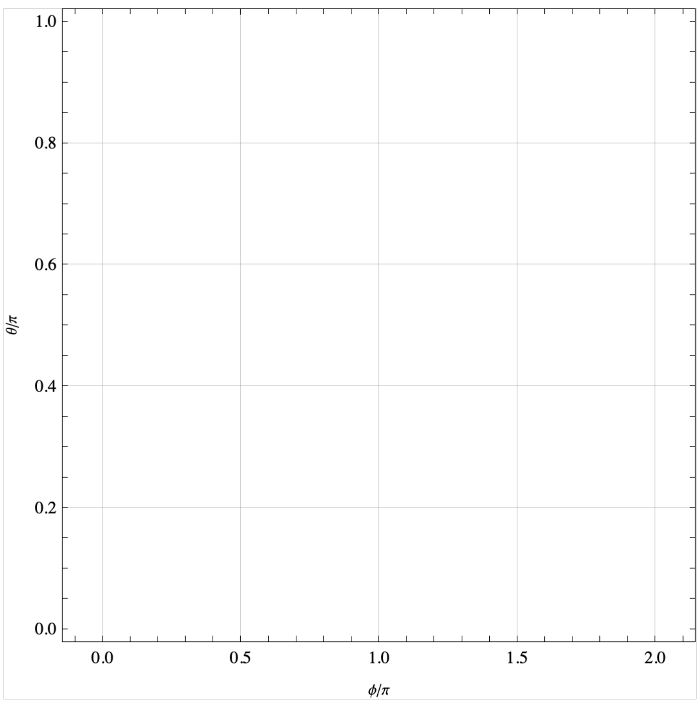
Part: Part specification unequalmassxij⟦3⟧ is longer than depth of object.

General: Further output of Part::partd will be suppressed during this calculation.

In[∘]:=
```
c12[β_] := ContourPlot[Evaluate[unequalmassxij[[1]] == 0], {f, -0.1, 2.1}, {t, 0, 1},
  GridLines → Automatic, ContourStyle → Black, FrameLabel → {"ϕ/π", "θ/π"}]
c13[β_] := ContourPlot[Evaluate[unequalmassxij[[2]] == 0], {f, -0.1, 2.1}, {t, 0, 1},
   GridLines → Automatic, ContourStyle → Red, FrameLabel → {"ϕ/π", "θ/π"}];
c14[β_] := ContourPlot[Evaluate[unequalmassxij[[3]] == 0], {f, -0.1, 2.1}, {t, 0, 1},
   GridLines → Automatic, ContourStyle → Blue, FrameLabel → {"ϕ/π", "θ/π"}];
c23[β_] := ContourPlot[Evaluate[unequalmassxij[[4]] == 0], {f, -0.1, 2.1}, {t, 0, 1},
   GridLines → Automatic, ContourStyle → Green, FrameLabel → {"ϕ/π", "θ/π"}];
c24[β_] := ContourPlot[Evaluate[unequalmassxij[[5]] == 0], {f, -0.1, 2.1}, {t, 0, 1},
   GridLines → Automatic, ContourStyle → Orange, FrameLabel → {"ϕ/π", "θ/π"}];
c34[β_] := ContourPlot[Evaluate[unequalmassxij[[6]] == 0], {f, -0.1, 2.1}, {t, 0, 1},
   GridLines → Automatic, ContourStyle → Yellow, FrameLabel → {"ϕ/π", "θ/π"}];
```

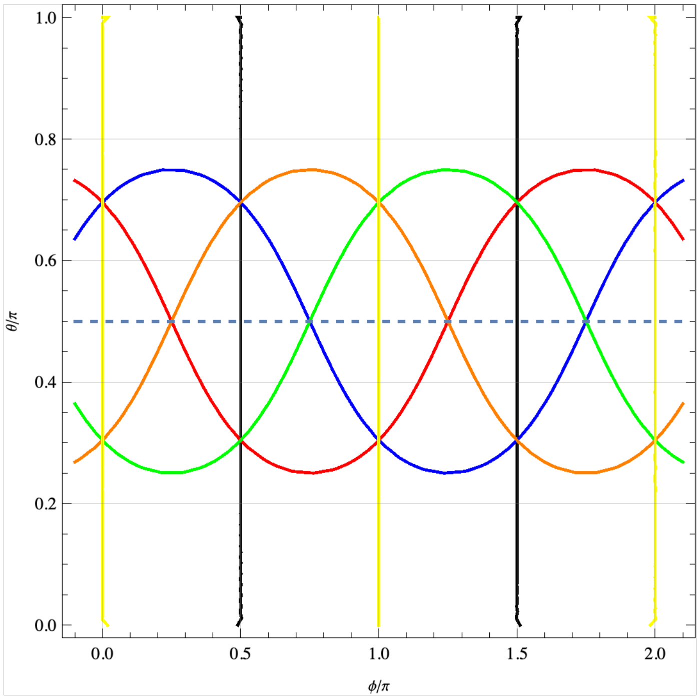
In[∘]:=
```
c12[1.0]
```

Out[∘]=



In[ ]:=
```
pall = Show[{c12, c13, c14, c23, c24, c34}, Plot[0.5, {x, -.1, 2.1}, PlotStyle → Dashed]]
```

Out[ ]=



In[ ]:=
```
cptable = Table[ContourPlot[Evaluate[unequalmassxij[[i]] == 0 /. {R → 1, α → 1}],
    {f, 0, .5}, {t, 0, .5}, GridLines → Automatic, ContourStyle → Black], {i, 1, 6}];
```

In[ ]:=
```
Show[pall, PlotRange → {{0, 0.5}, {0, 0.5}}]
```

Out[ ]=