Fits to Brody distribution, and calculations for density of states

In[31]:=

```
PBrody[x_, w_] := (1 + w) (Gamma[(2 + w)/(1 + w)])^(1+w) x^w Exp[- (Gamma[(2 + w)/(1 + w)] x)^(1+w)];
bins = Table[x, {x, 0, 5, 0.2}];
bFit = Table[0.5 * (bins[[i]] + bins[[i + 1]]), {i, 1, Length[bins] - 1}];
SampleRange = # < Ecutoff1 &
```

Out[34]=

```
#1 < Ecutoff1 &
```

- **Run 0**

In[41]:=

```
SampleRange = # < Ecutoff0 &
curvestemp0 =
  Transpose[Import["/Users/niravmehta/Documents/GitHub/4BodySVD/AdiabaticCurves.dat"]];
Curves0 = Table[Table[{curvestemp0[[1, i]], curvestemp0[[j, i]]},
    {i, 1, Length[curvestemp0[[1]]]}], {j, 2, Length[curvestemp0]}];
Evals0 = Flatten[Import["/Users/niravmehta/Documents/GitHub/4BodySVD/Eigenvals.dat"]];
Ecutoff0 = Evals0[[4]]
Evals0 = Sort[Drop[Evals0, 1 ;; 4]];
eValsBound0 = Sort[Select[Evals0, SampleRange]];
```

Out[41]=

```
#1 < Ecutoff0 &
```

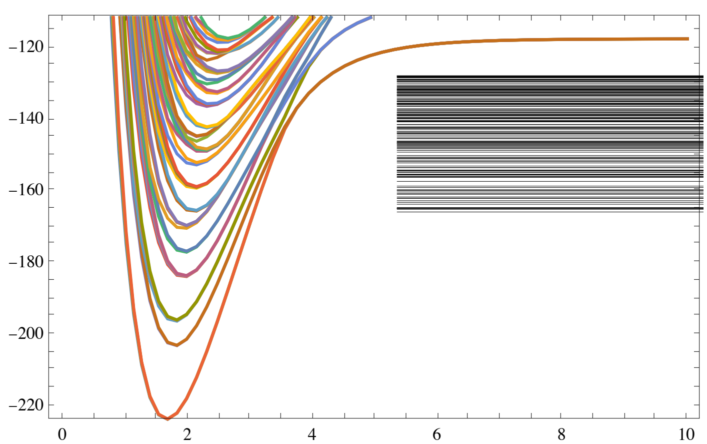
Out[45]=

```
-117.562
```

In[∘]:=

```
pcurves0 = ListPlot[Curves0, Frame → True, PlotMarkers → None,
   Joined → True, PlotRange → {Min[curvestemp0[[2]]], Ecutoff0 + 6}];
penergies0 = ListPlot[Table[{8, eValsBound0[[i]]}, {i, 1, Length[eValsBound0]}],
   Frame → True, PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{0, 0}, {5, 0}}]}]];
Show[pcurves0, penergies0, PlotRange → Automatic]
```

Out[∘]=



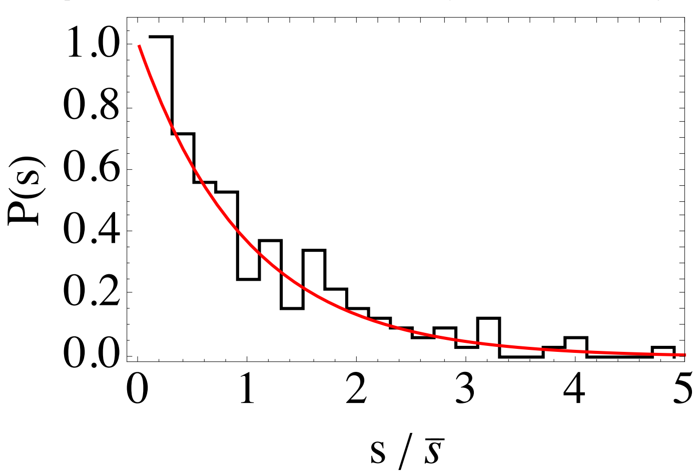
In[∘]:=
```
Es0 = Table[eValsBound0[[i + 1]] - eValsBound0[[i]], {i, 1, Length[eValsBound0] - 1}];
eTrim0 = Select[Es0, # > 0.0000001 &];
avg0 = Mean[eTrim0];
eTrim0 = eTrim0 / avg0;
(*Sort[eTrim0];*)
Length[Es0];
Length[eTrim0];
ρ0 = 1 / avg0;
EspaceBin0 = BinCounts[eTrim0, {bins}];
NormBrodyBinCs0 = Table[EspaceBin0[[i]] / Total[EspaceBin0] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist0 = Table[{bFit[[i]], NormBrodyBinCs0[[i]]}, {i, 1, Length[bFit]}]
phist0 = ListPlot[brodyPdist0, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars0 = FindFit[brodyPdist0, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit0 = Plot[PBrody[s, w /. pars0], {s, 0, 5}, PlotRange → All, PlotStyle → Red];
```

Out[∘]=
```
{{0.1, 1.03125}, {0.3, 0.71875}, {0.5, 0.5625}, {0.7, 0.53125},
 {0.9, 0.25}, {1.1, 0.375}, {1.3, 0.15625}, {1.5, 0.34375}, {1.7, 0.21875},
 {1.9, 0.15625}, {2.1, 0.125}, {2.3, 0.09375}, {2.5, 0.0625}, {2.7, 0.09375},
 {2.9, 0.03125}, {3.1, 0.125}, {3.3, 0.}, {3.5, 0.}, {3.7, 0.03125},
 {3.9, 0.0625}, {4.1, 0.}, {4.3, 0.}, {4.5, 0.}, {4.7, 0.03125}, {4.9, 0.}}
```

Out[∘]= $\{w \to 4.32273\times10^{-7}\}$

In[∘]:=
```
Show[phist0, pfit0, FrameLabel → {"s / s̄", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[∘]=



Run1

In[383]:=
```
rundata1 = Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run1/4BodySVD.par", "text"]
```

Out[383]=
```
xPoints (phi)         yPoints (theta)
30             60

LobattoPoints         NumChannels    Order
64             40        5

PotentialDepth         Rmin         Rmax         alpha (turns V on and off)
40.d0            0d0        10.d0        1.d0

m1        m2        m3        m4
1.d0        1.d0        1.d0        1.d0

xMin        xMax        yMin        yMax   (enter in units of pi)
0d0        0.25d0        0d0         0.5d0

Left        Right        Bot        Top
0        0        2        1



!  At top -- Phi(theta=pi/2,phi)=0 for odd parity so choose Top = 0
               Phi'(theta=pi/2,phi)=0 for even parity so choose Top = 1
```

```
SampleRange = Ecutoff1 - 30.0 < # < Ecutoff1 - 1.0 &
curvestemp1 = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run1/AdiabaticCurves.dat"]];
Curves1 = Table[Table[{curvestemp1[[1, i]], curvestemp1[[j, i]]},
    {i, 1, Length[curvestemp1[[1]]]}], {j, 2, Length[curvestemp1]}];
Evals1 = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run1/Eigenvals.dat"]];
Ecutoff1 = Evals1[[4]]
Evals1 = Sort[Drop[Evals1, 1 ;; 4]];
eValsBound1 = Sort[Select[Evals1, SampleRange]]
```

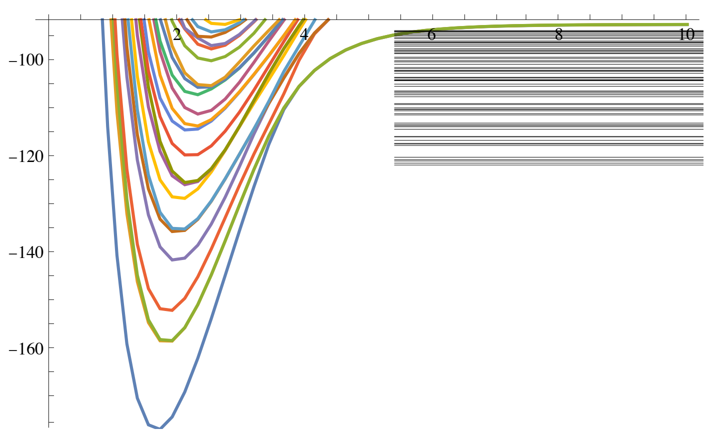
Out[289]= Ecutoff1 - 30. < #1 < Ecutoff1 - 1. &

Out[293]= -92.4717

Out[295]= {-121.61, -121.216, -120.765, -120.5, -119.99, -117.553, -117.246, -117.118, -116.484, -115.777, -115.681, -114.184, -113.676, -113.449, -113.215, -112.903, -111.217, -110.945, -110.367, -110.101, -110.014, -109.749, -109.055, -108.859, -107.449, -107.042, -107.022, -106.686, -106.434, -106.232, -105.278, -104.819, -104.126, -104.017, -103.859, -103.542, -103.394, -102.559, -102.159, -102.087, -101.971, -101.602, -101.375, -100.701, -100.207, -100.181, -99.9092, -99.4754, -99.2255, -98.5014, -98.3682, -98.0902, -97.8463, -97.7517, -97.4842, -96.9466, -96.666, -96.3003, -96.2263, -96.0905, -95.8742, -95.3924, -95.1579, -94.8176, -94.574, -94.2601, -94., -93.9054, -93.8451, -93.7054}

In[296]:=
```
pcurves1 = ListPlot[Curves1, PlotMarkers → None,
   Joined → True, PlotRange → {Min[curvestemp1[[2]]], Ecutoff1 + 1}];
penergies1 = ListPlot[Table[{8, eValsBound1[[i]]}, {i, 1, Length[eValsBound1]}],
   PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{15, 0}, {20, 0}}]}]];
Show[pcurves1, penergies1]
```

Out[298]=



In[299]:=
```
Es1 = Table[eValsBound1[[i + 1]] - eValsBound1[[i]], {i, 1, Length[eValsBound1] - 1}];
eTrim1 = Select[Es1, # > 0.0000001 &];
avg1 = Mean[eTrim1];
eTrim1 = eTrim1 / avg1;
(*Sort[eTrim1];*)
Length[Es1];
Length[eTrim1];
ρ1 = 1 / avg1;
EspaceBin1 = BinCounts[eTrim1, {bins}];
NormBrodyBinCs1 = Table[EspaceBin1[[i]] / Total[EspaceBin1] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist1 = Table[{bFit[[i]], NormBrodyBinCs1[[i]]}, {i, 1, Length[bFit]}]
phist1 = ListPlot[brodyPdist1, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars1 = FindFit[brodyPdist1, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit1 = Plot[PBrody[s, w /. pars1], {s, 0, 5}, PlotRange → All, PlotStyle → Red];
```

Out[308]=
```
{{0.1, 0.367647}, {0.3, 0.882353}, {0.5, 0.514706}, {0.7, 1.25},
 {0.9, 0.441176}, {1.1, 0.367647}, {1.3, 0.294118}, {1.5, 0.147059},
 {1.7, 0.367647}, {1.9, 0.}, {2.1, 0.0735294}, {2.3, 0.0735294}, {2.5, 0.},
 {2.7, 0.}, {2.9, 0.}, {3.1, 0.}, {3.3, 0.}, {3.5, 0.0735294}, {3.7, 0.0735294},
 {3.9, 0.}, {4.1, 0.0735294}, {4.3, 0.}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.}}
```

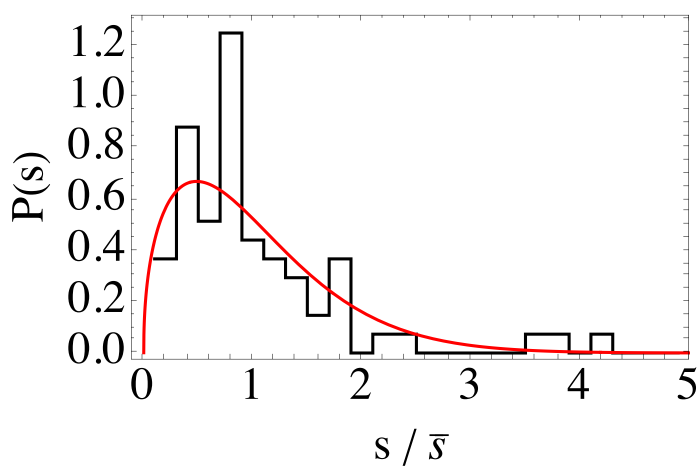
Out[310]=
```
{w → 0.440578}
```

In[312]:=
```
ρ1
```

Out[312]=
```
2.47269
```

In[313]:=
```
Show[phist1, pfit1, FrameLabel → {"s / s̄", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[313]=



Run2

In[386]:=
```
rundata2 = Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run2/4BodySVD.par", "text"]
```

Out[386]=
```
xPoints (phi)         yPoints (theta)
30            60

LobattoPoints         NumChannels    Order
64            80        5

PotentialDepth         Rmin         Rmax         alpha (turns V on and off)
40.d0            0d0        10.d0        1.d0

m1        m2        m3        m4
1.d0        1.d0        1.d0        1.d0

xMin        xMax        yMin        yMax   (enter in units of pi)
0d0        0.25d0        0d0         0.5d0

Left        Right        Bot        Top
0        1        2        1



!  At top -- Phi(theta=pi/2,phi)=0 for odd parity so choose Top = 0
               Phi'(theta=pi/2,phi)=0 for even parity so choose Top = 1
```

In[314]:=
```
SampleRange = Ecutoff2 - 30.0 < # < Ecutoff2 - 1 &
curvestemp2 = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run2/AdiabaticCurves.dat"]];
Curves2 = Table[Table[{curvestemp2[[1, i]], curvestemp2[[j, i]]},
    {i, 1, Length[curvestemp2[[1]]]}], {j, 2, Length[curvestemp2]}];
Evals2 = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run2/Eigenvals.dat"]];
Ecutoff2 = Evals2[[4]]
Evals2 = Sort[Drop[Evals2, 1 ;; 4]];
eValsBound2 = Sort[Select[Evals2, SampleRange]]
```

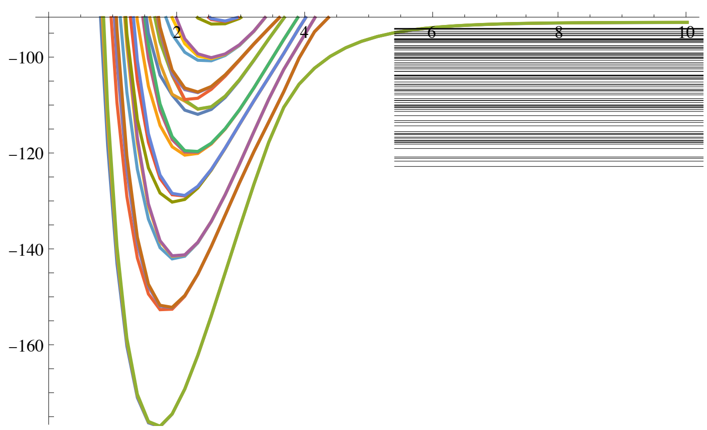
Out[314]= `Ecutoff2 - 30. < #1 < Ecutoff2 - 1 &`

Out[318]= `-92.4717`

Out[320]=
```
{-122.413, -121.312, -120.73, -120.439, -118.683, -117.761, -117.45,
 -117.288, -117.048, -116.962, -116.542, -116.306, -115.805, -115.616,
 -115.541, -115.17, -113.967, -113.265, -112.881, -111.855, -110.973, -110.58,
 -110.215, -110.1, -109.867, -109.704, -109.611, -109.134, -108.734, -108.663,
 -108.309, -107.528, -107.188, -106.715, -106.577, -106.345, -105.85, -105.486,
 -105.296, -104.848, -104.334, -104.104, -103.972, -103.621, -103.533, -103.509,
 -103.394, -102.646, -102.632, -102.13, -101.855, -101.504, -101.13, -100.824,
 -100.347, -99.9505, -99.8741, -99.7308, -99.6589, -99.3399, -99.163, -98.9669,
 -98.786, -98.1867, -97.881, -97.7348, -97.67, -97.3511, -97.2895, -96.6686,
 -96.4676, -96.287, -96.0921, -96.0597, -95.988, -95.9237, -95.8149, -95.2412,
 -95.0797, -94.7664, -94.6698, -94.3024, -93.898, -93.8351, -93.795, -93.6754}
```

In[321]:=
```
pcurves2 = ListPlot[Curves2, PlotMarkers → None,
   Joined → True, PlotRange → {Min[curvestemp2[[2]]], Ecutoff2 + 1}];
penergies2 = ListPlot[Table[{8, eValsBound2[[i]]}, {i, 1, Length[eValsBound2]}],
   PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{15, 0}, {20, 0}}]}]];
Show[pcurves2, penergies2]
```

Out[323]=



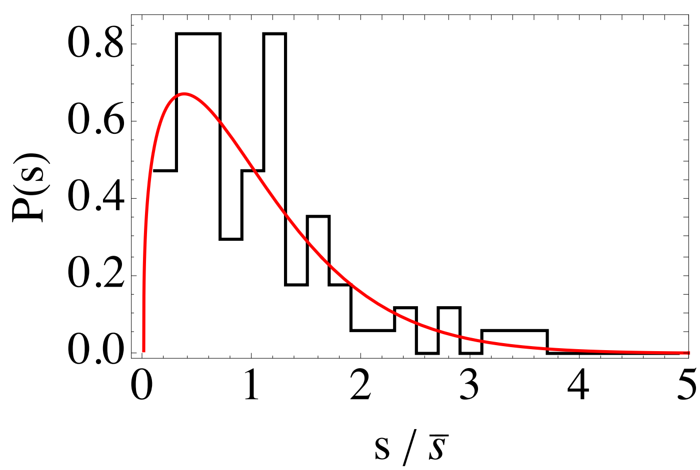
In[324]:=
```
Es2 = Table[eValsBound2[[i + 1]] - eValsBound2[[i]], {i, 1, Length[eValsBound2] - 1}];
eTrim2 = Select[Es2, # > 0.0000001 &];
avg2 = Mean[eTrim2];
eTrim2 = eTrim2 / avg2;
(*Sort[eTrim2];*)
Length[Es2];
Length[eTrim2];
ρ2 = 1 / avg2;
EspaceBin2 = BinCounts[eTrim2, {bins}];
NormBrodyBinCs2 = Table[EspaceBin2[[i]] / Total[EspaceBin2] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist2 = Table[{bFit[[i]], NormBrodyBinCs2[[i]]}, {i, 1, Length[bFit]}]
phist2 = ListPlot[brodyPdist2, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars2 = FindFit[brodyPdist2, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit2 = Plot[PBrody[s, w /. pars2], {s, 0, 5}, PlotRange → All, PlotStyle → Red];
```

Out[333]=
```
{{0.1, 0.47619}, {0.3, 0.833333}, {0.5, 0.833333}, {0.7, 0.297619},
 {0.9, 0.47619}, {1.1, 0.833333}, {1.3, 0.178571}, {1.5, 0.357143},
 {1.7, 0.178571}, {1.9, 0.0595238}, {2.1, 0.0595238}, {2.3, 0.119048}, {2.5, 0.},
 {2.7, 0.119048}, {2.9, 0.}, {3.1, 0.0595238}, {3.3, 0.0595238}, {3.5, 0.0595238},
 {3.7, 0.}, {3.9, 0.}, {4.1, 0.}, {4.3, 0.}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.}}
```

Out[335]=
```
{w → 0.319464}
```

In[337]:=
```
Show[phist2, pfit2, FrameLabel → {"s / s̅", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[337]=



Run3

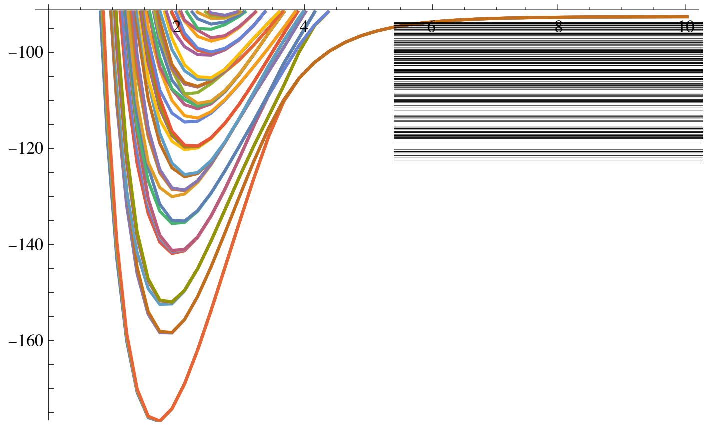
In[387]:=
```
rundata3 = Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run3/4BodySVD.par", "text"]
```

Out[387]=
```
xPoints (phi)          yPoints (theta)
60              60

LobattoPoints          NumChannels    Order
64              80        5

PotentialDepth          Rmin          Rmax          alpha (turns V on and off)
40.d0             0d0         10.d0         1.d0

m1         m2         m3         m4
1.d0         1.d0         1.d0         1.d0

xMin         xMax         yMin         yMax    (enter in units of pi)
0d0         0.5d0         0d0          0.5d0

Left         Right         Bot         Top
0         0         2         1



!  At top -- Phi(theta=pi/2,phi)=0 for odd parity so choose Top = 0
               Phi'(theta=pi/2,phi)=0 for even parity so choose Top = 1
```

```
In[338]:= SampleRange = Ecutoff3 - 30.0 < # < Ecutoff3 - 1 &
curvestemp3 = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run3/AdiabaticCurves.dat"]];
Curves3 = Table[Table[{curvestemp3[[1, i]], curvestemp3[[j, i]]},
    {i, 1, Length[curvestemp3[[1]]]}], {j, 2, Length[curvestemp3]}];
Evals3 = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run3/Eigenvals.dat"]];
Ecutoff3 = Evals3[[4]]
Evals3 = Sort[Drop[Evals3, 1 ;; 4]];
eValsBound3 = Sort[Select[Evals3, SampleRange]]

Out[338]= Ecutoff3 - 30. < #1 < Ecutoff3 - 1 &

Out[342]= -92.4718

Out[344]= {-122.413, -121.61, -121.312, -121.216, -120.765, -120.73, -120.5, -120.439, -119.99,
  -118.683, -117.761, -117.553, -117.45, -117.288, -117.246, -117.118, -117.048,
  -116.962, -116.542, -116.483, -116.306, -115.805, -115.776, -115.681, -115.616,
  -115.541, -115.169, -114.183, -113.966, -113.675, -113.449, -113.265, -113.215,
  -112.903, -112.881, -111.854, -111.217, -110.972, -110.945, -110.58, -110.367,
  -110.214, -110.101, -110.1, -110.013, -109.866, -109.749, -109.704, -109.611, -109.134,
  -109.054, -108.859, -108.734, -108.663, -108.308, -107.526, -107.449, -107.187,
  -107.042, -107.021, -106.714, -106.686, -106.576, -106.434, -106.345, -106.232,
  -105.85, -105.485, -105.295, -105.277, -104.847, -104.819, -104.332, -104.125,
  -104.103, -104.017, -103.972, -103.858, -103.62, -103.542, -103.532, -103.507,
  -103.394, -103.393, -102.646, -102.631, -102.558, -102.159, -102.129, -102.087,
  -101.971, -101.855, -101.601, -101.503, -101.374, -101.13, -100.818, -100.701,
  -100.346, -100.205, -100.18, -99.947, -99.9089, -99.873, -99.729, -99.6572, -99.4747,
  -99.3371, -99.2252, -99.1617, -98.9652, -98.785, -98.5003, -98.368, -98.185, -98.089,
  -97.8786, -97.8458, -97.7511, -97.7328, -97.6679, -97.484, -97.3496, -97.2859,
  -96.9458, -96.6676, -96.6653, -96.4657, -96.2998, -96.2844, -96.2252, -96.0917,
  -96.0903, -96.0572, -95.9872, -95.9228, -95.8734, -95.814, -95.3921, -95.2395,
  -95.1576, -95.0783, -94.8173, -94.7656, -94.6652, -94.5738, -94.3002, -94.2596,
  -93.9996, -93.9048, -93.8965, -93.8431, -93.8305, -93.7943, -93.7049, -93.6707}
```

In[345]:=
```
pcurves3 =
  ListPlot[Curves3, Joined → True, PlotRange → {Min[curvestemp3[[2]]], Ecutoff3 + 1}];
penergies3 = ListPlot[Table[{8, eValsBound3[[i]]}, {i, 1, Length[eValsBound3]}],
   PlotMarkers → Graphics[{Thickness[.001], Black, Line[{{15, 0}, {20, 0}}]}]];
Show[pcurves3, penergies3]
```

Out[347]=



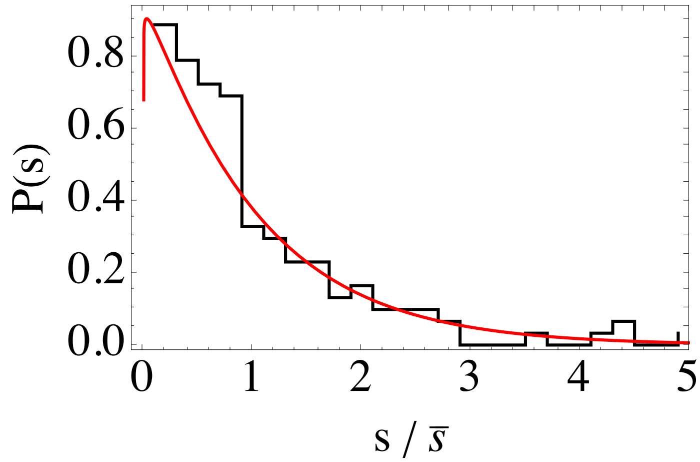
In[348]:=
```
Es3 = Table[eValsBound3[[i + 1]] - eValsBound3[[i]], {i, 1, Length[eValsBound3] - 1}];
eTrim3  = Select[Es3, # > 0.0000001 &];
avg3  = Mean[eTrim3];
eTrim3  = eTrim3 / avg3;
Sort[eTrim3];
Length[Es3];
Length[eTrim3];
ρ3 = 1 / avg3;
EspaceBin3 = BinCounts[eTrim3, {bins}];
NormBrodyBinCs3 = Table[EspaceBin3[[i]] / Total[EspaceBin3] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdist3  = Table[{bFit[[i]], NormBrodyBinCs3[[i]]}, {i, 1, Length[bFit]}]
phist3 = ListPlot[brodyPdist3, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars3  = FindFit[brodyPdist3, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfit3 = Plot[PBrody[s, w /. pars3], {s, 0, 5}, PlotRange → All, PlotStyle → Red];
```

Out[358]=
```
{{0.1, 0.888158}, {0.3, 0.789474}, {0.5, 0.723684}, {0.7, 0.690789}, {0.9, 0.328947},
 {1.1, 0.296053}, {1.3, 0.230263}, {1.5, 0.230263}, {1.7, 0.131579}, {1.9, 0.164474},
 {2.1, 0.0986842}, {2.3, 0.0986842}, {2.5, 0.0986842}, {2.7, 0.0657895},
 {2.9, 0.}, {3.1, 0.}, {3.3, 0.}, {3.5, 0.0328947}, {3.7, 0.}, {3.9, 0.},
 {4.1, 0.0328947}, {4.3, 0.0657895}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.0328947}}
```

Out[360]=
```
{w → 0.0248492}
```

In[380]:= `pall3 = Show[phist3, pfit3, FrameLabel → {"s / s̅", "P(s)"}, Frame → True, LabelStyle → Large]`

Out[380]=



Analysis so far

Runs 1 and 2 have taken advantage of a possible symmetry by further reducing the coordinate space. If my guess is right, then the density of states for run3 should be roughly twice that of run 1 and 2 since I expect both “even and odd” states about phi = $\pi/4$ to be present in run 3, but only even states in run1 and odd states in run2

In[363]:=
```
ρ1
ρ2
ρ3
```

Out[363]= 2.47269

Out[364]= 2.95779

Out[365]= 5.39274

In[242]:= `ρ1 + ρ2`

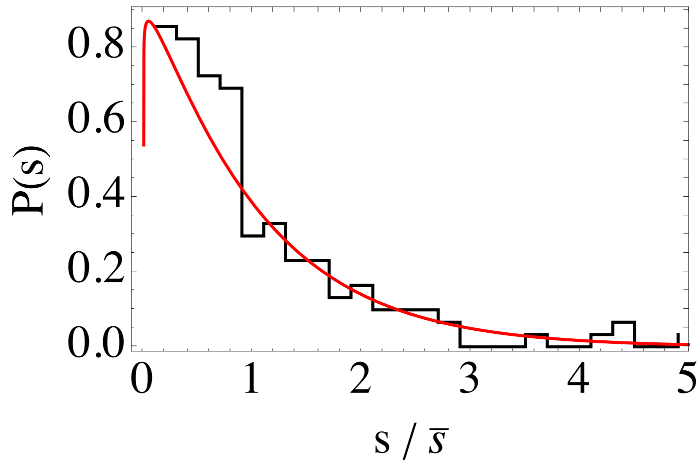
Out[242]= 5.68733

Further, I expect that if these states (even and odd) do not couple since the Hamiltonian is symmetric about $\phi=\pi/4$, then overlaying these two distributions will result in a distribution similar to that of run3

```
In[366]:= eValsBoundCombined = Sort[ Flatten[{eValsBound1, eValsBound2}]]
EsCombined = Table[eValsBoundCombined[[i + 1]] - eValsBoundCombined[[i]],
   {i, 1, Length[eValsBoundCombined] - 1}];
eTrimCombined = Select[EsCombined, # > 0.0000001 &];
avgCombined = Mean[eTrimCombined];
eTrimCombined = eTrimCombined / avgCombined;
(*Sort[eTrimCombined];*)
Length[eTrimCombined];
ρCombined = 1 / avgCombined;
EspaceBinCombined = BinCounts[eTrimCombined, {bins}];
NormBrodyBinCsCombined =
  Table[EspaceBinCombined[[i]] / Total[EspaceBinCombined] / (bins[[i + 1]] - bins[[i]]),
   {i, 1, Length[bins] - 1}];
brodyPdistCombined = Table[{bFit[[i]], NormBrodyBinCsCombined[[i]]}, {i, 1, Length[bFit]}]
phistCombined = ListPlot[brodyPdistCombined,
   InterpolationOrder → 0, Joined → True, PlotRange → All, PlotStyle → Black];
parsCombined = FindFit[brodyPdistCombined, {PBrody[s, w], w > 0, w < 1}, {w}, s]
pfitCombined =
  Plot[PBrody[s, w /. parsCombined], {s, 0, 5}, PlotRange → All, PlotStyle → Red];

Out[366]= {-122.413, -121.61, -121.312, -121.216, -120.765, -120.73, -120.5, -120.439, -119.99,
 -118.683, -117.761, -117.553, -117.45, -117.288, -117.246, -117.118, -117.048,
 -116.962, -116.542, -116.484, -116.306, -115.805, -115.777, -115.681, -115.616,
 -115.541, -115.17, -114.184, -113.967, -113.676, -113.449, -113.265, -113.215,
 -112.903, -112.881, -111.855, -111.217, -110.973, -110.945, -110.58, -110.367,
 -110.215, -110.101, -110.1, -110.014, -109.867, -109.749, -109.704, -109.611, -109.134,
 -109.055, -108.859, -108.734, -108.663, -108.309, -107.528, -107.449, -107.188,
 -107.042, -107.022, -106.715, -106.686, -106.577, -106.434, -106.345, -106.232,
 -105.85, -105.486, -105.296, -105.278, -104.848, -104.819, -104.334, -104.126,
 -104.104, -104.017, -103.972, -103.859, -103.621, -103.542, -103.533, -103.509,
 -103.394, -103.394, -102.646, -102.632, -102.559, -102.159, -102.13, -102.087,
 -101.971, -101.855, -101.602, -101.504, -101.375, -101.13, -100.824, -100.701,
 -100.347, -100.207, -100.181, -99.9505, -99.9092, -99.8741, -99.7308, -99.6589,
 -99.4754, -99.3399, -99.2255, -99.163, -98.9669, -98.786, -98.5014, -98.3682,
 -98.1867, -98.0902, -97.881, -97.8463, -97.7517, -97.7348, -97.67, -97.4842, -97.3511,
 -97.2895, -96.9466, -96.6686, -96.666, -96.4676, -96.3003, -96.287, -96.2263,
 -96.0921, -96.0905, -96.0597, -95.988, -95.9237, -95.8742, -95.8149, -95.3924,
 -95.2412, -95.1579, -95.0797, -94.8176, -94.7664, -94.6698, -94.574, -94.3024,
 -94.2601, -94., -93.9054, -93.898, -93.8451, -93.8351, -93.795, -93.7054, -93.6754}

Out[375]= {{0.1, 0.855263}, {0.3, 0.822368}, {0.5, 0.723684}, {0.7, 0.690789}, {0.9, 0.296053},
 {1.1, 0.328947}, {1.3, 0.230263}, {1.5, 0.230263}, {1.7, 0.131579}, {1.9, 0.164474},
 {2.1, 0.0986842}, {2.3, 0.0986842}, {2.5, 0.0986842}, {2.7, 0.0657895},
 {2.9, 0.}, {3.1, 0.}, {3.3, 0.}, {3.5, 0.0328947}, {3.7, 0.}, {3.9, 0.},
 {4.1, 0.0328947}, {4.3, 0.0657895}, {4.5, 0.}, {4.7, 0.}, {4.9, 0.0328947}}

Out[377]= {w → 0.039776}
```

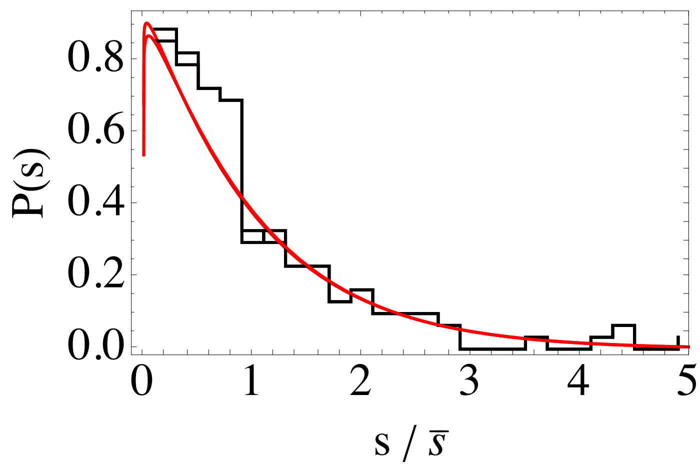
In[381]:=
```
pcomb = Show[phistCombined, pfitCombined,
  FrameLabel → {"s / s̅", "P(s)"}, Frame → True, LabelStyle → Large]
```

Out[381]=



In[382]:=
```
Show[pall3, pcomb]
```

Out[382]=



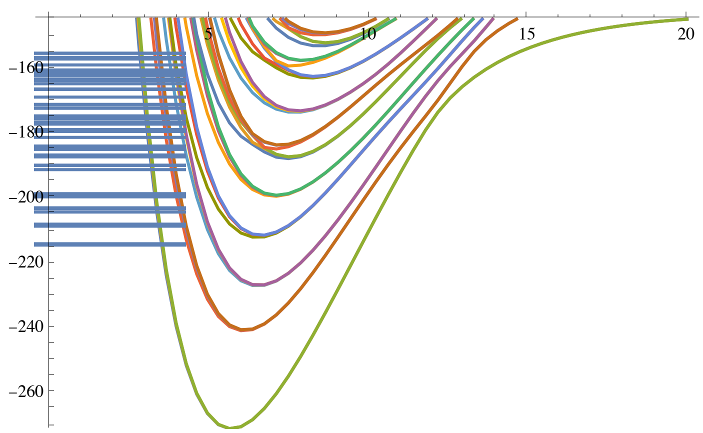
Run4

In[∘]:=
```
curvestemp = Transpose[
   Import["/Users/niravmehta/Documents/GitHub/4BodySVD/run4/AdiabaticCurves.dat"]];
Curves = Table[Table[{curvestemp[[1, i]], curvestemp[[j, i]]},
    {i, 1, Length[curvestemp[[1]]]}], {j, 2, Length[curvestemp]}];
Evals = Flatten[Import[
    "/Users/niravmehta/Documents/GitHub/4BodySVD/run4/Eigenvals.dat"]];
Ecutoff = Evals[[4]]
Evals = Sort[Drop[Evals, 1 ;; 4]];
eValsBound = Sort[Select[Evals, SampleRange]]
```

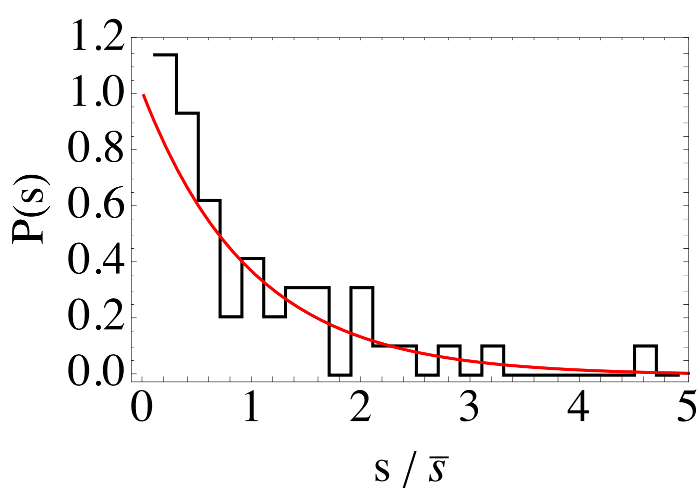
Out[∘]= -144.859

Out[∘]= {-214.521, -214.161, -214.11, -208.69, -208.285, -208.057, -204.329, -203.296, -203.014, -199.634, -198.797, -198.654, -191.245, -189.972, -189.868, -187.314, -186.918, -186.729, -185.014, -184.381, -183.966, -181.303, -179.486, -178.868, -177.223, -176.599, -175.546, -175.482, -175.09, -174.665, -172.339, -171.313, -171.123, -168.884, -166.53, -166.37, -164.765, -163.807, -163.43, -162.371, -162.06, -161.402, -161.202, -161.146, -160.503, -158.962, -157.144, -156.634, -155.37, -155.358}

In[∘]:=
```
pcurves4 = ListPlot[Curves, Joined → True, PlotRange → {Min[curvestemp[[2]]], Ecutoff}];
penergies4 = ListPlot[Table[{1, eValsBound[[i]]}, {i, 1, Length[eValsBound]}],
   PlotMarkers → Graphics[Line[{{0, 0}, {1, 0}}]]];
Show[pcurves4, penergies4]
```

Out[∘]=


```
In[∘]:= Es = Table[eValsBound[[i + 1]] - eValsBound[[i]], {i, 1, Length[eValsBound] - 1}];
eTrim = Select[Es, # > 0.0000001 &];
avg = Mean[eTrim];
eTrim = eTrim / avg;
Sort[eTrim];
Length[Es];
Length[eTrim];
ρ = 1 / avg;
EspaceBin = BinCounts[eTrim, {bins}];
NormBrodyBinCs = Table[
   EspaceBin[[i]] / Total[EspaceBin] / (bins[[i + 1]] - bins[[i]]), {i, 1, Length[bins] - 1}];
brodyPdist = Table[{bFit[[i]], NormBrodyBinCs[[i]]}, {i, 1, Length[bFit]}]
phist4 = ListPlot[brodyPdist, InterpolationOrder → 0,
   Joined → True, PlotRange → All, PlotStyle → Black];
pars = FindFit[brodyPdist, {PBrody[s, q], q > 0, q < 1}, {q}, s]
pfit4 = Plot[PBrody[s, q /. pars], {s, 0, 5}, PlotRange → All, PlotStyle → Red];

Out[∘]= {{0.1, 1.14583}, {0.3, 0.9375}, {0.5, 0.625}, {0.7, 0.208333}, {0.9, 0.416667},
 {1.1, 0.208333}, {1.3, 0.3125}, {1.5, 0.3125}, {1.7, 0.}, {1.9, 0.3125}, {2.1, 0.104167},
 {2.3, 0.104167}, {2.5, 0.}, {2.7, 0.104167}, {2.9, 0.}, {3.1, 0.104167}, {3.3, 0.},
 {3.5, 0.}, {3.7, 0.}, {3.9, 0.}, {4.1, 0.}, {4.3, 0.}, {4.5, 0.104167}, {4.7, 0.}, {4.9, 0.}}

Out[∘]= {q → 1.86974 × 10^-7}

In[∘]:= Show[phist4, pfit4, FrameLabel → {"s / s̄", "P(s)"}, Frame → True, LabelStyle → Large]

Out[∘]=
```



We need to more closely examine the identical particle symmetries and any other symmetries in the Hamiltonian in order to understand these spectra.

We defined the Jacobi coordinates in the H-tree above so that:

$$\vec{y}^{(12)} = S_{12}\,\vec{x} \tag{1}$$

where:

$$\frac{1}{\sqrt{\mu}}\begin{pmatrix} \sqrt{\mu_{12}} & -\sqrt{\mu_{12}} & 0 & 0 \\ 0 & 0 & \sqrt{\mu_{34}} & -\sqrt{\mu_{34}} \\ \frac{\sqrt{\mu_{12,34}}\ m_1}{m_1+m_2} & \frac{\sqrt{\mu_{12,34}}\ m_2}{m_1+m_2} & -\frac{\sqrt{\mu_{12,34}}\ m_3}{m_3+m_4} & -\frac{\sqrt{\mu_{12,34}}\ m_4}{m_3+m_4} \\ \frac{m_1}{\sqrt{M}} & \frac{m_2}{\sqrt{M}} & \frac{m_3}{\sqrt{M}} & \frac{m_4}{\sqrt{M}} \end{pmatrix} \tag{2}$$

To keep things as general as possible I'll assume for now that only particle 1 is of different mass and let $m_2 = m_3 = m_4 = m$, and $m_1 = \gamma\, m$
Then

$$\mu_{12} = \frac{\gamma\, m\, m}{\gamma\, m + m} = m\, \frac{\gamma}{1+\gamma} \longrightarrow \frac{m}{2} \tag{3}$$

$$\mu_{34} = \frac{m}{2} \tag{4}$$

$$\mu_{12,34} = \frac{(\gamma\, m + m)\,(2\, m)}{3\, m + \gamma\, m} = 2\, m\, \frac{(1+\gamma)}{(3+\gamma)} \longrightarrow m \tag{5}$$

$$\mu = \left(\frac{\gamma\, m\, m\, m\, m}{\gamma\, m + 3\, m}\right)^{1/3} = m\left(\frac{\gamma}{3+\gamma}\right)^{1/3} \longrightarrow \left(\frac{m}{2^{2/3}}\right) \tag{6}$$

So for the equal-mass case:

$$\frac{2^{1/3}}{\sqrt{m}}\begin{pmatrix} \sqrt{\frac{m}{2}} & -\sqrt{\frac{m}{2}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{m}{2}} & -\sqrt{\frac{m}{2}} \\ \frac{\sqrt{m}}{2} & \frac{\sqrt{m}}{2} & -\frac{\sqrt{m}}{2} & -\frac{\sqrt{m}}{2} \\ \frac{m}{\sqrt{4\, m}} & \frac{m}{\sqrt{4\, m}} & \frac{m}{\sqrt{4\, m}} & \frac{m}{\sqrt{4\, m}} \end{pmatrix} = 2^{1/3}\begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \tag{7}$$

and

$$\vec{x} = S_{12}{}^{-1}\, \vec{y}^{(12)} \tag{8}$$

Then

$$\vec{y}^{(13)} = S_{13}\, \vec{x} = S_{13}\, S_{12}{}^{-1}\, \vec{y}^{(12)} \tag{9}$$

Now note that we could use the same matrix S_{12} to construct $\vec{y}^{(13)}$ if the matrix were to act on a column vector $\begin{pmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{pmatrix}$ instead of the usual vector $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$. We can then say:

$$\begin{pmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = P_{23}\, \vec{x} \tag{10}$$

Hence:

$$S_{13} = S_{12}\, P_{23} \tag{11}$$

and

$$\vec{y}^{(13)} = S_{13}\,\vec{x} = S_{13}\,S_{12}^{-1}\,\vec{y}^{(12)} = S_{12}\,P_{23}\,S_{12}^{-1}\,\vec{y}^{(12)} \tag{12}$$

Similarly,

$$\vec{y}^{(14)} = S_{14}\,\vec{x} = S_{14}\,S_{12}^{-1}\,\vec{y}^{(12)} = S_{12}\,P_{24}\,S_{12}^{-1}\,\vec{y}^{(12)} \tag{13}$$

where

$$P_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{14}$$

In[∘]:= $\mathbf{S12} = 2^{1/3} \begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix};$

In[∘]:= $\mathbf{P24} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$

$\mathbf{P23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$

$\mathbf{P34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix};$

$\mathbf{P14} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix};$

$\mathbf{P13} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$

$\mathbf{P12} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$

In[∘]:=
```
𝒫12 = Simplify[S12.P12.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫12]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
In[∘]:= 𝒫34 = Simplify[S12.P34.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫34]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
In[∘]:= 𝒫14 = Simplify[S12.P14.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫14]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

```
In[∘]:= 𝒫13 = Simplify[S12.P13.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫13]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

```
In[∘]:= 𝒫23 = Simplify[S12.P23.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫23]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

```
In[∘]:= 𝒫24 = Simplify[S12.P24.Inverse[S12]][[1 ;; 3, 1 ;; 3]]; TraditionalForm[𝒫24]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

```
In[∘]:= TraditionalForm[FullSimplify[𝒫23.𝒫14]]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
In[∘]:= TraditionalForm[FullSimplify[𝒫24.𝒫13]]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
In[∘]:= TraditionalForm[FullSimplify[𝒫23.𝒫14.𝒫24.𝒫13]]
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4} \tag{15}$$

$$\rho_1^{(12)} = x_1 - x_2 \tag{16}$$

$$\rho_2^{(12)} = x_3 - x_4 \tag{17}$$

$$\rho_3^{(12)} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - \frac{m_3 x_3 + m_4 x_4}{m_3 + m_4} \tag{18}$$

and *mass-scaled* Jacobi coordinates as:

$$y_1^{(12)} = \sqrt{\frac{\mu_{12}}{\mu}}\,(x_1 - x_2) \tag{19}$$

$$y_2^{(12)} = \sqrt{\frac{\mu_{34}}{\mu}}\,(x_3 - x_4) \tag{20}$$

$$y_3^{(12)} = \sqrt{\frac{\mu_{12,34}}{\mu}}\left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - \frac{m_3 x_3 + m_4 x_4}{m_3 + m_4}\right) \tag{21}$$

$$y_4^{(12)} = \sqrt{\frac{M}{\mu}}\,X_{\text{cm}} \tag{22}$$

The convention here is that the superscript (12) indicates this coordinate system is convenient for treating the interaction between particles 1 and 2. In matrix notation:

$$\begin{pmatrix} y_1^{(12)} \\ y_2^{(12)} \\ y_3^{(12)} \\ y_4^{(12)} \end{pmatrix} = \frac{1}{\sqrt{\mu}} \begin{pmatrix} \sqrt{\mu_{12}} & -\sqrt{\mu_{12}} & 0 & 0 \\ 0 & 0 & \sqrt{\mu_{34}} & -\sqrt{\mu_{34}} \\ \frac{\sqrt{\mu_{12,34}}\,m_1}{m_1+m_2} & \frac{\sqrt{\mu_{12,34}}\,m_2}{m_1+m_2} & -\frac{\sqrt{\mu_{12,34}}\,m_3}{m_3+m_4} & -\frac{\sqrt{\mu_{12,34}}\,m_4}{m_3+m_4} \\ \frac{m_1}{\sqrt{M}} & \frac{m_2}{\sqrt{M}} & \frac{m_3}{\sqrt{M}} & \frac{m_4}{\sqrt{M}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{23}$$

Now we define the hyperspherical coordinates as:

$$y_1^{(12)} = \cos\theta_{12} \tag{24}$$

$$y_2^{(12)} = \sin\theta_{12}\cos\phi_{12} \tag{25}$$

$$y_3^{(12)} = \sin\theta_{12}\sin\phi_{12} \tag{26}$$

```
In[∘]:= Clear[μ4]
```

```
In[∘]:= μ4 = (μ12 μ34 μ1234)^(1/3) /.
          {μ12 → (m1 m2/(m1 + m2)), μ34 → (m3 m4/(m3 + m4)), μ1234 -> ((m1 + m2) (m3 + m4)/(m1 + m2 + m3 + m4)), M → m1 + m2 + m3 + m4}
```

Out[∘]= $\left(\frac{\text{m1 m2 m3 m4}}{\text{m1} + \text{m2} + \text{m3} + \text{m4}}\right)^{1/3}$

In[∘]:=
$$A = \frac{1}{\sqrt{\mu 4}}\begin{pmatrix} \sqrt{\mu 12} & -\sqrt{\mu 12} & 0 & 0 \\ 0 & 0 & \sqrt{\mu 34} & -\sqrt{\mu 34} \\ \frac{\sqrt{\mu 1234}\ m1}{m1+m2} & \frac{\sqrt{\mu 1234}\ m2}{m1+m2} & -\frac{\sqrt{\mu 1234}\ m3}{m3+m4} & -\frac{\sqrt{\mu 1234}\ m4}{m3+m4} \\ \frac{m1}{\sqrt{M}} & \frac{m2}{\sqrt{M}} & \frac{m3}{\sqrt{M}} & \frac{m4}{\sqrt{M}} \end{pmatrix} \text{ /. } \left\{\mu 12 \to \left(\frac{m1\ m2}{m1 + m2}\right),\right.$$
$$\left.\mu 34 \to \left(\frac{m3\ m4}{m3 + m4}\right),\ \mu 1234 \text{ -> } \left(\frac{(m1 + m2)\ (m3 + m4)}{m1 + m2 + m3 + m4}\right),\ M \to m1 + m2 + m3 + m4\right\} \text{ // FullSimplify}$$

Out[∘]=
$$\left\{\left\{\frac{\sqrt{\frac{m1\ m2}{m1+m2}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}},\ -\frac{\sqrt{\frac{m1\ m2}{m1+m2}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}},\ 0,\ 0\right\},\right.$$
$$\left\{0,\ 0,\ \frac{\sqrt{\frac{m3\ m4}{m3+m4}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}},\ -\frac{\sqrt{\frac{m3\ m4}{m3+m4}}}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}\right\},\ \left\{\frac{m1\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m1+m2)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}},\right.$$
$$\left.\frac{m2\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m1+m2)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}},\ -\frac{m3\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m3+m4)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}},\ -\frac{m4\sqrt{\frac{(m1+m2)\ (m3+m4)}{m1+m2+m3+m4}}}{(m3+m4)\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}}\right\},$$
$$\left\{\frac{m1}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}},\ \frac{m2}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}},\right.$$
$$\left.\left.\frac{m3}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}},\ \frac{m4}{\left(\frac{m1\ m2\ m3\ m4}{m1+m2+m3+m4}\right)^{1/6}\sqrt{m1+m2+m3+m4}}\right\}\right\}$$

Let’s just treat the equal mass case here...

In[∘]:=
$$\begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \text{FullSimplify}\left[\text{Inverse[A]}.\begin{pmatrix} x \\ y \\ z \\ XCM \end{pmatrix} \text{ /. } \{m1 \to m,\ m2 \to m,\ m3 \to m,\ m4 \to m\}\right];$$

In[∘]:=
```
Clear[R]
```

In[∘]:=
```
x12 = Simplify[x1 - x2]; FullSimplify[x12, m > 0]
x12hyp = Simplify[x12, m > 0] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x12hyp, m > 0]
```

Out[∘]= $2^{1/6}\ x$

Out[∘]= $2^{1/6}\ R\ \text{Cos}[\phi]\ \text{Sin}[\theta]$

In[∘]:=
```
x13 = FullSimplify[x1 - x3, m > 0];
x13hyp = Simplify[x13] /. {x → R Sin[θ] Cos[ϕ], y → R Sin[θ] Sin[ϕ], z → R Cos[θ]};
FullSimplify[x13hyp, m > 0]
```

Out[∘]=
$$\frac{R\left(\sqrt{2}\ \text{Cos}[\theta] + \text{Sin}[\theta]\ (\text{Cos}[\phi] - \text{Sin}[\phi])\right)}{2^{5/6}}$$

In[∘]:=
```
x14 = FullSimplify[x1 - x4, m > 0]
x14hyp = Simplify[x14] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x14hyp]
```

Out[∘]= $\frac{x + y + \sqrt{2}\ z}{2^{5/6}}$

Out[∘]= $\frac{R\left(\sqrt{2}\ \text{Cos}[\theta] + \text{Sin}[\theta]\ (\text{Cos}[\phi] + \text{Sin}[\phi])\right)}{2^{5/6}}$

In[∘]:=
```
x23 = Simplify[x2 - x3, m > 0];
x23hyp = Simplify[x23] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x23hyp]
```

Out[∘]= $\frac{R\left(\sqrt{2}\ \text{Cos}[\theta] - \text{Sin}[\theta]\ (\text{Cos}[\phi] + \text{Sin}[\phi])\right)}{2^{5/6}}$

In[∘]:=
```
x24 = Simplify[x2 - x4, m > 0];
x24hyp = Simplify[x24] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x24hyp]
```

Out[∘]= $\frac{R\left(\sqrt{2}\ \text{Cos}[\theta] + \text{Sin}[\theta]\ (-\text{Cos}[\phi] + \text{Sin}[\phi])\right)}{2^{5/6}}$

In[∘]:=
```
x34 = Simplify[x3 - x4, m > 0];
x34hyp = Simplify[x3 - x4, m > 0] /. {x → R Sin[θ] Cos[ϕ], y -> R Sin[θ] Sin[ϕ], z -> R Cos[θ]};
FullSimplify[x34hyp]
```

Out[∘]= $2^{1/6}\ R\ \text{Sin}[\theta]\ \text{Sin}[\phi]$

In[∘]:=
```
equalmassxij = FullSimplify[{x12hyp, x13hyp, x14hyp, x23hyp, x24hyp, x34hyp}]
```

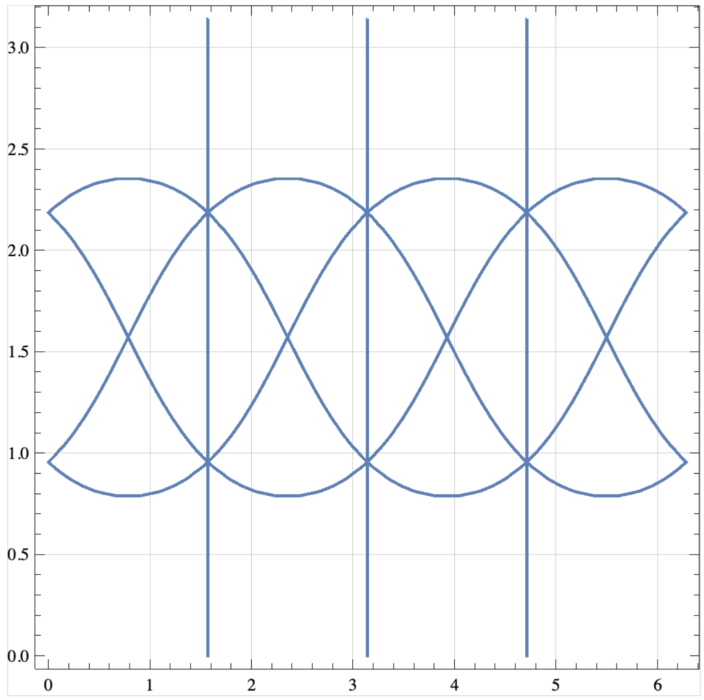
Out[∘]= $\left\{2^{1/6}\ R\ \text{Cos}[\phi]\ \text{Sin}[\theta],\ \frac{R\left(\sqrt{2}\ \text{Cos}[\theta] + \text{Sin}[\theta]\ (\text{Cos}[\phi] - \text{Sin}[\phi])\right)}{2^{5/6}},\ \frac{R\left(\sqrt{2}\ \text{Cos}[\theta] + \text{Sin}[\theta]\ (\text{Cos}[\phi] + \text{Sin}[\phi])\right)}{2^{5/6}},\ \frac{R\left(\sqrt{2}\ \text{Cos}[\theta] - \text{Sin}[\theta]\ (\text{Cos}[\phi] + \text{Sin}[\phi])\right)}{2^{5/6}},\ \frac{R\left(\sqrt{2}\ \text{Cos}[\theta] + \text{Sin}[\theta]\ (-\text{Cos}[\phi] + \text{Sin}[\phi])\right)}{2^{5/6}},\ 2^{1/6}\ R\ \text{Sin}[\theta]\ \text{Sin}[\phi]\right\}$

In[∘]:=
```
equalmassxij[[1]] == 0 /. R → 1
```

Out[∘]= $2^{1/6}\ \text{Cos}[\phi]\ \text{Sin}[\theta] == 0$

```
In[•]:= ContourPlot[Evaluate[equalmassxij == 0 /. R → 1], {ϕ, 0, 2 π}, {θ, 0, π}, GridLines → Automatic]
```

Out[•]=




```
In[•]:= unequalmassxij = FullSimplify[{x12hyp, x13hyp, x14hyp, x23hyp, x24hyp, x34hyp} /.
    {m1 → 1, m2 → 1, m3 → α, m4 → α, R → 1, θ → t π, ϕ → f π}]
```

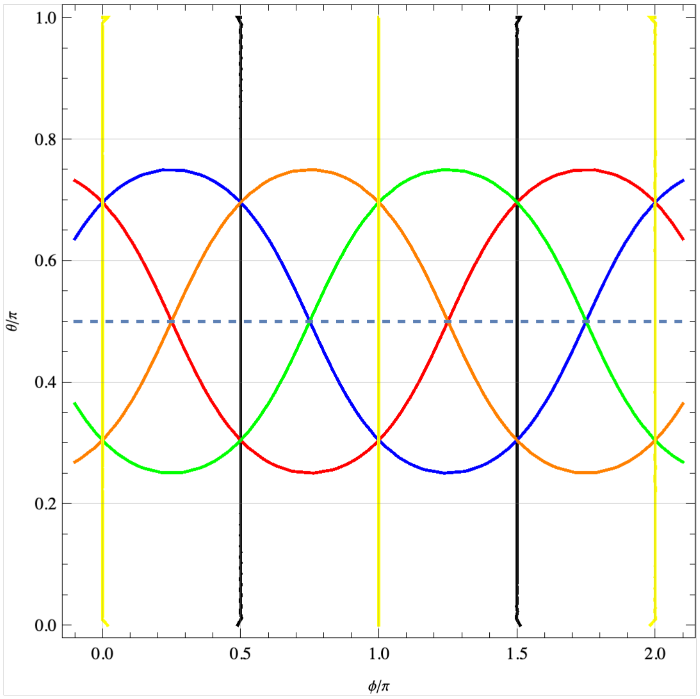
Out[•]=

$$\left\{2^{1/6}\,\mathrm{Cos}[f\,\pi]\,\mathrm{Sin}[\pi\,t],\ \frac{\sqrt{2}\,\mathrm{Cos}[\pi\,t]+(\mathrm{Cos}[f\,\pi]-\mathrm{Sin}[f\,\pi])\,\mathrm{Sin}[\pi\,t]}{2^{5/6}},\ \frac{\sqrt{2}\,\mathrm{Cos}[\pi\,t]+(\mathrm{Cos}[f\,\pi]+\mathrm{Sin}[f\,\pi])\,\mathrm{Sin}[\pi\,t]}{2^{5/6}},\ \frac{\sqrt{2}\,\mathrm{Cos}[\pi\,t]-(\mathrm{Cos}[f\,\pi]+\mathrm{Sin}[f\,\pi])\,\mathrm{Sin}[\pi\,t]}{2^{5/6}},\ \frac{\sqrt{2}\,\mathrm{Cos}[\pi\,t]+(-\mathrm{Cos}[f\,\pi]+\mathrm{Sin}[f\,\pi])\,\mathrm{Sin}[\pi\,t]}{2^{5/6}},\ 2^{1/6}\,\mathrm{Sin}[f\,\pi]\,\mathrm{Sin}[\pi\,t]\right\}$$

```
In[•]:= c12 = ContourPlot[Evaluate[unequalmassxij[[1]] == 0 /. {R → 1, α → 1}], {f, -0.1, 2.1},
   {t, 0, 1}, GridLines → Automatic, ContourStyle → Black, FrameLabel → {"ϕ/π", "θ/π"}];
c13 = ContourPlot[Evaluate[unequalmassxij[[2]] == 0 /. {R → 1, α → 1}], {f, -0.1, 2.1},
   {t, 0, 1}, GridLines → Automatic, ContourStyle → Red, FrameLabel → {"ϕ/π", "θ/π"}];
c14 = ContourPlot[Evaluate[unequalmassxij[[3]] == 0 /. {R → 1, α → 1}], {f, -0.1, 2.1},
    {t, 0, 1}, GridLines → Automatic, ContourStyle → Blue, FrameLabel → {"ϕ/π", "θ/π"}];
c23 = ContourPlot[Evaluate[unequalmassxij[[4]] == 0 /. {R → 1, α → 1}], {f, -0.1, 2.1},
    {t, 0, 1}, GridLines → Automatic, ContourStyle → Green, FrameLabel → {"ϕ/π", "θ/π"}];
c24 = ContourPlot[Evaluate[unequalmassxij[[5]] == 0 /. {R → 1, α → 1}], {f, -0.1, 2.1},
    {t, 0, 1}, GridLines → Automatic, ContourStyle → Orange, FrameLabel → {"ϕ/π", "θ/π"}];
c34 = ContourPlot[Evaluate[unequalmassxij[[6]] == 0 /. {R → 1, α → 1}], {f, -0.1, 2.1},
    {t, 0, 1}, GridLines → Automatic, ContourStyle → Yellow, FrameLabel → {"ϕ/π", "θ/π"}];
```

In[◦]:=
```
pall = Show[{c12, c13, c14, c23, c24, c34}, Plot[0.5, {x, -.1, 2.1}, PlotStyle → Dashed]]
```

Out[◦]=



In[◦]:=
```
cptable = Table[ContourPlot[Evaluate[unequalmassxij[[i]] == 0 /. {R → 1, α → 1}],
    {f, 0, .5}, {t, 0, .5}, GridLines → Automatic, ContourStyle → Black], {i, 1, 6}];
```

In[◦]:=
```
Show[pall, PlotRange → {{0, 0.5}, {0, 0.5}}]
```

Out[◦]=