CSCI 570 6498-8280-04

Homework 11

Q1 First, we will prove that k-Spanning Tree is NP-Hard.

Certificate: A set of path edges where the nodes have a degree less than or equal to k

Certifier: All nodes in visited by the edges having degree $\leq k$. ~~and~~

Thus, k-spanning tree is NP ~~complete~~.

Now, we select Hamiltonian Path and prove Hamiltonian Path $\leq_p$ k-spanning tree.

To prove this, we will take the Hamiltonian Path's graph and use it for k-spanning tree with $k=2$. If there is a solution, this means that there exists a spanning tree whose vertices have degree $\leq k$ and the tree is a path that goes through all vertices which is a Hamiltonian Path. Conversely, if there exists a Hamiltonian Path, the path is exactly a tree with degree ≤ 2.

This proves the k-spanning tree problem

Q2 First, we prove that Zero-Weight-Cycle is NP.
Certificate: A set of edges forming a cycle
Certifier: Check if sum of the edges is zero.

Thus, Zero-Weight-Cycle is NP.

Now, we prove that TSP $\leq_p$ Zero-Weight-Cycle.
We take the input graph of TSP and feed it as is to the Zero-Weight-Cycle.

~~We~~ If there is a path in TSP and its cost is zero, this means that we have a Zero-Weight-Cycle. Similarly, ~~I~~ if we have a Zero-Weight-Cycle, we send those nodes to the TSP for which we have a cycle. Since the cycle has a cost of zero, ~~the~~ we find the path in the TSP.

Hence, we proved that the Zero-Weight-Cycle is NP-Complete.

Q3 First, we prove that Redundant Clubs problem is NP.

<u>Certificate</u>: A list of K clubs that cover each of the adults in the set.

<u>Certifier</u>: Check if all the adults are covered and check if size of clubs is K, and check whether each person is a member of another set

Hence, Redundant Clubs Problem is NP.

Now, we prove that ~~Red~~
Set Cover $\leq_p$ Redundant Clubs Problem

We translate the inputs of Set Cover to Redundant Clubs. We use the elements of Set cover as ~~peo~~ list of people and make a list of clubs, one for each element of the Set Cover group. The Redundant Set input is thus:

$$K = G - K_{sc}$$

where G = No. of groups in Set Cover
K_{sc} = Value of k in the Set Cover problem

If we get a 'yes' from Set Cover i.e. an instance covering K_{sc} subsets, the remaining K subsets form the Redundant Clubs. Conversely, if we have K ~~sets~~ Redundant Club subsets and remaining K_{sc} clubs form a cover, we receive a 'Yes' from the Set Cover only if we get a 'Yes' from Redundant club.

Thus, our problem is NP-Hard and in turn it is NP-Complete.

Q4 First, we prove that HALF-IS is in NP. We have a set $S = |V|/2$ and verify if two nodes are not adjacent in polynomial time.

We prove Independent Set(IS) $\leq_p$ Half-IS. Consider an instance of IS where in set $A \subset V$, $|A| = k$..

(i) If $k = |V|/2$, we have the Half-IS in IS itself.

(ii) If $k < |V|/2$, we add m new disconnected nodes such that $k+m = (|V|+m)/2$ i.e. $m = |V| - 2k$. Thus, V' has an even no. of nodes ($V' = V + m$). Since the new nodes are disconnected, they must be present in the independent set and thus we have a new graph G' where it will have an independent set of size $|V|/2$ if we have an independent set of size k.

(iii) If $k > |V|/2$, then again add m new nodes to graph G' and connect them to all the nodes in V'. Since these nodes are connected to every other, none of them would belong to an independent set. Thus, the new graph has an independent set of size $|V'|/2$.

Thus, the problem is NP-Hard.
Hence, it is NP-Complete.

Q5 To check if the problem is NP, we can check if the no. of courses in the solution is larger or equal to k, and they don't have time overlap.

Given an independent set problem, suppose the graph has n nodes and has an independent set of size atleast k.

Suppose we take the courses as the nodes, and we draw an edge if there is an overlap between those edges. If there are K nodes in the independent set, this means that we have K courses which do not overlap. Conversely, if we have K courses that do not overlap, this means that we have an independent set of size k.

Thus, the problem is NP-Hard.

Thus, the problem is NP-Complete.