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Homework 10

Q1 To prove 3-SAT (15/16) is in NP, we can count how many clauses are satisfied given a truth value assignment and compare it to 15k/16.

We select general 3-SAT problem and prove it is deductible to 3-SAT (15k/16) to prove 3-SAT (15/16) is NP-Hard.

3-SAT contains 8 clauses, and to match the 3-SAT (15/16), we create 8 new clauses using 3 new variable. In total, we have 16 clauses out of which we need to prove that atleast 7/8 clauses would be fulfilled because we added exactly 8 clauses. If the new no. of clauses must not have been a multiple of 8, we would satisfy more than 15/16 of the clauses, but if one of the original ones weren't satisfied, then we would satisfy less than 15/16 cases. If the new no. of clauses is a multiple of 8. any assignment will satisfy 7/8 new clauses. so we require to satisfy all the original clauses in order to satisfy exactly 15/16 clauses.

Since 3-SAT (15/16) is both NP and NP-Hard it is NP-complete.

If there are 8 original cases, we add 8 more cases. Even if the original no. of clauses were not a multiple of 8, it would have been fine since we need atleast 15/16 clauses.

Q2 Proof for NP classification of Dense Subgraph Problem is trivial.

∴ We would be proving

Independent set problem $\leq_P$ Dense Subgraph Problem

In the decision problem of Independent Set, given a graph G, and an integer k it will output yes if the graph contains an independent set of size k.
For an arbitrary graph of G, we get the complementary graph G' of G.

A clique is a subset of vertices and an undirected graph such that every two distinct vertices in the clique are adjacent i.e. its induced subgraph is complete. We know that a clique will contain $k(k-1)/2$ edges if there are k vertices in G ~~and~~. Along with that, an independent set in G is a clique in G' and vice versa.

Thus, we would set m to $k(k-1)/2$ and test the dense subgraph problem.

Claim: There exists an independent set of size k in G iff there exists a subgraph of G' with at most k vertices and at least $k(k-1)/2$ edges.

(1) If there exists a clique in G' of size atleast k, then there exists a subgraph of G' with at most k vertices & $k(k-1)/2$ edges.

Here, a clique would be of size exactly k and it would have $k(k-1)/2$ edges.

(2) If there exists a subgraph of G' with at most k vertices, and $k(k-1)/2$ edges, then we have a clique of size atleast k.

Here, $k(k-1)/2$ edges in a graph implies that we have k edges.

Q3 First, we prove that SAT' belongs to NP. The certificate is the assignment values, and using this we can verify that SAT' instance and verify if it is satisfied. The condition of checking exactly m-2 clauses are satisfied can be done in linear time.

Next, we prove that SAT $\leq_p$ SAT'. To prove this, we add four more clauses, $x_1, \bar{x}_1, x_2, \bar{x}_2$.

Our claim: For the formula obtained at SAT', F has an assignment which satisfies SAT' iff the formula of SAT, F has an assignment which satisfies SAT.

(1) If F has an assignment that satisfies SAT, then F' also satisfies SAT'.

→ If an assignment proves F, then it satisfies exactly two of the four new clauses, giving us m-2 satisfied clauses for F.

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(2) If F' has an assignment which satisfies SAT', then F has an assignment which satisfies SAT.

→ The only unsatisfied clauses for F' would be x_1 or $\bar{x}_1$ and x_2 or $\bar{x}_2$. Here, all the original m clauses must be satisfied.

If a given SAT instance F is:
$(a \vee b) \wedge (\bar{b} \vee c)$
Then SAT' instance F' would be:
$(a \vee b) \wedge (\bar{b} \vee c) \wedge (x_1) \wedge (\bar{x}_1) \wedge (x_2) \wedge (\bar{x}_2)$

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Q4 ~~V.~~ We will prove that Vertex Cover with even degrees belongs to NP.

Certificate: A set of nodes that can cover all vertices

Certifier: All the constraints for vertex cover ~~+ Check for~~

Now that we are done proving it is in NP, we choose vertex cover and show that it is deductible to even degree version of vertex cover.

To prove this, we add a new vertex v and add one edges to nodes that have an odd degree. With this edge, the degree becomes even.

We create a triangle with one vertex as V. So, even if V is not a part of the vertex cover, the new vertices can be a part of the vertex cover. In a triangle, any two nodes must be included so as to cover all the edges of a triangle.