

Homework 12

Q1 Here, we make an assumption that no clause can have a variable and its complement together.

We have the following 2-approximation algorithm:

Let I be an instance of MIN-2SAT consisting of clause set C_I and variable set S_I .

We construct an auxiliary graph $G_I(V_I, E_I)$ corresponding to I where the node set V_I corresponds with the clause set C_I . For any two nodes v_i and v_j , the edge (v_i, v_j) is in E_I iff the corresponding clauses are such that there is a variable s in S_I such that it appears as it is in C_i and in complemented form in C_j , or viceversa.

To construct a truth assignment, we construct an approximate vertex cover V' for G_I such that $|V'|$ is at most 2 times of a minimum vertex cover for G_I . Then, we can construct a truth assignment that causes all clauses in $V_I - V'$ to be false:

Q2 Here, our objective is:
Minimize $\sum_{\substack{(u,v) \\ \in E}} c(u,v) \cdot x(u,v)$

Constraints:

$$x_v - x_u + x_{(u,v)} \geq 0 \quad \forall (u,v) \in E$$

$$x_u \in \{0, 1\} \quad \forall u \in V$$

Except $u=s$ and $u=t$

$$x_{(u,v)} \in \{0, 1\} \quad \forall (u,v) \in E.$$

$$x_s = 1, \quad x_t = 0.$$

Here, we partition the vertices in the cuts.

If $x_i = 0$, then i is on side S of cut.

Also, we need to clearly set $x_s = 1$ and $x_t = 0$ for correct partitioning and s & t are separate!

With the first constraint, we ensure that edges travelling across the cut are included.

Q3 Objective Fn: Maximize $\sum_{i=1}^{16} H_i$

Constraints:

$$H_i = \alpha_i (D_i - S_i)$$

$$\sum_{i=1}^{16} S_i = 720$$

$$(D_i - S_i) \geq 0 \quad \text{for } 0 < i \leq 16$$

$$S_i \geq 0 \quad \text{for } 0 < i \leq 16$$

Our variable is S_i

Q4 (a) p_i = Power of the i^{th} transmitter.
 $i = 1 \dots n$.

(b) Minimize $p_1 + p_2 + \dots + p_n$

(c) $p_i + p_j \geq d_{ij}$ where d_{ij} = distance of stations i and j .

We need $\sum_{i=1}^{n-1} (n^2 - n)/2$ constraints of inequality.