

Applied to Autonomous Drone Detection and Tracking

1 PROBLEM FORMULATION (Big Picture)

We observe a **sequence of video frames** of the sky.

Each pixel value and each object position is treated as a **random variable** evolving over time.

Our goal is to:

1. Detect statistically rare events (Detection)
 2. Track object motion over time (Trajectory Tracking)
 3. Infer threat behavior probabilistically (Threat Classification)
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◆ FEATURE 1: DETECTION

(Probabilistic Anomaly Detection)

1.1 Random Variable Definition

For each pixel at position (x, y) and time t :

$$X_t(x, y) = (H_t, S_t, V_t)$$

where:

- H = Hue
- S = Saturation
- V = Value

These are **random variables** because:

- Illumination
 - Atmospheric noise
 - Camera sensor noise
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1.2 Background Modeling (PDF Estimation)

We assume the **sky background** follows a **stationary probability distribution** during initial frames.

Using the first N frames:

$$p_H(h), \quad p_S(s), \quad p_V(v)$$

These are **empirical PDFs**, estimated using histograms:

$$p_H(h) \approx \frac{\text{Number of pixels with hue } h}{\text{Total pixels}}$$

1.3 Independence Assumption (Important)

We assume:

$$p(H, S, V) = p(H) \cdot p(S) \cdot p(V)$$

↗ This is a **naive Bayes assumption**, justified because:

- Hue, saturation, value represent different physical aspects
 - Keeps computation simple (academic constraint)
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1.4 Anomaly Detection Rule

For each pixel:

$$P(x, y) = p(H) \cdot p(S) \cdot p(V)$$

Decision rule:

Pixel is anomalous if $P(x, y) < \varepsilon$

Where:

- ε is a **probability threshold**

↗ Interpretation:

If a pixel has very low probability under the sky model, it likely belongs to a drone or foreign object.

1.5 Detection as Hypothesis Testing

This can be seen as:

- H_0 : Pixel belongs to background sky
- H_1 : Pixel belongs to a foreign object

Decision:

$$\text{Reject } H_0 \text{ if } P(x, y) < \varepsilon$$

◆ FEATURE 2: TRAJECTORY TRACKING

(Random Process & Markov Modeling)

2.1 State Definition

We define the object state at time t :

$$X_t = (x_t, y_t)$$

This is a **random process indexed by time**.

2.2 Motion Model

We assume **first-order Markov property**:

$$P(X_t | X_{t-1}, X_{t-2}, \dots) = P(X_t | X_{t-1})$$

Meaning:

The current position depends only on the previous position.

2.3 Velocity as a Random Variable

$$V_t = X_t - X_{t-1}$$

Velocity captures:

- Speed
- Direction

Noise arises from:

- Detection uncertainty
 - Sensor noise
 - Environmental disturbances
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2.4 Optical Flow as Estimation

Optical flow estimates V_t using local intensity changes.

Mathematically:

$$I_x v_x + I_y v_y + I_t = 0$$

This is solved using **least squares**, which is a **minimum error estimator**.

2.5 Trajectory as a Sample Path

The tracked trajectory:

$$\{X_1, X_2, \dots, X_T\}$$

is a **sample realization** of the random process $\{X_t\}$.

2.6 Missing Observations

When detection fails:

- X_t is unobserved
- Tracking continues using previous estimates

This mimics **hidden state estimation**, similar to HMM ideas (without full complexity).

◆ FEATURE 3: THREAT CLASSIFICATION

(Stochastic Behavior Analysis)

3.1 Motion Features as Random Variables

From the trajectory, we define:

- Speed:

$$|V_t| = \sqrt{v_x^2 + v_y^2}$$

- Direction:

$$\theta_t = \arctan\left(\frac{v_y}{v_x}\right)$$

- Direction change:

$$\Delta\theta_t = |\theta_t - \theta_{t-1}|$$

These form **time series of random variables**.

3.2 Statistical Behavior Over Time

We analyze statistics over a window:

- Mean speed: $E[|V_t|]$
 - Speed variance: $\text{Var}(|V_t|)$
 - Mean direction change: $E[\Delta\theta_t]$
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3.3 Probabilistic Interpretation of Classes

Bird / Noise

- High variance
- Large direction changes
- Unpredictable motion

$$\text{Var}(|V_t|) \uparrow, \quad E[\Delta\theta_t] \uparrow$$

Recon Drone

- Low speed
- Nearly constant direction

$$E[|V_t|] \downarrow, \quad E[\Delta\theta_t] \downarrow$$

Attack Drone

- High speed

- Downward motion
- Stable direction

$$E[|V_t|] \uparrow, \quad E[v_x] > 0$$

3.4 Decision Rule (MAP Intuition)

We choose the class C that best explains the observed motion:

$$C^* = \arg \max P(\text{motion features} \mid C)$$

Instead of explicit PDFs, we use **heuristic thresholds** — acceptable for constrained academic settings.

◆ OVERALL SYSTEM VIEW (Professor-Friendly)

Component	PRP Concept
HSV modeling	Random variables
Histogram PDFs	Empirical distributions
Thresholding	Hypothesis testing
Tracking	Random process
Optical flow	Least squares estimation
Trajectory	Sample path
Classification	Statistical inference



ONE-LINE SUMMARY (USE THIS)

"The system models the sky and object motion probabilistically, detects statistically rare events, tracks motion as a random process, and classifies threats using stochastic behavior analysis."
