Assig - 1

Q4

~~a~~ a) Assumming 'N' microphone array.

$\therefore$ We can define

$$Z_t = [(X_1)_t, (X_2)_t, (X_3)_t \cdots (X_N)_t]$$

where ~~X~~ $(X_i)_t$ is the data from ith microphone ~~at~~ upto time instant 't'

i.e.

$$Z_t = \begin{bmatrix} x_{10} & x_{20} & \cdots & x_{N0} \\ x_{11} & x_{21} & & x_{N1} \\ \vdots & \vdots & & \vdots \\ x_{1t} & x_{2t} & & x_{Nt} \end{bmatrix}_{(t \times N)} \quad \text{—— (1)}$$

where shape of Z_t is $(t \times N)$

Now

$$Cov(x,y) = E[(X-E(X))(Y-E(Y))]$$

Assuming X, Y are sampled uniformly from a gaussian distribution with $\mu = 0$ & $\sigma = 1$

$$Cov(X,Y) = E[(XY)] = \frac{\sum_{i=0}^{N} x_i y_i}{N}$$

Furthermore, one can write Covariance Matrix (Cov_M) as

$$Cov_M(X,Y) = \left(\frac{\begin{bmatrix} X \\ Y \end{bmatrix} \begin{bmatrix} X^T & Y^T \end{bmatrix}}{N} \right)$$

Let $A = \begin{bmatrix} X \\ Y \end{bmatrix}$

$\therefore$ $Cov_M(X,Y) = \frac{AA^T}{N}$ where

$A = \begin{bmatrix} X \\ Y \end{bmatrix}$

— (2)

$\therefore$ From (1) & (2)

$$Cov_M(X_1, X_2, X_3 \cdots X_N) = Z^T Z / t$$

where

$Z = [X_1, X_2 \sim X_N]$

— (3)

Now using relation

$$R(X_1, X_2 \dots X_N) = \frac{COV_M(X_1 \dots X_n)}{\sigma_{X_1} \dots \sigma_{X_N}}$$

As, we assumed $\sigma = 1$

$$\therefore \underbrace{R(X_1, X_2, \dots X_N)}_{\text{Correlation Matrix}} = COV_M(X_1 \dots X_N)$$

$$\therefore \boxed{R_t(X_1, X_2, \dots X_N)_t = \frac{Z_t^T Z_t}{t}}$$

where $Z = [X_1, X_2 \dots X_N]$

(b)

Now, as we know.

$$Z_{t+1}^T = \underbrace{(Z_t^T \mid V_{t+1}^T)}_{\text{Augment Matrix}} \quad — (4)$$

where V_{t+1}^T denotes. $\begin{bmatrix} x_{1,t+1} \\ x_{2,t+1} \\ \vdots \\ x_{N,t+1} \end{bmatrix}$.

Now for any Augment matrix $a = (b \mid c)$

$$aa^T = bb^T + cc^T \quad — (5)$$

$\therefore$ from (4), (5)

$$Z_{t+1}^T Z_{t+1} = Z_t^T Z_t + V_{t+1}^T V_{t+1} \quad — (6)$$

$$\therefore \quad R_{t+1}(X_1 -- X_N) = \frac{Z_{t+1}^T Z_{t+1}}{t \cancel{\ } +1}$$

Subsituting $Z_{t+1}^T Z_{t+1}$ from (6) above.

$$R_{t+1}(X_1 -- X_N) = \frac{Z_t^T Z_t}{t \cancel{\ } +1} + \left(\frac{1}{t \cancel{\ } +1}\right)(V_{t+1}^T V_{t+1})$$

As we know $R_t = \frac{Z_t^T Z_t}{\cancel{\ } t}$

$$\therefore \quad R_{t+1}(X_1 -- X_N) = \frac{\cancel{\ } t R_t(X_1 -- X_N)}{t \cancel{\ } +1} + \frac{1}{t \cancel{\ } +1}(V_{t+1}^T V_{t+1})$$

$$\cancel{\therefore \quad R_{t+1}(X_1 -- X_N) = N R_t(X_1 -- X_N)}$$

Ans

$$\therefore \quad R_{t+1}(X_1 -- X_N) = \frac{t R_t(X_1 -- X_N)}{t+1} + \frac{1}{(t+1)}(V_{t+1}^T V_t)$$

where

$R_t(X_1 -- X_N) \Rightarrow$ Correlation at time 't'

$V_t(X_1 -- X_N) \Rightarrow [x_{1,t+1} \; x_{2,t+1} -- x_{n,t+1}]$