Artificial Intelligence
Assig - 1

Q1 (a) Given

n faults are recorded in time T

Assuming each fault occurred as independent

$\therefore$ Probablity of a fault in a unit time

$= n/T$

$\therefore$ Probablity of a fault in $(t_2 - t_1)$ interval

is $p = \dfrac{n(t_2 - t_1)}{T}$

Therefore for k fault to be in $(t_2 - t_1)$ interval given that n fault has occured in T interval will be

$${}^{n}C_{k}\, p^{k}\, (1-p)^{n-k}$$

where p is the probablity of a fault in $t_2 - t_1$ interval

$$\boxed{\therefore \text{Probablity} = {}^{n}C_{k} \left[\frac{n(t_2 - t_1)}{T}\right]^{k} \left[1 - \frac{n(t_2 - t_1)}{T}\right]^{n-k}}$$

(b) Given, fault rate $(\lambda) = n/T$

$\therefore$ $E(x)$ in time interval $t_a = \frac{nt_a}{T} - \lambda t_a$

$\therefore$ $$p(k_a \in t_a) = \frac{(\lambda t_a)^{k_a} e^{-\lambda t_a}}{k_a!}$$

$$= \frac{\left(\frac{nt_a}{T}\right)^{k_a} e^{-(nt_a/T)}}{(k_a)!} \quad -①$$

$\therefore$ $$p(k_b \in t_b) = \frac{(\lambda t_b)^{k_b} e^{-\lambda t_b}}{(k_b)!}$$

$$= \frac{\left(\frac{nt_b}{T}\right)^{k_b} e^{-(nt_b/T)}}{(k_b)!} \quad -②$$

Assuming both events to be independent

$$p(k_a \in t_a, k_b \in t_b) = p(k_a \in t_a) \cdot p(k_b \in t_b)$$

$$= \frac{\left(\left(\frac{nt_a}{T}\right)^{k_a} e^{-(nt_a/T)}\right)\left(\left(\frac{nt_b}{T}\right)^{k_b} e^{-nt_b/T}\right)}{(k_a)!(k_b)!}$$

(c) From ~~[illegible]~~ Product rule

$$P(x/y) = \frac{P(x,y)}{P(y)}$$

$$\therefore \; P(k_a \in t_a / k_b \in t_b) = \frac{P(k_a \in t_a, k_c \in t_c)}{P(k_c \in t_c)}$$

Let

$$k_b = k_c - k_a$$

and $t_b = t_c - t_a$

$$\therefore \; P(k_a \in t_a, k_c \in t_c) = P(k_a \in t_a, k_b \in t_b)$$

As k_a & k_b are non-overlapping

$\therefore$ k_a & k_b are independent (iid)

$$P(k_a \in t_a, k_c \in t_c) = P(k_a \in t_a, k_b \in t_b) = P(k_a \in t_a) \times P(k_b \in t_b) \quad \text{—(1)}$$

Now, from previous question

$$P(k_a \in t_a) = \frac{(\lambda t_a)^{k_a} e^{-\lambda t_a}}{k_a!} \quad \text{—(2)}$$

From ①

$\therefore P(k_a \in t_a, k_c \in t_c) = P(k_a \in t_a)\, P(k_b \in t_b)$

$$\therefore P\left(k_a \in t_a / k_c \in t_c\right) = \frac{P(k_a \in t_a) \cdot P(k_b \in t_b)}{P(k_c \in t_c)}$$

Subsituting $k_b = k_c - k_a$ & $t_b = t_c - t_a$

$$\frac{P(k_a \in t_a) \cdot P(k_c - k_a \in t_c - t_a)}{P(k_c \in t_c)}$$

$$= \frac{\left[\frac{(\lambda t_a)^{k_a} e^{-\lambda t_a}}{k_a!}\right]\left[\frac{[\lambda(t_c - t_a)]^{k_c - k_a} e^{-\lambda(\cancel{t_c} - \cancel{t_a})}}{(k_c - k_a)!}\right]}{\frac{(\lambda t_c)^{k_c} e^{-\lambda t_c}}{(k_c)!}}$$

$$= \frac{(k_c)!}{(k_a)!\,(k_c - k_a)!} \quad \frac{t_a^{k_a} (t_c - t_a)^{k_c - k_a}}{t_c^{k_c}}$$

$$\boxed{P\left(k_a \in t_a / k_c \in t_c\right) = {}^{k_c}C_{k_a} \; \frac{t_a^{k_a} (t_c - t_a)^{k_c - k_a}}{t_c^{k_c}}}$$