(1) $f_1 \rightarrow$ outside scene $\quad f_2 \rightarrow$ reflection

$$g_1 = f_1 + h_2 * f_2 \quad [\text{blur } f_2]$$
$$g_2 = h_1 * f_1 + f_2 \quad [\text{blur } f_1]$$

Apply fourier transform,

$$G_1(\mu) = F_1(\mu) + H_2(\mu) F_2(\mu)$$
$$G_2(\mu) = F_2(\mu) + H_1(\mu) F_1(\mu)$$

Eliminating $F_2(\mu)$, we get

$$G_1(\mu) = F_1(\mu) + H_2(\mu)\left[G_2(\mu) - H_1(\mu)F_1(\mu)\right]$$

$$\Rightarrow \boxed{F_1(\mu) = \frac{G_1(\mu) - H_2(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu)}}$$

This gives:

$$F_2(\mu) = G_2(\mu) - H_1(\mu)F_1(\mu)$$
$$= G_2(\mu) - H_1(\mu)\frac{G_1(\mu) - H_1(\mu)H_2(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu)}$$

$$\boxed{F_2(\mu) = \frac{G_2(\mu) - H_1(\mu)G_1(\mu)}{1 - H_1(\mu)H_2(\mu)}}$$

By taking the inverse fourier transform (IDFT), we get back f_1 and f_2

The only problem will be when denominators become zero.

i.e. $H_1(\mu)H_2(\mu) = 1$

This happens when

$H_1(\mu) = 1$ and $H_2(\mu) = 1$

Since these are used for blurring the image, they must be low pass filters.

$$\int_a^b h_1(x)\,dx = 1 \quad \text{for any } a, b$$

$\Rightarrow H_1(0) = 1$, Similarly, $H_2(0) = 1$

Hence $H_1(\mu)H_2(\mu) = 1$ for $\mu = 0$

Thus the lower frequency components (specially $\mu = 0$) are not reconstructed well.

To address this proble, the denominator should be ensured non-zero.

We can add a small term (ε) to the denominator to ensure this.

The new reconstructed images are

$$F_1(\mu) = \frac{G_1(\mu) - H_2(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu) + \varepsilon}$$

and $$F_2(\mu) = \frac{G_2(\mu) - H_1(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu) + \varepsilon}$$

These are not exact images, because of the additional term ε, but are approximations

They solve the problem of $F(\cdot)$ shooting up.