Q.2

$g = h * f$

Apply DFT,

$G(\mu) = H(\mu)\, F(\mu)$

$\Rightarrow F(\mu) = \dfrac{G(\mu)}{H(\mu)}$

Again when $H(\mu) = 0$, $F(\mu)$ will shoot up

$H(\cdot)$ is a convolutional kernel such that $g(x) = f(x+1) - f(x)$ for $1 \le x \le N$

Applying DFT, we know that

$G(\mu) = (e^{\frac{j2\pi\mu}{N}} - 1)\, F(u)$

$\Rightarrow F(\mu) = \dfrac{G(\mu)}{(e^{\frac{j2\pi\mu}{N}} - 1)}$

Hence for $\mu = 0$, $F(\mu)$ shoots up.

To solve this, we need to assume some boundary condition

Let $f(N+1) = 0$

Then starting from the rightmost pixel N, obtain $f(\cdot)$ by reverse integration

For the 2D images, we have

$G_x(\mu,\sigma) = H_x(\mu,\sigma)\, F(\mu,\sigma) = (e^{\frac{j2\pi\mu}{N}} - 1)\, F(\mu,\sigma)$

$G_y(\mu,\sigma) = H_y(\mu,\sigma)\, F(\mu,\sigma) = (e^{\frac{j2\pi\sigma}{N}} - 1)\, F(\mu,\sigma)$

Knowing G_x, $\mu = 0 \rightarrow$ shoot up

Knowing G_y, $\sigma = 0 \rightarrow$ shoot up

Hence, we can rewrite as:

$$F(\mu, \sigma) = \frac{G_x - G_y}{H_x - H_y}$$

Hence, even when $\mu = 0$, $\sigma \neq 0$ won't shoot up
Similarly $\sigma = 0$, $\mu \neq 0$ won't shoot up
But $\mu = 0$ and $\sigma = 0$ will still be problematic

Here, the use of integration based on boundary conditions will be ineffective because integration across rows (f_x) will be inconsistent with integration across columns (f_y)