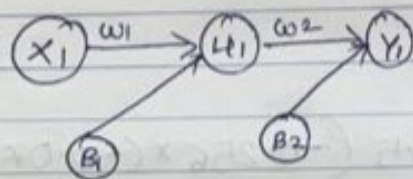


Network



Assumption

$$\begin{aligned}
 x_1 &= 1 & w_1 &= 0.2 \\
 b_1 &= 0.5 & w_2 &= 0.4 \\
 b_2 &= 0.8 & \text{Actual output} &= 1 \\
 \eta &= 0.5 & \text{Activation function} &= \frac{1}{1+e^{-x}}
 \end{aligned}$$

forward pass

$$\begin{aligned}
 h_1 &= x_1 w_1 + b_1 \\
 &= 0.2 + 0.5 \\
 &= \underline{\underline{0.7}}
 \end{aligned}$$

$$h_{1, \text{out}} = \frac{1}{1+e^{-0.7}} = \underline{\underline{0.6681}}$$

$$\begin{aligned}
 y_1 &= h_1 w_2 + b_2 \\
 &= 0.6681(0.4) + 0.8 \\
 &= \underline{\underline{1.06724}}
 \end{aligned}$$

$$\text{Out}_{y_1} = \frac{1}{1+e^{-1.06724}} = \underline{\underline{0.744}}$$

$$\begin{aligned}
 \text{Total error} &= \frac{1}{2} (\text{Actual output} - \text{Out}_{y_1})^2 \\
 &= \frac{1}{2} (1 - 0.744)^2 \\
 &= \underline{\underline{0.03276}}
 \end{aligned}$$

Backward pass (updating weights)

$$w_{2 \text{ up}} = w_2 - \eta \left(\frac{\partial \text{total error}}{\partial w_2} \right)$$

$$\frac{\partial \text{total error}}{\partial w_2} = \frac{\partial \text{total error}}{\partial \text{out}_{y_1}} \cdot \frac{\partial \text{out}_{y_1}}{\partial y} \cdot \frac{\partial y}{\partial w_2}$$

$$\begin{aligned}
 \frac{\partial \text{total error}}{\partial w_2} &= -(1 - 0.744) \\
 &= \underline{\underline{-0.256}} \quad \text{--- ①}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \text{out}_{y_1}}{\partial y_1} &= y_1(1 - y_1) = 1.067(1 - 0.1.067) \\
 &= \underline{\underline{-0.0717}} \quad \text{--- ②}
 \end{aligned}$$

$$\frac{\partial y}{\partial w_2} = \underline{0.6681}$$

$$w_{2\text{update}} = 0.4 - 0.5 (-0.256 \times (-0.0717) \times 0.6681) \\ = \underline{0.3938}$$

$w_{2\text{update}}$ weight is 0.3938

$$w_{1\text{update}} = \text{total } w_1 - \eta \left(\frac{\partial \text{total error}}{\partial w_1} \right)$$

$$= \frac{\partial \text{total error}}{\partial w_1} = \frac{\partial \text{total error}}{\partial \text{out}_{H_1}} \cdot \frac{\partial \text{out}_{H_1}}{\partial H_1} \cdot \frac{\partial H_1}{\partial w_1}$$

$$\frac{\partial \text{total error}}{\partial \text{out}_{H_1}} = \frac{\partial \text{total error}}{\partial \text{out}_y} \cdot \frac{\partial \text{out}_y}{\partial \text{out}_{H_1}} \cdot \cancel{\frac{\partial y}{\partial \text{out}_{H_1}}}$$

$$= -0.0256 \times 0.6681$$

$$= \underline{-0.0171}$$

$$\frac{\partial \text{out}_{H_1}}{\partial H_1} = -(1 - 0.6681) \\ = \underline{-0.3319}$$

$$\frac{\partial H_1}{\partial w_1} = 0.2$$

$$w_{1\text{update}} = (0.2) - 0.5 (-0.0171) \times (-0.3319) \times 0.2 \\ = \underline{0.1994}$$

w_1 update weight is 0.1994

$$\begin{array}{ll} w_1 = 0.2 & w_{1\text{ up}} = 0.1994 \\ w_2 = 0.4 & w_{2\text{ up}} = 0.3938 \end{array}$$