Tutorial 2

1.

i	j
0	1
1	2
3	3
6	4
10	5
⋮	⋮

Let k^{th} term $= n$

$$T_k = \Sigma k = \frac{k(k-1)}{2}$$

$$\frac{k(k-1)}{2} = n$$

$$\Rightarrow \frac{k^2}{2} - \frac{k}{2} = n$$

Ignoring lower order terms and constants

$$\Rightarrow k^2 = n$$

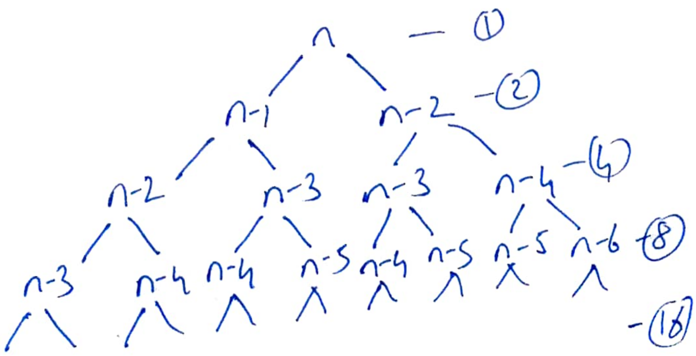
$$k = \sqrt{n}$$

$$\boxed{T(n) = O(\sqrt{n})}$$

2. $T(n) = T(n-1) + T(n-2) + O(1)$

For recursive fibonacci solution

Recursion Tree



No. of times function is running will be sum of the series

$S = 1 + 2 + 4 + 8 + 16 + \ldots + 2^n$

$$S = \frac{2^{n+1} - 1}{2-1} = 2^{n+1} - 1$$

$$\boxed{T(n) = O(2^n)}$$

3. $O(n(\log n))$

```
for (i=1; i<=n; i=i*2)
{
 for (j=1; j<=n; j++)
 {
   O(1) statements
 }
}
```

$O(n^3)$

```
for (i=1; i<=pow(n,3); ++i)
{
  O(1) statement
}
```

$O(\log(\log n))$

```
for (i=2; i<=n; i=pow(i,k))
{
  O(1) statement
}
```

4. $T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + cn^2$

Ignoring lower order terms:

$T(n) = T\left(\frac{n}{2}\right) + cn^2$

$a=1 \quad b=2 \quad c=0 \quad f(n)=n^2$

$\boxed{n^0 < n^2}$ True

$\boxed{T(n) = O(n^2)}$

5.

i	j
1	n times
2	$n/2$ times.
3	$n/3$ times
4	$n/4$ times.
⋮	
n	1 time

$$S = n + n/2 + n/3 + n/4 + \text{----} \; 1$$

$$S = \frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cancel{n} \; \text{----} \; \frac{n}{n}$$

$$S = n\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \text{----} \frac{1}{n}\right)$$

$$S = n\left(\sum_{i=1}^{n} \frac{1}{i}\right)$$

$$S = n\log n$$

8. $2, 2^k, (2^k)^k, ((2^k)^k)^k$ ----

Let there be n terms

$2^{k^0}, 2^{k^1}, 2^{k^2}, 2^{k^3}$ ---- $2^{k^{n-1}}$

$$2^{k^{n-1}} = n$$

$$k^{n-1} \log 2 = \log n$$

$$(n-1)\log k = \log(\log n) \quad \text{(ignoring constants)}$$

$$n = \log(\log n)$$

Time complexity $\boxed{T(n) = O(\log(\log n))}$

8. a) $$100 < \log(\log n) < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$$

b) $$1 < \log(\log n) < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{8n}$$

c) $$96 < \log_8 n < \log 2n < 5n < n\log_6 n < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$$