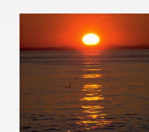
Chapter 5: Other Relational Languages

Database System Concepts, 5th Ed.



Chapter 5: Other Relational Languages

- Tuple Relational Calculus
- Domain Relational Calculus
- Query-by-Example (QBE)
- Datalog

Tuple Relational Calculus

- A nonprocedural query language, where each query is of the form

$$\{t \mid P(t)\}$$

- It is the set of all tuples *t* such that predicate *P* is true for *t*
- *t* is a *tuple variable*, $t[A]$ denotes the value of tuple *t* on attribute *A*
- $t \in r$ denotes that tuple *t* is in relation *r*
- *P* is a *formula* similar to that of the predicate calculus

Predicate Calculus Formula

1. Set of attributes and constants
2. Set of comparison operators: (e.g., $<$, $\leq$, $=$, $\neq$, $>$, $\geq$)
3. Set of connectives: and ($\wedge$), or (v), not ($\neg$)
4. Implication ($\Rightarrow$): x $\Rightarrow$ y, if x if true, then y is true

$$x \Rightarrow y \equiv \neg x \vee y$$

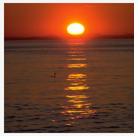
5. Set of quantifiers:

   $\exists\ t \in\ r\ (Q\ (t\ )) \equiv$ "there exists" a tuple in t in relation r such that predicate $Q\ (t\ )$ is true

   $\forall t \in\ r\ (Q\ (t\ )) \equiv Q$ is true "for all" tuples t in relation r

Banking Example

- *branch* (*branch_name, branch_city, assets*)
- *customer* (*customer_name, customer_street, customer_city*)
- *account* (*account_number, branch_name, balance*)
- *loan* (*loan_number, branch_name, amount*)
- *depositor* (*customer_name, account_number*)
- *borrower* (*customer_name, loan_number*)



Example Queries

- Find the *loan_number, branch_name,* and *amount* for loans of over $1200

$$\{t \mid t \in loan \wedge t[amount] > 1200\}$$

- Find the loan number for each loan of an amount greater than $1200

$$\{t \mid \exists\, s \in \text{loan}\ (t[loan\_number] = s[loan\_number] \wedge s[amount] > 1200)\}$$

Notice that a relation on schema [*loan_number*] is implicitly defined by the query

Example Queries

- Find the names of all customers having a loan, an account, or both at the bank

$$\{t \mid \exists s \in borrower\,(\,t\,[customer\_name\,] = s\,[customer\_name\,])$$
$$\lor \exists u \in depositor\,(\,t\,[customer\_name\,] = u\,[customer\_name\,])$$

- Find the names of all customers who have a loan and an account at the bank

$$\{t \mid \exists s \in borrower\,(\,t\,[customer\_name\,] = s\,[customer\_name\,])$$
$$\land \exists u \in depositor\,(\,t\,[customer\_name\,] = u\,[customer\_name]\,)$$

Example Queries

- Find the names of all customers having a loan at the Perryridge branch

$$
\begin{aligned}
\{t \mid \exists s \in \textit{borrower}\,(&t\,[\textit{customer\_name}\,] = s\,[\textit{customer\_name}\,] \\
&\wedge \exists u \in \textit{loan}\,(u\,[\textit{branch\_name}\,] = \text{“Perryridge”} \\
&\qquad \wedge\ u\,[\textit{loan\_number}\,] = s\,[\textit{loan\_number}\,]))\}
\end{aligned}
$$

- Find the names of all customers who have a loan at the Perryridge branch, but no account at any branch of the bank

$$
\begin{aligned}
\{t \mid \exists s \in \textit{borrower}\,(&t\,[\textit{customer\_name}\,] = s\,[\text{customer\_name}\,] \\
&\wedge \exists u \in \textit{loan}\,(u\,[\textit{branch\_name}\,] = \text{“Perryridge”} \\
&\qquad \wedge\ u\,[\textit{loan\_number}\,] = s\,[\text{loan\_}\textit{number}\,])) \\
&\wedge \textbf{not}\ \exists v \in \textit{depositor}\,(v\,[\textit{customer\_name}\,] = \\
&\qquad\qquad t\,[\textit{customer\_name}\,])\}
\end{aligned}
$$

Example Queries

- Find the names of all customers having a loan from the Perryridge branch, and the cities in which they live

$$
\begin{aligned}
\{t \mid \exists s \in \mathit{loan}\ (&s\,[\mathit{branch\_name}\,] = \text{“Perryridge”} \\
&\wedge \exists u \in \mathit{borrower}\ (u\,[\mathit{loan\_number}\,] = s\,[\mathit{loan\_number}\,] \\
&\qquad \wedge\ t\,[\mathit{customer\_name}\,] = u\,[\mathit{customer\_name}\,]) \\
&\wedge \exists\ v \in \mathit{customer}\ (u\,[\mathit{customer\_name}\,] = v\,[\mathit{customer\_name}\,] \\
&\qquad\qquad \wedge\ t\,[\mathit{customer\_city}\,] = v\,[\mathit{customer\_city}\,])))\}
\end{aligned}
$$

Example Queries

- Find the names of all customers who have an account at all branches located in Brooklyn:

$$\{t \mid \exists\, r \in customer\,(t[customer\_name] = r[customer\_name]) \wedge$$
$$(\forall\, u \in branch\,(u[branch\_city] = \text{“Brooklyn”} \Rightarrow$$
$$\exists\, s \in depositor\,(t[customer\_name] = s[customer\_name]$$
$$\wedge \exists\, w \in account\,(\,w[account\_number] = s\,[account\_number]$$
$$\wedge\,(\,w[branch\_name] = u[branch\_name])))))\}$$

Safety of Expressions

- It is possible to write tuple calculus expressions that generate infinite relations.
- For example, $\{ t \mid \neg\, t \in r \}$ results in an infinite relation if the domain of any attribute of relation *r* is infinite
- To guard against the problem, we restrict the set of allowable expressions to safe expressions.
- An expression $\{t \mid P(t)\}$ in the tuple relational calculus is *safe* if every component of *t* appears in one of the relations, tuples, or constants that appear in *P*
  - NOTE: this is more than just a syntax condition.
    - E.g. $\{ t \mid t[A] = 5 \lor \textbf{true} \}$ is not safe --- it defines an infinite set with attribute values that do not appear in any relation or tuples or constants in *P*.

Domain Relational Calculus

- A nonprocedural query language equivalent in power to the tuple relational calculus
- Each query is an expression of the form:

$$\{ < x_1, x_2, \ldots, x_n > \mid P(x_1, x_2, \ldots, x_n)\}$$

  - $x_1, x_2, \ldots, x_n$ represent domain variables
  - P represents a formula similar to that of the predicate calculus

Example Queries

- Find the *loan_number, branch_name,* and *amount* for loans of over $1200

$$\{< l, b, a > \mid < l, b, a > \in loan \land a > 1200\}$$

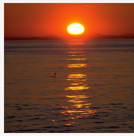
- Find the names of all customers who have a loan of over $1200

$$\{< c > \mid \exists\ l, b, a\ (< c, l > \in borrower \land < l, b, a > \in loan \land a > 1200)\}$$

- Find the names of all customers who have a loan from the Perryridge branch and the loan amount:

$$\{< c, a > \mid \exists\ l\ (< c, l > \in borrower \land \exists b\ (< l, b, a > \in loan \land b = \text{“Perryridge”}))\}$$

$$\{< c, a > \mid \exists\ l\ (< c, l > \in borrower \land < l, \text{“Perryridge”}, a > \in loan)\}$$



Example Queries

- Find the names of all customers having a loan, an account, or both at the Perryridge branch:

$$
\begin{aligned}
\{< c > \mid \exists\, l\, (< c, l > \in\ & borrower \\
& \quad \wedge \exists\, b,a\, (< l, b, a > \in\ loan \wedge b = \text{“Perryridge”})) \\
\vee\ \exists\, a\, (< c, a > \in\ & depositor \\
& \quad \wedge \exists\, b,n\, (< a, b, n > \in\ account \wedge b = \text{“Perryridge”}))\}
\end{aligned}
$$

- Find the names of all customers who have an account at all branches located in Brooklyn:

$$
\begin{aligned}
\{< c > \mid \exists\, \text{s},n\, (< c, s, n > \in\ \text{customer}) \wedge \\
\forall\, x,y,z\, (< x, y, z > \in\ branch \wedge y = \text{“Brooklyn”}) \Rightarrow \\
\exists\, a,b\, (< x, y, z > \in\ account \wedge < c,a > \in\ depositor)\}
\end{aligned}
$$

Safety of Expressions

The expression:

$$\{ < x_1, x_2, \ldots, x_n > \mid P\,(x_1, x_2, \ldots, x_n)\}$$

is safe if all of the following hold:

4. All values that appear in tuples of the expression are values from *dom* (*P*) (that is, the values appear either in *P* or in a tuple of a relation mentioned in *P*).
5. For every "there exists" subformula of the form $\exists\, x\,(P_1(x))$, the subformula is true if and only if there is a value of *x* in *dom* (P_1) such that $P_1(x)$ is true.
6. For every "for all" subformula of the form $\forall_x\,(P_1\,(x))$, the subformula is true if and only if $P_1(x)$ is true for all values *x* from *dom* (P_1).

Query-by-Example (QBE)

- Basic Structure
- Queries on One Relation
- Queries on Several Relations
- The Condition Box
- The Result Relation
- Ordering the Display of Tuples
- Aggregate Operations
- Modification of the Database

QBE — Basic Structure

- A graphical query language which is based (roughly) on the domain relational calculus
- **Two dimensional syntax** – system creates templates of relations that are requested by users
- Queries are expressed “by example”

QBE Skeleton Tables for the Bank Example

branch	branch-name	branch-city	assets

customer	customer-name	customer-street	customer-city

loan	loan-number	branch-name	amount

QBE Skeleton Tables (Cont.)

branch	branch_name	branch_city	assets

customer	customer_name	customer_street	customer_city

loan	loan_number	branch_name	amount

borrower	customer_name	loan_number

account	account_number	branch_name	balance

depositor	customer_name	account_number

Queries on One Relation

- Find all loan numbers at the Perryridge branch.

loan	loan_number	branch_name	amount
	P._x	Perryridge	

  - _x is a variable (optional; can be omitted in above query)
  - P. means print (display)
  - duplicates are removed by default
  - To retain duplicates use P.ALL

loan	loan_number	branch_name	amount
	P.ALL.	Perryridge	

Queries on One Relation (Cont.)

- Display full details of all loans
  - Method 1:

loan	*loan-number*	*branch-name*	*amount*
	P._x	P._y	P._z

  - Method 2: Shorthand notation

loan	*loan_number*	*branch_name*	*amount*
P.			

Queries on One Relation (Cont.)

- Find the loan number of all loans with a loan amount of more than $700

loan	*loan_number*	*branch_name*	*amount*
	P.		>700

- Find names of all branches that are not located in Brooklyn

branch	*branch_name*	*branch_city*	*assets*
	P.	¬ Brooklyn	

Queries on One Relation (Cont.)

- Find the loan numbers of all loans made jointly to Smith and Jones.

borrower	customer_name	loan_number
	Smith	P._x
	Jones	_x

- Find all customers who live in the same city as Jones

customer	customer_name	customer_street	customer_city
	P._x		_y
	Jones		_y

Queries on Several Relations

- Find the names of all customers who have a loan from the Perryridge branch.

loan	*loan_number*	*branch_name*	*amount*
	_*x*	Perryridge	

borrower	*customer_name*	*loan_number*
	P._*y*	_*x*

Queries on Several Relations (Cont.)

- Find the names of all customers who have both an account and a loan at the bank.

depositor	*customer_name*	*account_number*
	P._*x*	

borrower	*customer_name*	*loan_number*
	_*x*	

Negation in QBE

- Find the names of all customers who have an account at the bank, but do not have a loan from the bank.

depositor	*customer_name*	*account_number*
	P._*x*	

borrower	*customer_name*	*loan_number*
¬	_*x*	

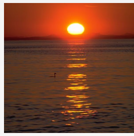
¬ means "there does not exist"

Negation in QBE (Cont.)

- Find all customers who have at least two accounts.

depositor	customer_name	account_number
	P._x	_y
	_x	¬ _y

¬ means “not equal to”



The Condition Box

- Allows the expression of constraints on domain variables that are either inconvenient or impossible to express within the skeleton tables.
- Complex conditions can be used in condition boxes
- Example: Find the loan numbers of all loans made to Smith, to Jones, or to both jointly

borrower	*customer_name*	*loan_number*
	$\_n$	P.$\_x$

conditions
$\_n$ = Smith **or** $\_n$ = Jones

Condition Box (Cont.)

- QBE supports an interesting syntax for expressing alternative values

branch	*branch_name*	*branch_city*	*assets*
	P.	$\_x$	

conditions
$\_x$ = (Brooklyn **or** Queens)

Condition Box (Cont.)

- Find all account numbers with a balance greater than \$1,300 and less than \$1,500

account	*account_number*	*branch_name*	*balance*
	P.		_x

conditions
$\_x \geq 1300$
$\_x \leq 1500$

- Find all account numbers with a balance greater than \$1,300 and less than \$2,000 but not exactly \$1,500.

account	*account_number*	*branch_name*	*balance*
	P.		_x

conditions
$\_x = (\geq 1300 \textbf{ and } \leq 2000 \textbf{ and } \neg\ 1500)$

Condition Box (Cont.)

- Find all branches that have assets greater than those of at least one branch located in Brooklyn

branch	*branch_name*	*branch_city*	*assets*
	P._*x*		_*y*
		Brooklyn	_*z*

conditions
_*y* > _*z*

The Result Relation

- Find the *customer_name*, *account_number,* and *balance* for all customers who have an account at the Perryridge branch.
  - We need to:
    - Join *depositor* and *account.*
    - Project *customer_name, account_number* and *balance.*
  - To accomplish this we:
    - Create a skeleton table, called *result,* with attributes *customer_name, account_number,* and *balance.*
    - Write the query.

The Result Relation (Cont.)

- The resulting query is:

account	account_number	branch_name	balance
	_y	Perryridge	_z

depositor	customer_name	account_number
	_x	_y

result	customer_name	account_number	balance
P.	_x	_y	_z

Ordering the Display of Tuples

- AO = ascending order; DO = descending order.
- Example: list in ascending alphabetical order all customers who have an account at the bank

depositor	*customer_name*	*account_number*
	P.AO.	

- When sorting on multiple attributes, the sorting order is specified by including with each sort operator (AO or DO) an integer surrounded by parentheses.
- Example: List all account numbers at the Perryridge branch in ascending alphabetic order with their respective account balances in descending order.

account	*account_number*	*branch_name*	*balance*
	P.AO(1).	Perryridge	P.DO(2).

Aggregate Operations

- The aggregate operators are AVG, MAX, MIN, SUM, and CNT
- The above operators must be postfixed with “ALL” (e.g., SUM.ALL. or AVG.ALL._x) to ensure that duplicates are not eliminated.
- Example: Find the total balance of all the accounts maintained at the Perryridge branch.

account	*account_number*	*branch_name*	*balance*
		Perryridge	P.SUM.ALL.

Aggregate Operations (Cont.)

- UNQ is used to specify that we want to eliminate duplicates
- Find the total number of customers having an account at the bank.

depositor	*customer_name*	*account_number*
	P.CNT.UNQ.	

Query Examples

- Find the average balance at each branch.

account	*account_number*	*branch_name*	*balance*
		P.G.	P.AVG.ALL._x

- The "G" in "P.G" is analogous to SQL's **group by** construct
- The "ALL" in the "P.AVG.ALL" entry in the *balance* column ensures that all balances are considered
- To find the average account balance at only those branches where the average account balance is more than $1,200, we simply add the condition box:

conditions
AVG.ALL._x > 1200

Query Example

- Find all customers who have an account at all branches located in Brooklyn.
  - Approach: for each customer, find the number of branches in Brooklyn at which they have accounts, and compare with total number of branches in Brooklyn
  - QBE does not provide subquery functionality, so both above tasks have to be combined in a single query.
    - Can be done for this query, but there are queries that require subqueries and cannot always be expressed in QBE.
- In the query on the next page

  CNT.UNQ.ALL._*w* specifies the number of distinct branches in Brooklyn. Note: The variable _*w* is not connected to other variables in the query

  CNT.UNQ.ALL._*z* specifies the number of distinct branches in Brooklyn at which customer *x* has an account.

Query Example (Cont.)

depositor	customer_name	account_number
	P.G._x	_y

account	account_number	branch_name	balance
	_y	_z	

branch	branch_name	branch_city	assets
	_z	Brooklyn	
	_w	Brooklyn	

conditions
CNT.UNQ._z = CNT.UNQ._w

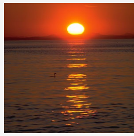
Modification of the Database – Deletion

- Deletion of tuples from a relation is expressed by use of a D. command. In the case where we delete information in only some of the columns, null values, specified by –, are inserted.
- Delete customer Smith

customer	*customer_name*	*customer_street*	*customer_city*
D.	Smith		

- Delete the *branch_city* value of the branch whose name is “Perryridge”.

branch	*branch_name*	*branch_city*	*assets*
	Perryridge	D.	



Deletion Query Examples

- Delete all loans with a loan amount greater than $1300 and less than $1500.
  - For consistency, we have to delete information from loan and borrower tables

loan	*loan_number*	*branch_name*	*amount*
D.	$\_y$		$\_x$

borrower	*customer_name*	*loan_number*
D.		$\_y$

conditions
$\_x = (\geq 1300 \textbf{ and } \leq 1500)$

Deletion Query Examples (Cont.)

- Delete all accounts at branches located in Brooklyn.

account	*account_number*	*branch_name*	*balance*
D.	_y	_x	

depositor	*customer_name*	*account_number*
D.		_y

branch	*branch_name*	*branch_city*	*assets*
	_x	Brooklyn	

Modification of the Database – Insertion

- Insertion is done by placing the I. operator in the query expression.
- Insert the fact that account A-9732 at the Perryridge branch has a balance of $700.

account	*account_number*	*branch_name*	*balance*
I.	A-9732	Perryridge	700

Modification of the Database – Insertion (Cont.)

- Provide as a gift for all loan customers of the Perryridge branch, a new \$200 savings account for every loan account they have, with the loan number serving as the account number for the new savings account.

account	*account_number*	*branch_name*	*balance*
I.	_*x*	Perryridge	200

depositor	*customer_name*	*account_number*
I.	_*y*	_*x*

loan	*loan_number*	*branch_name*	*amount*
	_*x*	Perryridge	

borrower	*customer_name*	*loan_number*
	_*y*	_*x*

Modification of the Database – Updates

- Use the U. operator to change a value in a tuple without changing *all* values in the tuple. QBE does not allow users to update the primary key fields.
- Update the asset value of the Perryridge branch to $10,000,000.

branch	*branch_name*	*branch_city*	*assets*
	Perryridge		U.10000000

- Increase all balances by 5 percent.

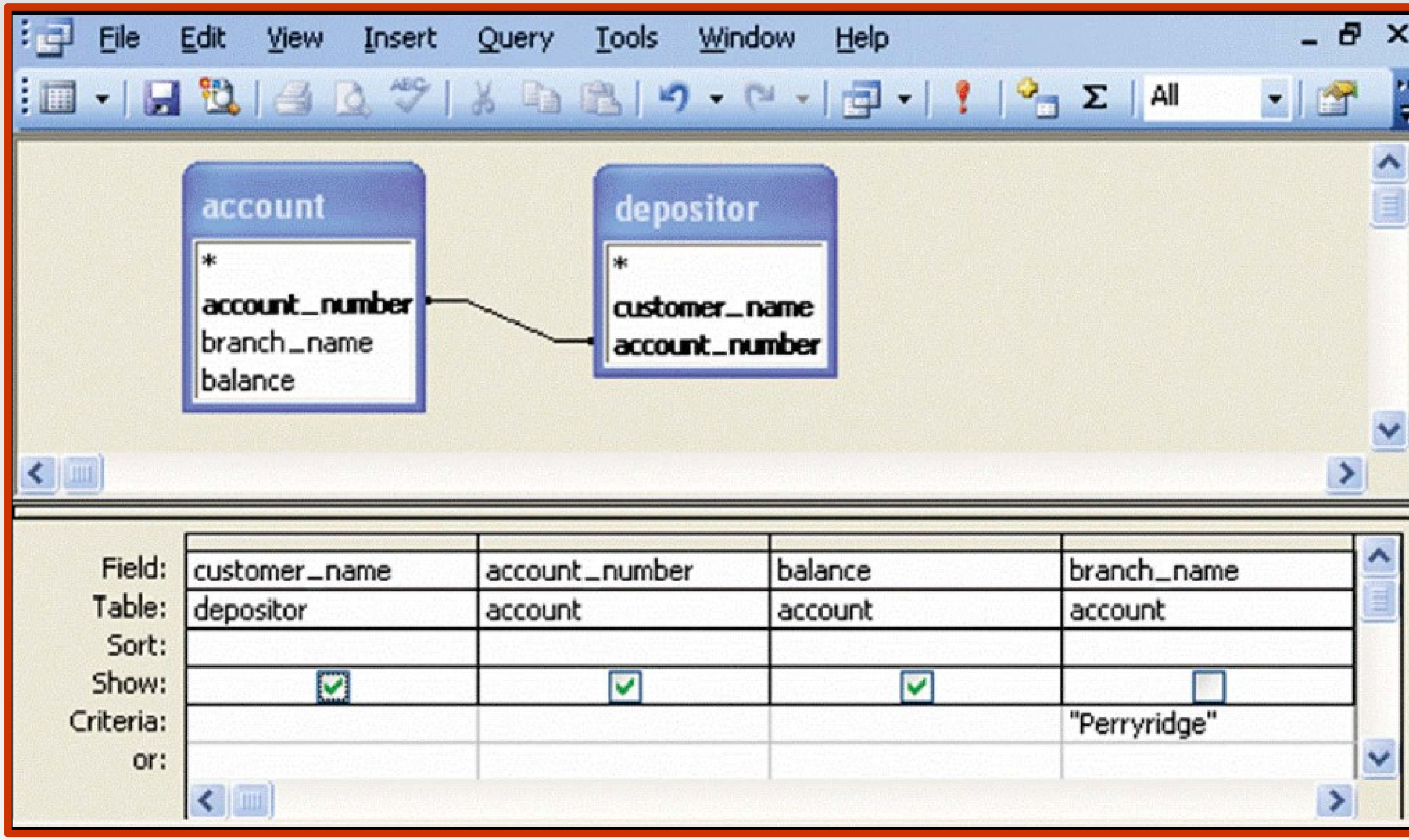
account	*account_number*	*branch_name*	*balance*
			U._*x* * 1.05

Microsoft Access QBE

- Microsoft Access supports a variant of QBE called Graphical Query By Example (GQBE)
- GQBE differs from QBE in the following ways
  - Attributes of relations are listed vertically, one below the other, instead of horizontally
  - Instead of using variables, lines (links) between attributes are used to specify that their values should be the same.
    - Links are added automatically on the basis of attribute name, and the user can then add or delete links
    - By default, a link specifies an inner join, but can be modified to specify outer joins.
  - Conditions, values to be printed, as well as group by attributes are all specified together in a box called the **design grid**

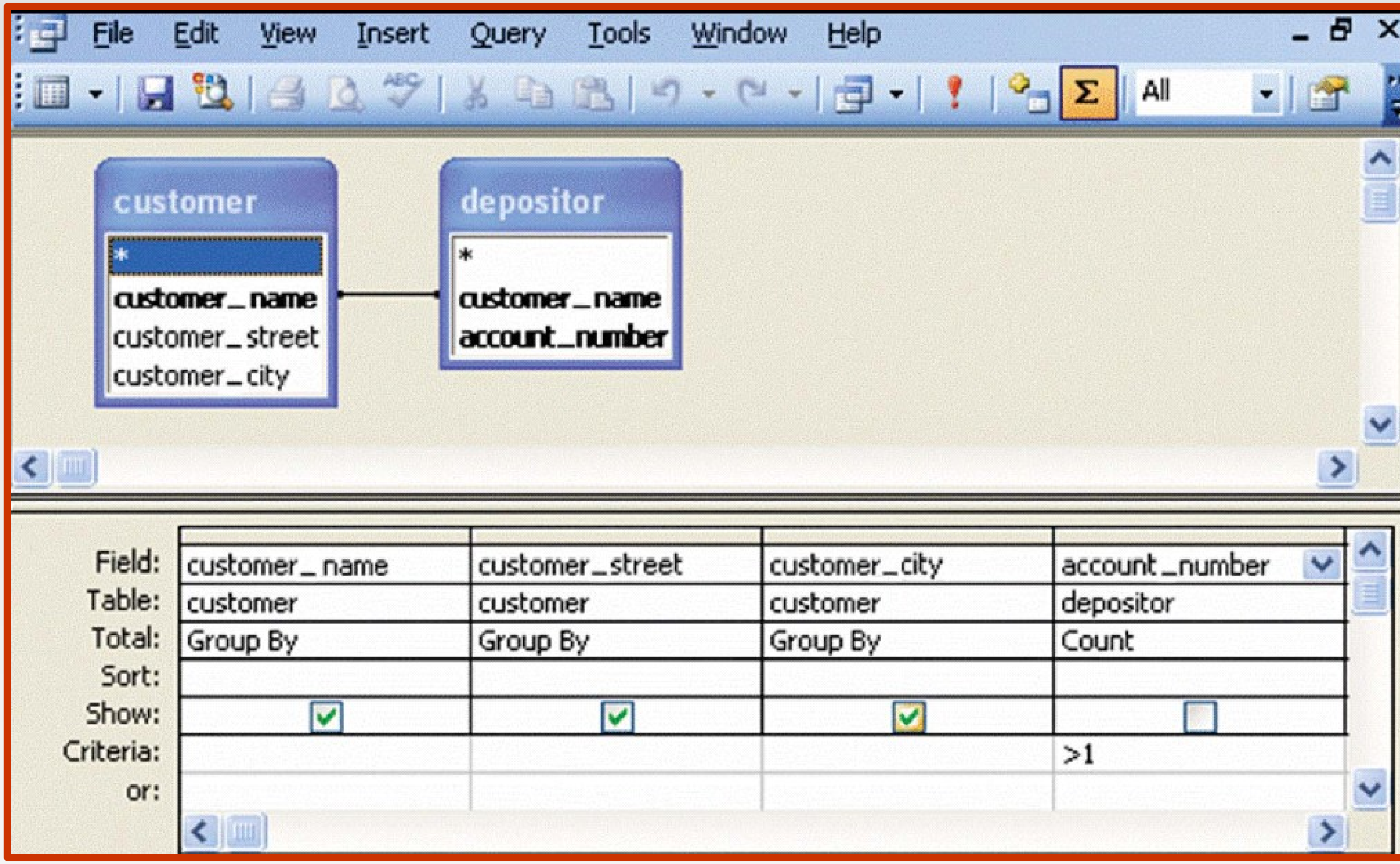
An Example Query in Microsoft Access QBE

- Example query: Find the *customer_name*, *account_number* and *balance* for all accounts at the Perryridge branch



An Aggregation Query in Access QBE

- Find the *name, street* and *city* of all customers who have more than one account at the bank

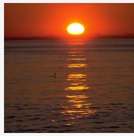


Aggregation in Access QBE

- The row labeled **Total** specifies
  - which attributes are group by attributes
  - which attributes are to be aggregated upon (and the aggregate function).
  - For attributes that are neither group by nor aggregated, we can still specify conditions by selecting **where** in the Total row and listing the conditions below
- As in SQL, if group by is used, only group by attributes and aggregate results can be output

Datalog

- Basic Structure
- Syntax of Datalog Rules
- Semantics of Nonrecursive Datalog
- Safety
- Relational Operations in Datalog
- Recursion in Datalog
- The Power of Recursion



Basic Structure

- Prolog-like logic-based language that allows recursive queries; based on first-order logic.
- A Datalog program consists of a set of *rules* that define views.
- Example: define a view relation *v1* containing account numbers and balances for accounts at the Perryridge branch with a balance of over $700.

  v1 (*A*, *B*) *:– account* (*A*, "Perryridge", *B*), *B* > 700.

- Retrieve the balance of account number "A-217" in the view relation *v1.*

  ? v1 ("A-217", *B*).

- To find account number and balance of all accounts in *v1* that have a balance greater than 800

  ? *v1* (*A*,*B*), *B* > 800

Example Queries

- Each rule defines a set of tuples that a view relation must contain.
  - E.g. *v1* (*A*, *B*) :– *account* (*A*, "Perryridge", *B*), *B* > 700 is read as

    for all *A, B*

    if (*A*, "Perryridge", *B*) $\in$ *account* **and** *B* > 700

    then (*A, B*) $\in$ *v1*
- The set of tuples in a view relation is then defined as the union of all the sets of tuples defined by the rules for the view relation.
- Example:

  interest_rate (*A*, 5) :– *account* (*A, N, B*) , *B* < 10000
  interest_rate (*A*, 6) :– *account* (*A, N, B*), *B* >= 10000

Negation in Datalog

- Define a view relation *c* that contains the names of all customers who have a deposit but no loan at the bank:

  c(*N*) :– *depositor* (*N, A*), **not** *is_borrower* (*N*).
  is_borrower (*N*) :–*borrower* (*N,L*).

- NOTE: using **not** *borrower* (*N, L*) in the first rule results in a different meaning, namely there is some loan L for which N is not a borrower.
  - To prevent such confusion, we require all variables in negated "predicate" to also be present in non-negated predicates

Named Attribute Notation

- Datalog rules use a positional notation that is convenient for relations with a small number of attributes
- It is easy to extend Datalog to support named attributes.
  - E.g., *v1* can be defined using named attributes as

    v1 (*account_number A*, balance *B*) *:–*
      account (*account_number A*, *branch_name* "Perryridge", balance *B*),
      B > 700.

Formal Syntax and Semantics of Datalog

- We formally define the syntax and semantics (meaning) of Datalog programs, in the following steps
  1. We define the syntax of predicates, and then the syntax of rules
  2. We define the semantics of individual rules
  3. We define the semantics of non-recursive programs, based on a layering of rules
  4. It is possible to write rules that can generate an infinite number of tuples in the view relation. To prevent this, we define what rules are “safe”. Non-recursive programs containing only safe rules can only generate a finite number of answers.
  5. It is possible to write recursive programs whose meaning is unclear. We define what recursive programs are acceptable, and define their meaning.

Syntax of Datalog Rules

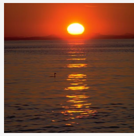
- A positive literal has the form

$$p\,(t_1,\ t_2\ \ldots,\ t_n)$$

  - p is the name of a relation with n attributes
  - each t_i is either a constant or variable
- A negative literal has the form

$$\textbf{not}\ p\,(t_1,\ t_2\ \ldots,\ t_n)$$

- Comparison operations are treated as positive predicates
  - E.g. $X > Y$ is treated as a predicate $>(X,Y)$
  - ">" is conceptually an (infinite) relation that contains all pairs of values such that the first value is greater than the second value
- Arithmetic operations are also treated as predicates
  - E.g. A = B + C is treated as +(B, C, A), where the relation "+" contains all triples such that the third value is the sum of the first two



Syntax of Datalog Rules (Cont.)

- **Rules** are built out of literals and have the form:

$$p\ \underbrace{(t_1,\ t_2,\ \ldots,\ t_n)}_{\textit{head}}\ :-\ \underbrace{L_1,\ L_2,\ \ldots,\ L_m}_{\textit{body}}.$$

  - each L_i is a literal
  - head – the literal $p\ (t_1,\ t_2,\ \ldots,\ t_n)$
  - body – the rest of the literals
- A **fact** is a rule with an empty body, written in the form:

$$p\ (v_1,\ v_2,\ \ldots,\ v_n).$$

  - indicates tuple $(v_1,\ v_2,\ \ldots,\ v_n)$ is in relation p
- A **Datalog program** is a set of rules

Semantics of a Rule

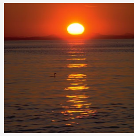
- A **ground instantiation** *of a rule* (or simply **instantiation**) is the result of replacing each variable in the rule by some constant.
  - Eg. Rule defining *v1*

    v1(*A*,*B*) :– *account* (*A*,“Perryridge”, *B*), *B* > 700.

  - An instantiation above rule:

    v1 (“A-217”, 750) :–*account* (“A-217”, “Perryridge”, 750),
    750 > 700.

- The body of rule instantiation *R’* is **satisfied** in a set of facts (database instance) *I* if
  1. For each positive literal $q_i(v_{i,1}, ..., v_{i,ni})$ in the body of *R’*, *I* contains the fact $q_i(v_{i,1}, ..., v_{i,ni})$.
  2. For each negative literal **not** $q_j(v_{j,1}, ..., v_{j,nj})$ in the body of *R’*, *I* does not contain the fact $q_j(v_{j,1}, ..., v_{j,nj})$.



Semantics of a Rule (Cont.)

- We define the set of facts that can be **inferred** from a given set of facts I using rule R as:

  $$infer(R, I) = \{\ p\ (t_1, \ldots, t_n) \mid \text{there is a ground instantiation } R' \text{ of } R \text{ where } p\ (t_1, \ldots, t_n) \text{ is the head of } R', \text{ and the body of } R' \text{ is satisfied in } I\ \}$$

- *Given an set of rules* $\Re = \{R_1, R_2, \ldots, R_n\}$, we define

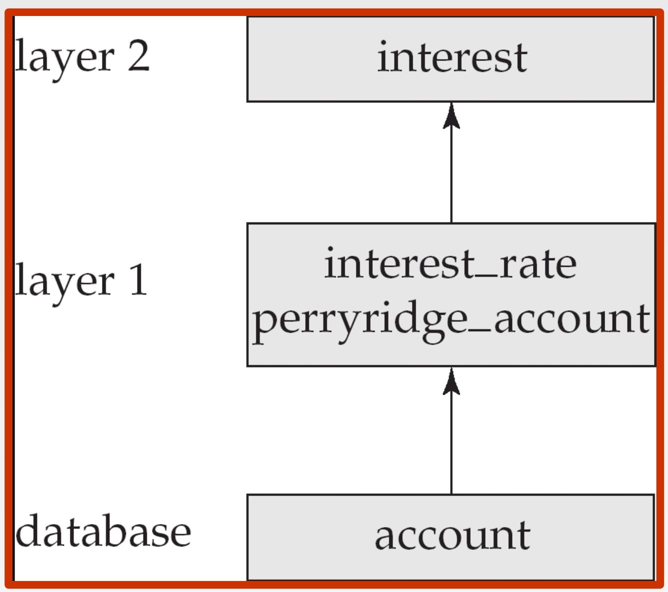
  $$infer(\Re, I) = infer\ (R_1, I\ ) \cup infer\ (R_2, I\ ) \cup \ldots \cup infer\ (R_n, I\ )$$

Layering of Rules

- Define the interest on each account in Perryridge

  interest(A, I) :– *perryridge_account* (*A,B*),
  interest_rate(A,R), *I* = *B* * *R*/100.
  perryridge_account(A,B) :– *account* (*A*, “Perryridge”, *B*).
  interest_rate (*A*,5) :– account (*N*, *A*, *B*), *B* < 10000.
  interest_*rate* (*A*,6) :– account (*N*, A, B), *B* >= 10000.

- Layering of the view relations



Layering Rules (Cont.)

Formally:

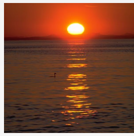
- A relation is a layer 1 if all relations used in the bodies of rules defining it are stored in the database.
- A relation is a layer 2 if all relations used in the bodies of rules defining it are either stored in the database, or are in layer 1.
- A relation *p* is in layer *i* + 1 if
  - it is not in layers 1, 2, ..., *i*
  - all relations used in the bodies of rules defining a *p* are either stored in the database, or are in layers 1, 2, ..., *i*

Semantics of a Program

Let the layers in a given program be 1, 2, ..., *n.* Let $\Re_i$ denote the set of all rules defining view relations in layer *i.*

- Define I_0 = set of facts stored in the database.
- Recursively define $I_{i+1} = I_i \cup infer(\Re_{i+1}, I_i)$
- The set of facts in the view relations defined by the program (also called the semantics of the program) is given by the set of facts I_n corresponding to the highest layer *n.*

Note: Can instead define semantics using view expansion like in relational algebra, but above definition is better for handling extensions such as recursion.



Safety

- It is possible to write rules that generate an infinite number of answers.

  $gt(X, Y) \text{ :– } X > Y$

  not_in_loan (*B*, *L*) :– **not** *loan* (*B*, *L*)

  To avoid this possibility Datalog rules must satisfy the following conditions.

  - Every variable that appears in the head of the rule also appears in a non-arithmetic positive literal in the body of the rule.
    - This condition can be weakened in special cases based on the semantics of arithmetic predicates, for example to permit the rule

      $p\,(A)\ \text{ :- }\ q\,(B),\, A = B + 1$
  - Every variable appearing in a negative literal in the body of the rule also appears in some positive literal in the body of the rule.

Relational Operations in Datalog

- Project out attribute *account_name* from account.

  $$query\,(A)\ :\!-\ account\,(A,\ N,\ B\,).$$

- Cartesian product of relations r_1 and r_2.

  $$query\,(X_1,\ X_2,\ \ldots,\ X_n,\ Y_1,\ Y_1,\ Y_2,\ \ldots,\ Y_m\,)\ :\!-$$
  $$r_1\,(X_1,\ X_2,\ \ldots,\ X_n),\ r_2\,(Y_1,\ Y_2,\ \ldots,\ Y_m).$$

- *Union of relations* r_1 and r_2.

  $$query\,(X_1,\ X_2,\ \ldots,\ X_n)\ :\!-r_1\,(X_1,\ X_2,\ \ldots,\ X_n),$$
  $$query\,(X_1,\ X_2,\ \ldots,\ X_n)\ :\!-r_2\,(X_1,\ X_2,\ \ldots,\ X_n),$$

- Set difference of r_1 and r_2.

  $$query\,(X_1,\ X_2,\ \ldots,\ X_n)\ :\!-r_1(X_1,\ X_2,\ \ldots,\ X_n),$$
  $$\textbf{not}\ r_2\,(X_1,\ X_2,\ \ldots,\ X_n),$$

Recursion in Datalog

- Suppose we are given a relation

  manager (*X, Y*)

  containing pairs of names X, Y such that Y is a manager of X (or equivalently, X is a direct employee of Y).
- Each manager may have direct employees, as well as indirect employees
  - Indirect employees of a manager, say Jones, are employees of people who are direct employees of Jones, or recursively, employees of people who are indirect employees of Jones
- Suppose we wish to find all (direct and indirect) employees of manager Jones. We can write a recursive Datalog program.

  empl_jones (*X*) :- *manager* (*X, Jones*).

  empl_jones (*X*) :- *manager* (*X, Y*), *empl_jones* (*Y*).

Semantics of Recursion in Datalog

- Assumption (for now): program contains no negative literals
- The view relations of a recursive program containing a set of rules $\mathfrak{R}$ are defined to contain exactly the set of facts *I*
  computed by the iterative procedure *Datalog-Fixpoint*

procedure Datalog-Fixpoint
    I = set of facts in the database
    repeat
        $Old\_I = I$
        $I = I \cup infer(\mathfrak{R}, I)$
    until $I = Old\_I$

- At the end of the procedure, $infer(\mathfrak{R}, I) \subseteq I$
  - $Infer(\mathfrak{R}, I) = I$ if we consider the database to be a set of facts that are part of the program
- *I* is called a **fixed point** of the program.

Example of Datalog-FixPoint Iteration

employee_name	*manager_name*
Alon	Barinsky
Barinsky	Estovar
Corbin	Duarte
Duarte	Jones
Estovar	Jones
Jones	Klinger
Rensal	Klinger

Iteration number	Tuples in *empl_jones*
0	
1	(Duarte), (Estovar)
2	(Duarte), (Estovar), (Barinsky), (Corbin)
3	(Duarte), (Estovar), (Barinsky), (Corbin), (Alon)
4	(Duarte), (Estovar), (Barinsky), (Corbin), (Alon)

A More General View

- Create a view relation *empl* that contains every tuple (*X, Y*) such that *X* is directly or indirectly managed by *Y.*

  empl (*X, Y*) :– *manager* (*X, Y*).
  empl (*X, Y*) :– *manager* (*X, Z*), *empl* (*Z, Y*)

- Find the direct and indirect employees of Jones.

  ? empl (X, “Jones”).

- Can define the view *empl* in another way too:

  empl (*X, Y*) :– *manager* (*X, Y*).
  empl (*X, Y*) :– empl (X, Z), *manager* (*Z, Y).*

The Power of Recursion

- Recursive views make it possible to write queries, such as transitive closure queries, that cannot be written without recursion or iteration.
  - Intuition: Without recursion, a non-recursive non-iterative program can perform only a fixed number of joins of manager with itself
    - This can give only a fixed number of levels of managers
    - Given a program we can construct a database with a greater number of levels of managers on which the program will not work

Recursion in SQL

- Starting with SQL:1999, SQL permits recursive view definition
- E.g. query to find all employee-manager pairs

```
with recursive empl (emp, mgr ) as (
        select emp, mgr
        from   manager
   union
        select manager.emp, empl.mgr
        from   manager, empl
        where manager.mgr = empl.emp       )
select *
from   empl
```

Monotonicity

- A view V is said to be **monotonic** if given any two sets of facts I_1 and I_2 such that $I_1 \subseteq I_2$, then $E_v(I_1) \subseteq E_v(I_2)$, where E_v is the expression used to define V.
- A set of rules R is said to be monotonic if
  $I_1 \subseteq I_2$ implies $\mathit{infer}(R, I_1) \subseteq \mathit{infer}(R, I_2)$,
- Relational algebra views defined using only the operations: Π, σ, $\times$, $\cup$, |X|, $\cap$, and ρ (as well as operations like natural join defined in terms of these operations) are monotonic.
- Relational algebra views defined using set difference (–) may not be monotonic.
- Similarly, Datalog programs without negation are monotonic, but Datalog programs with negation may not be monotonic.

Non-Monotonicity

- Procedure *Datalog-Fixpoint* is sound provided the rules in the program are monotonic.
  - Otherwise, it may make some inferences in an iteration that cannot be made in a later iteration. E.g. given the rules

    a :- **not** *b.*
    b :- *c.*
    c.

    Then *a* can be inferred initially, before b is inferred, but not later.
- We can extend the procedure to handle negation so long as the program is "stratified": intuitively, so long as negation is not mixed with recursion

Non-Monotonicity (Cont.)

- There are useful queries that cannot be expressed by a stratified program
  - Example: given information about the number of each subpart in each part, in a part-subpart hierarchy, find the total number of subparts of each part.
  - A program to compute the above query would have to mix aggregation with recursion
  - However, so long as the underlying data (part-subpart) has no cycles, it is possible to write a program that mixes aggregation with recursion, yet has a clear meaning
  - There are ways to evaluate some such classes of non-stratified programs

End of Chapter 5

Figure 5.1

customer_name
Adams
Hayes

Figure 5.2

branch	*branch_name*	*branch_city*	*assets*

customer	*customer_name*	*customer_street*	*customer_city*

loan	*loan_number*	*branch_name*	*amount*

borrower	*customer_name*	*loan_number*

account	*account_number*	*branch_name*	*balance*

depositor	*customer_name*	*account_number*

Figure 5.5

account_number	*branch_name*	*balance*
A-101	Downtown	500
A-215	Mianus	700
A-102	Perryridge	400
A-305	Round Hill	350
A-201	Perryridge	900
A-222	Redwood	700
A-217	Perryridge	750

Figure 5.6

account_number	*balance*
A-201	900
A-217	750

Figure 5.9

account_number	*balance*
A-201	900
A-217	750

Figure in-5.2

loan	*loan_number*	*branch_name*	*amount*
	P.	Perryridge	

Figure in-5.15

conditions
$x \neg = \text{Jones}$

Figure in-5.18

conditions
$\_y \geq 2 * \_z$

Figure in-5-31

branch	*branch_name*	*branch_city*	*assets*
	Perryridge	D.	

Figure in-5.36

branch	*branch_name*	*branch_city*	*assets*
I.	Capital	Queens	