

Numerical solution key for Tutorial 1

Q1. a. $\lambda = h/mv = 4.8 \times 10^{-34} \text{ m}$
b. $\lambda = 0.729 \text{ \AA}$

Q2. $\lambda = h/p = h/\sqrt{2mE_k}$
 $= 0.02866 \text{ \AA}$

$E_k = \frac{1}{2}mv^2$
(Kinetic energy)

Q3. $\text{H}_2 \text{ molecule} = 2 \text{ protons} + 2 \text{ electrons}$
 $m_{\text{H}_2} = 2 m_{\text{proton}} + 2 m_{\text{electron}}$
 $\lambda = h/m_{\text{H}_2}v = 1.685 \text{ \AA}$

Q4. For electron
 $E_e = p^2/2m$
 $\Rightarrow p = \sqrt{2mE_e}$
 $\Rightarrow \lambda_e = h/\sqrt{2mE_e}$
 $\frac{\lambda_e}{\lambda_p} = ?$

For Photon
 $E_p = hc/\lambda$
 $\lambda = hc/E_p$
given $E_e = E_p$

Q5. a. $E = eV = p^2/2m \Rightarrow p = \sqrt{2meV}$
 $\lambda = h/p = \frac{1.228}{\sqrt{V}} \text{ nm} = 0.01228 \text{ nm}$
b. $\lambda = h/mv = 0.2429 \text{ nm}$

Q6. $\Delta E \cdot \Delta t = \hbar/2$
distance travelled = $\Delta t \cdot v$
 $= 5.86 \text{ km}$

$\Delta t \rightarrow \text{life time}$
 $\Delta E = 567 \text{ eV}$
 $1 \text{ MeV} = 10^6 \text{ eV}$

Q7. (i) $p = \sqrt{2mE} = 3.77 \times 10^{-23} \text{ kg m/s}$
(ii) $\lambda = 1.228 / \sqrt{V} \text{ nm} = 0.0175 \text{ nm}$
(iii) $|\vec{k}| = 2\pi/\lambda = 359 \text{ nm}^{-1}$

Q8. $\Delta E \cdot \Delta t = \hbar/2$
 $\Delta E = \hbar/2\Delta t = 0.5275 \text{ J}$
show it in eV

Q9. $\Delta p_{\text{max}} = p = mv$
 $\Delta p_{\text{max}} \cdot \Delta x_{\text{min}} = \hbar/2$
 $\Rightarrow \Delta x_{\text{min}} = \hbar/2\Delta p_{\text{max}} = 0.0193 \text{ nm}$

Q10. $\Delta x = 0.02 \text{ nm}$
 $\Delta p_x = \hbar/2\Delta x = 2.638 \times 10^{-24} \text{ kg m/s}$

Numerical solution key for tutorial 2

Q1. a., d. & e. are well behaved.

Q2. $\nabla^2 \psi = - \underbrace{(k_1^2 + k_2^2 + k_3^2)}_{\substack{\downarrow \\ \text{eigen value}}} \psi$

Q3. (i) $\int_{-\infty}^{\infty} \psi^* \psi dx = 1$ $a = \sqrt{3}$

(ii) $\int_{-0.45}^{0.55} \psi^* \psi dx$

(iii) $\langle x \rangle = \int_{-\infty}^{\infty} \psi^* x \psi dx$
 $= 3/4$

Q4. (i) yes if $\hat{p}_x \psi = p \psi$ otherwise no.

(ii) $\langle \hat{p}_x \rangle = \int_{-\infty}^{\infty} \psi^* \hat{p}_x \psi dx$
 $= -3ik$

Q5. If $\int_{-\infty}^{\infty} \psi^* \psi dx$ is finite, it is normalizable
otherwise no.

Q6. $\int_{-\infty}^{\infty} \psi^* \psi dx = 1$

$$A = (2a/\pi)^{1/4}$$

Q7. use $\hat{H} \psi = E \psi$

or $\left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \psi = E \psi$ to find E

$$E = a^2 \hbar^2 / \sqrt{2m}$$

Q8. use Schrodinger eqⁿ to find $V(x)$

Q9. consult ppt.

Numerical Solution Key for tutorial 3

Q1. $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

(i) find $\int_{-0.45}^{0.55} \psi^* \psi dx = 0.64\%$
(n=2)

(ii) $\langle n \rangle = \int_{-\infty}^{\infty} \psi^* x \psi dx = \int_0^L \psi^* x \psi dx = L/2$

Q2. $E = \frac{1}{2} m v^2 = \frac{\hbar^2 \pi^2}{2mL^2} n^2$ $n=?$

Q3. (i) $E_2 - E_1 = 113.27 \text{ eV}$
(ii) $\lambda = (E_2 - E_1)/h = 0.11 \text{ nm}$
(iii) UV region

Q4. $E_4 - E_2 = h\nu \Rightarrow L = \sqrt{\frac{6\hbar^2 \pi^2}{mL\nu}} = 1.784 \text{ nm}$

Q5. $T = e^{-2K_2 L}$ $K_2 = \sqrt{2m(V-E)}/\hbar$

a. $\frac{T_1}{T_2} = ?$

Q6. $T = e^{-2k_2 L}$

$$k_2 = \frac{\sqrt{2m(V-E)}}{\hbar}$$

$E = ?$

Q7. $T = 2.83\%$

Q8. $T = 3.1\%$

Q9. $|F| = \frac{dE}{dx}$ $E = \frac{\hbar^2 \pi^2}{2mL^2} n^2$

$$= \frac{\hbar^2 \pi^2}{mL^3} \Big|_{L=0.1 \text{ nm}}$$

$$= 1.21 \times 10^{-7} \text{ eV}^2 \text{ N}$$

$$= 1.21 \times 10^{-7} \text{ N}$$