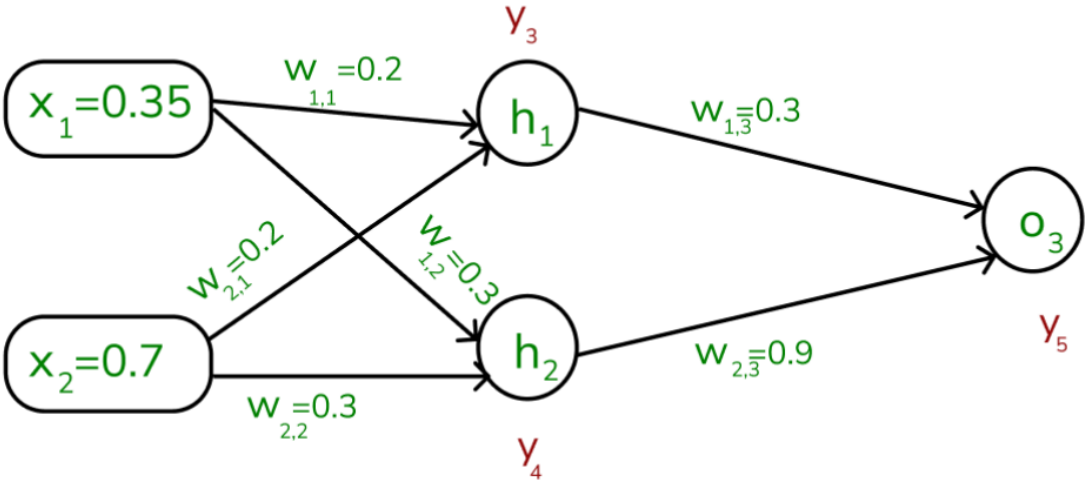


1. Forward Propagation

1. Initial Calculation

The weighted sum at each node is calculated using:

$$aj = \sum(wi, j * xi)$$

Where,

- a_j is the weighted sum of all the inputs and weights at each node
- $w_{i,j}$ represents the weights between the i^{th}input and the j^{th} neuron
- x_i represents the value of the i^{th} input

o (output): After applying the activation function to a, we get the output of the neuron:

o_j = activation function(a_j)

2. Sigmoid Function

The sigmoid function returns a value between 0 and 1, introducing non-linearity into the model.

$$y_j = \frac{1}{1 + e^{-a_j}}$$

3. Computing Outputs

At h1 node

$$\begin{aligned} a_1 &= (w_{1,1}x_1) + (w_{2,1}x_2) \\ &= (0.2 * 0.35) + (0.2 * 0.7) \\ &= 0.21 \end{aligned}$$

Once we calculated the a1 value, we can now proceed to find the y3 value:

$$y_j = F(a_j) = \frac{1}{1 + e^{-a_1}}$$

$$y_3 = F(0.21) = \frac{1}{1 + e^{-0.21}}$$

$$y_3 = 0.56$$

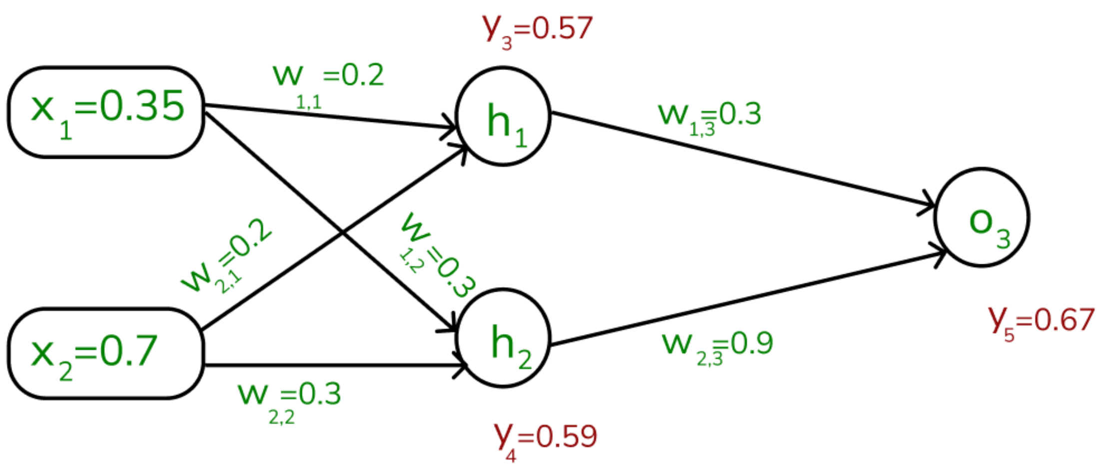
Similarly find the values of y4 at **h2** and y5 at O3

$$a_2 = (w_{1,2} * x_1) + (w_{2,2} * x_2) = (0.3 * 0.35) + (0.3 * 0.7) = 0.315$$

$$y_4 = F(0.315) = \frac{1}{1 + e^{-0.315}}$$

$$a3 = (w_{1,3} * y_3) + (w_{2,3} * y_4) = (0.3 * 0.57) + (0.9 * 0.59) = 0.702$$

$$y_5 = F(0.702) = \frac{1}{1 + e^{-0.702}} = 0.67$$



Values of y3, y4 and y5

4. Error Calculation

Our actual output is 0.5 but we obtained 0.67. To calculate the error we can use the below formula:

$$Error_j = y_{target} - y_5$$

$$Error = 0.5 - 0.67 = -0.17$$

Using this error value we will be backpropagating.

2. Backpropagation

1. Calculating Gradients

The change in each weight is calculated as:

$$\Delta w_{ij} = \eta \times \delta_j \times O_j$$

Where:

- δ_j is the error term for each unit,
- η is the learning rate.

2. Output Unit Error

For O3:

$$\delta_5 = y_5(1 - y_5)(y_{target} - y_5)$$

$$= 0.67(1 - 0.67)(-0.17) = -0.0376$$

3. Hidden Unit Error

For h1:

$$\delta_3 = y_3(1 - y_3)(w_{1,3} \times \delta_5)$$

$$= 0.56(1 - 0.56)(0.3 \times -0.0376) = -0.0027$$

For h2:

$$\delta_4 = y_4(1 - y_4)(w_{2,3} \times \delta_5)$$

$$= 0.59(1 - 0.59)(0.9 \times -0.0376) = -0.0819$$

3. Weight Updates

For the weights from hidden to output layer:

$$\Delta w_{2,3} = 1 \times (-0.0376) \times 0.59 = -0.022184$$

New weight:

$$w_{2,3}(\text{new}) = -0.022184 + 0.9 = 0.877816$$

For weights from input to hidden layer:

$$\Delta w_{1,1} = 1 \times (-0.0027) \times 0.35 = 0.000945$$

New weight:

$$w_{1,1}(\text{new}) = 0.000945 + 0.2 = 0.200945$$

Similarly other weights are updated:

- $w_{1,2}(\text{new}) = 0.273225$
- $w_{1,3}(\text{new}) = 0.278568$
- $w_{2,1}(\text{new}) = 0.269445$
- $w_{2,2}(\text{new}) = 0.18534$

The updated weights are illustrated below

Through backward pass the weights are updated

After updating the weights the forward pass is repeated yielding:

- $y_3 = 0.57$
- $y_4 = 0.56$
- $y_5 = 0.6571$

Since $y_5 = 0.6571$ is still not the target output the process of calculating the error and backpropagating continues until the desired output is reached.

This process demonstrates how backpropagation iteratively updates weights by minimizing errors until the network accurately predicts the output.

$$Error = y_{target} - y_5$$

$$= 0.5 - 0.6571 = -0.1571$$

This process is said to be continued until the actual output is gained by the neural network. Backpropagation is a technique that makes neural network learn. By propagating errors backward and adjusting the weights and biases neural networks can gradually improve their predictions.