# Scope of optimization

- Optimization means the mathematical process through which best possible results are obtained under the given set of Conditions/Constraints.
- During the second world war, the subject "Optimization Techniques" in the name of "Operations Research" gained a momentum.

# Mathematical Programming Problem (MPP)

- The mathematical formulation of a given real-life problem to optimize the objective function under given set of conditions is called mathematical Programming Problem.





## Mathematical formulation ✓

**Step 1:** Study the given situation and find the key decision to made.
(e.g. maximize the profit, minimize the cost etc.)

**Step 2:** Identify the decision variables of the problem.

**Step 3:** Formulate the objective function to be optimized

**Step 4:** Formulate the constraints (Conditions) of the problem

**Step 5:** Add the non-negativity restrictions.

Mathematically, MPP is given by

$$\underset{\text{Optimize}}{\text{Max / Min}} \quad f(X) \rightarrow \text{objective function}$$

$$\text{subject to} \quad g_i(X) \geq, =, \leq 0, \quad i = 1, 2, \dots, m \rightarrow \text{Constraint set}$$

$$X \geq 0 \rightarrow \text{Non-Negative restrictions}$$

where $X = (x_1, x_2, \dots, x_n)^T$

$x_1, x_2, x_3$

$x_1, x_2 \geq 0$ and $x_3$ integer

✓ $x_1, x_2, x_3 \geq 0$

✓ $x_1, x_2, x_3 \geq 0$ and integers

✓ $x_1, x_2 \geq 0, \; x_3 \geq 5,$

✓ $1 \leq x_1 \leq 3, \; x_2 \geq 0, \; x_3 \geq 5$





# Classification of Mathematical Programming Problem (MPP)

1. Linear Programming Problem (LPP)

If the objective function and all constraints are linear functions of the decision variables, then MPP is known as linear programming problem (LPP).

$f(x), \; g_i(x)$

2. Non-linear PP (NLPP)

If the objective function or at least one of the constraints or both are non-linear functions of decision variables in MPP, then the problem is termed as NLPP.

$$\left.\begin{array}{l} f(x) \rightarrow \text{Non-linear} \\ \text{All or some } g_i(x) \rightarrow \text{Non-linear} \end{array}\right\} \text{NLPP}$$

$$\left.\begin{array}{l} f(x) \rightarrow \text{Nonlinear} \\ \text{All } g_i(x) \rightarrow \text{linear} \end{array}\right\} \text{NLPP}$$

$$\left.\begin{array}{l} f(x) \rightarrow \text{Linear} \\ \text{All or some } g_i(x) \rightarrow \text{Non-linear} \end{array}\right\} \text{NLPP}$$

3. Integer programming problem (IPP)

It is a particular case of LPP or NLPP in which some or all decision variables are integers.

# Formulation to LPP models

Exp1 A company manufactures 3 products. Each product has to pass through 3 different operations. The time each product takes in each operation is shown in the table given below. Formulate the problem as an LPP model to maximize the profit.

| Operation | Time per unit (minutes) Product 1 | Product 2 | Product 3 | Operation Capacity (minutes per day) |
|---|---|---|---|---|
| 1 | 1 | 2 | 7 | 400 |
| 2 | 3 | 5 | 9 | 300 |
| 3 | 4 | 6 | 8 | 700 |
| Profit per unit (Rs.) | 5 $x_1$ | 3 $x_2$ | 9 $x_3$ | |

Decision: Max. profit

Decision Variables:
$x_1 \rightarrow$ Units produced of product 1
$x_2 \rightarrow$ ——— 2
$x_3 \rightarrow$ ——— 3

$$\text{Max } Z = 5x_1 + 3x_2 + 9x_3$$

$$\text{s.t. } x_1 + 2x_2 + 7x_3 \le 400$$
$$3x_1 + 5x_2 + 9x_3 \le 300$$
$$4x_1 + 6x_2 + 8x_3 \le 700$$
$$x_1, x_2, x_3 \ge 0 \text{ and integers}$$



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Exp2 A company manufactures two products A and B. The company makes a profit of Rs 40 and Rs 50 per unit of products A and B respectively. The two products are produced in a common production process and are sold in two different markets. The production process has a capacity of 35000 man-hours. It takes 5 hrs to produce one unit of A and 2 hrs to produce one unit of B. The market has been surveyed and company officials feel that the maximum number of unit of A that can be sold is 8000 and that of B is 12000. Formulate the problem as an LPP model so that company gets the maximum profit.

| | Product A | Product B | Capacity |
|---|---|---|---|
| Production Process | 5 | 2 | 35000 |
| Profit per unit | 40 $x_1$ | 50 $x_2$ | |

Decision: Max profit
Decision variables: $x_1, x_2$

$x_1 \rightarrow$ Units produced of product A
$x_2 \rightarrow$ ——— B

$$\text{Max } Z = 40x_1 + 50x_2$$

$$\text{s.t. } 5x_1 + 2x_2 \le 35000$$
$$x_1 \le 8000$$
$$x_2 \le 12000$$
$$x_1, x_2 \ge 0 \text{ and integers}$$

## Exp 3 (Reddy Mikks problem).

Reddy Mikks produces both interior and exterior paints from two raw materials $M_1$ and $M_2$. The following table provides the basic data:

| | Usage of raw material per ton of: Exterior paint $x_1$ | Interior paint $x_2$ | Maximum daily availability (tons) |
|---|---|---|---|
| $M_1$ | 6 | 4 | 24 |
| $M_2$ | 1 | 2 | 6 |
| Profit per ton ($) 1000 | 5 | 4 | |

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. Also the maximum daily demand of interior paint is 2 tons. Reddy Mikks wants to determine the optimum (Best) product mix of interior and exterior paints that maximizes the total daily profit.

Decision: Max. profit

Decision variables:

$x_1 \rightarrow$ Units produced of exterior paint

$x_2 \rightarrow$ Units ——— interior paint.

$$\text{Max } Z = 5x_1 + 4x_2$$

s.t.

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + x_2 \leq 6$$

$$x_2 \leq x_1 + 1 \quad \rightarrow \text{or } -x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$



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## Exp 4

For a 24 hour hospital, the following nurses are required:

6-periods

| Period of Day | 8-12 (1, $x_1$) | 12-16 (2, $x_2$) | 16-20 (3, $x_3$) | 20-24 (4, $x_4$) | 24-04 (5, $x_5$) | 04-08 (6, $x_6$) |
|---|---|---|---|---|---|---|
| Minimum No. of Nurses required | 20 | 15 | 25 | 10 | 5 | 8 |

Every nurse works continuously for 8 hrs and is paid the same salary. It is desired to find number of nurses to be employed at the beginning of each period so that the expenses on salary of nurses are minimized. Formulate the problem as an LPP model.

Decision: Min. expense or Min No. of nurses ✓

Decision variables: Let $x_j$ be the number of nurses employed at the beginning of the $j^{th}$ period, $j = 1, 2, 3, 4, 5, 6$

$$\text{Min } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

s.t.

$$x_1 + x_2 \geq 15 \qquad x_4 + x_5 \geq 5$$

$$x_2 + x_3 \geq 25 \qquad x_5 + x_6 \geq 8$$

$$x_3 + x_4 \geq 10 \qquad x_6 + x_1 \geq 20$$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ and integers.

Exp 5 (Trim Loss Problem)

20 feet Standard Roll

The pacific paper Company produces paper rolls with a standard width of 20 feet each. Special customer orders with different widths which are produced by cutting the standard rolls. Typical orders (which may vary daily) are summarized in the following table:

| Order | Desired width (ft) | Desired number of rolls |
|---|---|---|
| 1 | 5 | 150 |
| 2 | 7 | 200 |
| 3 | 9 | 300 |

Determine the Knife-setting Combinations that will fill the required orders with the least trim-loss area.

| Required width (ft) | Knife setting 1 | 2 | 3 | 4 | 5 | 6 | Minimum no. of rolls required |
|---|---|---|---|---|---|---|---|
| 5 | 0 | 2 | 2 | 4 | 1 | 0 | 150 |
| 7 | 1 | 1 | 0 | 0 | 2 | 0 | 200 |
| 9 | 1 | 0 | 1 | 0 | 0 | 2 | 300 |
| Trim-loss per Standard roll | 4 | 3 | 1 | 0 | 1 | 2 | |

Min $Z = 4x_1 + 3x_2 + x_3 + x_5 + 2x_6$

s.t. $2x_2 + 2x_3 + 4x_4 + x_5 \geq 150$

$x_1 + x_2 + 2x_5 \geq 200$

$x_1 + x_3 + 2x_6 \geq 300$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$



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objective function

| Minimize the use of standard rolls of 20 feet | Minimize the trim-loss area. |
|---|---|
| Min $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$ | Min $Z = 4x_1 + 3x_2 + x_3 + x_5 + 2x_6$ |

**Exp6** A company has two grades of inspectors, I and II, who are to be assigned for a quality control inspection. It is required that at least 2000 pieces be inspected per 8 hour day. Grade I inspectors can check pieces at the rate of 50 per hour with an accuracy of 97% (3%). Grade II inspectors can check pieces at the rate of 40 per hour with an accuracy of 95% (5%). The wage rate of grade I inspector is Rs. 4.50 per hour and that of grade II is Rs. 2.50 per hour. Each time an error is made by an inspector, the cost to the company is Rs 2.00. The company has available for inspection job, 10 grade I and 5 grade II inspectors. Formulate the problem to minimize the total cost of inspection.

Decision : Min. total cost → total cost < Cost of salary / Cost for inaccuracy

Decision variables : $x_1 \rightarrow$ No. of Grade I inspectors

$x_2 \rightarrow$ " " " II "

Cost of salary $= 4.5x_1 + 2.5x_2$ | Cost of inaccuracy $= 2\times 50\times 0.03\, x_1 + 2\times 40\times 0.05\, x_2$



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$$\text{Total Cost} = 4.5x_1 + 2.5x_2 + (2\times 50\times 0.03)x_1 + (2\times 40\times 0.05)x_2$$

$$= 7.5x_1 + 6.5x_2 \rightarrow \text{(per hour)}$$

$$\text{Min } Z = 8(7.5x_1 + 6.5x_2)$$

$$\text{s.t. } 8(50x_1 + 40x_2) \geq 2000$$

$$x_1 \leq 10,$$

$$x_2 \leq 5$$

$$x_1, x_2 \geq 0 \text{ and integers.}$$