GHZ state

GHZ Quantum State (Greenberger–Horne–Zeilinger State)

The standard **3-qubit GHZ state** is:

$$|\mathrm{GHZ}_3\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

This means:

- All qubits are **0 together**
- OR all qubits are **1 together**
- In perfect quantum superposition

⚠️ It is **not**:

$$|000\rangle \text{ OR } |111\rangle$$

It is both **at the same time**.

N Qubit GHZ state

For **n qubits**, the GHZ state is:

$$|\mathrm{GHZ}_n\rangle = \frac{1}{\sqrt{2}}\big(|00...0\rangle + |11...1\rangle\big)$$

Examples:

4 qubits

$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

5 qubits

$$\frac{1}{\sqrt{2}}(|00000\rangle + |11111\rangle)$$

So:

Arbitrary GHZ State

General GHZ-type state:

$$|\psi\rangle = \alpha|00...0\rangle + \beta e^{i\phi}|11...1\rangle$$

Where:

- α, β → amplitudes
- $\alpha^2 + \beta^2 = 1$
- ϕ → relative phase

Special case:

$$\alpha = \beta = \frac{1}{\sqrt{2}}, \phi = 0$$

→ Standard GHZ

How to create GHZ State

For n qubits:

Step 1: Apply Hadamard on first qubit

$$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Step 2: Apply CNOT gates

Connect first qubit to all others.

```markdown
|0> ──H──■──■──■──
         |  |  |
|0> ─────X──|──|──
            X──|──
|0> ────────────X───
```

Result → GHZ state