Quantum Entanglement

What is Entanglement

1. What is Entanglement?

- **Definition:** Entanglement is a quantum phenomenon where **two or more qubits are linked such that the state of one qubit instantaneously affects the state of the other**, no matter how far apart they are.
- **Key feature:** The state of the whole system **cannot be written as a product of individual qubit states**.

Mathematically:

For two qubits A and B:

$$|\psi\rangle_{AB} \neq |\phi\rangle_A \otimes |\chi\rangle_B$$

- If it **can** be factorized → **product state** (not entangled).
- If it **cannot** → **entangled state.**

Example:

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \quad \text{(entangled)}$$

How entanglement works?

- **Measurement effect:** Measuring one qubit **instantly determines the other**.
  - Example: Bell state $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$
    - Measure qubit 1 → if 0, qubit 2 is 0; if 1, qubit 2 is 1.
- **Contrast with product state:**
  - $|0\rangle \otimes |+\rangle$ → measurement of qubit 1 **does not affect** qubit 2.

Intuition: Entangled qubits are like “linked dancers” — one’s move affects the other instantly.

Bell States

- There are 4 Bell states.
- They form a complete basis for 2-qubit entangled states.
- Useful in quantum teleportation, superdense coding, and quantum cryptography.

The 4 Bell states:

1. **Φ^+ (Phi plus)**

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

2. **Φ^- (Phi minus)**

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

3. **Ψ^+ (Psi plus)**

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

4. **Ψ^- (Psi minus)**

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Understanding Bell States

1. Φ⁺ (Phi plus)

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

- **What it says:** The two qubits are **always the same.**
- If you measure the first qubit:
  - 0 → second qubit = 0
  - 1 → second qubit = 1
- **Phase:** + → constructive combination
- **Intuition:** "The qubits are twins"

Understanding Bell States…

2. Φ^- (Phi minus)

$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

- **What it says:** Qubits are **always the same**, but there's a **relative minus sign**.
- Measurement correlations are the same as Φ^+ ($0 \to 0$, $1 \to 1$).
- **Difference:** The minus sign shows **quantum phase difference**, important for interference in quantum circuits.

Understanding Bell States…

3. Ψ^+ (Psi plus)

$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

- **What it says:** The two qubits are **always opposite**.
- If you measure the first qubit:
  - 0 → second qubit = 1
  - 1 → second qubit = 0
- **Phase:** + → constructive combination
- **Intuition:** “The qubits are opposites”

Understanding Bell States…

4. Ψ^- (Psi minus)

$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

- **What it says:** The two qubits are **always opposite**, with a **relative minus sign**.
- Measurement correlations: first qubit 0 → second qubit 1, first qubit 1 → second qubit 0
- **Minus sign:** affects interference and quantum operations.
- **Intuition:** “Opposites with a twist”

Properties of Bell States

- **Maximally entangled:** Measurement of one qubit perfectly predicts the other
- **Orthogonal:** The four states are mutually orthogonal
- **Non-factorizable:** Cannot be written as a product of individual qubit states