

Roll No.

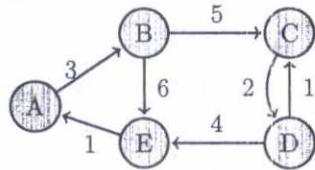
Thapar Institute of Engineering & Technology, Patiala
Department of Mathematics
End Semester Examination

BE (Generic Elective) (December 10, 2024)
Course Code/Name: UMA069/Graph Theory

Time: 9:00AM-12:00 PM ; M. Marks: 40
Course Instructor: Dr. Arun Maiti

Note : Attempt 5 out of 6 questions. No marks will be given for answer written with pencil. For any double attempt, only the first attempt will be considered.

1. (a) Write down the adjacency matrix of the following directed graph.



(Marks: 4, [CO1, L1])

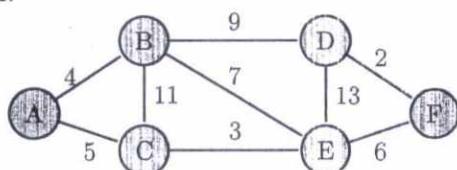
- (b) Determine the maximum number of edges of a graph that has 7 vertices and every vertex has degree ≤ 3 . Is such graph unique (up-to isomorphism)?
(Marks: 4, [CO1 L2])

2. (a) For what values of ℓ , m and n , the graphs K_ℓ and $K_{m,n}$ are Eulerian graphs.

(Marks: 4, [CO2, L2])

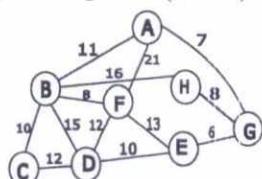
- (b) Determine if each statement is true. Justify your answer with a proof or counterexample.
(a) A graph of order $n \geq 4$ that contains a triangle cannot be Hamiltonian.
(b) If G with n vertices and $|E(G)| \geq \frac{1}{2}(n-1)(n-2) + 2$, then G is Hamiltonian.
(Hint: Ore's theorem)
(Marks: 4, [CO2, L2])

3. (a) Using the Dijkstra algorithm find the shortest path between the vertex A and F in the following graph.



(Marks: 4, [CO3, L2])

- (b) Determine the minimum spanning tree (MST) of the following weighted graph using Kruskal algorithm?

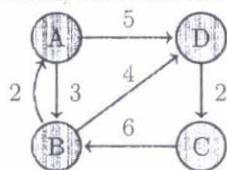


(Marks: 4, [CO3, L2])
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4. (a) A tree, T , with 35 vertices has 25 leaves, two vertices of degree 2, three vertices of degree 4, two vertices of degree 6 and three vertices of degree x . Solve for x .

(Marks: 4, [CO1, L1])

- (b) Using Floyd-Warshals algorithm, find the shortest path between the vertices of the following weighted graph.



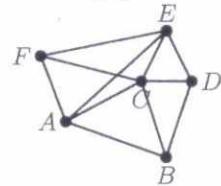
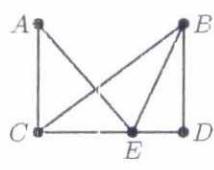
(Marks: 4, [CO3, L2])

5. (a) State Kuratowski theorem regarding planarity of a graph, and using it or otherwise verify whether the following graphs are planar.



(Marks: 4, [CO4, L3])

- (b) Determine the chromatic numbers of the following graphs.



(Marks: 4, [CO4, L1])

6. (a) For a planar graph G with no triangle in it, show that $|E(G)| \leq 2|G| - 4$. Further, use it to show that $K_{3,3}$ is not planar.

(Marks: 4, [CO4, L2])

- (b) Determine the chromatic polynomial of the following graph using deletion-contraction recursive algorithm.



(Marks: 4, [CO4, L2])

—————End of Question Paper—————

Answer sheets will be shown on 24th December at 4:30pm in G304