

Roll No. ....

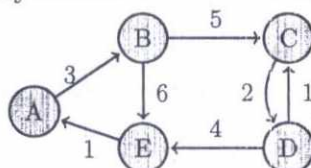
**Thapar Institute of Engineering & Technology, Patiala**  
**Department of Mathematics**  
**End Semester Examination**

BE (Generic Elective) (December 10, 2024)  
 Course Code/Name: UMA069/Graph Theory

Time: 9:00AM-12:00 PM ; M. Marks: 40  
 Course Instructor.: Dr. Arun Maht

**Note : Attempt 5 out of 6 questions. No marks will be given for answer written with pencil. For any double attempt, only the first attempt will be considered.**

1. (a) Write down the adjacency matrix of the following directed graph.



(Marks: 4, [CO1, L1])

- (b) Determine the maximum number of edges of a graph that has 7 vertices and every vertex has degree  $\leq 3$ . Is such graph unique (up-to isomorphism)?

(Marks: 4, [CO1 L3])

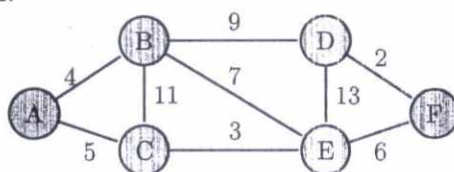
2. (a) For what values of  $\ell$ ,  $m$  and  $n$ , the graphs  $K_\ell$  and  $K_{m,n}$  are Eulerian graphs.

(Marks: 4, [CO2, L2])

- (b) Determine if each statement is true. Justify your answer with a proof or counterexample. (a) A graph of order  $n \geq 4$  that contains a triangle cannot be Hamiltonian. (b) If  $G$  with  $n$  vertices and  $|E(G)| \geq \frac{1}{2}(n-1)(n-2) + 2$ , then  $G$  is Hamiltonian. (Hint: Ore's theorem)

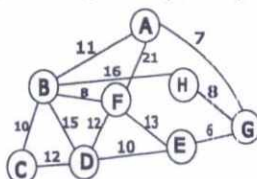
(Marks: 4, [CO2, L2])

3. (a) Using the Dijkstra algorithm find the shortest path between the vertex  $A$  and  $F$  in the following graph.



(Marks: 4, [CO3, L2])

- (b) Determine the minimum spanning tree (MST) of the following weighted graph using Kruskal algorithm?



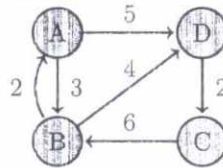
(Marks: 4, [CO3, L2])

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4. (a) A tree,  $T$ , with 35 vertices has 25 leaves, two vertices of degree 2, three vertices of degree 4, two vertices of degree 6 and three vertices of degree  $x$ . Solve for  $x$ .

(Marks: 4, [CO1, L1])

- (b) Using Floyd-Warshals algorithm, find the shortest path between the vertices of the following weighted graph.



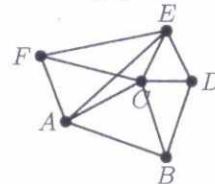
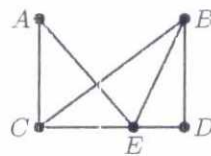
(Marks: 4, [CO3, L2])

5. (a) State Kuratowski theorem regarding planarity of a graph, and using it or otherwise verify whether the following graphs are planar.



(Marks: 4, [CO4, L3])

- (b) Determine the chromatic numbers of the following graphs.



(Marks: 4, [CO4, L1])

6. (a) For a planar graph  $G$  with no triangle in it, show that  $|E(G)| \leq 2|G| - 4$ . Further, use it to show that  $K_{3,3}$  is not planar.

(Marks: 4, [CO4, L2])

- (b) Determine the chromatic polynomial of the following graph using deletion-contraction recursive algorithm.



(Marks: 4, [CO4, L2])

—————End of Question Paper—————

Answer sheets will be shown on 24th December at 4:30pm in G304