Roll Number: _____

Thapar Institute of Engineering and Technology, Patiala

School of Mathematics

B.E. (Sem: V & VII)	Course Name: Graph Theory and Applications (UMA069)
Date: 16-12-2023	Time: 3 hours (09:00 AM–12:00 PM)
M. Marks: 100	Name of Faculty: Dr. Arun Maiti
Instruction:	**Attempt any 5 out of 6 questions**

Note: No marks for ambiguous answer and answer written with pencil.

1. a) Let G_1 and G_2 be two graphs with labelled vertices whose adjacency matrices are given below by A_{G_1} and A_{G_2}, respectively.

$$A_{G_1} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array} \qquad A_{G_2} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

i) Find $G_1 \oplus G_2$ ii) Find the shortest walk and longest trail between the vertices b and f in G_1 iii) are unlabelled graphs G_1 and G_2 isomorphic ? Justify your answer.

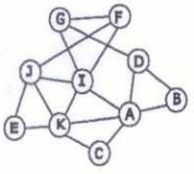
b) Out degree's of vertices of a directed graph are 1, 5, 5, 2, 0. Find the number of edges in the graph.

c) Can you construct an Euler graph with 6 vertices and 10 edges? Justify your answer.

[12 + 4 + 4 marks]

2. a) Let G be a graph with n vertices. Show that if G is a tree then G has exactly $n-1$ edges. Is the converse true?

b) Find the eccentricity of each of the vertices of the following graph, and further identify the centers?



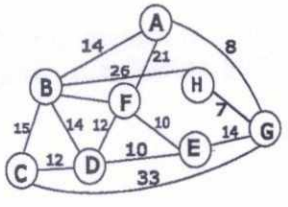
c) Find the number of internal vertices in a complete rooted binary tree with n-vertices.

[6 + 9 + 5 marks]

3. a) State Cayley's formula for trees. Draw the labelled tree with Prüfer sequence $\{10, 10, 6, 1, 2, 2, 1, 5\}$.

b) i) Determine the minimum spanning tree (MST) of the following weighted graph using Prim's algorithm starting from the vertex B? Comment on the optimality of the solution obtained. ii) Describe an algorithm that always produces an optimal solution for MST, and use it to determine a MST of this graph.

[8 + 12 marks]



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4. a) Let G be a planar simple graph with $\#V(G) \geq 3$, then show that $\#E(G) \leq 3\#V(G) - 6$. Use this to prove that G has a vertex of degree at most 5.

b) State Kuratowski theorem for planarity of graph. Determine if the following graphs are planar.

c) For a planar graph G and its geometric dual G^*, what is the relation between rank of G and nullity of G^*.

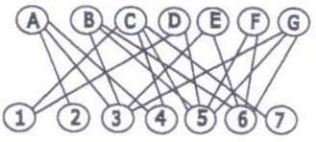
[7+ 9 + 4 marks]

5. a) Show using induction that a tree with two or more vertices is 2-chromatic.
b) Determine the chromatic number and the chromatic polynomial (using deletion-contraction recursive algorithm) of the following graph.

[6 + 14 marks]

6. a) For a graph G with n vertices and e edges, prove that edge connectivity $\leq \frac{2e}{n}$.
b) Find the vertex and edge connectivity of the following graph H.

H



B

c) Find a matching for the above bipartite graph B or use Hall's Marriage theorem to show that no matching exists.

[6 + 6 + 8 marks]

———————End ———————

Note: Answer sheets will be shown on 17th and 19th December at 4 PM in my room number 724 (CSE Building).