

GHZ state

GHZ Quantum State (Greenberger–Horne–Zeilinger State)

The standard 3-qubit GHZ state is:

$$|\text{GHZ}_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

This means:

- All qubits are **0 together**
- OR all qubits are **1 together**
- In perfect quantum superposition

⚠ It is not:

$$|000\rangle \text{ OR } |111\rangle$$

It is both at the same time.

N Qubit GHZ state

For n qubits, the GHZ state is:

$$|\text{GHZ}_n\rangle = \frac{1}{\sqrt{2}}(|00\dots0\rangle + |11\dots1\rangle)$$

Examples:

4 qubits

$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

5 qubits

$$\frac{1}{\sqrt{2}}(|00000\rangle + |11111\rangle)$$

So:

Arbitrary GHZ State

General GHZ-type state:

$$|\psi\rangle = \alpha|00\dots0\rangle + \beta e^{i\phi}|11\dots1\rangle$$

Where:

- $\alpha, \beta \rightarrow$ amplitudes
- $\alpha^2 + \beta^2 = 1$
- $\phi \rightarrow$ relative phase

Special case:

$$\alpha = \beta = \frac{1}{\sqrt{2}}, \phi = 0$$

→ Standard GHZ

How to create GHZ State

For n qubits:

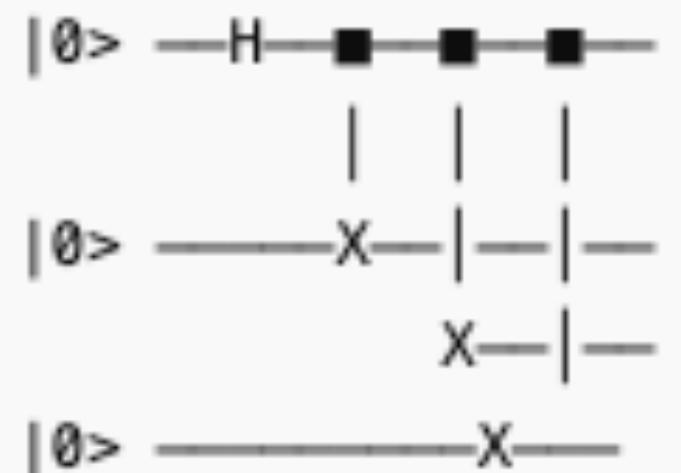
Step 1: Apply Hadamard on first qubit

$$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Step 2: Apply CNOT gates

Connect first qubit to all others.

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Result → GHZ state