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New method for feature extraction based on fractal behavior

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Abstract

In this paper, a novel approach to feature extraction based on fractal theory is presented as a powerful technique in pattern recognition. This paper presents a new fractal feature that can be applied to extract the feature of two-dimensional objects. It is constructed by a hybrid feature extraction combining wavelet analysis, central projection transformation and fractal theory. New fractal feature and fractal signatures are reported. A multiresolution family of the wavelets is also used to compute information conserving micro-features. We employed a central projection method to reduce the dimensionality of the original input pattern. A wavelet transformation technique to transform the derived pattern into a set of sub-patterns. Its fractal dimension can readily be computed, and to use the fractal dimension as the feature vectors. Moreover, a modified fractal signature is also used to distinguish the distinct handwritten signatures. We expect that the proposed fractal method can also be used for improving the extraction and classification of features in pattern recognition.

Keywords: Feature extraction; Fractal geometry; Box dimension; Wavelet transformation; Handwritten signature verification; Central projection transformation

1. Introduction

Fractal behavior and structure can be intuitively appreciated in a variety of ways. Fractal is mathematical sets with a high degree of geometrical complexity, which can model many classes of time-series data as well as images. The fractal dimension is an important characteristic of fractals; it contains information about their geometrical structure. As the interest in fractal geometry rises, the applications are getting more and more numerous in many domains. This paper aims at showing that these concepts can also be applied to feature extraction in pattern recognition. The motivation behind using fractal transformations is to develop a novel feature extraction technique, which can not only be used to recognize different two-dimensional objects, but also utilized to identify different handwritten signatures. In this paper, we are investigating the utility of several emerging techniques to extract additional information. These methods include utilizing fractal features to enhance pattern discrimination, central projection transformation to describe the shape of a region, and wavelet features to aid in segmentation and boundary identification. Use of these new techniques has yielded very promising results.

Fractal is a new discipline in the field of mathematics, which has been developed in last 20 years and is one of the most important scientific discoveries in this century. Fractals combined with the other nonlinear science theories bring us the dawn of scientific revolution. It proposes a powerful tool for human being to explore the complexity. Therefore, there are more and more researches focusing on the theoretic and applied

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investigations of this new subject. Since Mandelbrot [1] proposed this technique, the subject of fractal dimension has drawn a great deal of attention from mathematicians, physicists, chemists, biologists, geologists, and electrical and computer engineers in various disciplines. Specifically, in the area of pattern recognition and image processing, the fractal dimension has been used for image compression, texture segmentation and feature extraction [2–5], etc. The process of pattern recognition requires the extraction of features from regions of the image, and the processing of these features with a pattern classification algorithm. Many applications of fractal concepts rely on the ability to estimate the fractal dimension of objects. One of the basic characteristics of a fractal is its dimension. A fractal object can be characterized by its dimension which is a way of interpretation, determines how much "space" it occupies between arbitrary m and $m+1$ dimensional manifolds. This feature has been used in texture classification, segmentation, shape analysis and other problems [6–8]. All these approaches face and attempt to solve two basic problems: (1) accuracy of the estimate, (2) amount of information needed to obtain the estimate. In this paper, we utilize a method called " box counting dimension" to estimate the fractal dimension of an image. All of these features rely on computing the mass probability density function based on the central projection transformation and wavelet decomposition for the fractal set. We analyze the box counting methodology from the standpoint of computing fractal features. We focus on fractal dimension for computing the fractal signature. This concept can be useful in the measurement and classification of pattern's features.

2. Basic theory and algorithm

Fractal theory is based on geometry and dimension theories. Fractals are mathematical sets with a high degree of geometrical complexity, which can model many classes of time series data as well as images. The fractal dimension is an important characteristic of fractals because it contains information about their geometric structure. It has become an effective tool to study complex sets. There are many definitions for the fractal dimensions of a fractal set [9,10]. The simplest and most appealing way of assigning a dimension to a set that can yield a fractal dimension to certain kinds of sets is the so-called box dimension. In the section, the important concept and algorithm about box dimension and modified algorithm of fractal dimension will be introduced.

2.1. Fractal dimension

Fundamental to most definitions of dimension is the idea of measurement at scale δ. For each δ, a set can be measured in a way that ignores irregularities of size less than δ, and we see how these measurements behave as $\delta \to 0$.

Suppose F is a plane curve, the measurement $M_\delta(F)$ denotes the number of sets (with length δ) which divide the set F. A dimension of F is determined by the power law obeyed by $M_\delta(F)$ as $\delta \to 0$. If

$$M_\delta(F) \sim \mathscr{K}\delta^{-s}, \tag{1}$$

for constants $\mathscr{K}$ and s, we might say that F has dimension s, and $\mathscr{K}$ can be considered as "s-dimensional length" of F.

Taking the logarithm of both sides in Eq. (1) yields the formula:

$$\log_2 M_\delta(F) \simeq \log_2 \mathscr{K} - s\log_2 \delta,$$

in the sense that the difference of the two sides tends to 0 with δ, this suggests that the box dimension of F, denoted by $\dim(F)$, should satisfy

$$\dim(F) = \lim_{\delta \to 0} \frac{\log_2 M_\delta(F)}{-\log_2 \delta}. \tag{2}$$

If the limit exists, $\dim(F)$ is called the fractal dimension of set F.

2.2. Box counting dimension (BCD)

Box computing dimension or box dimension is one of the most widely used dimensions. Its popularity is largely due to its relative ease of mathematical calculation and empirical estimation.

Let F be a non-empty and bounded subset of $\mathbb{R}^n$, $\xi = \{\omega_i\colon i = 1, 2, 3, \ldots\}$ be covers of the set F. $N_\delta(F)$ denotes the number of covers, such that

$$N_\delta(F) = |\xi\colon d_i \leqslant \delta|,$$

where d_i stands for the diameter of the ith cover. This equation means that $N_\delta(F)$ is the smallest number of subsets which cover the set F, and their diameters d_i's are not greater than δ.

The upper and lower bounds of the box computing dimension of F can be defined by the following formulas:

$$\underline{\dim}_B F = \underline{\lim}_{\delta \to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta},$$

$$\overline{\dim}_B F = \overline{\lim}_{\delta \to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta}, \tag{3}$$

where the overline stands for the upper bound of dimension while the underline for lower bound.

If both the upper bound $\underline{\dim}_B F$ and the lower bound $\overline{\dim}_B F$ are equal, i.e.

$$\underline{\lim}_{\delta \to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta} = \overline{\lim}_{\delta \to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta},$$

the common value is called *box computing dimension* or *box dimension* of F, namely

$$\dim_B F = \lim_{\delta\to 0} \frac{\log_2 N_\delta(F)}{-\log_2 \delta}. \tag{4}$$

Further discussions on fractal theory can be found in Refs. [11,12,10].

The modified box computing dimension method gives a very good estimate of fractal dimension. It can be easily shown that computation complexity of other approaches, including the original box counting dimension, is much higher than that of this approach. Thus it has the advantages of simplicity in computation and improvement in efficiency.

2.3. Minkowski dimension and modified fractal signature

There is an important equivalent definition of box counting dimension of a rather different form that is the δ-parallel body F_δ of F.

$$F_\delta = \{x \in R^n: |x - y| \leqslant \delta, \text{ for } y \in F\}. \tag{5}$$

If F is a subset of R^n and, for some d, if the limit of $[Vol^n(F_\delta)]/\delta^{n-d}$ tends to be positive and finite as $\delta \to 0$, then it makes sense to regard F as d-dimension. The limiting value is called the Minkowski dimension of F, named $\dim_M F$. $Vol^n(F_\delta)$ is Lebesgue measure of F. Even if this limit does not exist, we may be able to extract the critical exponent of δ and this turns out to be related to the box dimension. The initial and key step for Minkowski dimension is to evaluate the $Vol(F_\delta)$ under different δ.

The relationship between the box counting dimension and Minkowski dimension can be provided by the following equation:

$$\underline{\dim_B} F = n - \overline{\lim_{\delta\to 0}} \frac{\log_2 Vol^n(F_\delta)}{-\log_2 \delta},$$

$$\overline{\dim_B} F = n - \underline{\lim_{\delta\to 0}} \frac{\log_2 Vol^n(F_\delta)}{-\log_2 \delta}, \tag{6}$$

where F_δ stands for δ-parallel body of F, and $Vol^n(F_\delta)$ denotes n-dimension area or volume of F_δ. For a nonempty and bounded set F in R^n, we have $\dim_M F = \dim_B F$.

Let $F = \{X_{i,j}\}$, $i = 0,1,\ldots,K$, $j = 0,1,\ldots,L$ be the image of a pattern with multi-gray level, and $X_{i,j}$ be the gray level of the (i,j)th pixel. In a certain measure range, the gray level surface of F can be viewed as a fractal. The surface area can be used to approximate its fractal dimension.

In particular, the gray level function F is a nonempty and bounded set in R^3 for either text areas, graphics areas or background areas in the pattern. We use a technique that is referred to as *Blanket Technique* [13] to thicken the function F. It leads to a set F_δ, which is still a nonempty and bounded set in R^3. According to the definition of *Minkowski Dimension* and Ref. [5], we can conclude that if

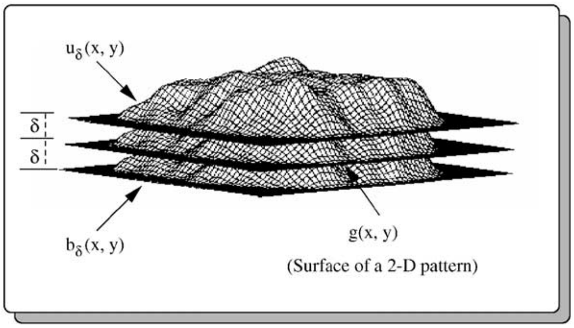

Fig. 1. Surface and its blankets.

$$\lim_{\delta\to 0} \frac{Vol^3(F_\delta)}{\delta^{3-D}} = \beta > 0,$$

then

$$D = \dim_M F = \dim_B F,$$

where β denotes a constant, $Vol^3(F_\delta)$ stands for the volume of the blanket F_δ.

The idea of the blanket technique is based on the equivalent definition of the box computing dimension shown in Eq. (5), i.e. the δ-parallel body. In the blanket technique, all points of the three-dimensional space at distance δ from the gray level surface are considered. These points construct a "blanket" of thickness 2δ covering this surface. A graphical illustration is shown in Fig. 1. The image is represented by a gray-level function $g(i,j)$. The covering blanket is defined by its upper surface $u_\delta(i,j)$ and its lower surface $b_\delta(i,j)$.

- Initially, $\delta = 0$ and given the gray-level function equals the upper and lower surfaces, namely: $g(i,j) = u_0(i,j) = b_0(i,j)$.
- For $\delta = 1,2,\ldots$, the blanket surfaces are defined as follows:

$$u_\delta(i,j) = \max\left\{u_{\delta-1}(i,j) + 1, \max_{|(m,n)-(i,j)|\leqslant 1} u_{\delta-1}(m,n)\right\}, \tag{7}$$

$$b_\delta(i,j) = \min\left\{b_{\delta-1}(i,j) - 1, \min_{|(m,n)-(i,j)|\leqslant 1} b_{\delta-1}(m,n)\right\}. \tag{8}$$

The image pixels (m,n) with distance less than one from (i,j) are taken to be the four immediate neighbors of (i,j). Similar expressions exist when the eight-neighborhood is desired. A point $f(x,y)$ will be included in the blanket for δ when $b_\delta(x,y) < f(x,y) < u_\delta(x,y)$. The blanket definition uses the fact that the blanket of the surface for radius δ includes all the points of the blanket for radius $\delta - 1$, together with all the points within radius 1 from the surfaces of that blanket. Eq. (7) ensures that the new upper surface u_δ is higher by at least 1 from $u_{\delta-1}$, and also at a distance at least 1 from $u_{\delta-1}$ in the horizontal and vertical directions.

The volume Vol_δ of the blanket is computed from u_δ and b_δ:

$$Vol_\delta = \sum_{i,j} (u_\delta(i,j) - b_\delta(i,j)). \tag{9}$$

As the volume Vol_δ of the blanket is measured with radius δ, the area of a fractal surface can be deduced as

$$A_\delta = \frac{Vol_\delta}{2\delta} \quad \text{or} \quad A_\delta = \frac{Vol_\delta - Vol_{\delta-1}}{2}. \tag{10}$$

- Let $A(\delta)$ be the area of the surface of the blanket. Based on the Minkowski dimension, we have

$$A(\delta) \approx \beta\delta^{2-D}, \quad \text{if } \delta \text{ is sufficiently small,} \tag{11}$$

where β denotes a constant and D stands for the fractal dimension of the image.

Since the dimension can be regarded as a slope on a log–log scale, only two points are needed to get the dimension. We use two values of δ to compute the fractal dimension, namely, we take $\delta = \delta_1$ and δ_2, then $A_{\delta_1} \approx \beta\delta_1^{2-D}$ and $A_{\delta_2} \approx \beta\delta_2^{2-D}$. Thus, we have $A_{\delta_1}/A_{\delta_2} \approx \delta_1^{2-D}/\delta_2^{2-D}$.

Taking the logarithm of both sides yields

$$2 - D \approx \frac{\log_2 A_{\delta_1} - \log_2 A_{\delta_2}}{\log_2 \delta_1 - \log_2 \delta_2},$$

$$D \approx 2 - \frac{\log_2 A_{\delta_1} - \log_2 A_{\delta_2}}{\log_2 \delta_1 - \log_2 \delta_2}. \tag{12}$$

It is used to approximate the fractal dimension of the signature images, i.e.,

$$D \approx 2 - \frac{\log_2 A(\delta)}{\log_2 \delta} \quad \text{if } \delta \text{ is sufficiently small.} \tag{13}$$

We call D as modified fractal signature of set F. It will be used to distinguish the different handwritten signatures.

Several points are worth noting:

1. Why do the different document images have different fractal dimensions? The essential distinction of handwritten signature images is their values of $A(\delta)$.
2. The value of $A(\delta)$ depends on the volume $Vol^3(F_\delta)$ of thickened blanket F_δ only.
3. In summary, they can be represented as

$$D \;\Leftrightarrow\; A(\delta) \;\Leftrightarrow\; Vol^3(F_\delta).$$

Consequently, in this paper, the volume $Vol^3(F_\delta)$ of the thickened blanket F_δ is applied to identify different handwritten signature, instead of using the fractal dimension. We call such technique of approximating the fractal dimension "Modified Fractal Signature".

3. Applications

From the specific point of view of shape characterization of patterns the main aim of the fractal approach is to find a "measure" to distinguish between curves with different, often very complicated, contours. The main idea is to describe the complexity of the curve through a new parameter, the fractal dimension, so as to fill in the gap between one- and two-dimensional objects (for objects on a plane). Two applications of the fractal theory are presented in this section, namely: (1) the fractal technique is used to extract the features for two-dimensional objects; (2) the fractal signature is employed to identify different handwritten signatures.

3.1. *Application of fractal technique for feature extraction*

Feature extraction is a crucial step in pattern recognition. It is responsible for measuring the features of the objects in an image. Pattern recognition requires the extraction of features from the regions of an image, and the processing of these features with a pattern classification technique. We employed a central projection method to reduce the problem of two-dimensional patterns into that of one-dimensional ones, and thereafter, utilize the well-established one-dimensional wavelet transform coupled with fractal theory to extract the one-dimensional pattern's feature vectors for the purpose of pattern recognition. The key steps of the experimental procedure consist of the following:

Step 1. *Central projection of two-dimensional patterns*: We denote each of the two-dimensional patterns in question by $p(x,y)$. Thus, the central projection of $p(x,y)$ can be expressed as follows:

$$f(x_k) = \sum_{i=0}^{M} p(x_k \cos\theta_i, x_k \sin\theta_i). \tag{14}$$

Step 2. *Wavelet transformation of the one-dimensional patterns*: Let $f(x_k) = c_{j,k}$, where $k = 0,1,\ldots,2N-1$ and

$$V_0 = \{c_{j,0}, c_{j,1}, \ldots, c_{j,2N-1}\}.$$

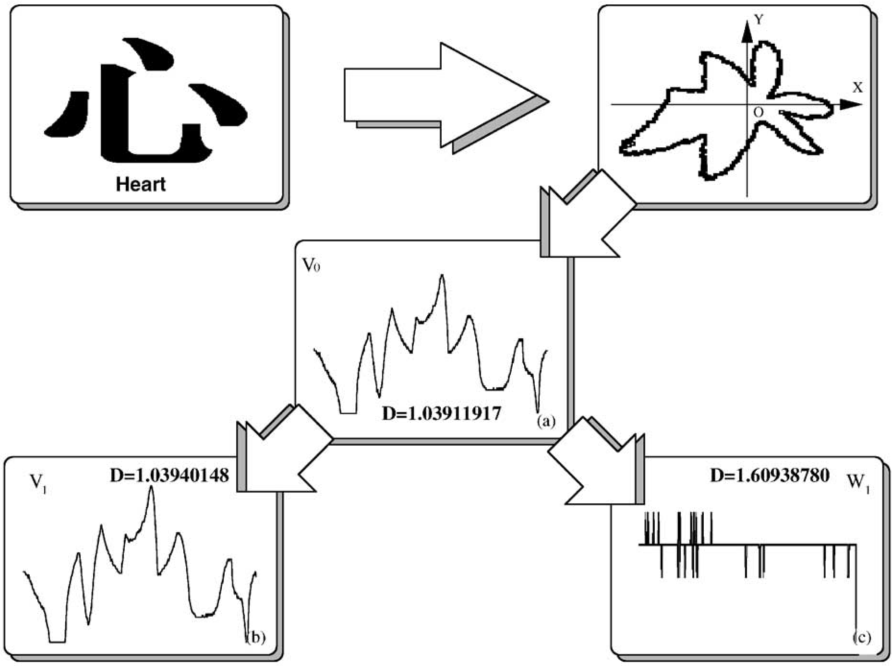

Fig. 2. Diagram of feature extraction by wavelet sub-patterns and divider dimensions for the Chinese character "Heart".

Thus, the expressions for the wavelet transformation of V_0 can be written as follows:

$$c_{j+1,m} = \sum_{k=0}^{5} h_k c_{j,k+2m}; \qquad d_{j+1,m} = \sum_{k=0}^{5} g_k c_{j,k+2m} \tag{15}$$

where $m = 0, 1, \ldots, N-1$.

Hence, the wavelet transformation sub-patterns of V_0 obtained using Eq. (16) will become

$$V_1 = \{c_{j+1,0}, c_{j+1,1}, \ldots, c_{j+1,N-1}\};$$

$$W_1 = \{d_{j+1,0}, d_{j+1,1}, \ldots, d_{j+1,N-1}\}. \tag{16}$$

Step 3. *Computation of divider dimensions for wavelet sub-patterns*: Since the divider dimensions of non-self-intersecting curves are asymptotic values, we can derive their approximations based on the following expression:

$$\frac{\log M_\delta(C)}{-\log \delta},$$

when δ is set to be small enough. $M_\delta(C)$ is the compass dimension computed with a ruler of size δ along the curve C.

In our experiments, we performed three consecutive wavelet transformations for each one-dimensional pattern. Hence, the one-dimensional pattern, such as the central projection of the Chinese character "Heart" (Fig. 2), will yield three non-self-intersecting curves; namely: V_0, V_1, W_1. By comparing these two frequency spectra, we can find that V_0 contains all frequency components of $f(x)$, while V_1 contains only one half of the frequency components of $f(x)$. The reason is that V_0 is divided into two parts: $V_0 = V_1 \oplus W_1$. As for the first part, V_1, only the low-frequency components of V_0 are kept, and the high-frequency components are lost. In addition, only the high-frequency components of V_0 are kept, and the low-frequency components are lost in W_1. For each of the three curves, we further computed its divider dimension, and therefore, related each pattern with a feature vector. These feature vectors are presented as follows:

$$D_{v_o} = 1.03911917, \quad D_{V_1} = 1.03940148,$$

$$D_{W_1} = 1.60938780.$$

These feature vectors are selected to be used as the fractal feature for extracting the features of object. More details about the central projection transformation and the orthonormal wavelet decomposition can be found in Ref. [14].

In our experiments, we have tested our new approach for two types of images: 52 upper and lower case printed alphabets and 3000 Chinese characters. We classify these features and perform the recognition of different types of patterns by using the least distance classification method. For each type of the patterns, the center

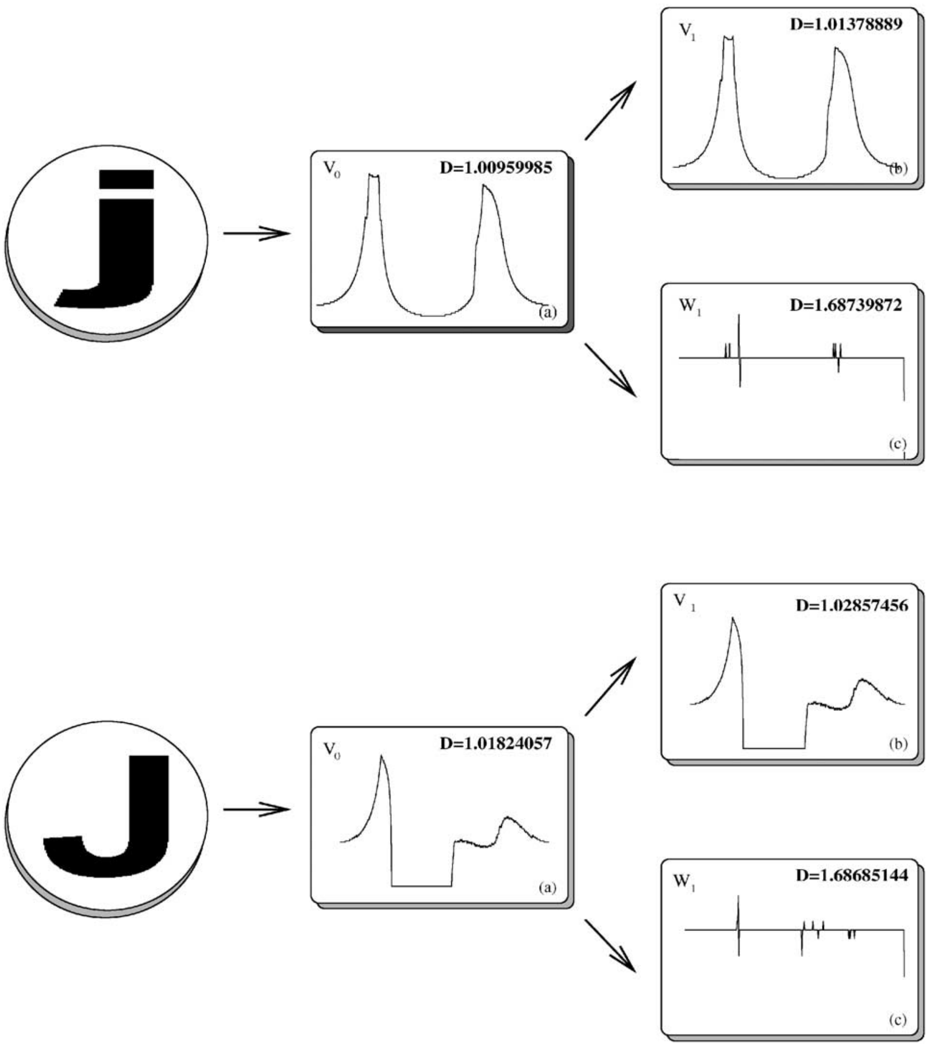

Fig. 3. Diagram of feature extraction by wavelet sub-patterns and divider dimensions for English letters.

signature is calculated. When an unknown pattern is input, at first, its fractal feature should be extracted and through calculating the distances between the signature and other center signatures, one can extract the feature of input pattern whose center signature has the least distance from its fractal feature. For the printed alphanumeric symbols such as Fig. 3, their feature vectors are presented in Tables 1 and 2, where the values in each row indicate the divider dimensions of three sub-patterns. The recognition rates obtained in the 3000 Chinese characters using our new approach is almost 98.48% in Table 3. The results of our experiments show that fractal technique is an efficient tool for analyzing complex patterns, such as Chinese characters and other patterns, as well as distinguishing similar Chinese characters (Fig. 4). The essential advantage of fractal technique descriptor is that it can greatly simplify the processing of multicontour objects and speed up computation.

3.2. Fractal signature and handwritten signature verification

The fractal behavior of handwriting, which was far from being evident, is first shown. This hypothesis being confirmed, as method for estimation of the fractal signature is elaborated. Fractal dimension is the basic parameter of a fractal set, which represents much information related to a signal's geometric features. Mathematically, a fractal dimension is a fraction greater than the topological dimension of a set and remains constant whatever the scale. The more the fractal dimension is close to the

Table 1
Divider dimensions computed for 26 English capital letters

Char.	$D_{V_0}(*)$	$D_{V_1}(*)$	$D_{W_1}(*)$
A	1.02428066	1.01922532	1.68744245
B	1.02257026	1.02645751	1.68744634
C	1.01511397	1.02131134	1.66728915
D	1.01174042	1.01188459	1.68747179
E	1.01595372	1.02556757	1.68742009
F	1.01509414	1.01771325	1.66508823
G	1.01971707	1.02393809	1.68736747
H	1.01308349	1.01330136	1.68743814
I	1.00604717	1.00609545	1.68740190
J	1.01824056	1.02857455	1.68685143
K	1.01951431	1.02323475	1.68740503
L	1.01131929	1.02259282	1.05470158
M	1.03798570	1.05001552	1.67312117
N	1.01758461	1.01944563	1.68742593
O	1.00836156	1.00933081	1.68747074
P	1.01240955	1.01961086	1.68746727
Q	1.01913383	1.02425047	1.68746961
R	1.01994148	1.01933705	1.68744517
S	1.02676442	1.02742447	1.68742514
T	1.00804798	1.00571412	1.68743990
U	1.01419675	1.00763265	1.67220975
V	1.03317001	1.03304412	1.66507245
W	1.03828123	1.02907946	1.68741857
X	1.01881230	1.01740113	1.68740933
Y	1.02730020	1.00695774	1.68730673
Z	1.01652159	1.01952465	1.68742569

Table 2
Divider dimensions computed for 26 lower English characters

Char.	$D_{V_0}(*)$	$D_{V_1}(*)$	$D_{W_1}(*)$
a	1.02336203	1.02459637	1.68746291
b	1.01208646	1.01447766	1.68746858
c	1.01431350	1.00269714	1.66832568
d	1.01432229	1.01465575	1.68742464
e	1.01776391	1.02482181	1.68745870
f	1.01330694	1.01098376	1.68742043
g	1.01715018	1.02665110	1.68743479
h	1.01578535	1.02036016	1.68746704
i	1.00993413	1.00588175	1.68740947
j	1.00959984	1.01378889	1.68739872
k	1.01677571	1.01900263	1.68741976
l	1.00610311	1.00590914	1.68739995
m	1.02466262	1.01742566	1.68744462
n	1.01677789	1.02164941	1.68746612
o	1.00653610	1.00469998	1.68747002
p	1.01376876	1.01504736	1.68746509
q	1.01406060	1.01374885	1.68743489
r	1.01475430	1.01255902	1.68741007
s	1.02708397	1.02936140	1.68742038
t	1.01532360	1.01459309	1.68739523
u	1.01650788	1.01338435	1.67400905
v	1.02522847	1.01855134	1.66508389
w	1.03117814	1.02527709	1.68741442
x	1.02080866	1.02288142	1.68741772
y	1.02701473	1.02071202	1.68738552
z	1.01295483	1.01706887	1.68743641

topological dimension, the more the fractal surface looks smooth. When fractal theory is used to analyse image texture, the fractal dimension represents the roughness of the texture. In this paper, the approach provides a new flexible approach to the classification and verification of handwritten signatures.

The basic idea of the experiment is that an image of a handwritten signature can be mapped onto a gray-level function. Furthermore, this function can be mapped on to a surface which can be used to approximate its fractal dimension. In order to extract the structure information of the handwritten signature, *fractal signature* will be used. From the definition of the fractal signature, i.e. Eq. (13), it is clear that the fractal signature is completely determined by the area of the surface. The surface is a mapping of the gray-level function which represents the handwritten signature image. Consequently, the fractal signature reflects certain characteristics of the image of the handwritten signature. In this experiment, the images of the handwritten signature are scanned with a resolution of 600 dpi. Examples of the image of the handwritten signature are shown in Fig. 5 with two samples.

Step 1. For $x = 1$ to $X_{\max}$ do
For $y = 1$ to $Y_{\max}$ do
F is mapped onto a gray-level function $g_k(x, y)$;

Table 3
Recognition results of the 3000 printed Chinese characters

Number of patterns	1000	2000	3000
Error rate (%)	0.48	0.50	0.58
Rejection rate (%)	0.52	0.69	0.94
Recognition rate (%)	99.00	98.81	98.48

Step 2. For $x = 1$ to $X_{\max}$ do
For $y = 1$ to $Y_{\max}$ do

Substep 1. Initially, taking $\delta = 0$, the upper layer $u_0^k(x, y)$ and lower layer $b_0^k(x, y)$ of the blanket are chosen as the same as the gray-level function $g_k(x, y)$, namely:

$$u_0^k(x, y) = b_0^k(x, y) = g_k(x, y);$$

Substep 2. Taking $\delta = \delta_1$,

(a) $u_{\delta_1}(x, y)$ is computed according to Eq. (7), i.e.

$$u_{\delta_1}(x, y) = \max\left\{u_0(x, y) + 1, \max_{|(i,j)-(x,y)| \leqslant 1} u_0(i, j)\right\};$$

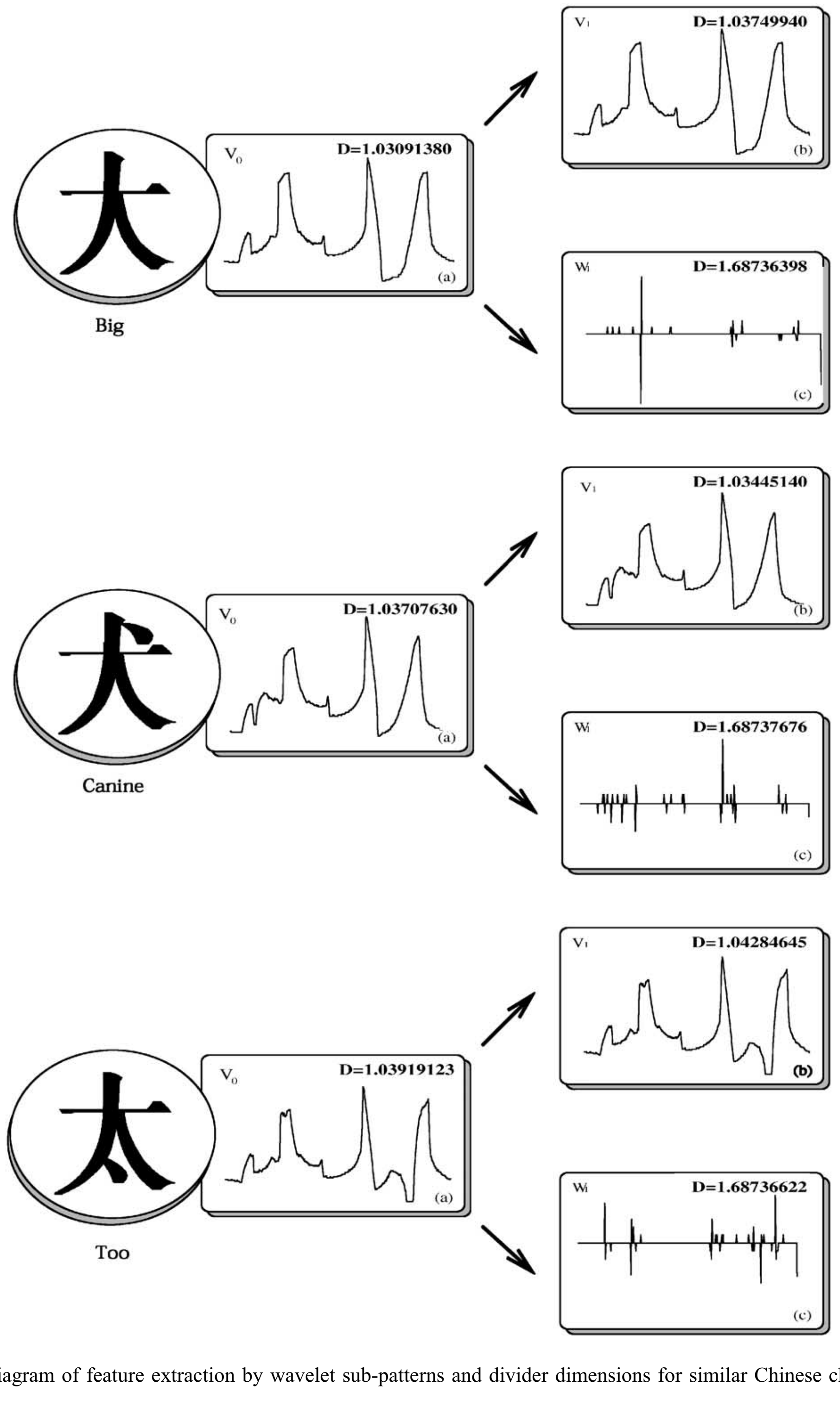

Fig. 4. Diagram of feature extraction by wavelet sub-patterns and divider dimensions for similar Chinese characters.

Fig. 5. Examples of signatures: (a) Six examples of A person's signature, (b) Six examples of another B person's signature.

(b) $b_{\delta_1}(x, y)$ is computed according to Eq. (8), i.e.

$$b_{\delta_1}(x, y) = \min\left\{b_0(x, y) - 1, \min_{|(i,j)-(x,y)| \leqslant 1} b_0(i, j)\right\};$$

(c) The volume Vol_{δ_1} of the blanket is computed by Eq. (9), i.e.

$$Vol_{\delta_1} = \sum_{x,y} (u_{\delta_1}(x, y) - b_{\delta_1}(x, y));$$

Substep 3. Taking $\delta = \delta_2$,

(a) $u_{\delta_2}(x, y)$ is computed according to

$$u_{\delta_2}(x, y) = \max\left\{u_{\delta_1}(x, y) + 1, \max_{|(i,j)-(x,y)| \leqslant 1} u_{\delta_1}(i, j)\right\};$$

(b) $b_{\delta_2}(x, y)$ is computed according to

$$b_{\delta_2}(x, y) = \min\left\{b_{\delta_1}(x, y) - 1, \min_{|(i,j)-(x,y)| \leqslant 1} b_{\delta_1}(i, j)\right\};$$

(c) The volume Vol_{δ_1} of the blanket is computed by

$$Vol_{\delta_2} = \sum_{x,y} (u_{\delta_2}(x, y) - b_{\delta_2}(x, y));$$

Step 3. The sub fractal signature A_δ^k is computed by Eq. (10), namely

$$A_\delta^k = \frac{Vol_{\delta_2} - Vol_{\delta_1}}{2}.$$

where k is a constant, and it is called the scale of fractal signature.

Step 4. Combining sub fractal signatures A_δ^k, $k = 1, 2, \ldots, n$, into the whole fractal signature:

$$A_\delta = \bigcup_{k=1}^{n} A_\delta^k,$$

we can obtain the results of classification of two different handwritten signatures from the fractal signature trend in Fig. 6. We can see that the fractal signatures of the two different handwritten signatures are clearly differentiated while the scale $k > 4$. In our experiments, 20 persons' signatures were trained and tested. These people's signatures are embodied six modes. Using this multiple-model, the technique can offer better classification results. The experimental results indicate that this method is effective and reliable.

4. Conclusions

Pattern recognition requires the extraction of features from regions of the image, and the processing of these features with a pattern recognition algorithm. In this work, we presented the results which aimed at showing that, within the field of image analysis, it is possible to use fractal feature based on the estimating fractal dimension to extract information that is relevant in object recognition tasks. The motivation behind using fractal transformation is to develop a high-speed feature extraction technique. A multiresolution family of the wavelets is also used to compute information conserving micro-features. Along the way, we have shown that

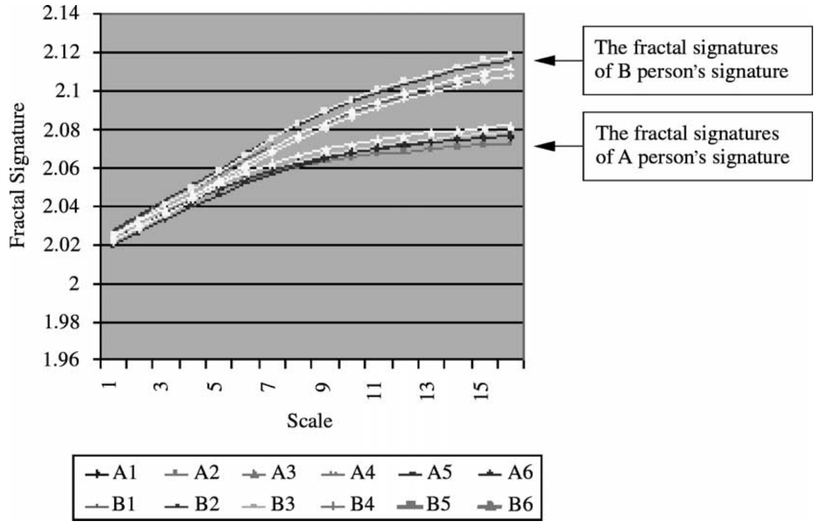

Fig. 6. An example of handwritten signature verification.

dimension tells us much about geometrical properties, such as the projection of sets, and we have seen something of the problems associated with calculating the fractal dimensions. Clearly, dimensions are intimately related to the study of fractal geometry. However, the experiment results show that this approach allows us to obtain new and interesting descriptions of complex patterns. In some situations, it has even already yielded better results than "classical" methods. For all the cases, differences in fractal dimension can be found the significative values. We expect that the proposed fractal method can also be used for improving the extraction and classification of features in a pattern recognition system.

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