MBA 524 (621) SPRG 2021: Quantitative Analysis Project

Abhijay Shukla, Adarsh Choubey, Edward Parrilla, Mehul Mehta

May 9th, 2021

Part I. Introduction

Defining Market Efficiency:

Market efficiency refers to the degree to which market prices reflect all available, relevant information. If markets are efficient, then all information is already incorporated into prices, and so there is no way to "beat" the market because there are no undervalued or overvalued securities available.

Defining Efficient Market Hypothesis:

According to the EMH, stocks always trade at their fair value on exchanges, making it impossible for investors to sell overvalued or purchase undervalued stocks. Therefore, it is impossible to outperform the overall market through market timing or expert stock selection, and the only way an investor can achieve higher returns is by purchasing riskier investments.

Defining Anomaly

In financial markets, anomalies are referred to a situation where a security or group of securities performs contrary to the notion of efficient markets. With the rapid dissemination and constant release of information, sometimes it is difficult to achieve efficient markets. Hence, there are different market anomalies that occur once and disappear, while others are continuous in nature.

In finance, two common types of anomalies are **pricing anomalies** and **market anomalies**. Pricing anomalies are when stocks are priced differently than how a model predicts it will be priced. Market anomalies are distortions in returns that contradict the efficient market hypothesis (EMH).

Procedure to Test an Anomaly

The below five steps can be used to test an anomaly in the market.

1. **Formation:** In this step, the stocks are sorted into 5 different portfolios based on their past month returns.

2. **Holding:** The stocks are held for a period of 6 or 12 months.

3. **Rebalance:** You must follow steps 1 and 2 again and reform your portfolios systematically over time. This is called **Rebalancing**. Rebalancing could be done after the holding period is over.

4. **Resulting Data:** You should have a time series of returns for each of the portfolios. Create a Long/short portfolio that is long good stocks and short the bad stocks (P1-P5)

5. **Analysis:** Examine average returns and alphas

Interpret the evidence of Stock Return Patterns like Value, Momentum, and Gross Profit within the context of market efficiency

Anomalies such as Value, Momentum, and Gross Profit occur because the models core assumptions of market efficiency get deviated. Interpreting the reasons behind these anomalies:

a) **Value Anomaly:** Value Anomaly arises because of 2 reasons i.e., behavioural and distress story.

   i) **Behavioral Anomaly:** People "like" growth companies and "dislike" value stocks. This leads to premium.

   ii) **One possible story for Distress** is that Value companies are less flexible (since there are high assets in place) than growth companies (high PVGO), so when times are tough value stocks do very poorly (i.e., there is a non-linear relationship with market)

b) **Momentum Anomaly:** The investors' behavioral characteristics, overreaction and the liquidity issues associated with the asset, leads the recent past winners –keep winning, losers keep losing for about 3-12 months, causing the momentum anomaly.

c) **Gross Profit Anomaly:** The impact of the prospective future dilution is ignored for the less profitable firms which leads to their overvaluation. (Explaining the Profitability Anomaly by Ryan Erhard and Richard G. Sloan June 2020)

Summarizing "Value and Momentum Everywhere" (Asness et al., 2013)

The paper *'Value and Momentum Everywhere'* found a consistent return premium when the value and momentum return strategy was used across different asset classes and markets. Value and momentum return correlate strongly across the asset classes and are noted to be **negatively correlated** with each other, both within and across asset classes.

Studying value and momentum jointly is more powerful than examining each in isolation. The negative correlation between value and momentum strategies and their high positive expected returns implies that a simple combination of the two is much closer to the efficient frontier than either strategy alone, and exhibits less variation across markets and over time.

The paper identifies the Global funding Liquidity Risk as a partial source of value and momentum anomalies, which are identifiable only when examining value and momentum jointly across markets. Other sources include behavioral like emotional attachment to the growth firms, over exuberance towards the momentum stocks etc.

Summarizing "The other side of value: The gross profitability premium" (Novy-Marx, 2013)

The paper *"The other side of value: The gross profitability premium"* measures Profitability, as the ratio of a firm's gross profits (revenues minus cost of goods sold) to its assets, which is also equivalent to the book-to-market (B/M), predicting the average returns. Gross profits-to-assets is also complimentary to book-to-market, but contributes economically significant information across various sectors over the B/M value.

Common wisdom states that firms for which investors require high rates of return are priced lower and, consequently, have higher book-to-markets than firms for which investors require lower returns. Because valuation ratios help identify variation in expected returns, with higher

book-to-markets indicating higher required rates, value firms generate higher average returns than growth firms.

Gross profitability represents the other side of value. The same basic philosophy underpins both the valuation ratios and profitability strategy. Both are designed to identify productive stocks cheaply. Value strategies do this by financing the purchase of inexpensive assets through the sale of expensive assets, and profitability strategies achieve the same end by financing the purchase of productive assets through the sale of unproductive assets. Both strategies generate significant premium over the normal returns.

Results of the Analysis

In this project, quantitative analysis has been attempted for the comparative study amongst the three strategies, 'The Gross Profit', 'The Value Plus Momentum' and 'Investing in Market Portfolio'. Based on the results obtained from our CAPM and the Fama French 3 Factor Models and comparing them with the industry adjusted value, momentum and profitable factors we observed that given our scenarios, **'The Value Plus Momentum' strategy performed the best**. We also conclude that the strategy can be employed for a wide range of different scenarios and it will perform better than both 'The Gross Profit' strategy and 'Investing in Market Portfolio' strategy.

Part II. Testing the Gross Profit Anomaly

Data and Methodology

We now describe our data and methodology for constructing the Gross Profit/Assets portfolios stocks globally across different equity markets. We began by examining the gross profitability of individual stocks in the portfolio. The U.S. stock universe consists of all common equity in CRSP (share codes 10 and 11) with a book value from compustat in the previous 6 months, and at least 12 months of past return history from 1985 to 2016.

Most studies of individual stocks examine a much broader and less liquid set of securities. We limit the universe of stocks to a very liquid set of securities that could be traded for reasonably low cost at reasonable trading volume size. To achieve this, we only use tickers of firms with valuations of more than 1 million in market capitalization. We also omit the penny stocks (lag price less than a dollar) and discard the tickers with missing values to avoid deviations.

The portfolio was tested on the Gross Profit Ratio and then 5 different portfolios were constructed i.e., Portfolio with High Gross Profit to Asset, Portfolio with Low Gross Profit to Asset Total and Long_Short: High_Low Portfolio etc.,. We then test the anomalies and performance of the portfolio by comparing the factors from the Fama and French 3 factor model and CAPM factors model.

Thereafter, out of the 5 portfolio, Gross Profit Long/Short portfolio (GPA) was taken and then merged with the Fama French Factors in order to evaluate and compare the following 3 strategies i.e. **Gross Profit plus Value** (GPA + HML); **Value plus Momentum** (HML + UMD); and Passive investing in the **Market Portfolio** (MKTRF factor).

Comparison of the CAPM and Fama French 3 Factor Model returns results to the results in Novy-Marx's paper

We now consider the performance of our portfolios which are sorted by profitability, and test the hypothesis that *profitability predicts average returns*. 'Gross Profit Effect Factor Regression Results Table' here and the *'Excess Returns to Portfolios Sorted on Profitability Table', from the Novy-Marx's paper,* both show the results of dependence of the univariate factors (alpha and betas) on gross profits-to-assets ratio. The table from the Novy-Marx's paper compares its portfolios' returns with the returns on the Three Fama and French factor model, while our model also goes on to include the CAPM factors.

In the Novy-Marx's paper, we observe a significant profitable minus unprofitable return spread despite the fact that the strategy is a growth strategy, with significant negative values of HML. The abnormal returns of the profitable minus unprofitable return spread (HML) relative to the Fama and French three-factor model is 0.52% per month, with a test-statistic of 4.49. For our 'Gross Profit Strategy', Fama and French three-factor model results are on the same line, with the spread being 0.62% and test-statistic value of 3.96 respectively.

The portfolio results presented in the papers and in our project, both suggest that the gross profits-to-assets ratio has high power to predict the average return. Profitable firms also tend to be growth firms, in the sense that they grow faster. Gross profitability is a powerful predictor of future growth in gross profitability, earnings, free cash flow. Because the profitability strategy is
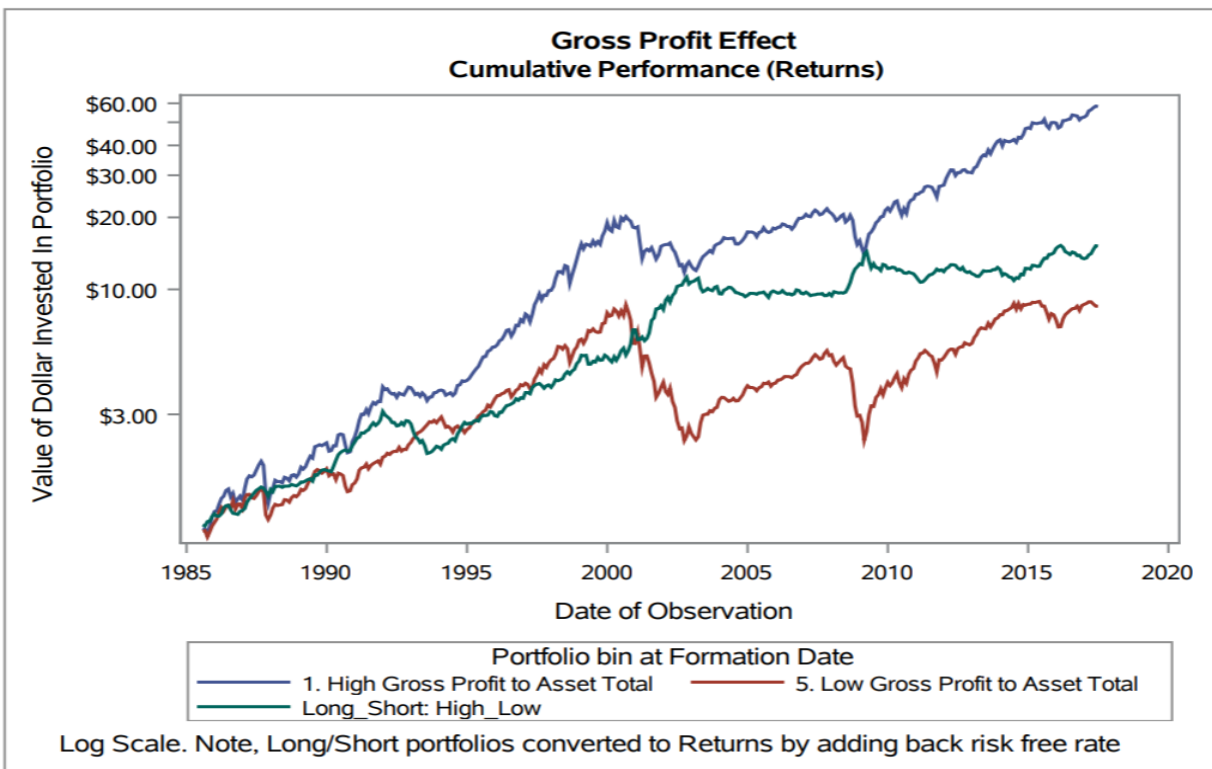
a growth strategy, it generally provides a great hedge for value strategies. **The Novy-Marx's paper finds that an investor running the two strategies together would capture both strategies' returns, 0.71% per month, but would face no additional risk, while our portfolio running a similar strategy would only return 0.275% per month.**

Portfolio	r^e	Alphas and three-factor loadings			
		α	MKT	SMB	HML
Panel A: Portfolios sorted on gross profits-to-assets					
Low	0.31	−0.18	0.94	0.04	0.15
	[1.65]	[−2.54]	[57.7]	[1.57]	[5.87]
2	0.41	−0.11	1.03	−0.07	0.20
	[2.08]	[−1.65]	[67.5]	[−3.13]	[8.51]
3	0.52	0.02	1.02	−0.00	0.12
	[2.60]	[0.27]	[69.9]	[−0.21]	[5.42]
4	0.41	0.05	1.01	0.04	−0.24
	[1.94]	[0.83]	[70.6]	[1.90]	[−11.2]
High	0.62	0.34	0.92	−0.04	−0.29
	[3.12]	[5.01]	[58.3]	[−2.03]	[−12.3]
High–low	0.31	0.52	−0.03	−0.08	−0.44
	[2.49]	[4.49]	[−0.99]	[−2.15]	[−10.8]
Panel B: Portfolios sorted on book-to-market					
Low	0.39	0.13	0.98	−0.09	−0.39
	[1.88]	[2.90]	[90.1]	[−5.62]	[−23.9]
2	0.45	−0.02	0.99	0.05	0.04
	[2.33]	[−0.29]	[78.1]	[2.61]	[2.23]
3	0.56	0.03	0.96	0.04	0.22
	[2.99]	[0.53]	[63.5]	[2.09]	[9.71]
4	0.67	−0.00	0.96	0.10	0.53
	[3.58]	[−0.03]	[74.8]	[5.66]	[27.1]
High	0.80	0.07	1.01	0.25	0.51
	[3.88]	[1.04]	[60.7]	[10.7]	[20.5]
High–low	0.41	−0.06	0.03	0.34	0.91
	[2.95]	[−0.71]	[1.44]	[12.0]	[30.0]

Gross Profit Effect
Factor Regression Results

Obs	bin	_TYPE_	alpha_capm	mktrf_capm	alpha_ff3	mktrf_ff3	smb	hml
1	1. High Gross Profit to Asset Total	PARMS	0.25	0.97	0.32	0.95	-0.12	-0.25
2	1. High Gross Profit to Asset Total	T	2.65	46.15	3.70	47.37	-4.14	-8.12
3	2	PARMS	0.03	1.05	0.12	1.01	-0.07	-0.32
4	2	T	0.27	46.80	1.38	48.85	-2.43	-10.05
5	3	PARMS	0.02	1.05	0.06	1.02	0.04	-0.11
6	3	T	0.29	55.75	0.75	53.80	1.36	-3.80
7	4	PARMS	-0.13	1.03	-0.20	1.07	-0.03	0.22
8	4	T	-1.31	46.88	-2.16	49.85	-1.13	6.75
9	5. Low Gross Profit to Asset Total	PARMS	-0.29	1.04	-0.30	1.04	0.04	0.04
10	5. Low Gross Profit to Asset Total	T	-2.74	44.14	-2.81	42.45	1.16	0.94
11	Long_Short: High_Low	PARMS	0.53	-0.07	0.62	-0.09	-0.16	-0.29
12	Long_Short: High_Low	T	3.34	-2.04	3.96	-2.51	-3.09	-5.14

Gross Profit Effect
Cumulative Performance (Returns)

Portfolio bin at Formation Date
— 1. High Gross Profit to Asset Total   — 5. Low Gross Profit to Asset Total
— Long_Short: High_Low

Log Scale. Note, Long/Short portfolios converted to Returns by adding back risk free rate

Gross Profit Effect
Sharpe Ratio by bin

Portfolio bin at Formation Date	Mean Excess Return	Standard Deviation of Excess Returns	Sharpe Ratio
1. High Gross Profit to Asset Total	0.90%	4.62%	0.19
2	0.73%	4.99%	0.15
3	0.73%	4.88%	0.15
4	0.57%	4.92%	0.12
5. Low Gross Profit to Asset Total	0.41%	5.01%	0.08
Long_Short: High_Low	0.48%	3.11%	0.16

Part III. Comparison of Investment Strategies

Summary of the Descriptive Statistics

The Gross Profit Long/Short portfolio performance is combined with the Fama French Factors to evaluate all the 3 strategies i.e., Gross Profit plus Value , Value plus Momentum and Passive investing in the Market Portfolio.

Summary stats of monthly returns for all 3 strategies

The MEANS Procedure

Portfolio bin at Formation Date	N Obs	Variable	Label	N	Mean	Median	Std Dev	Minimum	Maximum	1st Pctl
Long_Short: High_Low	384	value_momentum	Value plus momentum	384	0.0073031	0.0080000	0.0495532	-0.3038000	0.2509000	-0.1607000
		GPR_Value		384	0.0021539	0.000600000	0.0290198	-0.1112000	0.1258000	-0.0822000
		Market_Portfolio	Gross plus Value Investing_MKTRF	384	0.0067419	0.0115500	0.0440703	-0.2324000	0.1247000	-0.1072000

Portfolio bin at Formation Date	N Obs	Variable	Label	Skewness
Long_Short: High_Low	384	value_momentum	Value plus momentum	-1.0972278
		GPR_Value		0.3071067
		Market_Portfolio	Gross plus Value Investing_MKTRF	-0.9120727

Risk vs Return

The quantitative analysis reflects that the **value plus momentum strategy worked best for the Gross Profit Long/Short portfolio.** The momentum and value strategies are often combined since they tend to perform better at different phases of the market cycle – a characteristic which helps to smooth long-term performance and control volatility risk. Value plus momentum strategy targets superior returns and brings diversification and risk management to a portfolio. The analysis shows that the maximum return for this strategy is 25.09% per month while the worst return for this strategy is -30% per month. This makes sense since higher risk is associated with the higher return and lower risk is associated with a greater probability of smaller return. This is the tradeoff which an investor faces while considering the investment decisions.

Best Strategy for the Long/Short portfolio

The mean statistics indicate that for the Gross Profit Long/Short portfolio, **value plus momentum strategy worked the best**. It gave an average return of **0.73%** per month whereas the Gross Profit Plus Value Strategy and Investing in Market Portfolio did not work that well. Gross Profit plus Value strategy gave an average return of **0.21%** per month**.**

When using 'mean' to understand the descriptive statistics there is one major disadvantage i.e., it is particularly susceptible to the influence of outliers (these are values that are unusual compared to the rest of the data set by being especially small or large in numerical value). That is why it is

better to use median (to understand which strategy worked best) which doesn't get affected by the presence of outliers.

When observing median, it was found that Investing in Market Portfolio strategy was profitable as it gave a monthly return of 1.1% while the other 2 strategies i.e., Value plus momentum and Gross Profit plus Value gave a monthly return of 0.8% and 0.06% respectively.

Volatility of the Strategies

Value plus momentum strategy was the most volatile strategy as the monthly returns fluctuated about 4.95% and Gross Profit plus Value strategy was the 2nd most volatile strategy with a monthly return fluctuation of 2.90%. This makes sense because the higher the volatility, the greater the chances for significantly higher returns. Investors who trade in volatile strategies have a greater opportunity to make bigger profits than those who play it safe and stick to low-volatile strategies.

Returns of the Strategies

The **maximum or best monthly return was for value plus momentum strategy i.e., 25.09%** followed by Gross Profit plus Value strategy (12.58%) and Investing in Market Portfolio had the worst maximum returns (i.e., 12.47%).

Similarly, the **minimum or the lowest monthly return for value plus momentum strategy was as low as -30.38%** whereas the minimum monthly returns in the case of Gross Profit plus Value and Investing in Market Portfolio were -11.11% and -23.24%.

Skewness of the strategies and their relation with risk of the strategies

For skewness, if the value is greater than + 1, the distribution is right skewed. If the value is less than -1.0, the distribution is left skewed. Therefore, value plus momentum and investing in Market Portfolio are found to be left skewed and Gross Profit plus Value strategy is normally distributed (i.e., the skewness is very close to 0, indicating that the distribution of Gross Profit plus Value strategy is not skewed).

Investors use standard deviation to predict future returns of the portfolio, but the standard deviation assumes a normal distribution. The most returns are not normally distributed and that is when skewness comes in picture. Many financial models assume that the future performance of an asset is normally distributed. However, if the data are skewed, this kind of model will always underestimate skewness risk in its predictions. The more skewed the data, the less accurate this financial model will be.

Question 2: Graph the cumulative performance of all 3 strategies on the same line graph.

Briefly describe the graph and table

If $10,000 is invested at the beginning of the time period for each strategy, **Value plus momentum gives the best profits**. Moreover, investing in market strategy gives very close

profits as compared to value plus momentum strategy. Gross profit plus value strategy gives the lowest profits.

Cummulative performance of all 3 strategies
Cumulative Performance
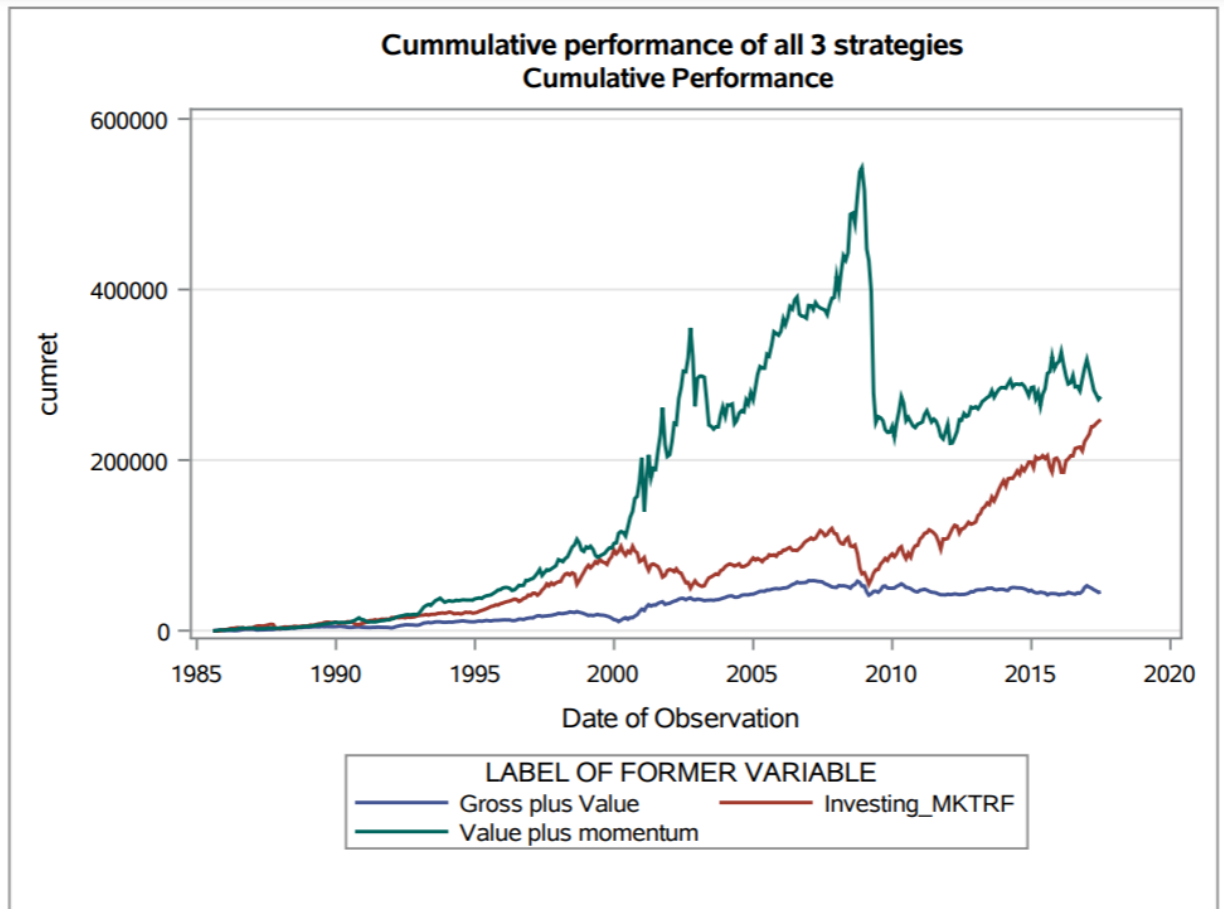
This means that if the investor would have invested $10,000 in 1985, the total investment would be worth approximately $52,806.03 for Gross Profit plus Value), $225,830.89 for Investing in Market Portfolio and $317,877.93 for Value plus momentum in December 2016.

End of the sample period- Portfolio values: Strategy wise

Obs	Strategies	Year	Month (December)	Cummulative Returns($)
1	Gross plus Value	2016	12	52806.025
2	Investing_MKTRF	2016	12	225830.888
3	Value plus momentum	2016	12	317877.929

Defining Sharpe Ratio.

In finance, the Sharpe ratio (also known as the Sharpe index, the Sharpe measure, and the reward-to-variability ratio) measures the performance of an investment (e.g., a security or portfolio) compared to a risk-free asset, after adjusting for its risk. It is defined as the difference between the returns of the investment and the risk-free return, divided by the standard deviation of the investment (i.e., its volatility). It represents the additional amount of return that an investor receives per unit of increase in risk.

Sunday, May

Mean excess return, standard deviation and Sharpe Ratio: Strategy wise

LABEL OF FORMER VARIABLE	Mean Excess Return	Standard Deviation of Excess Returns	Sharpe Ratio
Gross plus Value	0.490%	2.914%	0.168
Investing_MKTRF	0.949%	4.407%	0.215
Value plus momentum	1.005%	4.982%	0.202

Use of Sharpe Ratio in the Investment Analysis

The Sharpe ratio is a well-known and well-reputed measure of risk-adjusted return on an investment or portfolio, developed by the economist William Sharpe. The Sharpe ratio can be used to evaluate the total performance of an aggregate investment portfolio or the performance of an individual stock. The Sharpe ratio indicates how well an equity investment performs in comparison to the rate of return on a risk-free investment, such as U.S. government treasury bonds or bills.

Strategies with the most attractive Sharpe Ratio

Investing in Market Portfolio has the most attractive Sharpe Ratio of 0.22 whereas Gross Profit plus Value strategy has the lowest sharpe ratio. Good sharpe ratio indicates a high degree of expected return for a relatively low amount of risk.

Capital Asset Pricing Model:

The Capital Asset Pricing Model (CAPM) describes the relationship between systematic risk and expected return for assets, particularly stocks. CAPM is widely used throughout finance for pricing risky securities and generating expected returns for assets given the risk of those assets and cost of capital.

Interpretation of Alpha and Beta of CAPM Model:

For Gross plus Value strategy,

❏ Market beta is negative i.e., -0.145. This indicates that the strategy has an inverse relation to the market.

❏ Alpha of 0.59 means that the strategy outperformed its benchmark index by 0.59%.

For Investing in Market strategy,

❏ Market beta is approximately one i.e., 0.999. This means that the strategy price tends to move with the market.

❏ Alpha of 0.275 means that strategy was able to beat the market by 0.275%.

For Value plus momentum strategy,

❏ Market beta is negative. This indicates that the strategy has an inverse relation to the market.

❏ Alpha of 1.23 means that the strategy outperformed its benchmark index by 1.23%.

Factor Regression Results   Sunday, May 9, 202

Obs	bin	_TYPE_	alpha_capm	mktrf_capm	alpha_ff3	mktrf_ff3	smb	hml
1	Gross plus Value	PARMS	0.588	-0.145	0.275	-0.000	-0.008	0.999
2	Gross plus Value	T	3.999	-4.398	24.812	-0.046	-2.174	251.468
3	Investing_MKTRF	PARMS	0.275	0.999	0.275	1.000	-0.008	-0.001
4	Investing_MKTRF	T	24.876	402.150	24.812	389.046	-2.174	-0.288
5	Value plus momentum	PARMS	1.229	-0.333	1.032	-0.247	0.034	0.637
6	Value plus momentum	T	4.995	-6.027	4.483	-4.620	0.446	7.726

Fama French Three-Factor Model:

The Fama and French Three-Factor Model (or the Fama French Model for short) is an asset pricing model developed in 1992 that expands on the capital asset pricing model (CAPM) by adding size risk and value risk factors to the market risk factor in CAPM. This model considers the fact that value and small-cap stocks outperform markets on a regular basis. By including these two additional factors, the model adjusts for this outperforming tendency, which is thought to make it a better tool for evaluating manager performance.

Alpha: Alpha (α) is a term used in investing to describe an investment strategy's ability to beat the market, or it's "edge." Alpha is thus also often referred to as "excess return" or "abnormal rate of return," which refers to the idea that markets are efficient, and so there is no way to systematically earn returns that exceed the broad market as a whole.

Beta: Beta is a measure of the volatility—or systematic risk—of a security or portfolio compared to the market as a whole. Beta is used in the capital asset pricing model (CAPM), which describes the relationship between systematic risk and expected return for assets (usually stocks).

Interpretation of Alpha and Betas of Fama French Three-Factor Model:

The Fama and French Three-Factor model is used to understand the expected relation between profitability and expected return. The below table summarizes the Fama and French factors [the market factor (MKT), the size factor small-minus-large (SMB), and the value factor high-minus-low (HML)], with test-statistics.

For Gross plus Value strategy,

❏ Market beta is zero. A zero-beta portfolio is a portfolio constructed to have zero systematic risk, or in other words, a beta of zero. A zero-beta portfolio would have the same expected return as the risk-free rate.

❏ Moreover, Small Minus Big (SMB) is negative which indicates that the strategy worked for big caps stocks as compared to small caps stocks.

❏ Positive Beta HML means that the strategy worked well for a portfolio consisting of value stock companies.

❏ Gross Profit plus Value strategy was able to beat the market by 0.275%.

For Investing in Market strategy,

❏ Market beta is one. This means that the strategy price tends to move with the market.

❏ Moreover, Small Minus Big (SMB) is negative which indicates that the strategy worked for big caps stocks as compared to small caps stocks.

❏ Negative Beta HML means that the strategy worked well for a portfolio consisting of growth stock companies.

❏ Investing in Market Portfolio strategy were able to beat the market by 0.275%.

For Value plus momentum strategy,

❏ Market beta is negative. This indicates that the strategy has an inverse relation to the market.

❏ Moreover, Small Minus Big (SMB) is positive which indicates that the strategy worked for small caps stocks as compared to large caps stocks.

❏ Positive Beta HML means that the strategy worked well for a portfolio consisting of value stock companies.

❏ Value plus momentum strategy performed the best and was able to beat the market with an alpha of 1.032.

The regression models give the coefficient and the T statistics for all the strategies. The t statistic is the coefficient divided by its standard error. The standard error is an estimate of the standard deviation of the coefficient, the amount it varies across cases. Therefore, t statistic can be thought

of as a measure of the precision with which the regression coefficient is measured. Therefore, the positive the t-statistics, the more significant the strategies. In the analysis, it was found that all the 3 strategies have positive t value which indicates that they all are significant.

Reasoning for why the results of the market are different from the other two strategies.

The differences in the strategies can be explained by the following ways:

❏ Unlike CAPM which is a single factor model based on the relationship between returns and market factor, the Fama-French model is based on stock return having its basis in not one but three separate risk factors: market, size and value or book to market based factor. Therefore, it gives more accurate results than the CAPM model.

❏ CAPM is built on four major assumptions, including one that reflects an unrealistic real-world picture. This assumption—that investors can borrow and lend at a risk-free rate—is unattainable in reality. Individual investors are unable to borrow (or lend) at the same rate as the U.S. government. Therefore, the minimum required return line might actually be less steep (provide a lower return) than the model calculates.

Annual Performance of Each Strategy:

Historically it has been observed that strategy "Gross profit plus Value" performed in line with the strategy "Value plus Momentum". Although due to higher volatility of the latter strategy, its returns are likely to be higher than the former strategy. The passive investment market strategy has always underperformed among all. But after the 2007-08 financial crisis, when compared to the other two investment strategies, annual returns are slightly higher for passive investment strategies. Moreover, fluctuations in returns are slightly stable during the years after a financial crisis.

Further, the stock market has performed worse in the early years of 2000s. For example, many growth stocks and US large caps were at record low of -30% & -12 % respectively. Here in the table for annual returns, we can see that the investors who invested in one of the active market strategies i.e., either Gross profit plus value or Value plus momentum, they would have marked a profit in their portfolio. On the contrary, the passive market strategy didn't work well during that time as it showed a negative return in the portfolio.

Annual Returns: Strategy wise

year	Returns: Gross plus value	Returns: Value plus momentum	Returns: Investing_mktrf
1985	-1.761%	1.543%	7.910%
1986	7.783%	17.008%	9.580%
1987	-4.229%	-4.070%	-3.747%
1988	12.596%	6.747%	10.898%
1989	-3.572%	21.953%	19.049%
1990	-11.948%	6.777%	-13.043%
1991	-10.530%	-1.752%	27.784%
1992	22.625%	25.989%	6.022%
1993	15.508%	40.459%	7.995%
1994	-1.242%	2.432%	-3.953%
1995	4.573%	19.918%	29.718%
1996	5.814%	11.742%	15.232%
1997	15.139%	25.973%	24.783%
1998	-9.443%	10.612%	18.621%
1999	-24.758%	-0.767%	19.714%
2000	46.330%	79.329%	-16.700%
2001	15.056%	-2.216%	-14.690%
2002	9.989%	39.119%	-22.418%
2003	2.864%	-15.435%	30.471%
2004	6.601%	6.185%	10.631%
2005	9.078%	24.890%	2.998%
2006	10.660%	3.424%	10.159%
2007	-15.761%	3.913%	0.976%
2008	1.326%	21.597%	-37.776%
2009	-4.326%	-52.575%	28.235%
2010	-3.970%	1.405%	17.354%
2011	-8.164%	-0.537%	0.435%
2012	8.666%	8.108%	16.253%
2013	2.059%	8.244%	35.179%
2014	-1.799%	-0.045%	11.708%
2015	-9.778%	10.501%	0.079%
2016	20.728%	0.385%	13.266%

Best and Worst five Annual Returns for each strategy

Now among the best and worst performing strategies, we can articulate that when the market was performing at its best, the Value plus momentum resulted in the highest returns at each rank. Conversely, when the market was at low or performed badly, the value plus momentum strategy performs best, as it gives fewer negative returns when compared to other two strategies.

5 best annual returns for each strategy — Sunday, May 9, 2021

| Obs	Gross plus value	Ranking_Gross plus value	Value plus Momentum	Ranking_Value plus Momentum	Investing_mktrf	Ranking_Investing_mktrf
1	22.625%	2	25.989%	4	27.784%	5
2	15.508%	4	40.459%	2	29.718%	3
3	15.139%	5	25.973%	5	30.471%	2
4	46.330%	1	79.329%	1	28.235%	4
5	20.728%	3	39.119%	3	35.179%	1

5 worst annual returns for each strategy — Sunday, May 9, 2021

Obs	Gross plus value	Ranking_Gross plus value	Value plus Momentum	Ranking_Value plus Momentum	Investing_mktrf	Ranking_Investing_mktrf
1	-11.948%	30	-4.070%	30	-13.043%	28
2	-10.530%	29	-1.752%	28	-16.700%	30
3	-24.758%	32	-2.216%	29	-14.690%	29
4	-15.761%	31	-15.435%	31	-22.418%	31
5	-9.778%	28	-52.575%	32	-37.776%	32

Bar graph of the 5 best and worst Annual Returns of any of the 3 strategies.

In the bar graph, we have plotted 5 best and worst performance annual returns for "Gross profit plus Value" strategy, where the best return was marked at ~46% and the worst return was ~ -25%.
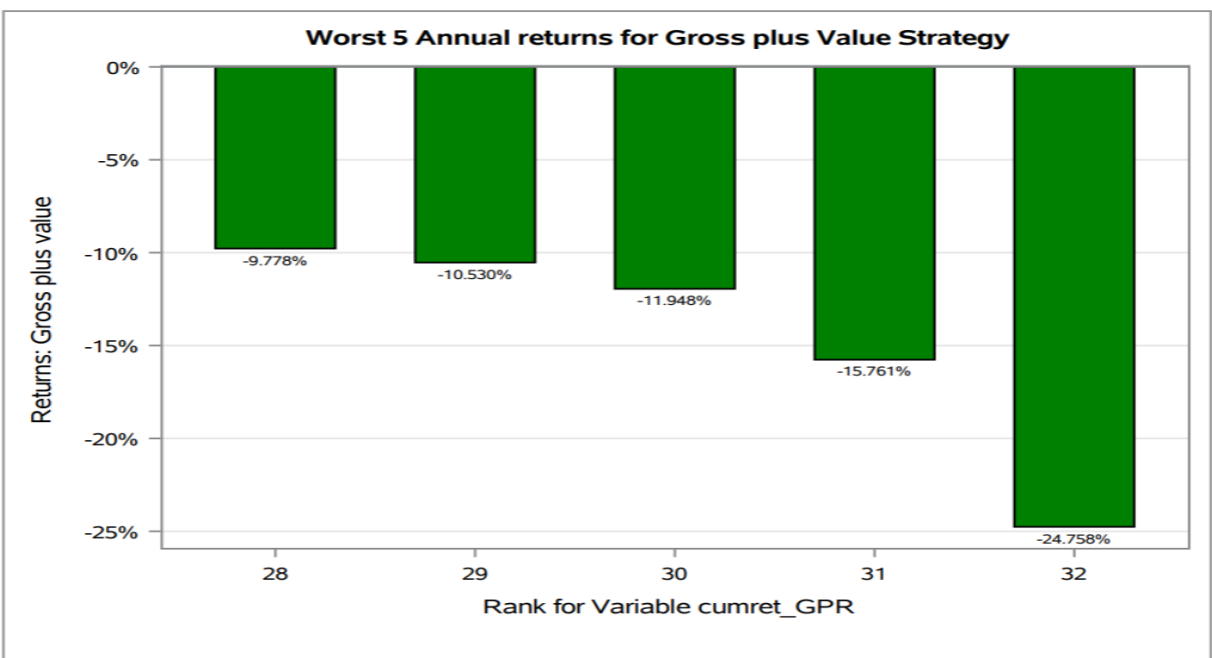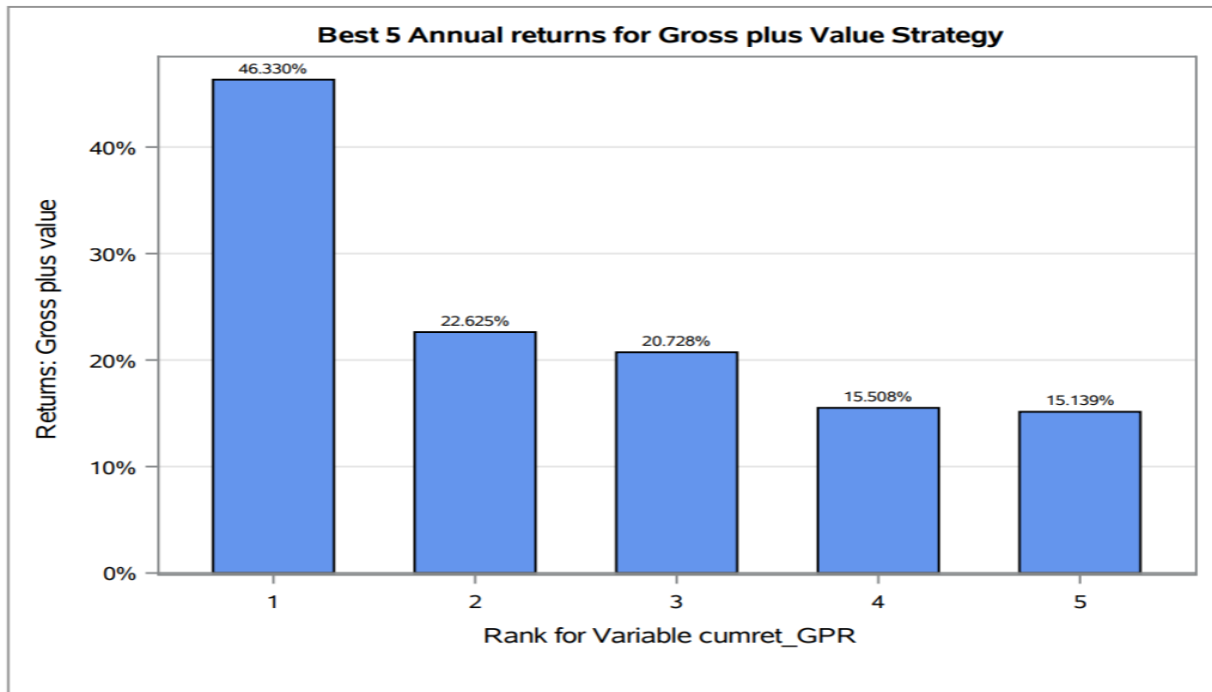
Likewise, for each of the strategies, we have checked the percentage of months and years with positive returns from 1985-2016. Here, we have analyzed that among the different strategies, Value plus Momentum & Passive strategy works well. In selecting value plus momentum strategy, ~60% of months and 75% of years have positive returns. On the other hand, in the passive strategy, ~62% of months and 78% of years have positive returns. Since there is not much difference between the positive return stats, but considering performance factor (best/worst performance) value plus momentum performs strategy well.

% of Months and years with positive returns (>0)

Strategy	% of positive months	% of positive years
Variable_GPR	51.30%	56.25%
Variable_ValMom	60.16%	75.00%
Variable_MrktPortf	62.76%	78.13%

Conclusion:

In this section, we have combined the Gross Profit Long/Short portfolio (GPA) performance with the Fama French Factors in order to evaluate and compare the following 3 strategies:

Active market strategies: (1) Gross Profit plus Value (GPA + HML); (2) Value plus Momentum (HML + UMD); Passive market strategies: (3) Passive investing in the Market Portfolio (MKTRF factor).

We have analyzed the performance of strategies based on several factors such as Risk & return, volatility, Cumulative returns, Sharpe ratio, Factor analysis (CAPM + Fama French) and performance matrix (based on annual returns). Based on the following parameters, we have selected the "Value plus Momentum" as the best strategy:

Risk and Reward

In the first part, we have evaluated the monthly returns of a portfolio for 3 strategies, where the maximum return was noted in "Value plus momentum" strategy. Further, while analyzing the volatilities, "Value plus Momentum" is highly volatile as compared to the other two strategies. Additionally, the active strategies are negatively skewed, it means that there are high chances of frequent gains, which makes it more attractive as compared to passive strategy. It concludes that if volatilities are higher and returns are negatively skewed, then the risk is higher in that strategy. Hence, this justifies our results that "Value plus momentum" is although a riskier strategy but historically it has recorded higher monthly returns.

Cumulative Returns & Sharpe Ratio

Here, we have examined the performance of our strategies by investing $10,000 in our portfolio (long/short) at the beginning of the period. After investing the aforementioned amount, the cumulative returns are higher in the "Value plus Momentum" strategy. Additionally, the annual returns are captured where, overall, the "Value plus Momentum" strategy performs quite well.

Further, to measure the functioning of strategies, we have used another parameter "Sharpe ratio", that represents the additional amount of return that an investor receives per unit of increase in risk. Here, among all the three strategies "Passive market" and "Value plus Momentum" has a higher Sharpe Ratio. "Passive market" strategy has a slightly higher value. However, we have seen that the "Value plus Momentum" holds higher risk than the "passive market" strategy, then selecting the latter strategy would be a good choice, as even the risk is higher but it gives the same Sharpe ratio than the former strategy.

Factor analysis (CAPM + Fama French)

Lastly, we have performed factor analysis for all the three strategies, where we have captured the various market factors. In the CAPM model, factors are favorable for "Value plus Momentum" strategy, like the alpha is 1.23%, which shows that it is able to outperform the market (explained in detail in the earlier section).

Further, in the FF3 model, the factors such as; alpha is higher than 1 which again beats the market, beta market is negative i.e., the strategy performs opposite to market, positive beta SMB shows that small cap stocks performed well in this strategy, and positive beta HML indicates that strategy worked well for value stock companies. **Hence, from the factor analysis matrices, we can conclude that overall, the "Value plus Momentum" strategy performed well.**

Recommendation to the Client

Here, we have also designed a rating matrix based on several parameters (discussed below in detail) to shortlist the recommendable strategy:

Parameters	Strategy wise: Rating matrix		
	Gross plus value	Value plus Momentum	Passive Market
Monthly returns	●●	●●●	●
Volatility & Skewness (Risk)	●	●●●	●●
Sharpe ratio	●	●●	●●●
Cumulative returns (on portfolio $10000)	●	●●●	●●
Annual returns (No. of best among top 5)	●	●●●	●●
Annual returns (No. of worst among top 5)	●●	●	●●●
% of months & years with positive returns	●	●●	●●●
CAPM factors	●	●●●	●●
FF3 factors	●	●●●	●●
Notation: **● – Worst rating; ●● – Good rating; ●●● – Best rating**			

Finally, considering our multi-factor analysis as an accurate tool for evaluating the performance of different investment strategies, we recommend investors to pursue "Value plus Momentum" strategy as it becomes more confident in the bullish market, and when the market is bearish, it still gives better returns than the other two strategies.