MA/FIM 548 Research Project:
Credit Suisse Structured Products – Auto callable Barrier Reverse Convertibles

Submitted To: Dr. Yerkin Kitapbayev

Submitted By: Mehul Mehta

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Auto Callable Barrier Reverse Convertible

The Auto callable Barrier Reverse Convertible is a structured product. It pays a guaranteed coupon throughout the product's lifetime (up to maturity or early redemption). This particular product will be redeemed before maturity under certain conditions. For early redemption to occur, each of the product's underlying assets must be above its specified barrier level.

Product Description

Sr. No.	Underlying's	Bloomberg Ticker	Strike (100%)	Barrier (59%)	Trigger Barrier (100%	Ratio
1	Allianz SE Registered Share	ALV GY EQUITY	EUR 217.175	EUR 128.1333	EUR 217.175	4.6046
2	AXA SA Bearer Share	CS FP EQUITY	EUR 22.9075	EUR 13.5154	EUR 22.9075	43.6538
3	Hannover Rück SE Registered Share	HNR1 GY EQUITY	EUR 156.575	EUR 92.3793	EUR 156.575	6.3867

Details of the Contract

Sr. No.	Particulars	Details	Date
1	**Issuer**	Credit Suisse AG	-
2	**Countries for Public Offering**	Switzerland	-
3	**Initial Fixing Date**	Date on which the Strike and the Barrier and the Trigger Barrier and the Ratio is fixed, and from which date the Complex Products may be traded.	12 April 2021
4	**Issue/Payment Date**	Date on which the Complex Products are issued and the Issue Price is paid.	19 April 2021
5	**Final Fixing Date**	Date on which the Final Level will be fixed.	13 July 2022
6	**Last Trading Date**	Until the official close of trading on SIX Swiss Exchange Ltd, being the last date on which the Complex Products may be traded.	13 July 2022
7	**Final Redemption Date**	Date on which each Complex Product will be redeemed at the Final Redemption Amount, unless previously redeemed, repurchased or cancelled.	19 July 2022
8	**Listing**	SIX Swiss Exchange Ltd	-
9	**Denomination**	EUR 1'000	-
10	**Minimum Investment**	EUR 1'000	-

Sr. No.	Particulars	Details	Date
11	**Interest Amount**	0.00% p.a. of the Denomination. The Issuer shall pay the Interest Amount per Complex Product to the holders, unless previously redeemed, repurchased or cancelled.	19 July 2021, 19 October 2021, 19 January 2022, 19 April 2022 and 19 July 2022
12	**Premium Amount**	8.00% p.a. of the Denomination. The Issuer shall pay the Interest Amount per Complex Product to the holders, unless previously redeemed, repurchased or cancelled.	19 July 2021, 19 October 2021, 19 January 2022, 19 April 2022 and 19 July 2022
13	**Early Redemption (Trigger Observation Date)**	If the level of each Underlying at the Valuation Time on any Trigger Observation Date is at or above its Trigger Barrier (100% of its Strike)	13 October 2021, 13 January 2022 or 11 April 2022
14	**Trigger Redemption Date**	An amount in cash equal to 100% of the Denomination.	19 October 2021, 19 January 2022 or 19 April 2022
15	**Final Redemption**	(i) If no Barrier Event has occurred, a cash amount equal to 100% of the Denomination; or (ii) If a Barrier Event has occurred, and (a) the Final Level of each Underlying is at or above its Strike, a cash amount equal to 100% of the Denomination; or (b) the Final Level of at least one Underlying is below its Strike, the number of Worst-Performing Underlying's specified in the Ratio of the Worst-Performing Underlying. Fractions will not be cumulated and will be paid in cash (based on the Final Level of the Worst-Performing Underlying). In this case, the investor will have to bear the usual transaction fees, based on the	At Maturity

Sr. No.	Particulars	Details	Date
		Strike of the Worst-Performing Underlying	

Steps followed to Price the Auto Callable Barrier Reverse Convertibles Product

1. Let us assume that the asset prices follow **correlated Geometric Brownian Motions**. For the stocks following the correlated geometric Brownian motion process, we simulate the stock price at different time period using Black-Scholes formula.
2. In this pricing applications, we need to check whether the price of the underlying crosses the barrier or not at the given time period (i.e., at T= 0.5, 0.75, 1 and 1.25). If the price of the underlying at the above-mentioned specific period is above the barrier price this means that no barrier event has occurred and if the price of the underlying is below the barrier price this means barrier event has occurred. If barrier is occurred at T=0.5, 0.75 and 1, early redemption will happen otherwise final redemption will happen.
3. The condition of **early redemption is met at T1**, if the level of each Underlying at the Valuation Time i.e., at T1=0.5 (13 October 2021) is at or above its 100% of its Strike. If this condition is met, the Payoff of the product will be early redeemed as a result of a Trigger Event. If the product is not redeemed at T1, it will check the early redemption condition at T2.
4. The condition of **early redemption is met at T2**, if the level of each Underlying at the Valuation Time i.e., at T2=0.75 (13 January 2022) is at or above its 100% of its Strike. If this condition is met, the Payoff of the product will be early redeemed as a result of a Trigger Event. If the product is not redeemed at T2, it will check the early redemption condition at T3.
5. The condition of **early redemption is met at T3**, if the level of each Underlying at the Valuation Time i.e., at T3=1.0 (11 April 2022) is at or above its 100% of its Strike. If this condition is met, the Payoff of the product will be early redeemed as a result of a Trigger Event. If the product is not redeemed at T3, it will check the final redemption condition at T4.
6. If none of the above-mentioned condition (step 3, 4 & 5) is met, the Complex Product will be **redeemed at the Final Redemption.** Therefore, we will check the **condition of final redemption at T4=1.25 (13 July 2022).**
    a. **At T4=1.25** (13 July 2022), if no Barrier Event (if the level of any Underlying at any time (observed continuously) on any Barrier Observation Date is at or below its Strike) has occurred, then the product will be redeemed.
    b. **At T4=1.25** (13 July 2022), if the Barrier Event has occurred and the Final Level of each Underlying is at or above its Strike then the product will be redeemed.

c. **At T4=1.25** (13 July 2022), if a Barrier Event has occurred and the Final Level of at least one Underlying is below its Strike then the product will be redeemed.

7. Finally, to calculate the payoff of the structured product, we average the payoff over N = 10^5 simulations and then compute the variance of the payoff.
8. To reduce the variance of the payoff, I have used **Antithetic Method**. The method of antithetic variates reduces the variance by introducing negative dependence between the pairs of replications.

Mathematical Equation for Payoff at Different Time Period.

Sr. No.	Condition	Time	Date	Payoff Equation
1	**Early Redemption at T1**	T=0.5	13 October 2021	$\text{Payoff} = e^{-r*T1} * (IV) + CP1 + CP2$
2	**Early Redemption at T2**	T=0.75	13 January 2022	$\text{Payoff} = e^{(-r*T2)} * (IV) + CP1 + CP2 + CP3$
3	**Early Redemption at T3**	T=1	11 April 2022	$\text{Payoff} = e^{(-r*T3)} * (IV) + CP1 + CP2 + CP3 + CP4$
4(i)	**Final Redemption at Maturity (Condition (i))**	T=1.25	13 July 2022	$\text{Payoff} = e^{(-r*T4)} * (IV) + CP1 + CP2 + CP3 + CP4 + CP5$
4(ii) (a)	**Final Redemption at Maturity (Condition (ii) (a)**	T=1.25	13 July 2022	$\text{Payoff} = e^{(-r*T4)} * (IV) + CP1 + CP2 + CP3 + CP4 + CP5$
4(ii) (b)	**Final Redemption at Maturity (Condition (ii) (b)**	T=1.25	13 July 2022	$\text{Payoff} = e^{(-r*T4)} * (IV * S_{worst}) + CP1 + CP2 + CP3 + CP4 + CP5$

Note: While calculating the Premium Amount, the difference in the price of 5 days is neglected i.e., the Issuer shall pay the Premium Amount on 13th July and not on 19 July 2021. The above difference in price is very small and therefore is neglected.

The variables used in the above Payoff equation means the following:

1. **Investment (IV):** IV means the denomination amount i.e., EUR 1'000.

2. **Premium Amount/Coupon Payment (CP)**: The Premium Amount given in the contract is 8% pa of the denomination. Therefore, in each quarter, the issuer shall pay 2% of the denomination amount.
   - **Coupon Payment at Time T=0.25 (CP1: 1st quarter) is equal to** 2% of denomination and then discounting it with time T=0.25. In the below equation, 0.25*PA*IV means 2% of the denomination.

$$CP1 = e^{(-r*0.25)} * (0.25 * PA * IV)$$

   - **Coupon Payment at Time T1=0.5 (CP2: 2nd quarter) is equal to** 2% of denomination and then discounting it with time T=0. 5. In the below equation, 0.25*PA*IV means 2% of the denomination.

$$CP2 = e^{(-r*T1)} * (0.25 * PA * IV)$$

   - **Coupon Payment at Time T2=0.75 (CP3: 3rd quarter) is equal to** 2% of denomination and then discounting it with time T=0.75. In the below equation, 0.25*PA*IV means 2% of the denomination.

$$CP3 = e^{(-r*T2)} * (0.25 * PA * IV)$$

   - **Coupon Payment at Time T3=1 (CP4: 4th quarter) is equal to** 2% of denomination and then discounting it with time T=1. In the below equation, 0.25*PA*IV means 2% of the denomination.

$$CP4 = e^{(-r*T3)} * (0.25 * PA * IV)$$

   - **Coupon Payment at Time T4=1.25 (CP5: Next Year, 1st quarter) is equal to** 2% of denomination and then discounting it with time T=1.25. In the below equation, 0.25*PA*IV means 2% of the denomination.

$$CP5 = e^{(-r*T4)} * (0.25 * PA * IV)$$

3. **Worst Performing Underlying (S_worst):** Out of all the Underlyings, the price of the Underlying at Maturity divided by its Strike results in the lowest value.

Payoff at Maturity for Worst Performing Underlying is worst performer times denomination.

$$Payoff\ at\ Maturity = e^{-r*T4} * (Conversion\ Ratio) * (Price\ of\ Underlying\ at\ Maturity) + CP1 + CP2 + CP3 + CP4 + CP5$$

$$Payoff\ at\ Maturity = e^{-r*T4} * (Investment/StrikePrice) * (Price\ of\ Underlying\ at\ Maturity) + CP1 + CP2 + CP3 + CP4 + CP5$$

$$\boldsymbol{Payoff\ at\ Maturity = e^{-r*T4} * (\text{Investment}) * (\text{S\_worst}) + CP1 + CP2 + CP3 + CP4 + CP5}$$

Therefore, Payoff at maturity will be the sum of discounted price of ((Investment) (S_worst)) and the Premium* Amounts *(CP1+CP2+CP3+CP4+CP5).*

Estimates for the Model Parameter

1. **Risk Free Rate (r = -0.746%):** Switzerland 1 year bond data is considered as the risk-free rate.

2. **Dividends Yield Rates:** To calculate the dividend, the Forward Dividend & Yield data from Yahoo Finance is taken.

- d1=0.0444; (Dividend for Stock 1)
- d2=0.0617; (Dividend for Stock 2)
- d3=0.0293; (Dividend for Stock 3)

3. **Volatilities of the Underlying:** Standard Deviation of the daily returns of each underlying were taken. Moreover, 3-year historical data was considered to calculate the volatilities of the underlying.

- sig1=0.00780911*sqrt (252);
- sig2=0.008202997*sqrt (252);
- sig3=0.007596542*sqrt (252);

4. **Correlation Matrix:**
   The daily returns of each underlying were used to calculate the correlation matrix. Moreover, 3-year historical data was considered to calculate the correlation matrix.

Particulars	Return S1	Return S2	Return S3
Return S1	1	0.874028799	0.798699928
Return S2	0.874028799	1	0.759168398
Return S3	0.798699928	0.759168398	1

Results

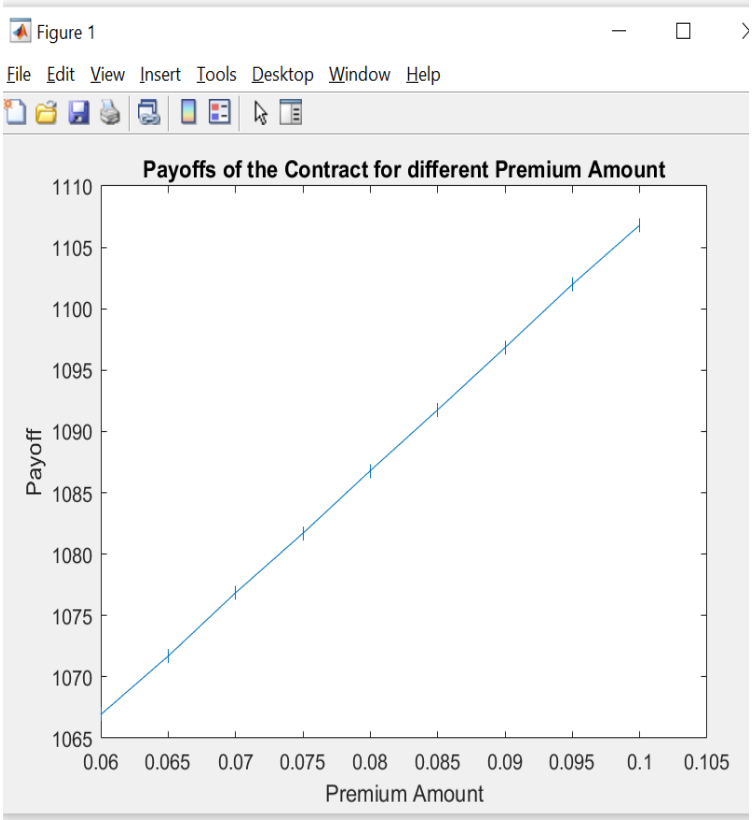
1. Payoff with different coupon rate

In the project, premium amount is taken as 8%. However, to understand how the payoff will change with change in coupon rate, we do a scenario analysis. The result of the scenario analysis is following.

Sr. No.	Premium Amount	Mean of the Payoff
1	6%	1066.935
2	6.5%	1071.694
3	7%	1076.850
4	7.5%	1081.698

Sr. No.	Premium Amount	Mean of the Payoff
5	8%	1086.799
6	8.5%	1091.758
7	9%	1096.832
8	9.5%	1101.986
9	10%	1106.816

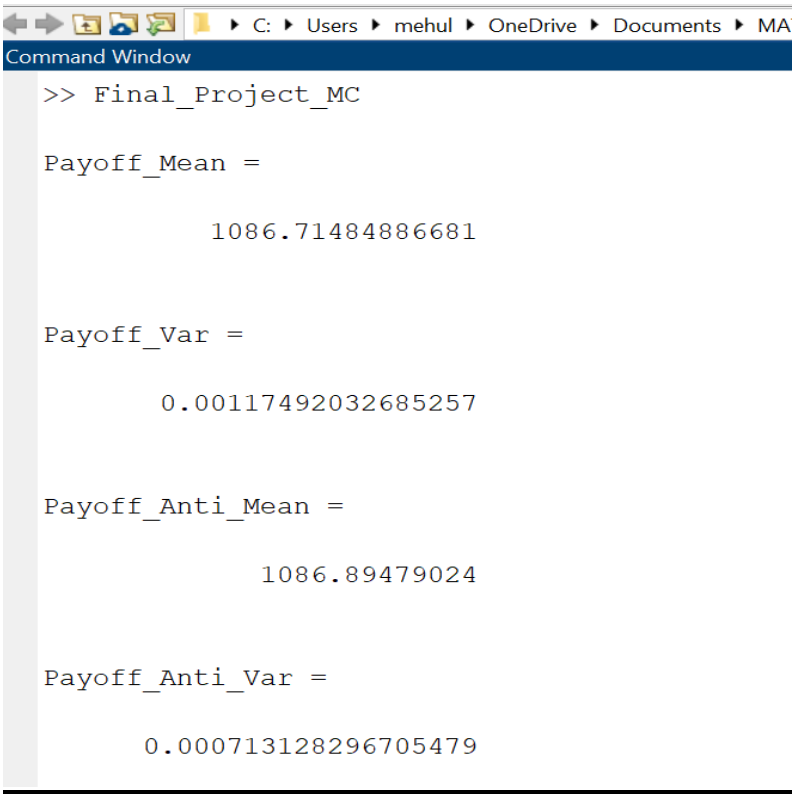
The below graphs show the variation of Payoffs with the Premium Amount.



2. Variance Reduction Technique (Antithetic Method)

The below price of the contract is calculated at 8 percent premium amount per annum.

Particulars	Before using Antithetic Method	After using Antithetic Method
Mean of the Payoff	1086.71484886681	1086.89479024
Variance of the Payoff	0.00117492032685257	0.000713128296705479
Total Reduction in Variance	**39.26%**	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Payoff_Anti_Mean =
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