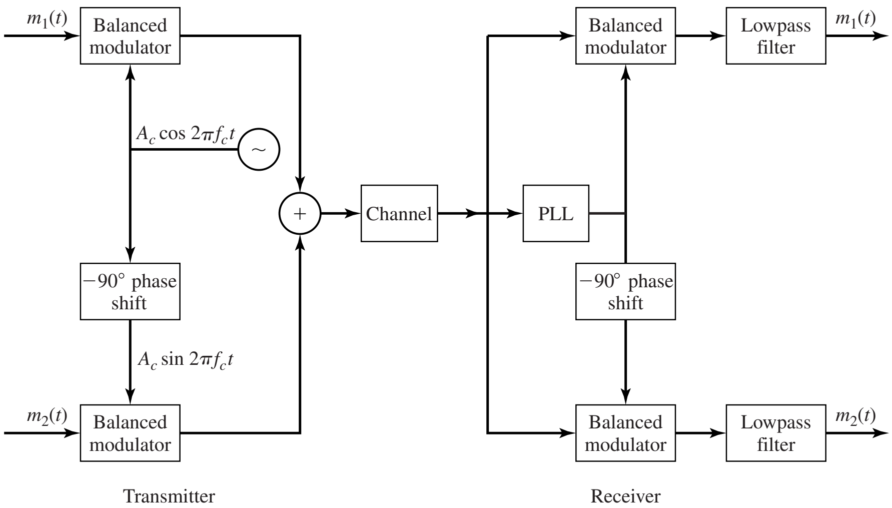

Figure 3.24 Quadrature-carrier multiplexing.

3.3 ANGLE MODULATION

In the previous section we considered amplitude modulation of the carrier as a means for transmitting the message signal. Amplitude-modulation methods are also called *linear-modulation methods,* although conventional AM is not linear in the strict sense.

Another class of modulation methods are frequency and phase modulation which are described in this section. In frequency-modulation (FM) systems, the frequency of the carrier f_c is changed by the message signal and in phase-modulation (PM) systems the phase of the carrier is changed according to the variations in the message signal. Frequency and phase modulation are obviously quite nonlinear, and very often they are jointly referred to as *angle-modulation methods*. As our analysis in the following sections will show, angle modulation, due to its inherent nonlinearity, is more complex to implement, and much more difficult to analyze. In many cases only an approximate analysis can be done. Another property of angle modulation is its bandwidth-expansion property. Frequency and phase-modulation systems generally expand the bandwidth such that the effective bandwidth of the modulated signal is usually many times the bandwidth of the message signal.† With a higher implementation complexity and a higher bandwidth occupancy, one would naturally raise a question as to the usefulness of these systems. As our analysis in Chapter 5 will show, the major benefit of these systems is their high degree of noise immunity. In fact these systems trade-off bandwidth for high noise immunity. That is the reason that FM systems are widely used in high-fidelity music broadcasting and point-to-point communication systems where the transmitter power is quite limited.

†Strictly speaking, the bandwidth of the modulated signal, as it will be shown later, is infinite. That is why we talk about the *effective bandwidth*.

3.3.1 Representation of FM and PM Signals

An angle-modulated signal in general can be written as

$$u(t) = A_c \cos(\theta(t))$$

$\theta(t)$ is the phase of the signal, and its instantaneous frequency $f_i(t)$ is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) \tag{3.3.1}$$

Since $u(t)$ is a bandpass signal, it can be represented as

$$u(t) = A_c \cos(2\pi f_c t + \phi(t)) \tag{3.3.2}$$

and, therefore,

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) \tag{3.3.3}$$

If $m(t)$ is the message signal, then in a PM system we have

$$\phi(t) = k_p m(t) \tag{3.3.4}$$

and in an FM system we have

$$f_i(t) - f_c = k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t) \tag{3.3.5}$$

where k_p and k_f are phase and frequency *deviation constants*. From the above relationships we have

$$\phi(t) = \begin{cases} k_p m(t), & \text{PM} \\ 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau, & \text{FM} \end{cases} \tag{3.3.6}$$

Equation (3.3.6) shows the close and interesting relation between FM and PM systems. This close relationship makes it possible to analyze these systems in parallel and only emphasize their main differences. The first interesting result observed from Equation (3.3.6) is that if we phase modulate the carrier with the integral of a message, it is equivalent to frequency modulation of the carrier with the original message. On the other hand, in Equation (3.3.6) the relation can be expressed as

$$\frac{d}{dt}\phi(t) = \begin{cases} k_p \frac{d}{dt} m(t), & \text{PM} \\ 2\pi m(t), & \text{FM} \end{cases} \tag{3.3.7}$$

which shows that if we frequency modulate the carrier with the derivative of a message, the result is equivalent to phase modulation of the carrier with the message itself. Figure 3.25 shows the above relation between FM and PM. Figure 3.26 illustrates a square-wave signal and its integral, a sawtooth signal, and their corresponding FM and PM signals.

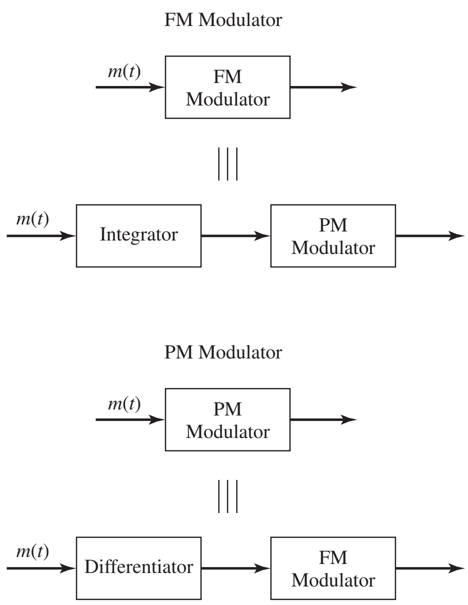

Figure 3.25 A comparison of FM and PM modulators.

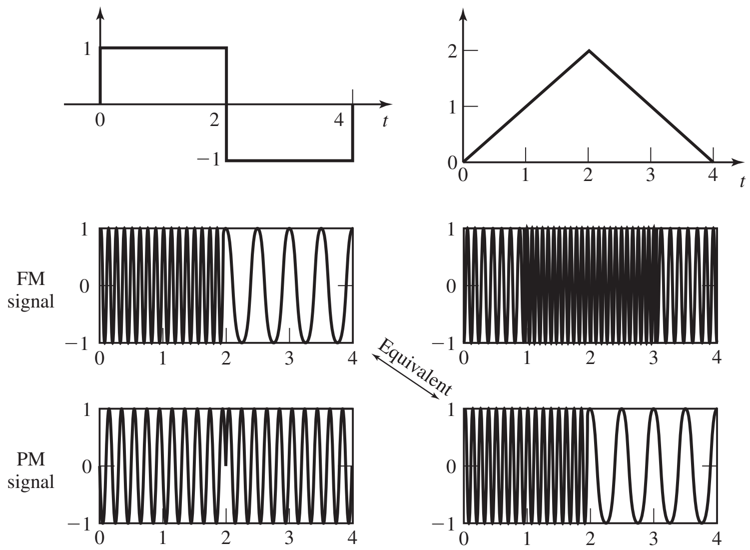

Figure 3.26 FM and PM of square and sawtooth waves.

The demodulation of an FM signal involves finding the instantaneous frequency of the modulated signal and then subtracting the carrier frequency from it. In the demodulation of PM, the demodulation process is done by finding the phase of the signal and then recovering $m(t)$. The maximum phase deviation in a PM system is given by

$$\Delta\phi_{\max} = k_p \max[|m(t)|] \tag{3.3.8}$$

and the maximum frequency-deviation in an FM system is given by

$$\Delta f_{\max} = k_f \max[|m(t)|] \tag{3.3.9}$$

Example 3.3.1

The message signal

$$m(t) = a\cos(2\pi f_m t)$$

is used to either frequency modulate or phase modulate the carrier $A_c\cos(2\pi f_c t)$. Find the modulated signal in each case.

Solution In PM we have

$$\phi(t) = k_p m(t) = k_p a\cos(2\pi f_m t) \tag{3.3.10}$$

and in FM we have

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau = \frac{k_f a}{f_m}\sin(2\pi f_m t) \tag{3.3.11}$$

Therefore, the modulated signals will be

$$u(t) = \begin{cases} A_c\cos(2\pi f_c t + k_p a\cos(2\pi f_m t)), & \text{PM} \\ A_c\cos\left(2\pi f_c t + \frac{k_f a}{f_m}\sin(2\pi f_m t)\right), & \text{FM} \end{cases} \tag{3.3.12}$$

By defining

$$\beta_p = k_p a \tag{3.3.13}$$

$$\beta_f = \frac{k_f a}{f_m} \tag{3.3.14}$$

we have

$$u(t) = \begin{cases} A_c\cos(2\pi f_c t + \beta_p\cos(2\pi f_m t)), & \text{PM} \\ A_c\cos(2\pi f_c t + \beta_f\sin(2\pi f_m t)), & \text{FM} \end{cases} \tag{3.3.15}$$

The parameters β_p and β_f are called the *modulation indices* of the PM and FM systems respectively.

We can extend the definition of the modulation index for a general nonsinusoidal signal $m(t)$ as

$$\beta_p = k_p \max[|m(t)|] \tag{3.3.16}$$

$$\beta_f = \frac{k_f \max[|m(t)|]}{W} \tag{3.3.17}$$

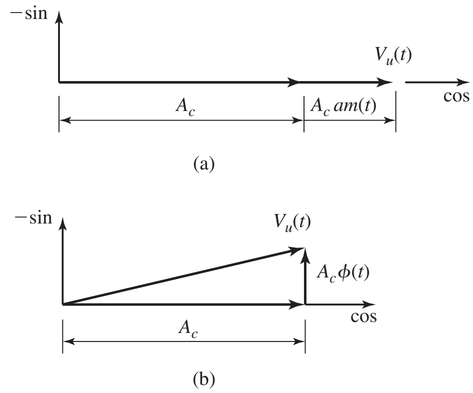

Figure 3.27 Phasor diagram for the conventional AM (a) and narrowband angle modulation (b).

where W denotes the bandwidth of the message signal $m(t)$. In terms of the maximum phase and frequency deviation $\Delta\phi_{\max}$ and $\Delta f_{\max}$, we have

$$\beta_p = \Delta\phi_{\max} \tag{3.3.18}$$

$$\beta_f = \frac{\Delta f_{\max}}{W} \tag{3.3.19}$$

Narrowband Angle Modulation.[†] If in an angle-modulation system the deviation constants k_p and k_f and the message signal $m(t)$ are such that for all t we have $\phi(t) \ll 1$, then we can use a simple approximation to expand $u(t)$ as

$$\begin{aligned} u(t) &= A_c \cos 2\pi f_c t \cos\phi(t) - A_c \sin 2\pi f_c t \sin\phi(t) \\ &\approx A_c \cos 2\pi f_c t - A_c\phi(t) \sin 2\pi f_c t \end{aligned} \tag{3.3.20}$$

This last equation shows that in this case the modulated signal is very similar to a conventional AM signal. The only difference is that the message signal $m(t)$ is modulated on a sine carrier rather than a cosine carrier. The bandwidth of this signal is similar to the bandwidth of a conventional AM signal, which is twice the bandwidth of the message signal. Of course this bandwidth is only an approximation to the real bandwidth of the FM signal. A phasor diagram for this signal and the comparable conventional AM signal are given in Figure 3.27. Note that compared to conventional AM, the narrowband angle-modulation scheme has far less amplitude variations. Of course, the angle-modulation system has constant amplitude and, hence, there should be no amplitude variations in the phasor-diagram representation of the system. The slight variations here are due to the first-order approximation that we have used for the expansions of $\sin(\phi(t))$ and $\cos(\phi(t))$. As we will see in Chapter 5, the narrowband angle-modulation method does not provide any better noise immunity compared to a conventional AM system. Therefore, narrowband angle modulation is seldom used in practice for communication purposes. However, these systems can be used as an intermediate stage for generation of wideband angle-modulated signals as we will discuss in Section 3.3.3.

[†]Also known as *low-index angle modulation*.

3.3.2 Spectral Characteristics of Angle-Modulated Signals

Due to the inherent nonlinearity of angle-modulation systems the precise characterization of their spectral properties, even for simple message signals, is mathematically intractable. Therefore, the derivation of the spectral characteristics of these signals usually involves the study of very simple modulating signals and certain approximations. Then the results are generalized to the more complicated messages. We will study the spectral characteristics of an angle-modulated signal in three cases: when the modulating signal is a sinusoidal signal, when the modulating signal is a general periodic signal, and when the modulating signal is a general nonperiodic signal.

Angle Modulation by a Sinusoidal Signal. Let us begin with the case where the message signal is a sinusoidal signal. As we have seen, in this case for both FM and PM, we have

$$u(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t) \tag{3.3.21}$$

where β is the modulation index that can be either β_p or β_f. Therefore, the modulated signal can be written as

$$u(t) = \mathrm{Re}\big(A_c e^{j2\pi f_c t} e^{j\beta \sin 2\pi f_m t}\big) \tag{3.3.22}$$

Since $\sin 2\pi f_m t$ is periodic with period $T_m = \frac{1}{f_m}$, the same is true for the complex exponential signal

$$e^{j\beta \sin 2\pi f_m t}$$

Therefore, it can be expanded in a Fourier series representation. The Fourier series coefficients are obtained from the integral

$$\begin{aligned} c_n &= f_m \int_0^{\frac{1}{f_m}} e^{j\beta \sin 2\pi f_m t} e^{-jn2\pi f_m t}\, dt \\ &\overset{u=2\pi f_m t}{=} \frac{1}{2\pi} \int_0^{2\pi} e^{j(\beta \sin u - nu)}\, du \end{aligned} \tag{3.3.23}$$

This latter integral is a well-known integral known as the *Bessel function of the first kind of order n* and is denoted by $J_n(\beta)$. Therefore, we have the Fourier series for the complex exponential as

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} \tag{3.3.24}$$

By substituting Equation (3.3.24) in Equation (3.3.22), we obtain

$$\begin{aligned} u(t) &= \mathrm{Re}\left(A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t}\right) \\ &= \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos\left(2\pi(f_c + n f_m)t\right) \end{aligned} \tag{3.3.25}$$

Equation (3.3.25) shows that even in this very simple case, where the modulating signal is a sinusoid of frequency f_m, the angle-modulated signal contains all frequencies of the form $f_c + n f_m$ for $n = 0, \pm 1, \pm 2, \ldots$. Therefore, the actual bandwidth of the modulated signal is infinite. However, the amplitude of the sinusoidal components of frequencies $f_c \pm n f_m$ for large n is very small. Hence, we can define a finite *effective bandwidth* for the modulated signal. A series expansion for the Bessel function is given by

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\beta}{2}\right)^{n+2k}}{k!(k+n)!} \tag{3.3.26}$$

The above expansion shows that for small β, we can use the approximation

$$J_n(\beta) \approx \frac{\beta^n}{2^n n!} \tag{3.3.27}$$

Thus for a small modulation index β, only the first sideband corresponding to $n = 1$ is of importance. Also, using the above expansion, it is easy to verify the following symmetry properties of the Bessel function.

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases} \tag{3.3.28}$$

Plots of $J_n(\beta)$ for various values of n are given in Figure 3.28, and a table of the values of the Bessel function is given in Table 3.1.

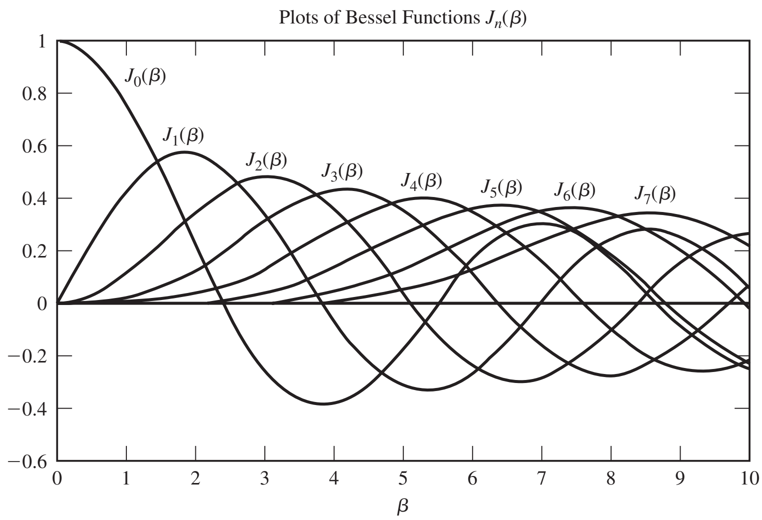

Figure 3.28 Bessel functions for various values of n.

TABLE 3.1 TABLE OF BESSEL FUNCTION VALUES

n	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 8$	$\beta = 10$
0	0.997	0.990	0.938	0.765	0.224	−0.178	0.172	−0.246
1	0.050	0.100	0.242	0.440	0.577	−0.328	0.235	0.043
2	0.001	0.005	0.031	0.115	0.353	0.047	−0.113	0.255
3				0.020	0.129	0.365	−0.291	0.058
4				0.002	0.034	0.391	−0.105	−0.220
5					0.007	0.261	0.186	−0.234
6					0.001	0.131	0.338	−0.014
7						0.053	0.321	0.217
8						0.018	0.223	0.318
9						0.006	0.126	0.292
10						0.001	0.061	0.207
11							0.026	0.123
12							0.010	0.063
13							0.003	0.029
14							0.001	0.012
15								0.004
16								0.001

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Example 3.3.2

Let the carrier be given by $c(t) = 10\cos(2\pi f_c t)$ and let the message signal be $\cos(20\pi t)$. Further assume that the message is used to frequency modulate the carrier with $k_f = 50$. Find the expression for the modulated signal and determine how many harmonics should be selected to contain 99% of the modulated signal power.

Solution The power content of the carrier signal is given by

$$P_c = \frac{A_c^2}{2} = \frac{100}{2} = 50 \tag{3.3.29}$$

The modulated signal is represented by

$$\begin{aligned} u(t) &= 10\cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} \cos(20\pi\tau)\, d\tau\right) \\ &= 10\cos\left(2\pi f_c t + \frac{50}{10}\sin(20\pi t)\right) \\ &= 10\cos(2\pi f_c t + 5\sin(20\pi t)) \end{aligned} \tag{3.3.30}$$

The modulation index is given by

$$\beta = k_f \frac{\max[|m(t)|]}{f_m} = 5 \tag{3.3.31}$$

and, therefore, the FM-modulated signal is

$$
\begin{aligned}
u(t) &= \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + nf_m)t) \\
&= \sum_{n=-\infty}^{\infty} 10 J_n(5) \cos(2\pi(f_c + 10n)t)
\end{aligned}
\tag{3.3.32}
$$

It is seen that the frequency content of the modulated signal is concentrated at frequencies of the form $f_c + 10n$ for various n. To make sure that at least 99% of the total power is within the effective bandwidth, we have to choose k large enough such that

$$
\sum_{n=-k}^{n=k} \frac{100 J_n^2(5)}{2} \geq 0.99 \times 50 \tag{3.3.33}
$$

This is a nonlinear equation and its solution (for k) can be found by trial and error and by using tables of the Bessel functions. Of course, in finding the solution to this equation we have to employ the symmetry properties of the Bessel function given in Equation (3.3.28). Using these properties we have

$$
50 \left[J_0^2(5) + 2 \sum_{n=1}^{k} J_n^2(5) \right] \geq 49.5 \tag{3.3.34}
$$

Starting with small values of k and increasing it, we see that the smallest value of k for which the left-hand side exceeds the right-hand side is $k = 6$. Therefore, taking frequencies $f_c \pm 10k$ for $0 \leq k \leq 6$ guarantees that 99% of the power of the modulated signal has been included and only one per cent has been left out. This means that, if the modulated signal is passed through an ideal bandpass filter centered at f_c with a bandwidth of at least 120 Hz, only 1% of the signal power will be eliminated. This gives us a practical way to define the *effective bandwidth* of the angle-modulated signal as being 120 Hz. Figure 3.29 shows the frequencies present in the effective bandwidth of the modulated signal.

In general the effective bandwidth of an angle-modulated signal, which contains at least 98% of the signal power, is given by the relation

$$
B_c = 2(\beta + 1) f_m \tag{3.3.35}
$$

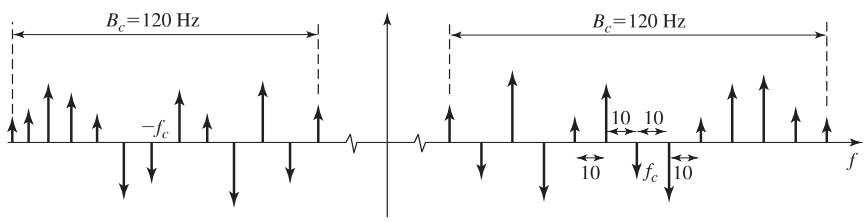

Figure 3.29 The harmonics present inside the effective bandwidth of Example 3.3.2.

where β is the modulation index and f_m is the frequency of the sinusoidal message signal. It is instructive to study the effect of the amplitude and frequency of the sinusoidal message signal on the bandwidth and the number of harmonics in the modulated signal. Let the message signal be given by

$$m(t) = a\cos(2\pi f_m t) \tag{3.3.36}$$

The bandwidth[†] of the modulated signal is given by

$$B_c = 2(\beta+1)f_m = \begin{cases} 2(k_p a + 1)f_m, & \text{PM} \\ 2\left(\frac{k_f a}{f_m} + 1\right)f_m, & \text{FM} \end{cases} \tag{3.3.37}$$

or

$$B_c = \begin{cases} 2(k_p a + 1)f_m, & \text{PM} \\ 2(k_f a + f_m), & \text{FM} \end{cases} \tag{3.3.38}$$

Equation (3.3.38) shows that increasing a, the amplitude of the modulating signal, in PM and FM has almost the same effect on increasing the bandwidth B_c. On the other hand, increasing f_m, the frequency of the message signal, has a more profound effect in increasing the bandwidth of a PM signal as compared to an FM signal. In both PM and FM the bandwidth B_c increases by increasing f_m, but in PM this increase is a proportional increase and in FM this is only an additive increase, which in most cases of interest, (for large β) is not substantial. Now if we look at the number of harmonics in the bandwidth (including the carrier) and denote it with M_c, we have

$$M_c = 2\lfloor\beta\rfloor + 3 = \begin{cases} 2\left\lfloor k_p a\right\rfloor + 3, & \text{PM} \\ 2\left\lfloor \frac{k_f a}{f_m}\right\rfloor + 3, & \text{FM} \end{cases} \tag{3.3.39}$$

Increasing the amplitude a increases the number of harmonics in the bandwidth of the modulated signal in both cases. However, increasing f_m, has no effect on the number of harmonics in the bandwidth of the PM signal and decreases the number of harmonics in the FM signal almost linearly. This explains the relative insensitivity of the bandwidth of the FM signal to the message frequency. On the one hand, increasing f_m decreases the number of harmonics in the bandwidth and, at the same time, it increases the spacing between the harmonics. The net effect is a slight increase in the bandwidth. In PM, however, the number of harmonics remains constant and only the spacing between them increases. Therefore, the net effect is a linear increase in bandwidth. Figure 3.30 shows the effect of increasing the frequency of the message in both FM and PM.

Angle Modulation by a Periodic Message Signal. To generalize the preceding results, we now consider angle modulation by an arbitrary periodic message signal $m(t)$. Let us consider a PM-modulated signal where

$$u(t) = A_c\cos(2\pi f_c t + \beta m(t)) \tag{3.3.40}$$

[†]From now on, by bandwidth we mean effective bandwidth unless otherwise stated.

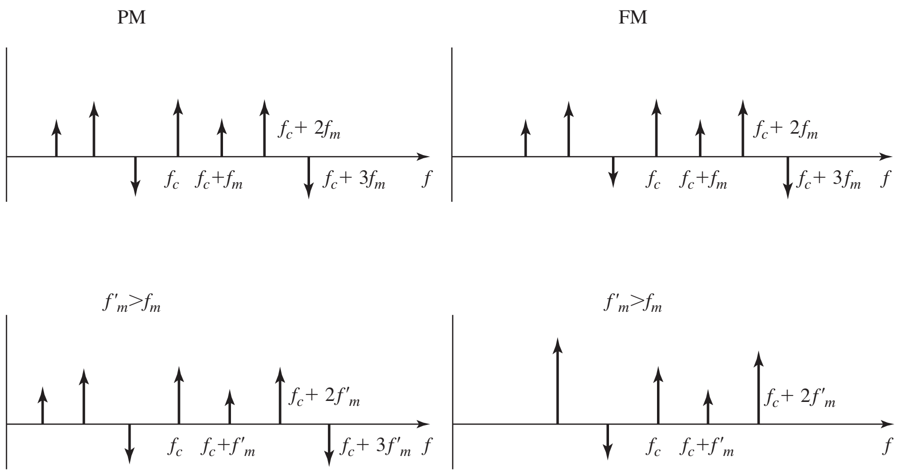

Figure 3.30 The effect of increasing bandwidth of the message in FM and PM.

We can write this as

$$u(t) = A_c \mathrm{Re}\left[e^{j2\pi f_c t} e^{j\beta m(t)}\right] \tag{3.3.41}$$

We are assuming that $m(t)$ is periodic with period $T_m = \frac{1}{f_m}$. Therefore, $e^{j\beta m(t)}$ will be a periodic signal with the same period, and we can find its Fourier series expansion as

$$e^{j\beta m(t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t} \tag{3.3.42}$$

where

$$\begin{aligned} c_n &= \frac{1}{T_m} \int_0^{T_m} e^{j\beta m(t)} e^{-j2\pi n f_m t}\, dt \\ &\stackrel{u=2\pi f_m t}{=} \frac{1}{2\pi} \int_0^{2\pi} e^{j[\beta m(\frac{u}{2\pi f_m}) - nu]}\, du \end{aligned} \tag{3.3.43}$$

and

$$\begin{aligned} u(t) &= A_c \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_c t} e^{j2\pi n f_m t}\right] \\ &= A_c \sum_{n=-\infty}^{\infty} |c_n| \cos(2\pi(f_c + n f_m)t + \angle c_n) \end{aligned} \tag{3.3.44}$$

It is seen again that the modulated signal contains all frequencies of the form $f_c + n f_m$.

The detailed treatment of the spectral characteristics of an angle-modulated signal for a general nonperiodic deterministic message signal $m(t)$ is quite involved due to the nonlinear nature of the modulation process. However, there exists an approximate relation for the effective bandwidth of the modulated signal, known as the *Carson's rule,* and given by

$$B_c = 2(\beta + 1)W \tag{3.3.45}$$

where β is the modulation index defined as

$$\beta = \begin{cases} k_p \max[|m(t)|], & \text{PM} \\ \frac{k_f \max[|m(t)|]}{W}, & \text{FM} \end{cases} \tag{3.3.46}$$

and W is the bandwidth of the message signal $m(t)$. Since in wideband FM the value of β is usually around 5 or more, it is seen that the bandwidth of an angle-modulated signal is much greater than the bandwidth of various amplitude-modulation schemes, which is either W (in SSB) or $2W$ (in DSB or conventional AM).

3.3.3 Implementation of Angle Modulators and Demodulators

Any modulation and demodulation process involves the generation of new frequencies that were not present in the input signal. This is true for both amplitude and angle-modulation systems. This means that, if we interpret the modulator as a system with the message signal $m(t)$ as the input and with the modulated signal $u(t)$ as the output, this system has frequencies in its output that were not present in the input. Therefore, a modulator (and demodulator) can not be modeled as a linear time-invariant system because a linear time-invariant system can not produce any frequency components in the output that are not present in the input signal.

Angle modulators are, in general, time-varying and nonlinear systems. One method for generating an FM signal directly is to design an oscillator whose frequency changes with the input voltage. When the input voltage is zero, the oscillator generates a sinusoid with frequency f_c, and when the input voltage changes, this frequency changes accordingly. There are two approaches to designing such an oscillator, usually called a VCO or *voltage-controlled oscillator*. One approach is to use a *varactor diode*. A varactor diode is a capacitor whose capacitance changes with the applied voltage. Therefore, if this capacitor is used in the tuned circuit of the oscillator and the message signal is applied to it, the frequency of the tuned circuit, and the oscillator, will change in accordance with the message signal. Let us assume that the inductance of the inductor in the tuned circuit of Figure 3.31 is L_0 and the capacitance of the varactor diode is given by

$$C(t) = C_0 + k_0 m(t) \tag{3.3.47}$$

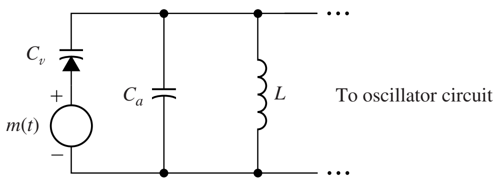

Figure 3.31 Varactor diode implementation of an angle modulator.

When $m(t) = 0$, the frequency of the tuned circuit is given by $f_c = \frac{1}{2\pi\sqrt{L_0 C_0}}$. In general, for nonzero $m(t)$, we have

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi\sqrt{L_0(C_0 + k_0 m(t))}} \\ &= \frac{1}{2\pi\sqrt{L_0 C_0}} \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}} \\ &= f_c \frac{1}{\sqrt{1 + \frac{k_0}{C_0} m(t)}} \end{aligned} \tag{3.3.48}$$

Assuming that

$$\epsilon = \frac{k_0}{C_0} m(t) \ll 1$$

and using the approximations

$$\sqrt{1+\epsilon} \approx 1 + \frac{\epsilon}{2} \tag{3.3.49}$$

$$\frac{1}{1+\epsilon} \approx 1 - \epsilon, \tag{3.3.50}$$

we obtain

$$f_i(t) \approx f_c \left(1 - \frac{k_0}{2C_0} m(t)\right) \tag{3.3.51}$$

which is the relation for a frequency-modulated signal.

A second approach for generating an FM signal is by use of a *reactance tube*. In the reactance-tube implementation, an inductor whose inductance varies with the applied voltage is employed and the analysis is very similar to the analysis presented for the varactor diode. It should be noted that although we described these methods for generation of FM signals, due to the close relation between FM and PM signals, basically the same methods can be applied for generation of PM signals (see Figure 3.25).

Another approach for generating an angle-modulated signal is to first generate a narrowband angle-modulated signal, and then change it to a wideband signal. This method is usually known as the *indirect method* for generation of FM and PM signals. Due to the similarity of conventional AM signals, generation of narrowband

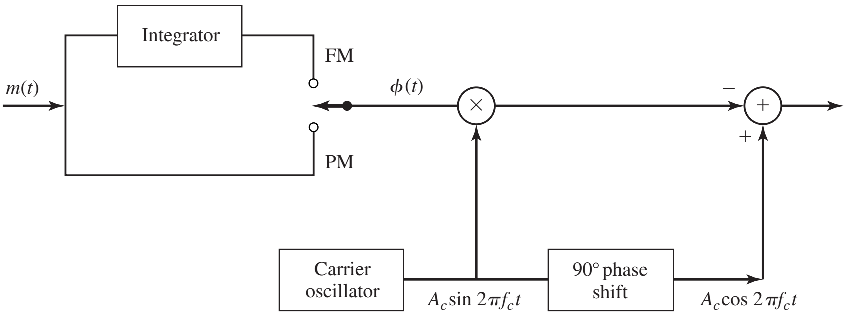

Figure 3.32 Generation of narrowband angle-modulated signal.

angle-modulated signals is straightforward. In fact any modulator for conventional AM generation can be easily modified to generate a narrowband angle-modulated signal. Figure 3.32 shows the block diagram of a narrowband angle modulator. The next step is to use the narrowband angle-modulated signal to generate a wideband angle-modulated signal. Figure 3.33 shows the block diagram of a system that generates wideband angle-modulated signals from narrowband angle-modulated signals. The first stage of such a system is, of course, a narrowband angle-modulator such as the one shown in Figure 3.32. The narrowband angle-modulated signal enters a frequency multiplier that multiplies the instantaneous frequency of the input by some constant n. This is usually done by applying the input signal to a nonlinear element and then passing its output through a bandpass filter tuned to the desired central frequency. If the narrowband-modulated signal is represented by

$$u_n(t) = A_c \cos(2\pi f_c t + \phi(t)) \tag{3.3.52}$$

the output of the frequency multiplier (output of the bandpass filter) is given by

$$y(t) = A_c \cos(2\pi n f_c t + n\phi(t)) \tag{3.3.53}$$

In general, this is, of course, a wideband angle-modulated signal. However, there is no guarantee that the carrier frequency of this signal, nf_c, will be the desired carrier frequency. The last stage of the modulator performs an up or down conversion to shift

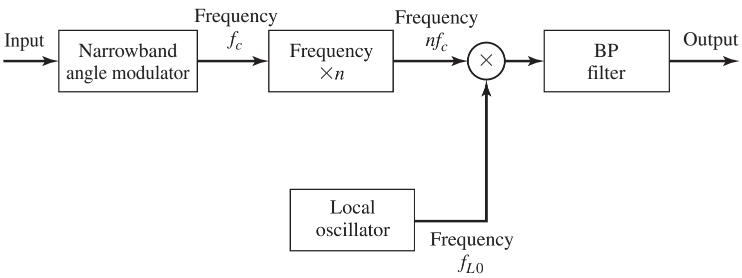

Figure 3.33 Indirect generation of angle-modulated signals.

the modulated signal to the desired center frequency. This stage consists of a mixer and a bandpass filter. If the frequency of the local oscillator of the mixer is f_{L0} and we are using a down converter, the final wideband angle-modulated signal is given by

$$u(t) = A_c \cos(2\pi(nf_c - f_{L0})t + n\phi(t)) \tag{3.3.54}$$

Since we can freely choose n and f_{L0}, we can generate any modulation index at any desired carrier frequency by this method.

FM demodulators are implemented by generating an AM signal whose amplitude is proportional to the instantaneous frequency of the FM signal, and then using an AM demodulator to recover the message signal. To implement the first step; i.e., transforming the FM signal into an AM signal, it is enough to pass the FM signal through an LTI system whose frequency response is approximately a straight line in the frequency band of the FM signal. If the frequency response of such a system is given by

$$|H(f)| = V_0 + k(f - f_c) \quad \text{for } |f - f_c| < \frac{B_c}{2} \tag{3.3.55}$$

and if the input to the system is

$$u(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right), \tag{3.3.56}$$

then, the output will be the signal

$$v_o(t) = A_c(V_0 + kk_f m(t)) \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right) \tag{3.3.57}$$

The next step is to demodulate this signal to obtain $A_c(V_0 + kk_f m(t))$, from which the message $m(t)$ can be recovered. Figure 3.34 shows a block diagram of these two steps.

There exist many circuits that can be used to implement the first stage of an FM demodulator; i.e., FM to AM conversion. One such candidate is a simple differentiator with

$$|H(f)| = 2\pi f \tag{3.3.58}$$

Another candidate is the rising half of the frequency characteristics of a tuned circuit as shown in Figure 3.35. Such a circuit can be easily implemented, but usually the linear region of the frequency characteristic may not be wide enough. To obtain a linear characteristic over a wider range of frequencies, usually two circuits tuned at two frequencies, f_1 and f_2, are connected in a configuration which is known as a *balanced discriminator*. A balanced discriminator with the corresponding frequency characteristics is shown in Figure 3.36.

The FM demodulation methods described here that transform the FM signal into an AM signal have a bandwidth equal to the channel bandwidth B_c occupied by the FM signal. Consequently, the noise that is passed by the demodulator is the noise contained within B_c.

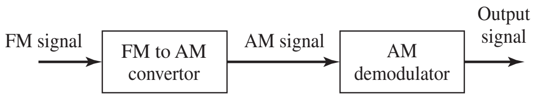

Figure 3.34 A general FM demodulator.

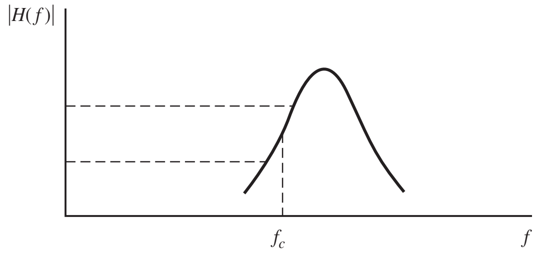

Figure 3.35 A tuned circuit used in an FM demodulator.

A totally different approach to FM signal demodulation is to use feedback in the FM demodulator to narrow the bandwidth of the FM detector and, as will be seen in Chapter 5, to reduce the noise power at the output of the demodulator. Figure 3.37 illustrates a system in which the FM discrimination is placed in the feedback branch of a feedback system that employs a voltage-controlled oscillator (VCO) path. The bandwidth of the discriminator and the subsequent lowpass filter is designed to match the bandwidth of the message signal $m(t)$. The output of the lowpass filter is the desired message signal. This type of FM demodulator is called an *FM demodulator with feedback (FMFB)*. An alternative to FMFB demodulator is the use of a phase-locked loop (PLL), as shown in Figure 3.38. The input to the PLL is the angle-modulated signal (we neglect the presence of noise in this discussion)

$$u(t) = A_c \cos[2\pi f_c t + \phi(t)] \tag{3.3.59}$$

where, for FM,

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau \tag{3.3.60}$$

The VCO generates a sinusoid of a fixed frequency, in this case the carrier frequency f_c, in the absence of an input control voltage.

Now, suppose that the control voltage to the VCO is the output of the loop filter, denoted as $v(t)$. Then, the instantaneous frequency of the VCO is

$$f_v(t) = f_c + k_v v(t) \tag{3.3.61}$$

where k_v is a deviation constant with units of Hz/volt. Consequently, the VCO output may be expressed as

$$y_v(t) = A_v \sin[2\pi f_c t + \phi_v(t)] \tag{3.3.62}$$

where

$$\phi_v(t) = 2\pi k_v \int_0^t v(\tau)\, d\tau \tag{3.3.63}$$

The phase comparator is basically a multiplier and filter that rejects the signal component centered at $2f_c$. Hence, its output may be expressed as

$$e(t) = \tfrac{1}{2} A_v A_c \sin[\phi(t) - \phi_v(t)] \tag{3.3.64}$$

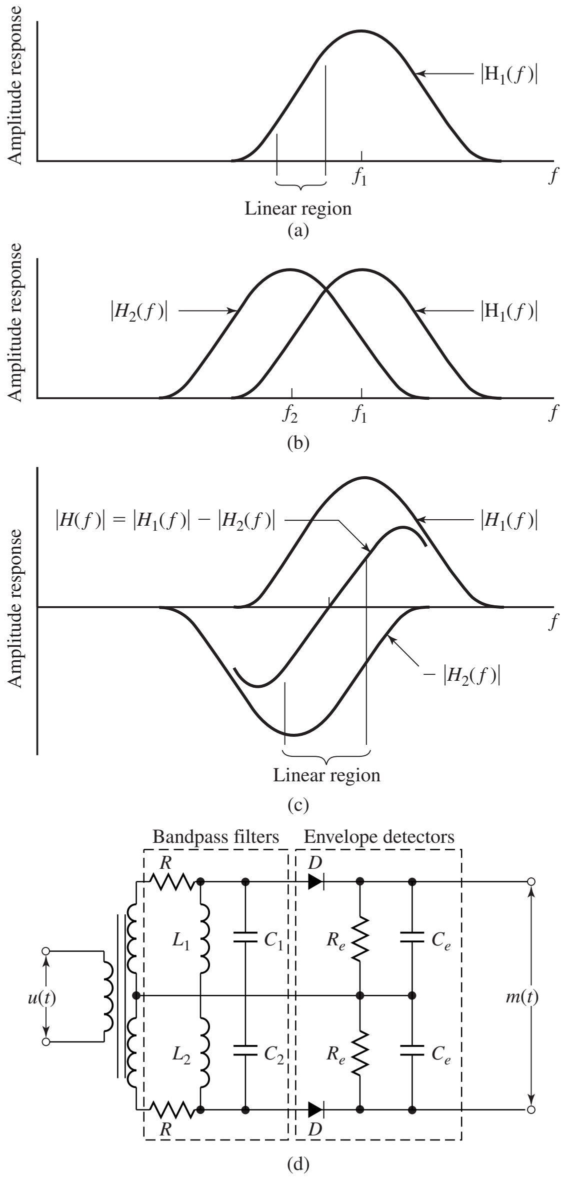

Figure 3.36 A balanced discriminator and the corresponding frequency response.

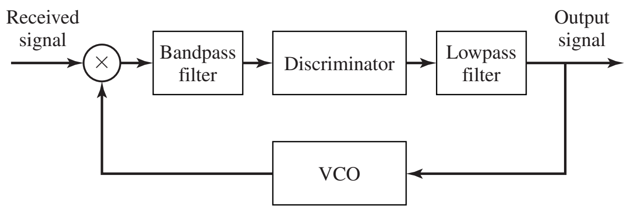

Figure 3.37 Block diagram of FMFB demodulator.

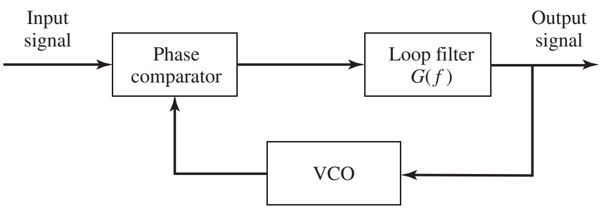

Figure 3.38 Block diagram of PLL-FM demodulator.

where the difference, $\phi(t) - \phi_v(t) \equiv \phi_e(t)$, constitutes the phase error. The signal $e(t)$ is the input to the loop filter.

Let us assume that the PLL is in lock, so that the phase error is small. Then,

$$\sin[\phi(t) - \phi_v(t)] \approx \phi(t) - \phi_v(t) = \phi_e(t) \tag{3.3.65}$$

Under this condition, we may deal with the linearized model of the PLL, shown in Figure 3.39. We may express the phase error as

$$\phi_e(t) = \phi(t) - 2\pi k_v \int_0^t v(\tau)\, d\tau \tag{3.3.66}$$

or, equivalently, either as

$$\frac{d}{dt}\phi_e(t) + 2\pi k_v v(t) = \frac{d}{dt}\phi(t) \tag{3.3.67}$$

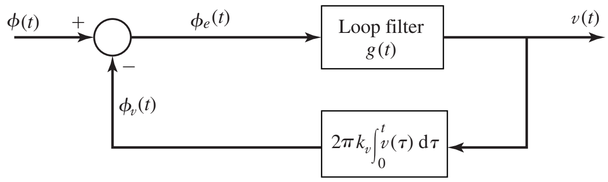

Figure 3.39 Linearized PLL.

or as

$$\frac{d}{dt}\phi_e(t) + 2\pi k_v \int_0^\infty \phi_e(\tau)g(t-\tau)\,d\tau = \frac{d}{dt}\phi(t) \tag{3.3.68}$$

The Fourier transform of the integro-differential equation in Equation (3.3.68) is

$$(j2\pi f)\Phi_e(f) + 2\pi k_v \Phi_e(f)G(f) = (j2\pi f)\Phi(f) \tag{3.3.69}$$

and, hence,

$$\Phi_e(f) = \frac{1}{1+\left(\frac{k_v}{jf}\right)G(f)}\Phi(f) \tag{3.3.70}$$

The corresponding equation for the control voltage to the VCO is

$$\begin{aligned} V(f) &= \Phi_e(f)G(f) \\ &= \frac{G(f)}{1+\left(\frac{k_v}{jf}\right)G(f)}\Phi(f) \end{aligned} \tag{3.3.71}$$

Now, suppose that we design $G(f)$ such that

$$\left| k_v \frac{G(f)}{jf} \right| \gg 1 \tag{3.3.72}$$

in the frequency band $|f| < W$ of the message signal. Then from Equation (3.3.71), we have

$$V(f) = \frac{j2\pi f}{2\pi k_v}\Phi(f) \tag{3.3.73}$$

or, equivalently,

$$\begin{aligned} v(t) &= \frac{1}{2\pi k_v}\frac{d}{dt}\phi(t) \\ &= \frac{k_f}{k_v}m(t) \end{aligned} \tag{3.3.74}$$

Since the control voltage of the VCO is proportional to the message signal, $v(t)$ is the demodulated signal.

We observe that the output of the loop filter with frequency response $G(f)$ is the desired message signal. Hence, the bandwidth of $G(f)$ should be the same as the bandwidth W of the message signal. Consequently, the noise at the output of the loop filter is also limited to the bandwidth W. On the other hand, the output from the VCO is a wideband FM signal with an instantaneous frequency that follows the instantaneous frequency of the received FM signal.

The major benefit of using feedback in FM signal demodulation is to reduce the threshold effect that occurs when the input signal-to-noise-ratio to the FM demodulator drops below a critical value. The threshold effect is treated in Chapter 5.

3.4 RADIO AND TELEVISION BROADCASTING

Radio and television broadcasting is the most familiar form of communication via analog signal transmission. Next, we describe three types of broadcasting, namely, AM radio, FM radio, and television.

3.4.1 AM Radio Broadcasting

Commercial AM radio broadcasting utilizes the frequency band 535–1605 kHz for transmission of voice and music. The carrier frequency allocations range from 540–1600 kHz with 10-kHz spacing.

Radio stations employ conventional AM for signal transmission. The baseband message signal $m(t)$ is limited to a bandwidth of approximately 5 kHz. Since there are billions of receivers and relatively few radio transmitters, the use of conventional AM for broadcast is justified from an economic standpoint. The major objective is to reduce the cost of implementing the receiver.

The receiver most commonly used in AM radio broadcast is the so called *superheterodyne receiver* shown in Figure 3.40. It consists of a radio frequency (RF) tuned amplifier, a mixer, a local oscillator, an intermediate frequency (IF) amplifier, an envelope detector, an audio frequency amplifier, and a loudspeaker. Tuning for the desired radio frequency is provided by a variable capacitor, which simultaneously tunes the RF amplifier and the frequency of the local oscillator.

In the superheterodyne receiver, every AM radio signal is converted to a common IF frequency of $f_{IF} = 455$ kHz. This conversion allows the use of a single tuned IF amplifier for signals from any radio station in the frequency band. The IF amplifier is designed to have a bandwidth of 10 kHz, which matches the bandwidth of the transmitted signal.

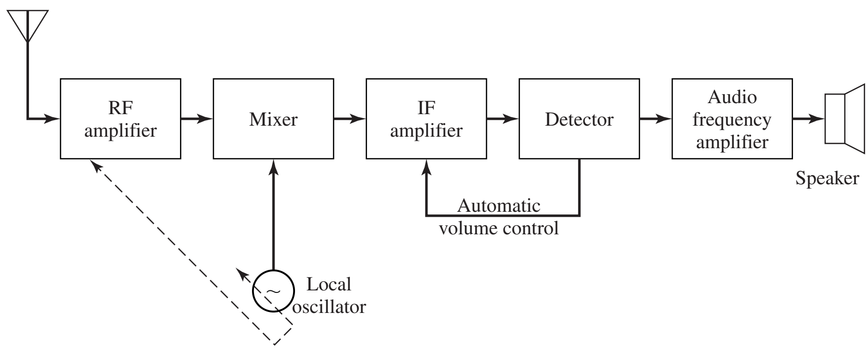

Figure 3.40 Superheterodyne AM receiver.