



# Location and Scale Parameters

- Normal PDF
  - A probability distribution is characterised by location and scale parameters
  - Location and scale parameters are typically used in modelling applications
  - For a Standard Normal Distribution, location = 0 and scale = 1
- Location Parameter
  - The effect of the location parameter is to translate the graph relative to the standard normal distribution
  - A location parameter of  $-x$  would have shifted the graph  $x$  units to the left on the horizontal axis
  - A location parameter of  $x$  would have shifted the graph  $x$  units to the right on the horizontal axis
  - A location parameter simply shifts the graph left or right on the horizontal axis
- Scale Parameter
  - The effect of a scale parameter greater than one is to stretch the Probability Distribution Function
  - The effect of a scale parameter lesser than one is to squeeze the Probability Distribution Function
  - The peak of the Probability Distribution Function is multiplied by the scale parameter
  - A non-positive scale parameter is not allowed
- Location and Scale Together

- When both location and scale parameter are applied then the position and the height of the peak will change
- Standard Form
  - The standard form of any distribution is the form that has location parameter zero and scale parameter one
- Formulas for Location and Scale Based on the Standard Form
  - Cumulative Distribution Function
    - $F(x; a, b) = F\left(\frac{x-a}{b}; 0, 1\right)$
  - Probability Density Function
    - $f(x; a, b) = \frac{1}{b}f\left(\frac{x-a}{b}; 0, 1\right)$
  - Percent Point Function
    - $G(\alpha; a, b) = a + bG(\alpha; 0, 1)$
  - Hazard Function
    - $h(x; a, b) = \frac{1}{b}h\left(\frac{x-a}{b}; 0, 1\right)$
  - Cumulative Hazard Function
    - $H(x; a, b) = H\left(\frac{x-a}{b}; 0, 1\right)$
  - Survival Function
    - $S(x; a, b) = S\left(\frac{x-a}{b}; 0, 1\right)$
  - Inverse Survival Function
    - $Z(\alpha; a, b) = a + bZ(\alpha; 0, 1)$
  - Random Numbers
    - $Y(a, b) = a + bY(0, 1)$
- Relationship to Mean and Standard Deviation
  - For the normal distribution, the location and scale parameters correspond to the mean and standard deviation, respectively