(i) Perspective division

$$w = 1$$

$$u = [\, x/1 \quad y/1 \quad z/1 \,]^T$$

$$u = [\, x \quad y \quad z \,]^T$$

$w = h$ $$v = [\, xh/h \quad yh/h \quad zh/h \,]^T$$

→ divide by w to do perspective division.

$$v = [\, x \quad y \quad z \,]^T = u = [\, x \quad y \quad z \,]^T$$

$$\therefore Mv = Mu = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

∴ b/c vectors v and u are the same after perspective division, multiplication by M gives the same result.

$4 \times \boxed{4 \quad 3} \times 1$

$$\begin{bmatrix} a_1x + a_2y + a_3z + a_4 \\ a_5x + a_6y + a_7z + a_8 \\ a_9x + a_{10}y + a_{11}z + a_{12} \\ 0 \quad 0 \quad 0 \quad 1 \end{bmatrix} = \underline{Mv = Mu}$$

(2)

(a)

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{bmatrix} \xrightarrow{-R3} \underbrace{\begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}}_{\text{rotation by } 45^\circ \text{ around y axis}}$$

$\cos^{-1}(1/\sqrt{2}) = 45^\circ$

(b)

$$1/\sqrt{29}\begin{bmatrix} 2 & 3 & -4 \\ 4 & 2 & -3 \\ 3 & -4 & 2 \end{bmatrix} \xrightarrow{R12} \begin{bmatrix} 12 & 18 & -24 \\ 12 & 6 & -9 \\ 12 & -16 & 8 \end{bmatrix}$$

This is not a rotation matrix because it doesn't fit the matrix structure of any given rotation matrix structure.

Plus the distance between 2 points isn't the same.

$$\sqrt{(2-4)^2+(3-2)^2+(-4-3)^2}$$

$$= \sqrt{4+1+49} = \sqrt{54}$$

$$\sqrt{(4-3)^2+(2--4)^2+(-3-2)^2}$$

$$= \sqrt{1+4+25} = \sqrt{30}$$