Portfolio Optimization

Data – Given Dow Jones index data containing 30 stocks with their monthly prices from 31st December 1990 to 31st December 2015.

```
           TRV      UTX      VZ       WMT      XOM      T
1990-12-31 7.683583 3.471237 7.835626 5.455634 6.233811 4.643250
1991-01-31 7.806031 3.444045 7.167901 5.951600 6.218753 4.387233
1991-02-28 8.112150 3.641321 7.241796 6.379935 6.728324 4.513183
1991-03-31 8.607386 3.549830 7.592801 6.996564 7.140262 4.712602
1991-04-30 8.715557 3.366849 7.480882 7.312538 7.262318 4.539377
1991-05-31 8.004714 3.466449 7.050730 7.741360 7.191589 4.401176
```

Returns were calculated from stock prices

```
                  AA          AAPL        AXP         BA          BAC         CAT
1991-02-28 -0.003884696  0.03370538  0.05494594 -0.01808654  0.03999988  0.097989291
1991-03-31  0.021443165  0.18777315  0.20312426 -0.02590673  0.19835795 -0.123568513
1991-04-30  0.037425309 -0.19117668 -0.12724754 -0.02659574  0.06498192  0.001183947
1991-05-31  0.053703048 -0.14326469  0.02500053  0.07676566  0.14237304  0.086614898
1991-06-30 -0.050966017 -0.11702172 -0.11308280 -0.06632650 -0.14381507 -0.045892790
1991-07-31  0.052223766  0.11445804  0.12777842  0.01366119 -0.02797213  0.001111518
```

Mean (Return) & Standard Deviation (Risk) is checked for all stocks: -

```
           AA         AAPL       AXP        BA         BAC        CAT        CVX
mean 0.007245182 0.02262399 0.01355588 0.01066564 0.01138973 0.01403217 0.009986238
sd   0.099077323 0.13032700 0.09122970 0.07754475 0.10813753 0.09075232 0.056907017
           DD         DIS        GE         HD         HPQ        INTC       IBM
mean 0.009930904 0.01186993 0.01088761 0.01648169 0.01279501 0.01764262 0.009429797
sd   0.073565598 0.07342657 0.07208348 0.07585694 0.10255132 0.10626401 0.079033695
           JNJ        JPM        KO         MCD        MMM        MRK        MSFT
mean 0.0114069   0.01603536 0.01009693 0.01282586 0.01053742 0.009674201 0.01781904
sd   0.0562112   0.09325048 0.05921784 0.06075465 0.05780476 0.073695649 0.09209781
           NKE        PFE        PG         TRV        UTX        VZ         WMT
mean 0.01706877 0.01172667 0.01057837 0.01145600 0.01341299 0.008228268 0.009873398
sd   0.08680258 0.06636159 0.05660330 0.07250083 0.06637466 0.065158146 0.064776105
           XOM         T
mean 0.009558079 0.00892252
sd   0.047296698 0.06516727
```

Optimum Portfolio is obtained using tseries package and portfolio.optim function in R

Weights for optimum portfolio without any constraint

```
          AA            AAPL         AXP          BA           BAC          CAT
-8.774074e-18  4.493958e-02  1.178979e-17  2.681595e-19 -7.188126e-18  3.378018e-17
          CVX           DD           DIS          GE           HD           HPQ
-2.238229e-18 -2.790544e-17 -6.059949e-19  1.703876e-17  9.029541e-02  6.885407e-18
```

INTC	IBM	JNJ	JPM	KO	MCD
2.348037e-02	0.000000e+00	7.950762e-02	4.017237e-17	1.373303e-02	4.609330e-02
MMM	MRK	MSFT	NKE	PFE	PG
6.112024e-02	9.384516e-04	3.257729e-02	5.359473e-02	1.560676e-02	1.719049e-01
TRV	UTX	VZ	WMT	XOM	T
3.520446e-02	-7.038430e-17	2.143405e-03	4.392272e-02	2.450169e-01	3.992076e-02

Expected Return for this portfolio
1.225857%
Standard Deviation or Risk
3.537248%

No Constraints Portfolio Weights
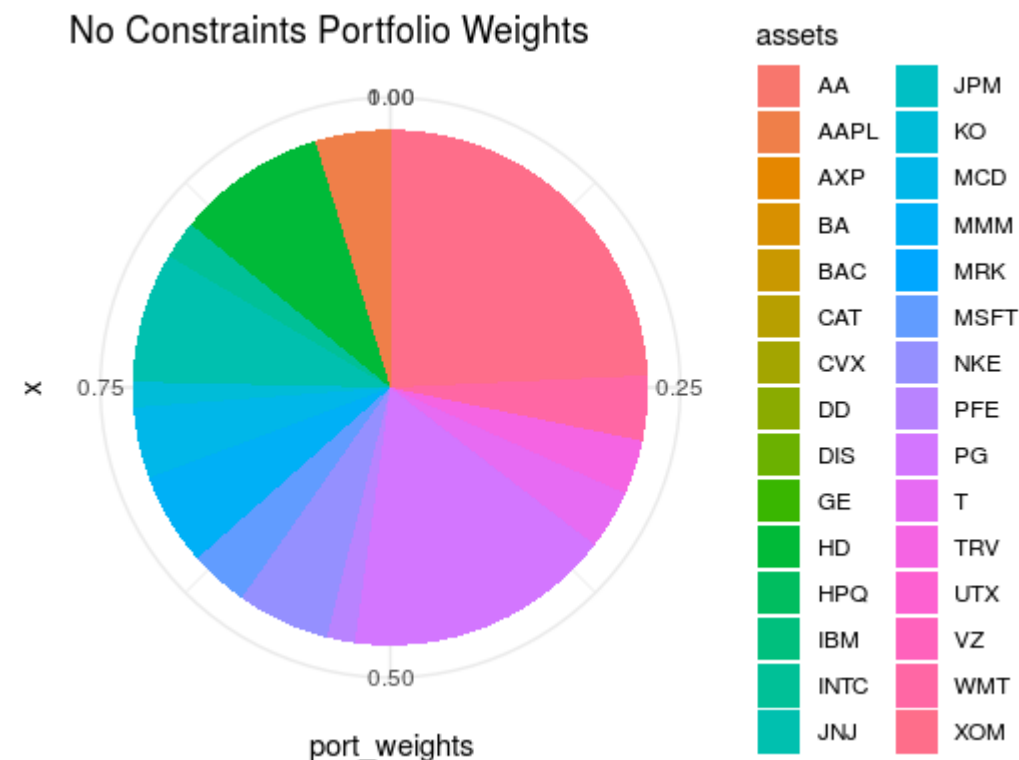
Setting the constraint that portfolio weights should be greater than 1%.

Optimum portfolio weights with constraints.

AAPL	HD	INTC	JNJ	KO	MCD	MMM	MSFT
0.04493958	0.09029541	0.02348037	0.07950762	0.01373303	0.04609330	0.06112024	0.03257729
NKE	PFE	PG	TRV	WMT	XOM	T	
0.05359473	0.01560676	0.17190494	0.03520446	0.04392272	0.24501694	0.03992076	

Now setting constraint for selecting optimum portfolio where **target return is equal to 1.1x of mean** returns and with **max weight constraint of 10%** for a constituent.

Portfolio Weights for these constraints

AA	AAPL	AXP	BA	BAC	CAT
-2.000785e-17	7.186646e-02	1.732583e-17	-2.060169e-18	8.007438e-20	-2.394257e-18

CVX	DD	DIS	GE	HD	HPQ
3.002775e-02	8.410385e-19	-1.780994e-17	-4.677415e-18	1.000000e-01	-1.250519e-17

INTC	IBM	JNJ	JPM	KO	MCD
2.612038e-02	-3.422651e-18	1.000000e-01	7.990097e-03	4.634627e-02	9.557666e-02

MMM	MRK	MSFT	NKE	PFE	PG
7.760438e-02	1.015140e-18	6.167193e-02	9.294919e-02	2.893401e-02	1.000000e-01

TRV	UTX	VZ	WMT	XOM	T
3.369654e-02	-4.280943e-18	2.067032e-18	2.117089e-03	1.000000e-01	2.509925e-02

Expected Return for this portfolio
1.348443%
**Risk / Standard Deviation for this portfolio**
3.865292%

Target return 1.1% mean & 10% max weights Portfolio Weights

port2_weights
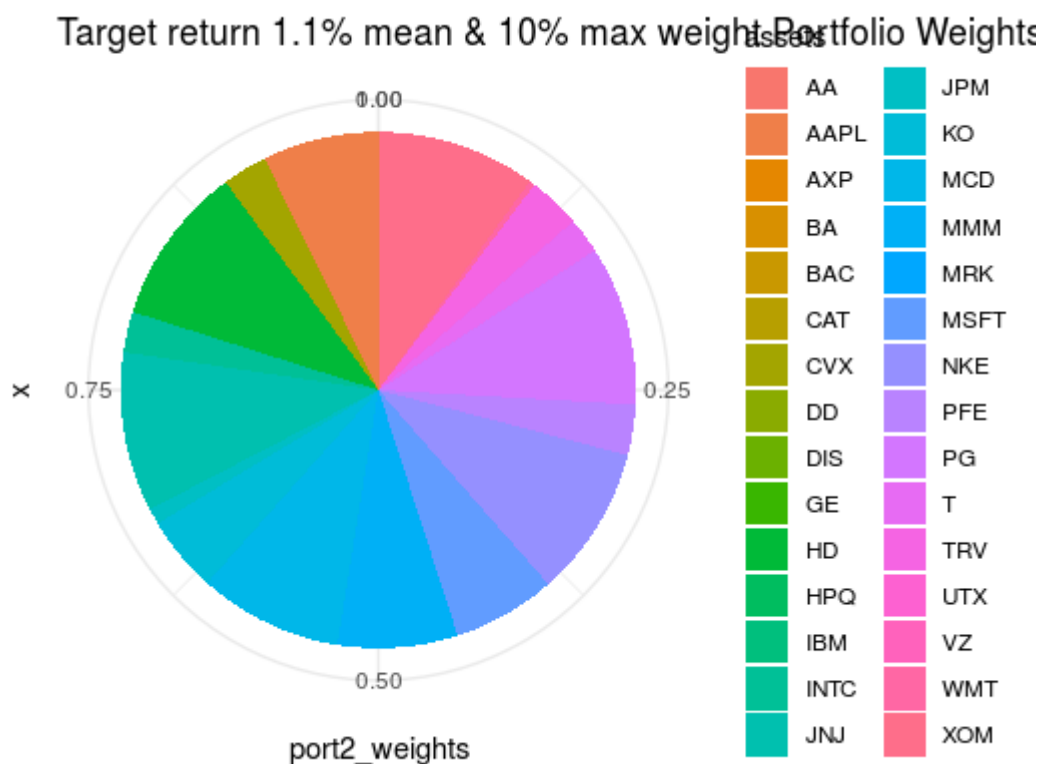
Efficient Frontier

For this three empty vectors are made for storing returns, standard of deviation and portfolio weights. Portfolio mean is varied from 1% (taking it as risk free rate) upto maximum return that can be achieved through any stock.
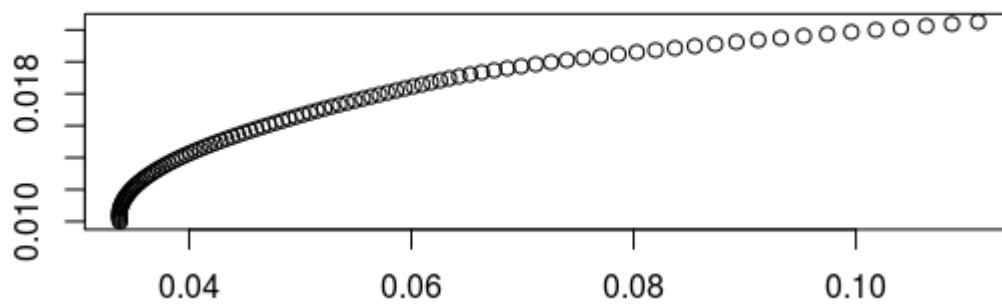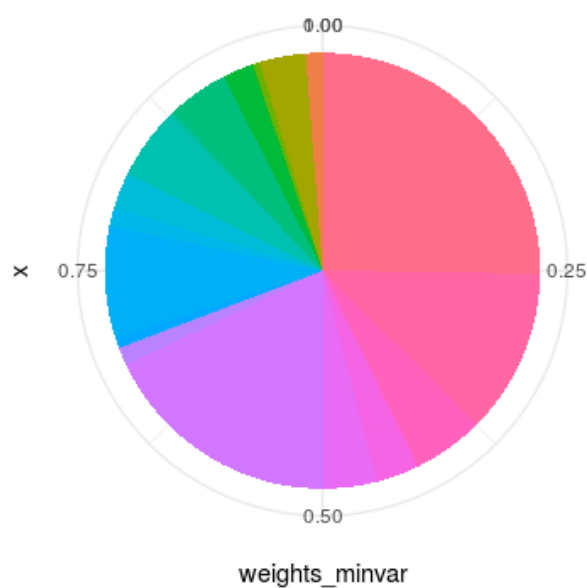
Creating minimum variance portfolio i.e. the portfolio with least risk. This is achieved by selecting that portfolio from efficient frontier for which standard of
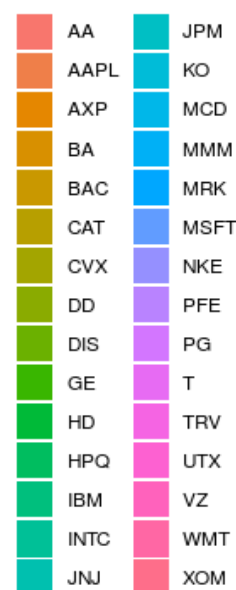deviation is minimum.

Minimum Variance portfolio

Weights

AA	AAPL	AXP	BA	BAC	CAT	CVX
-7.247035e-18	1.227318e-02	1.451541e-17	4.370624e-20	2.279622e-18	2.195571e-17	3.470991e-02

DD	DIS	GE	HD	HPQ	INTC	IBM
-1.450190e-17	5.404810e-03	-2.601505e-17	2.264499e-02	-2.124985e-18	2.849934e-03	4.519279e-02

JNJ	JPM	KO	MCD	MMM	MRK	MSFT
5.306458e-02	-2.164495e-20	2.669361e-02	1.376140e-02	8.330633e-02	7.513871e-03	8.462132e-19

NKE	PFE	PG	TRV	UTX	VZ	WMT
7.347500e-04	1.216935e-02	1.805310e-01	3.233291e-02	-1.089880e-17	5.428611e-02	1.217176e-01

| XOM	T
2.519193e-01	3.889358e-02

Maximum Sharpe Ratio Portfolio

Sharpe Ratio = Return over risk free rate of return / volatility. Therefore, a portfolio having greatest return and least standard of deviation will have maximum Sharpe ratio.

Maximum Sharpe ratio portfolio weights

AA	AAPL	AXP	BA	BAC	CAT	CVX
-4.304996e-17	2.364241e-01	-5.261283e-17	7.652262e-18	-1.978274e-17	-1.300791e-18	-4.927950e-16

DD	DIS	GE	HD	HPQ	INTC	IBM
3.270253e-17	4.547363e-17	9.909675e-17	2.442641e-01	9.741418e-18	2.769621e-02	1.260543e-16

JNJ	JPM	KO	MCD	MMM	MRK	MSFT
-1.119912e-17	4.920898e-02	-1.026131e-17	-1.985529e-18	-1.249557e-17	1.188996e-17	1.695169e-01

NKE	PFE	PG	TRV	UTX	VZ	WMT
2.728897e-01	-4.656686e-18	0.000000e+00	-1.521030e-17	2.768968e-17	-7.448637e-17	7.399158e-17

XOM	T
6.524095e-17	-5.980679e-17

Maximum Sharpe Ratio Portfolio Weights   assets

weights_max_sr
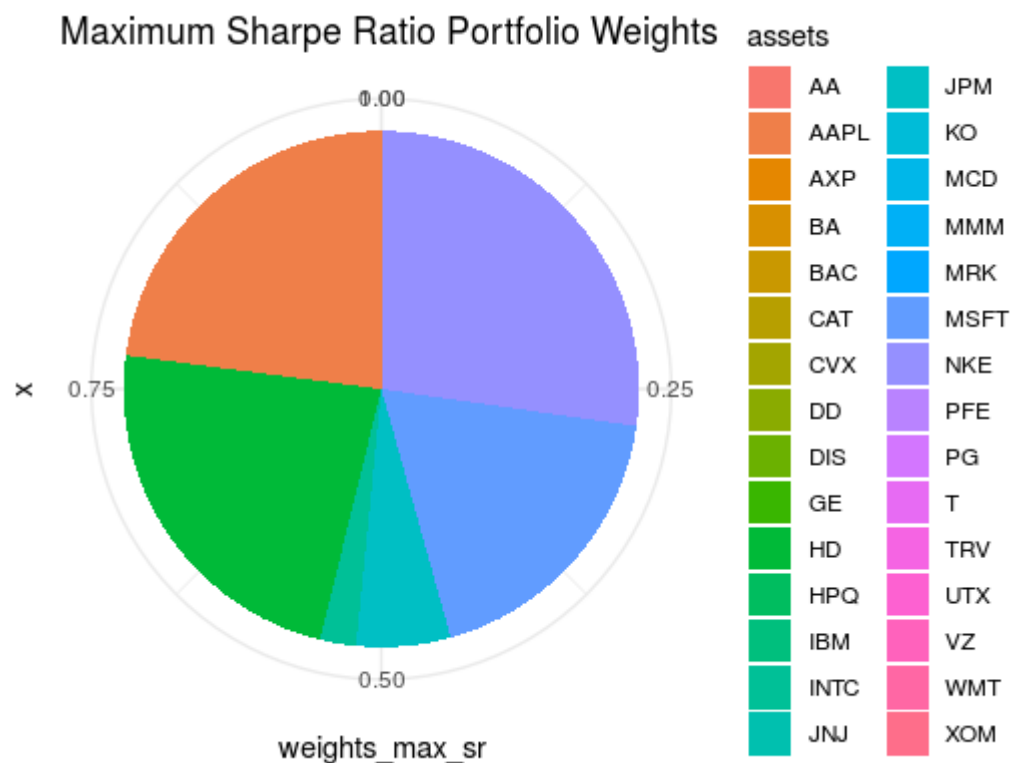
Variance & Expected Shortfall

Variance is calculated using formula = **Mean(portfolio) + Volatility(portfolio)*Z(α)**

We have taken confidence level to be 95% therefore taking z-score corresponding to it as 1.645

VaR (95%) for Minimum Variance Portfolio
6.592583%
Expected Shortfall for Minimum Variance Portfolio
-0.1195465%

VaR (95%) for Maximum Sharpe Ratio Portfolio
 13.15327%
Expected Shortfall for Maximum Sharpe Ratio Portfolio
-0.1195465%