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Week 4 – L'Hôpital's Rule and Integration

Theory of L'Hôpital's rule:

Suppose we have one of the following cases:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \qquad \text{OR} \qquad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$$

(where "*f(x)*" and "*g(x)*" are continuous differential functions and "*a"* can be any real number)

Then in these cases we have

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

So, L'Hôpital's Rule tells us that if we have an indeterminate form 0/0 or ∞/∞ all we need to do is differentiate the numerator and denominator and then take the limit. If it still comes to an indeterminate form, again apply L'Hôpital's rule until you can take its limit.

Example 1: Find $\lim_{x \to 0} \frac{x-\sin(x)}{x^3}$

Solution: -

i. This is a $\frac{0}{0}$ form, so L'Hôpital's rule applies. Thus,

$$\lim_{x \to 0} \frac{x-\sin(x)}{x^3} = \lim_{x \to 0} \frac{1-\cos(x)}{3x^2}$$

ii. This is again $\frac{0}{0}$, so we again apply L'Hôpital's rule. Thus, the limit becomes,

$$\lim_{x \to 0} \frac{x-\sin(x)}{x^3} = \lim_{x \to 0} \frac{1-\cos(x)}{3x^2} = \lim_{x \to 0} \frac{\sin(x)}{6x}$$

iii. This is once again $\frac{0}{0}$. Let's try L'Hôpital's rule again:

$$\lim_{x \to 0} \frac{\sin(x)}{6x} = \lim_{x \to 0} \frac{\cos(x)}{6} = \frac{\cos(0)}{6} = \frac{1}{6}$$

Thus, the limit $\lim_{x \to 0} \frac{x-\sin(x)}{x^3} = \frac{1}{6}$ (Using L'Hôpital's rule 3 times)

Example 2: Find $\lim\limits_{x \to 0} \frac{2\,\text{si}\ \ (x) - \sin(2x)}{x - \sin(x)}$

Solution: -

$\lim\limits_{x \to 0} \frac{2\sin(x) - \sin(2x)}{x - \sin(x)}$ $\left(= \frac{0}{0}\text{, } So\ we\ apply\ L'Hôspital's\ rule\right)$

$= \lim\limits_{x \to 0} \frac{2\cos(x) - 2\cos(2x)}{1 - \cos(x)}$ $\left(= \frac{0}{0}\text{, } So\ we\ again\ apply\ L'Hôspital's\ rule\right)$

$= \lim\limits_{x \to 0} \frac{-2\sin(x) + 4\text{si}\ \ (2x)}{\sin(x)}$ $\left(= \frac{0}{0}\text{, } So\ we\ apply\ L'Hôspital's\ rule\ for\ the\ third\ time\right)$

$= \lim\limits_{x \to 0} \frac{-2\cos(x) + 8\cos(2x)}{cos(x)}$ $\left(\neq \frac{0}{0}\ (or\ any\ other\ indeterminent\ form), So\ wesolve\ the\ limit\right)$

$= \frac{-2+8}{1}$

= 6

Thus, the limit $\lim\limits_{x \to 0} \frac{2\sin(x) - \sin(2x)}{x - \sin(x)}$ = 6 (Using L'Hôpital's rule 3 times)

Reference from:

Theory of L'Hôpital's rule: - http://tutorial.math.lamar.edu/Classes/CalcI/LHospitalsRule.aspx