Modeling the Relationship between Software Effort and Size Using Deming Regression

STAINS

STAtistics in INformation Systems Group

Nikolaos Mittas
Makrina Viola Kosti
Basiliki Argyropoulou
Lefteris Angelis

Programming Languages & Software Engineering Lab
Department of Informatics
Aristotle University of Thessaloniki, Greece
{nmittas, mkosti, vargyrop, lef}@csd.auth.gr

Presentation Outline

- Motivation and Objectives
  - Research questions
- Definitions and Description
- Methodology of Experimentation
  - Dataset
  - Evaluation method
- Results and Conclusions
- Directions of Future Research

The Research Problem in Software Cost Estimation (SCE)

- Accurate prediction of software cost needed
- Plethora of prediction methods
  - Expert judgment to statistical models and machine learning techniques
- The General Research problem
  - Identification of the “best” prediction technique for a certain dataset
- Findings from literature contradictory - no global answer
  - Lack of standardization in SCE methodologies, measurement, reporting techniques, terminology, etc
  - Accuracy depends on dataset

Motivation

- *Regression Analysis* (RA) (especially *Ordinary Least Squares* OLS) → Well-known modeling technique:
  - Various forms of regression for modeling the relationship between effort and size
- Despite the popularity of OLS in SCE → Several restrictions:
  - OLS it is assumed that the values of the independent variable (i.e. *size*) are measured without errors
- The assumption of error-free measurement is not so realistic in SCE:
  - Software size is the result of a counting and estimating process derived from a tool or an expert (*Source Lines of Code* (SLOC) or *Function Points* (FP))

Related Work

- Literature Regression forms
  - Miyazaki et al.(1991)
    - Regression based on relative errors
  - Chen & Stromberg (1997)
    - Heteroscedasticity (LMS, Least Median of Squares – LTS, Least Trimmed Squares)
  - Pickard et al. (1999)
    - RR, Robust Regression – LAD, Least Absolute Deviation
- Inaccuracy of measurement
  - Miyazaki et al.(1994)
  - Foss et al. (2001)

Proposed Model

- Deming regression → Improvement of the process for modeling the relationship between *effort* and *size*
- Deming regression → General class of *errors-in-variables* models:
  - Appropriate in situations where random errors exist in the measurements of both the independent and the dependent variable

Modeling the SCE Procedure

- SCE is the procedure of predicting the cost of a new project
- Let:
  - $Y \rightarrow$ Real random dependent variable representing the **cost** of projects
  - $X \rightarrow$ Real random variable representing the **size** of projects
- Goal to find a regression function

$$y_i = f(x_i) + \varepsilon_i \qquad (i = 1,\ldots,n)$$

  - $\varepsilon_i \rightarrow$ Real random error

Parametric Estimation Technique

- *Ordinary Least Squares* (OLS) Regression

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

- Objective:
  - Minimization of the overall *Sum of Squared Residuals* (SSR)

$$SSR = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

- Final estimation of *constant* (β_0) and *slope* (β_1)

$$\hat{\beta}_1 = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \qquad \hat{\beta}_0 = \bar{y} - \beta_1 \bar{x}$$

where $$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Proposed Model

- Deming Regression:
  - A form of *errors-in-variables* model
  - Takes into account:
    - The error arising from the **dependent** variable (*effort*) (similarly OLS)
    - The error in measurement of the **independent** variable (*size*)

Theoretical Model (1/2)

- Notation
  - $(X_i, Y_i) \rightarrow$ True **unknown** observations
  - $(x_i, y_i) \rightarrow$ Erroneously measured observations
  - $(\varepsilon_i, \delta_i) \rightarrow$ Error terms

- Constant ratio λ of error variances

$$\lambda = \frac{S^2_{e_x}}{S^2_{\delta_y}}$$

- Deming Regression Model

$$x_i = X_i + \varepsilon_i \qquad y_i = Y_i + \delta_i$$

Theoretical Model (2/2)

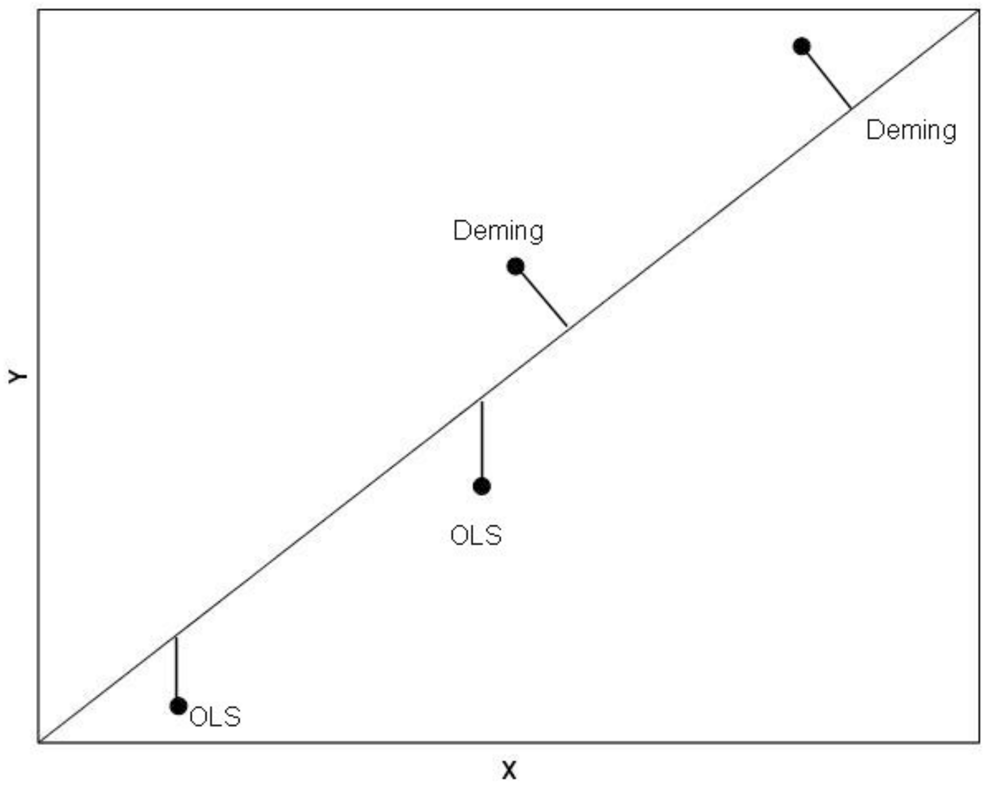
- Deming Regression

$$Y_i = \beta_0 + \beta_1 X_i$$

- Objective:
  - Minimization of the overa weighted *Sum of Square Residuals* (SSR)

$$SSR = \sum_{i=1}^{n}\left(\frac{\varepsilon_i^{\,2}}{S_{\varepsilon_x}^2} + \frac{\delta_i^{\,2}}{S_{\delta_y}^2}\right) =$$

$$\sum_{i=1}^{n}\left((y_i - \beta_0 - \beta_1 X_i)^2 + \lambda(x_i - X_i)^2\right)$$



Estimation of Coefficients

- Final estimation of *constant* (β_0), *slope* (β_1) and X

$$\hat{\beta}_1 = \frac{s_{yy} - \lambda s_{xx} + \sqrt{(s_{yy} - \lambda s_{xx})^2 + 4\lambda s^2_{xy}}}{2 s_{xy}} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$X_i = x_i + \frac{\hat{\beta}_1}{\hat{\beta}_1^2 + \lambda}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$$

where $$s_{xx} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad s_{yy} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

Methodology (Datasets)

- Desharnais
  - 77 completed software projects from a Canadian Software house
  - The project size measured with *FP*s
- COCOMO81
  - COCOMO81 → Public domain database
  - The project size is measured with *SLOC*s
- Maxwell
  - 62 projects from a commercial Finnish bank
  - The project size measured with *FP*s
- Nasa93
  - 93 projects from different centers
  - The project size measured with *SLOC*s

→ For better fitting, size and effort were logarithmically transformed

Methodology (Accuracy measures)

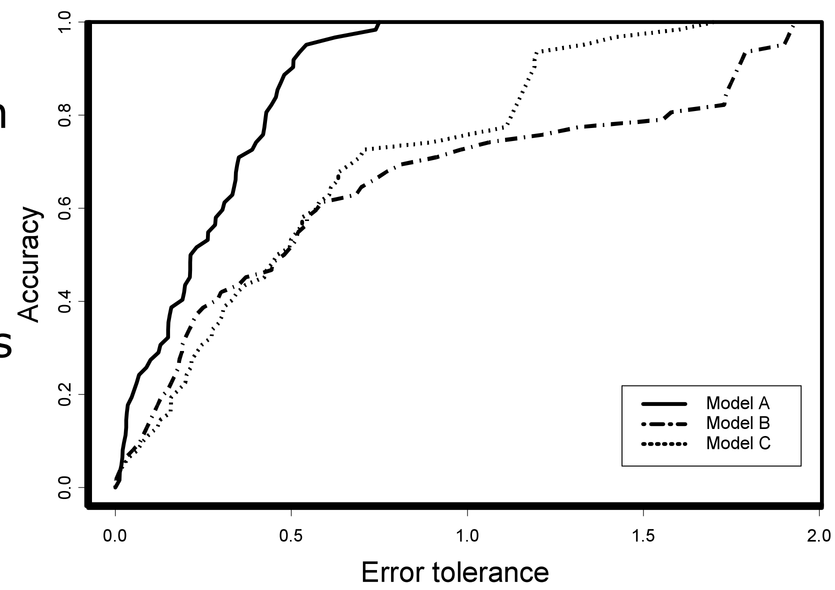
- The predictive accuracy of the cost model is usually based on local measures of error
  - Absolute Error (AE) $$AE_i = \left| Y_{A_i} - Y_{E_i} \right|$$
  - The Magnitude of Relative Error (MRE) $$MRE_i = \frac{\left| Y_{A_i} - Y_{E_i} \right|}{Y_{A_i}}$$
- The global accuracy measures are:
  - Mean and Median Magnitude of Relative Error (MMRE, MdMRE)
  - Mean and Median of Absolute Error (MAE, MdAE)
  - Percentage of projects with MRE≤25% (pred25)

Methodology (Graphical Comparison)

- Graphical analysis through *Regression Errror Characteristic* (REC) curves

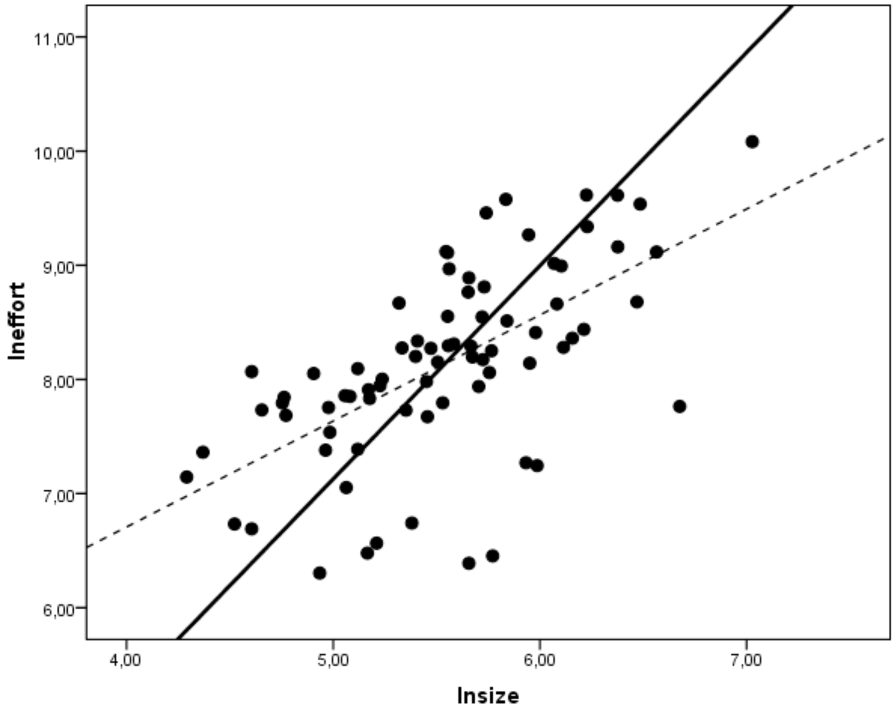
$$accuracy(e) = \frac{\#(\text{projects with error} \leq e)}{\#(\text{projects})}$$

- 2-dimensional plot:
  - x-axis → the error tolerance of a predefined accuracy measure
  - y-axis → the accuracy of a prediction model
- Trade-off between accuracy and tolerance:
  - The accuracy of a model increases as the error tolerance becomes higher
  - *e*=0 → only the predictions that are identical to actual values considered accurate



Desharnais Dataset

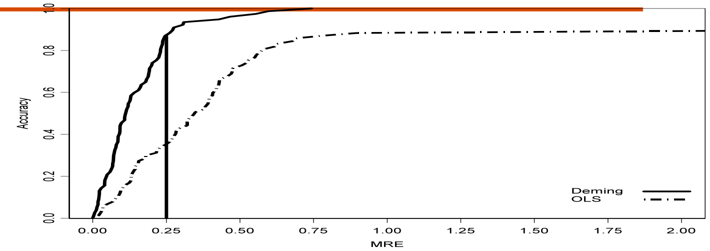
OLS		Deming	
intercept	slope	intercept	slope
2.993	0.929	-2.212	1.868



Deming (solid line) vs. OLS (dashed line)

Accuracy Measures (Desharnais)

	OLS	Deming	Improvement (%)
MAE	2101.13	614.25	70.77%
MdAE	1107.46	335.59	69.70%
MMRE (%)	66.88	15.28	77.15%
MdMRE (%)	34.88	11.30	67.60%
pred25 (%)	35.06	87.01	148.17%

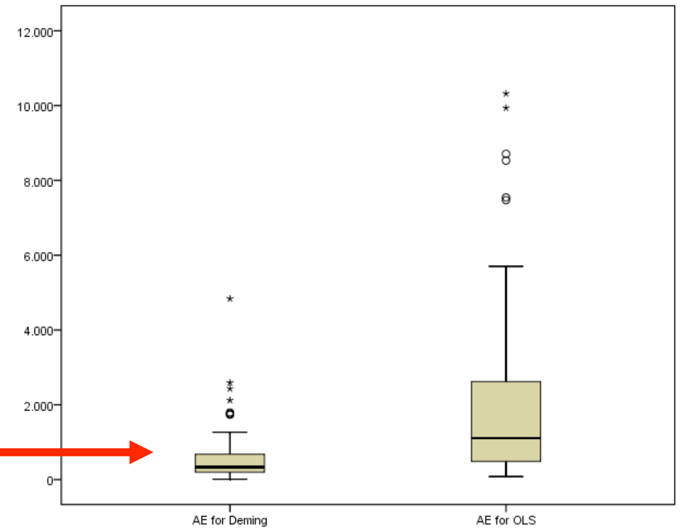


- Deming outperforms for all accuracy measures

- The improvement ranges from 67.60% (MdMRE) up to 148.17% (pred25)

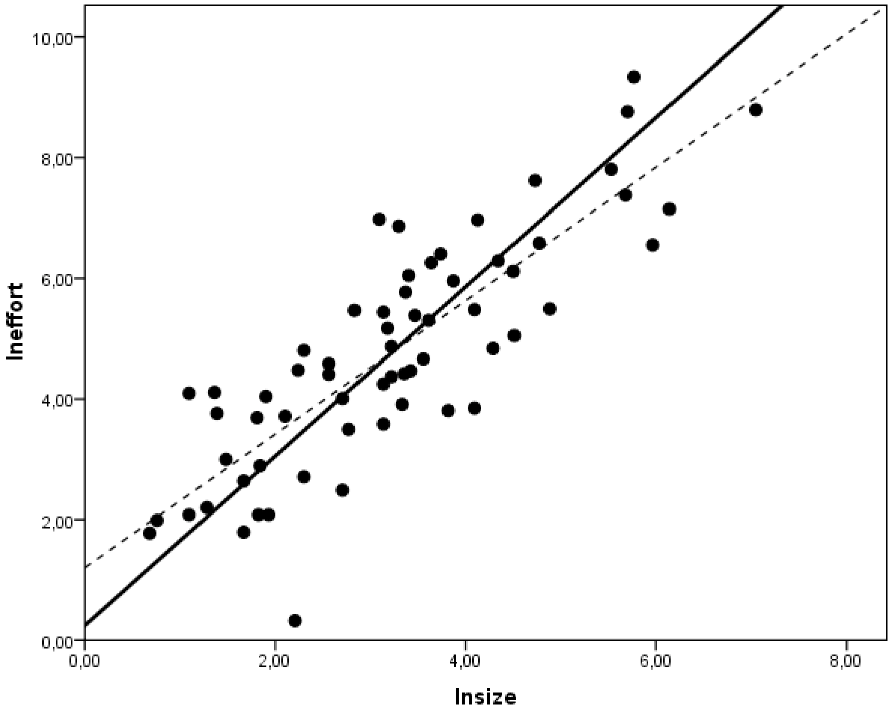
- The Wilcoxon test statistically signifies the difference for AEs

- Error reduction achieved by Deming



COCOMO81 Dataset

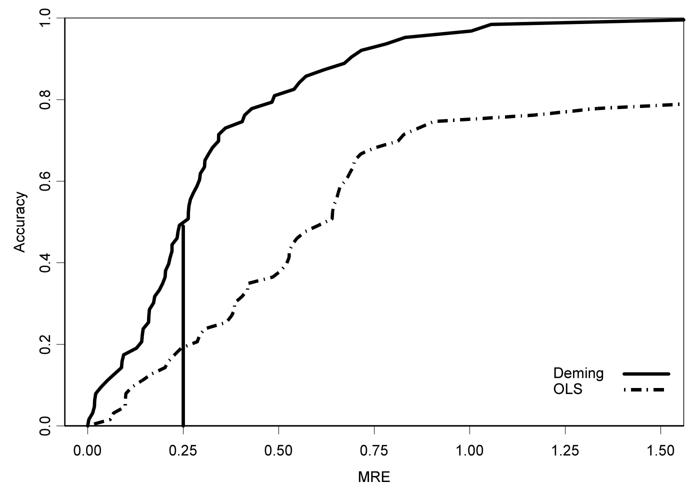
OLS		Deming	
intercept	slope	intercept	slope
1.204	1.106	0.243	1.404



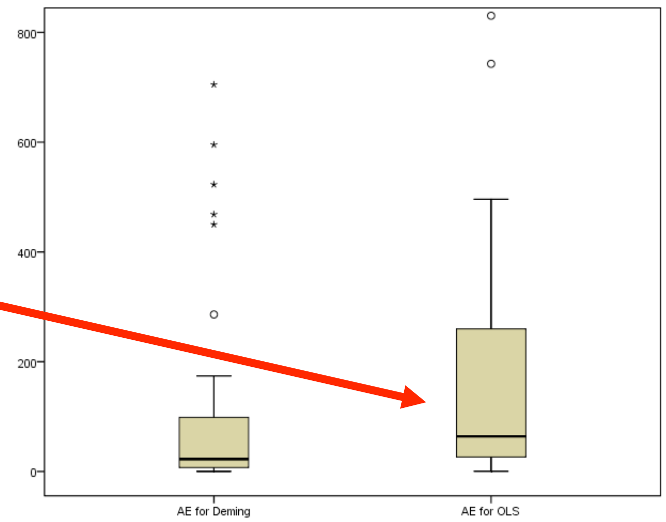
Deming (solid line) vs. OLS (dashed line)

Accuracy Measures (COCOMO81)

	OLS	Deming	Improvement (%)
MAE	455.37	222.22	51.20%
MdAE	63.90	22.47	64.84%
MMRE (%)	137.38	32.99	75.99%
MdMRE (%)	63.97	26.31	58.87%
pred25 (%)	19.05	49.21	158.32%

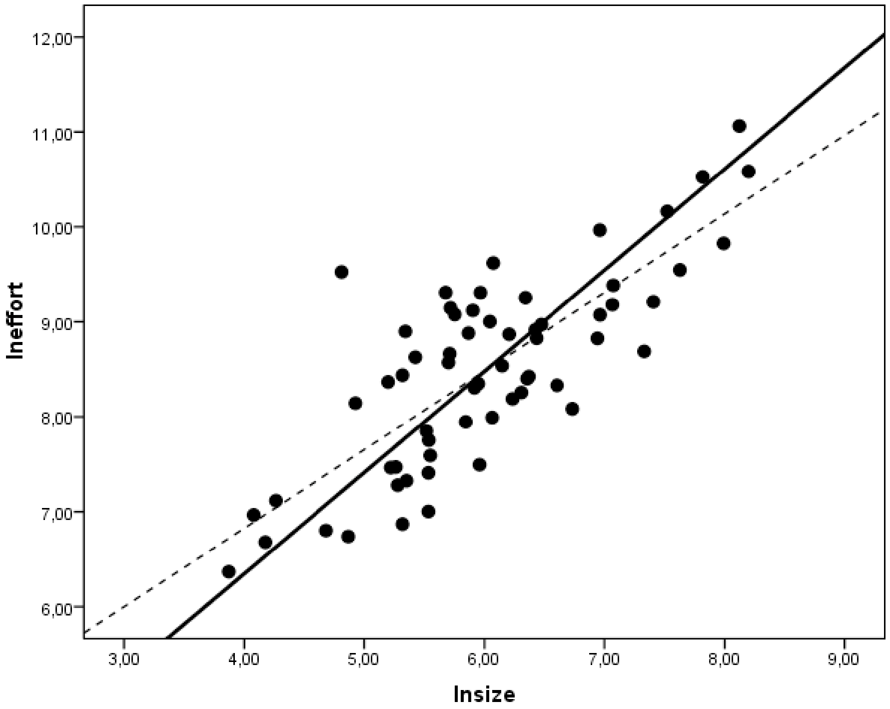


- Deming outperforms for all accuracy measures
- The improvement ranges from 51.20% (MAE) up to 158.32% (pred25)
- The Wilcoxon test statistically signifies the difference for AEs
- AE distribution of OLS → High variability
- REC curve for Deming dominates



Maxwell Dataset

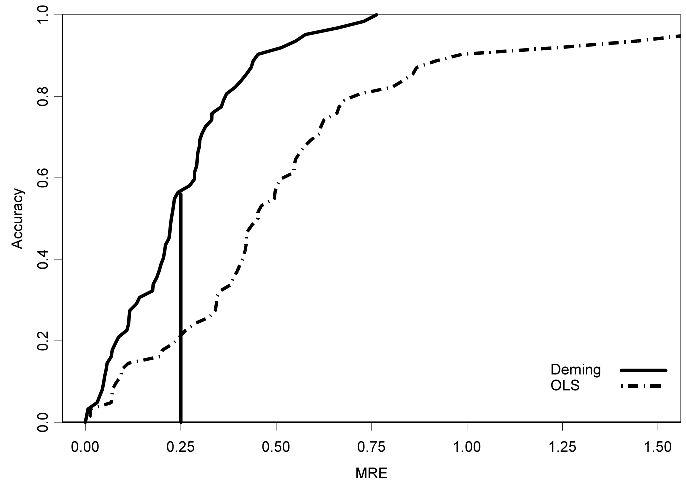
OLS		Deming	
intercept	slope	intercept	slope
3.517	0.827	2.088	1.065



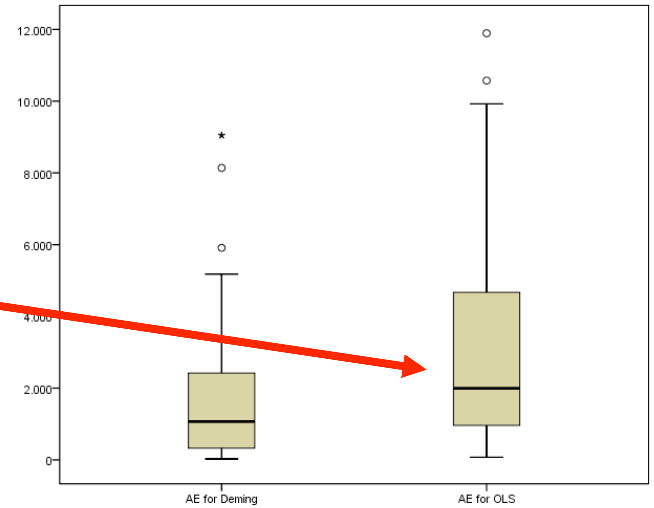
Deming (solid line) vs. OLS (dashed line)

Accuracy Measures (Maxwell)

	OLS	Deming	Improvement (%)
MAE	3766.83	1856.38	50.72%
MdAE	1997.54	1068.19	46.52%
MMRE (%)	55.33	25.46	53.99%
MdMRE (%)	45.22	22.67	49.87%
pred25 (%)	20.97	56.45	169.19%

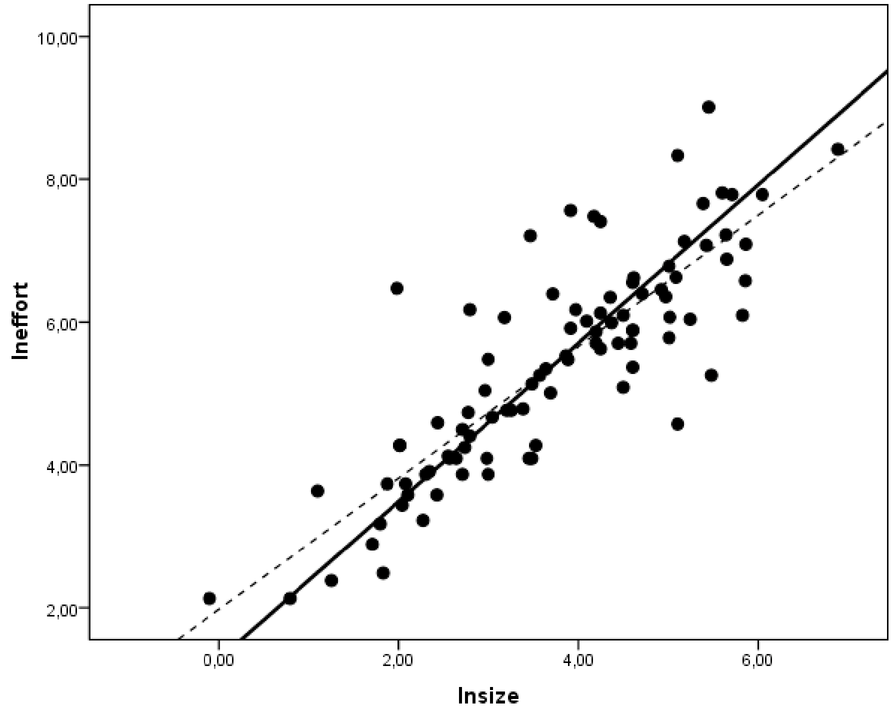


- Deming outperforms for all accuracy measures
- The improvement ranges from 46.52% (MdAE) up to 169.19% (pred25)
- The Wilcoxon test statistically signifies the difference for AEs
- AE distribution of OLS → High variability with a long upper tail
- REC curve for Deming dominates → Solid line climbs rapidly to 1



NASA93 Dataset

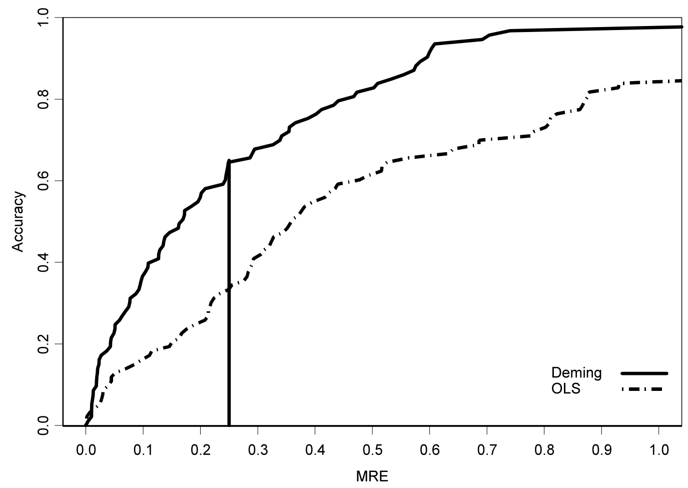
OLS		Deming	
intercept	slope	intercept	slope
1.977	0.920	1.277	1.107



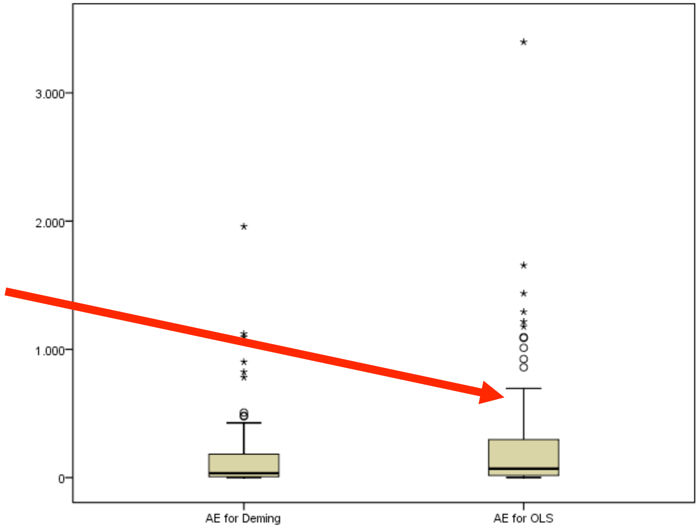
Deming (solid line) vs. OLS (dashed line)

Accuracy Measures (NASA93)

	OLS	Deming	Improvement (%)
MAE	346.51	198.94	42.59%
MdAE	70.34	34.21	51.36%
MMRE (%)	65.79	26.77	59.31%
MdMRE (%)	36.08	16.02	55.60%
pred25 (%)	33.33	64.52	93.58%



- Deming outperforms for all accuracy measures
- The improvement ranges from 42.59% (MdAE) up to 93.58% (pred25)
- The Wilcoxon test statistically signifies the difference for AEs
- AE distribution of OLS → Slightly higher variability
- REC curve for Deming dominates



Conclusions (1/2)

- Study of modeling the relationship between effort and size
- Main idea:
  - OLS is applied under the assumption that the observed values of the variables are measurements which coincide with the true values
  - Not realistic assumption in SCE → Heterogeneous projects with respect to:
    - Nature
    - The way they were measured

Conclusions (2/2)

- Goal of this paper:
  - Application of Deming regression
  - Alternative robust technique → Beneficial:
    - Counting process of the size is characterized by uncertainty due to:
      - Subjective decisions of the practitioners
      - Tools
- Significant improvement compared to OLS:
  - Several accuracy measures
  - Graphical inspection (REC curves, boxplots)
  - Statistical tests (Wilcoxon matched paired)

Future Work (1/2)

- Method deserves a deeper and thorough study
- Construction of *Prediction Intervals* (PI) → “optimistic” and “pessimistic” guess for the true magnitude of the cost:
  - Researchers suggest that PI → Realistic estimate accounting for both uncertainty and risk
  - Under the assumption of error in measurement → Point estimate is meaningless:
    - Expresses not the response to the true size value, but the response to the measured value

Future Work (2/2)

- Introduction of more explanatory (or independent) variables in the model → Increase the percent of variability of the effort that is explained by the cost function
- Examination of the performance of the comparative models to different situations:
  - Systematic treatment trough simulation
  - Errors of the independent variable ranges from a small amount into a high source of variability

Thank You