Computer Vision II - Assignment 1

Group 23: Gustavo Willner 2708177, Pedro Campana 2461919, Helge Meier

Problem 1.1)

Show that: $p(z|y,x) = \frac{p(y|z,x)p(z|x)}{p(y|x)}$
Chain rule
$p(z,y,x) = p(z|y,x)p(y,x)$
$p(z|y,x) = \frac{p(z,y,x)}{p(y,x)}$
$p(y,z,x) = p(y|z,x)p(z,x)$
$p(y|z,x) = \frac{p(y,z,x)}{p(z,x)}$
Insert into initial equation
$\frac{p(z,y,x)}{p(y,x)} = \frac{p(y,z,x)p(z|x)}{p(z,x)p(y|x)}$
Re-arange
$\frac{p(z,x)}{p(y,x)} = \frac{p(z|x)}{p(y|x)}$
Multiply with $\frac{p(x)}{p(x)}$
$\frac{p(z,x)}{p(y,x)}\frac{p(x)}{p(x)} = \frac{p(z|x)}{p(y|x)}\frac{p(x)}{p(x)}$
Chain rule and simplify
$\frac{p(z,x)}{p(y,x)} = \frac{p(z,x)}{p(y,x)}$
q.e.d.

Problem 1.2)

$p(x) = \int p(x,y)dy$

Problem 1.3)

$\mathbb{E}[X,Y] = \int_Y \int_X yxp(x,y)dxdy = \int_Y \int_X yxp(x)p(y)dxdy = \int_Y yp(y) \int_X xp(x)dxdy = \int_Y yp(y)dy \int_X xp(x)dx = \mathbb{E}[Y]\mathbb{E}[X]$

Problem 1.4)

$p(C=R|B=1) = 0.4$
$p(C=W|B=1) = 0.6$
$p(C=R|B=2) = \frac{3}{7}$
$p(C=R|B=2) = \frac{4}{7}$
$p(B=1) = 0.5$
$p(B=2) = 0.5$
$p(C=R) = p(C=R|B=1)\cdot p(B=1) + p(C=R|B=2)\cdot p(B=2) = 0.4\cdot 0.5 + \frac{3}{7}\cdot 0.5 = 0.41428$
$p(B=1|C=R) = \frac{p(C=R|B=1)p(B=1)}{p(C=R)} = \frac{0.4\cdot 0.5}{0.41428} = 0.482758$

Problem 2.1)

Here we make a key assumption about independence. We are assuming that given the disparity, there is a one-to-one correspondence between the pixels from image 0 and the pixels from image 1. We can justify it because in real life each point in the scene typically projects to one point in each of the two images. Certain exceptions such as occlusions are in this case ignored.

Problem 2.2)

One problem is the sensitivity to outliers. In the particular case of the disparity map, this could mean our model would have a hard time with occlusions. Another problem is the assumption of Gaussian noise, which could not be the case. We could be dealing with a particular lighting condition noise in one part of the image or other effects against our brightness constancy assumption. Ideally we would look for more robust likelihood functions. We could for example build a likelihood function based on the General Adaptive Robust Loss Function [Barron, CVPR 2019] by expressing the loss function as a negative log-likelihood of our likelihood function, shifting and normalizing. There are also patch-based likelihood functions such as the normalized cross correlation.

Problem 2.3)

We assume a kind of disparity smoothness, where neighboring pixels should have similar disparity values. That assumption is based on the real life knowledge that points in the same surface tend to have similar depth. In particular, the Markov assumption is that a pixel's disparity value is independent of all other pixels given its 4 neighbours. In the figure d_3 is not in the Markov blanket of d_1 because they are not direct neighbours. We would need d_0, d_2, d_5, c_1, c_3, e_1, e_3. Where "c" is the row above d and "e" the row below.

Problem 2.4)

The problem with the Pott's model is that it's not differentiable. Which means we cannot use the gradient to optimize the log-prior for probabilistic inference of the disparity posterior. We could instead use other robust functions such as the Lorentzian or the Student-t, those are differentiable.