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LAGFD-WAM numerical wave model— I. Basic physical model

Yuan Yeli,[1] Hua Feng,[1] Pan Zengdi[1] and Sun Letao[1]

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Abstract——The LAGFD-WAM wave model is a third generation wave model. In the present paper the physical aspect of the model was shown in great detail including energy spectrum balance equation, complicated characteristics equations and source functions.

INTRODUCTION

Sea waves are a kind of surface undulation with certain wide spectral range of 50~100 m typical space scale and 5~10 s typical time scale. When wind blows over the sea, waves are excited on the surface, they develop in propagating process and tend towards a stationary situation as a result of strong nonlinearity and breaking. According to the achievement of modern sea wave theory the main mechanisms of wave generation and development are as follows:

I) Resonance generation is due to the forcing of air pressure fluctuation (Phillips, 1957).

II) Shear flow instability near the air-sea interface leads to an energy flux from mean flow to waves (Miles, 1957).

III) The nonlinear instability of waves results in an energy transfer from the main frequency wave to the neighbouring ones (Benjamin, 1967; Yuan, 1986).

IV) An energy flux in phase space is born of wave-wave weak interaction (Hasselmann, 1963).

V) Wave breaking is the most typical feature of strong nonlinearity and the main mechanism of wave energy loss (Longuet-Higgins, 1969; Yuan, 1981).

Even though the above theories bring to light the main law of sea wave generation, development and decay, and set up the basis of numerical modelling of sea waves, field experiment is still a key link between modelling and theories. The most important experiments in past years are the air-sea energy input source function (Snyder, 1981) and the duration and fetch dependence of wave spectrum (JONSWAP, 1973).

Sea waves are a random elevation of sea surface with certain spectral structure, and the logical objects of research are its statistical quantities. According to the modelling object, there are two vari-

1. First Institute of Oceanography, State Oceanic Administration, Qingdao 266003, China

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eties of numerical moldels of sea waves: one simulates the statistical parameters such as characteristic wave height and period, the other models the wave spectrum directly.

Wave spectrum modelling approach is based on direct integration of the energy spectrum balance equation:

$$\frac{\partial E}{\partial t} + \boldsymbol{v}_g \nabla E = SS. \tag{1}$$

Phillips' derivation of equilibrium spectrum form (1985) shows the ability of the energy spectrum balance equation which reveals the law of wave spectrum evolution. Hasselmann's parameterization approach for the 6-D Boltzman integral (Hasselmann, 1985) makes the timely computation possible. These have laid foundation for developing the third-generation numerical wave model and its application in operational forecast.

Proceeding from the actual situation of the China seas and computer condition in China, the Laberatory of Geophysical Fluid Dynamics developed a third generation numerical sea wave model—LAGFD-WAM model. In the model, the wave number spectrum balance equation was corrected, more really unsteady ray equation was considered, new dissipation and bottom friction source functions and wave-current interaction source function were introduced and a new computation scheme—characteristic inlaid scheme was designed. The purpose of the paper is to clarify the physical basis of LAGFD-WAM model.

DERIVATION OF THE ENERGY SPECTRUM BALANCE EQUATION

To proceed from the motion equations and boundary conditions of incompressible viscous fluid, the total mean wave energy balance equation can be derived as follows:

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_\alpha}(U_\alpha E + F_\alpha) = - S_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha} + \in_{sb} - \in_{ds}, \tag{2}$$

on the right side of the above equation, the first term is the product of wave momentum flux tensor and deformation tensor of ambient current, this is a measure of energy transfer of wave-current interaction; the second term $\in_{sb}$ shows the rate of the work done by external forces on the surface and bottom; and the third term $\in_{ds}$ shows the energy loss rate due to internal viscosity. In fact the strong nonlinear uncontinuous process such as wave breaking is the main energy dissipation mechanism. In the present paper we will give a mathematical and physical description of the dissipation on the basis of the wave breaking theory. On the left side of the equation, the F_α is the wave energy flux transfered by waves.

In consideration of a wave energy packet $E = E(\boldsymbol{k})\delta A$ on base element

$$\delta A = |\delta \boldsymbol{k}' \times \delta \boldsymbol{k}''|, \tag{3}$$

the corresponding energy balance equation can be written as

$$\frac{\partial}{\partial t}[E(\boldsymbol{k})\delta A] + \nabla[(C_g \mathbf{L}_1]U)E(\boldsymbol{k})\delta A] \\ = - (S(\boldsymbol{k}) : \nabla U)\delta A + \Gamma_N(\boldsymbol{k})\delta A + \Gamma_+(\boldsymbol{k})\delta A - \Gamma_-(\boldsymbol{k})\delta A, \tag{4}$$

where $\mathbf{L}_1 = \{L_{1\alpha}\} = \{\cos\theta_1, \sin\theta_1\}$ is the wave unit vector, $C_g = \frac{1}{2}\frac{\sigma}{K}\left(1 + \frac{2Kd}{\operatorname{sh}2Kd}\right)$, $\sigma =$

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$(gK\mathrm{th}Kd)^{1/2}$, the wave energy flux F_a is $F_a = C_g L_{1a} E$. On the right side of Eq. (4), the first term is the wave-current interaction source function; the second one is the energy redistribution source function due to wave-wave interaction, whose integral on the wave-number space is zero; the third the energy input function and the forth the energy dissipation function.

It is obvious that the base element should be dispersive when the wave packet propagates. Using the motion equations of waves:

$$\frac{\partial \boldsymbol{k}}{\partial t} + \nabla(\sigma + \boldsymbol{k}U) = 0, \tag{5}$$

$$\nabla \times \boldsymbol{k} = 0, \tag{6}$$

we have the relation:

$$\frac{\partial}{\partial t}(\delta \boldsymbol{k}' \times \delta \boldsymbol{k}'') + \nabla \cdot [(C_g \boldsymbol{L}_1 + U)(\delta \boldsymbol{k}' \times \delta \boldsymbol{K}'')] = 0, \tag{7}$$

and its vertical projection equation:

$$\frac{\partial}{\partial t}\delta A + \nabla[(C_g \boldsymbol{L}_1 + U)\delta A] = 0, \tag{8}$$

with (4) $\sim$ (8) $\cdot\, E(\boldsymbol{k})$ and eliminating the base element, we can get the energy spectrum balance equation in wave-number space:

$$\{\frac{\partial}{\partial t} + (C_g \boldsymbol{L}_1 + U)\nabla\}E(\boldsymbol{k}) = - S(\boldsymbol{k}) : \nabla U + \Gamma_{\mathrm{N}} + \Gamma_{+}(\boldsymbol{k}) - \Gamma_{-}(\boldsymbol{k}). \tag{9}$$

THE COMPLICATED CHARACTERISTIC EQUATION

Characteristic equation describes the propagation law of the waves, it can be written as

$$\frac{\mathrm{d}x_a}{\mathrm{d}t} = (C_g L_{1a} + U_a). \tag{10}$$

The variation law of the mode and angle of the wave- number is governed by the following equations:

$$\frac{\partial k_1}{\partial t} + (C_g L_{1a} + U_a)\frac{\partial k_1}{\partial x_a} = -\left\{\frac{\partial \sigma}{\partial D}\frac{\partial D}{\partial x_1} + k_a \frac{\partial U_a}{\partial x_1}\right., \tag{11}$$

$$\frac{\partial k_2}{\partial t} + (C_g L_{1a} + U_a)\frac{\partial k_2}{\partial x_a} = -\left\{\frac{\partial \sigma}{\partial D}\frac{\partial D}{\partial x_2} + k_a \frac{\partial U_a}{\partial x_2}\right., \tag{12}$$

which can be derived by using wave motion Eqs (5) and (6).

Noticing $k_1 = K\cos\theta_1$, $k_2 = K\sin\theta_1$,with (11) $\cos\theta_1 +$ (12) $\sin\theta_1$, the variation equations of wave-number mode and angle can be reduced as below:

$$\frac{\partial K}{\partial t} + (C_g \boldsymbol{L}_1 + U_a)\nabla K = -\left\{\frac{\partial \sigma}{\partial D}\frac{\partial D}{\partial s} + K\boldsymbol{L}_1 \frac{\partial U}{\partial s}\right., \tag{13}$$

$$\frac{\partial \theta_1}{\partial t} + (C_g \boldsymbol{L}_1 + \boldsymbol{U}_a)\nabla\theta_1 = -\frac{1}{K}\left\{\frac{\partial \sigma}{\partial D}\frac{\partial D}{\partial n_1} + K\boldsymbol{L}_1 \frac{\partial U}{\partial n_1}\right., \tag{14}$$

where an increases with $\boldsymbol{L}_1 = \cos\theta_1 i + \sin\theta_1 j$ and n_1 with $N_1 = -\sin\theta_1 i + \cos\theta_1 j$.

Equations (10), (13) and (14) are the complicated characteristic equations describing the effect of unsteady depth and current on the wave propagation.

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DETERMINATION OF THE SOURCE FUNCTIONS

As we have pointed out above, with the recent development of wave spectrum theory and the third generation wave model, people firmly believe it possible to simulate the wave generation and development by directly integrating the energy balance equation. In the following section, we should introduce the source functions of four aspects: input, dissipation, wave-wave interaction and wave-current interaction, which are used in the LAGFD-WAM model.

Input source function

On the basis of the Phillips' resonance and Miles' shear flow instability mechanisms, the input source function can be written as

$$S_{in} = \alpha + \beta E(\boldsymbol{k}). \tag{15}$$

According to Willmarth's measurement (1962) in wind-wave tank, the coefficient a in (15) is

$$\alpha(k) = 80\left(\frac{\rho_a}{\rho_w}\right)^2 \frac{u_*^4 \sigma}{g^2 K^2} \cos^4(\theta_1 - \Theta) H[\cos(\theta_1 - \Theta)], \tag{16}$$

where $\rho_a/\rho_W = 1.25 \times 10^{-3}$, u_* is the friction velocity on the sea surface, Θ is the wind going derection, $H(\cdot)$ is the Heaviside function. According to Wu's measurement (1982), the drag coefficient C_d is

$$C_d = \left(\frac{u_*}{W}\right)^2 = (0.8 + 0.065W) \times 10^{-3}. \tag{17}$$

In light of Snyder's field measurement, the coefficient β can be written as

$$\beta = 0.25\frac{\rho_a}{\rho_w}\sigma[28\frac{u_*}{c}\cos(\theta_1 - \Theta) - 1]H[28\frac{u_*}{c}\cos(\theta_1 - \Theta) - 1]. \tag{18}$$

Dissipation source function

The energy dissipation of sea waves is mainly caused by wave breaking and bottom friction.

Wave breaking is a very complicated process, theoretical derivation of the energy loss rate is an outstanding problem. On the basis of similarity theory, Komen (1984), choosing nondimensional frequency ($\omega/\tilde{\omega}$) and normalized wave slope ($\tilde{\alpha}/\tilde{\alpha}_{pm}$) as the controlling nondimensional paramters, took the source function with the following form:

$$S_{ds}(\boldsymbol{k}) = - a_{ds}\tilde{\sigma}\left(\frac{\sigma}{\tilde{\sigma}}\right)^n \left(\frac{\tilde{\alpha}}{\tilde{\alpha}_{pm}}\right)^m E(\boldsymbol{k}). \tag{19}$$

Through computational fitting he got the coefficients below: $n = 2$, $m = 2$, $a_{ds} = 2.33 \times 10^{-5}$, $\tilde{\alpha}_{pm} = 3.02 \times 10^{-3}$, the practical form of dissipation source function is

$$S_{dsk}(\boldsymbol{k}) = - 2.59\tilde{\sigma}\left(\frac{K}{\tilde{K}}\right)(\tilde{E}\tilde{K}^2)^2 E(\boldsymbol{k}), \tag{20}$$

where

$$\tilde{\sigma} = \left[\iint E(\boldsymbol{k})\sigma^{-1}\mathrm{d}\boldsymbol{k}\frac{\theta}{\tilde{E}}\right]^{-1}, \quad \tilde{K} = \frac{\widetilde{\sigma^2}}{g},$$

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$$\bar{E}=\iint E(\boldsymbol{k})\mathrm{d}\boldsymbol{k}, \qquad \tilde{\alpha}=\bar{E}\tilde{\sigma}^4 g^{-2}=g\bar{E}\tilde{K}^2. \tag{21}$$

In line with our breaking spectrum theory (Yuan *et al.*, 1981), supposing that a field breaking event statistically occurs in the mean period between successive maxima and using the spectral relation derived by us, we can get the breaking energy dissipation source function as follows:

$$S_{\mathrm{ds}}=-d_1\tilde{\sigma}\left(\frac{\sigma}{\tilde{\sigma}}\right)^2\left(\frac{\tilde{\alpha}}{\tilde{\alpha}_{\mathrm{pm}}}\right)^{1/2}\exp\left\{-d_2(1-\epsilon^2)\frac{\tilde{\alpha}_{\mathrm{pm}}}{\tilde{\alpha}}\right\}E(\boldsymbol{k}). \tag{22}$$

It is very interesting that besides the nondimensional frequency ($\sigma/\tilde{\sigma}$) and the normalized wave slope ($\tilde{\alpha}/\tilde{\alpha}_{\mathrm{pm}}$), the expression depends on the spectral width ϵ as well, the relationship with ($\tilde{\sigma}/\tilde{\sigma}$) is the same as Komen's and that with ($\tilde{\alpha}/\tilde{\alpha}_{\mathrm{pm}}$) is in 1/2 but 2 power. This is a good improvement for the weakness of WAM model which underestimates of the large waves, we will discuss this in part Ⅲ of our series papers.

The second main dissipation mechanism is the bottom friction effect in shallow water. In wave number space the quasi-linear form of the source function corresponding to the bottom friction $\tau_{\mathrm{b}}=C_{\mathrm{b}}V|V|$ (Yuan, 1990) is

$$S_{\mathrm{bo}}=-C_{\mathrm{b}}\frac{8K}{\mathrm{sh}2Kd}\tilde{\sigma}\,\bar{E}^{1/2}E(\boldsymbol{k}), \tag{23}$$

in the LAGFD-WAM model we take $C_{\mathrm{b}}=2.5\times10^{-3}$.

Wave-wave nonlinear interaction source function

In 1962 Hasselmann first discussed the nonlinear energy transfer among the wave spectral components in a growing sea. The expression of the source function was derived as

$$\begin{aligned}S_{\mathrm{nl1}}(\boldsymbol{k})=\sigma\iiint &A(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k})[N_1N_2(N_3+N)\\&-N_3N(N_1+N^2)]\times\delta(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k})\times\\&\delta(\sigma_1+\sigma_2-\sigma_3-\sigma)\mathrm{d}\boldsymbol{k}_2\mathrm{d}\boldsymbol{k}_2\mathrm{d}\boldsymbol{k}_3,\end{aligned} \tag{24}$$

where $A(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k})$ is the wave-wave interaction function, $N(\boldsymbol{k})\equiv E(\boldsymbol{k})/\sigma$ is the action spectrum density.

$R(KD)$ is the shallow water factor obtained by JONSWAP data fitting:

$$R(KD)=1+\frac{5.5}{KD}\left(1-\frac{5KD}{6}\right)\exp\left(-\frac{5KD}{4}\right). \tag{25}$$

The practical nonlinear interaction source function is

$$S_{\mathrm{nl}}=R(KD)S_{\mathrm{nl1}}(\boldsymbol{k}). \tag{26}$$

Owing to the fact that (24) is a 6-D Boltzman integral its computation needs about $o(10^2)$ times of the computer hour for the other part of the model. On the basis of the Hasselmann's computational test, we designed a parameterized approach in wave number space in part Ⅱ of our series papers.

Wave-current interaction source function

Current, as an ambient field of wave propagation, not only refracts waves but also transfers energy to waves. According to Eq. (4) it is

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$$S_{cu} = - S_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha}, \tag{27}$$

where
$$S_{\alpha\beta} = - \tau_{\alpha\beta} + \left[\frac{C_g}{c}l_\alpha l_\beta + \frac{1}{2}\left(\frac{2C_g}{c} - 1\right)\delta_{\alpha\beta}\right]E\ (\boldsymbol{k}) \tag{28}$$

in the case of small amplitude (Phillips, 1977).

So the practical form of wave-current interaction source function is

$$\begin{aligned} S_{cu} = &- \left\{ \left[\frac{C_g}{c}(1 + \cos^2\theta_1) - \frac{1}{2}\right]\frac{\partial U_x}{\partial x} \right. \\ &+ \frac{C_g}{c}\sin\theta_1\cos\theta_1\left(\frac{\partial U_x}{\partial y} + \frac{\partial U_y}{\partial x}\right) \\ &+ \left. \left[\frac{C_g}{c}(1 + \sin^2\theta_1) - \frac{1}{2}\right]\frac{\partial U_y}{\partial y}\right\} E(\boldsymbol{k}) \end{aligned} \tag{29}$$

when the ambient current field has 25 km typical space scale and 10 s time scale, the source function for the waves of 100 m typical space scale and 10 s time scale has the same order of magnitude as that of dissipation term.

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