Problem 1

Question:

Calculate and compare the expected value and standard deviation of price at time t(Pt) , given each of the 3 types of price returns, assuming $rt \sim N$(0,σ2). Simulate each return equation using $rt \sim N$(0,σ2)and show the mean and standard deviation match your expectations.

Solution:

The three types are:

Classic Brownian Motion: P1 ~ N(P0,σ^2) , mean: P0 , std: σ

Arithmetic Return System: P1 ~ N(P0,P0^2σ^2) , mean: P0 , std: P0*σ

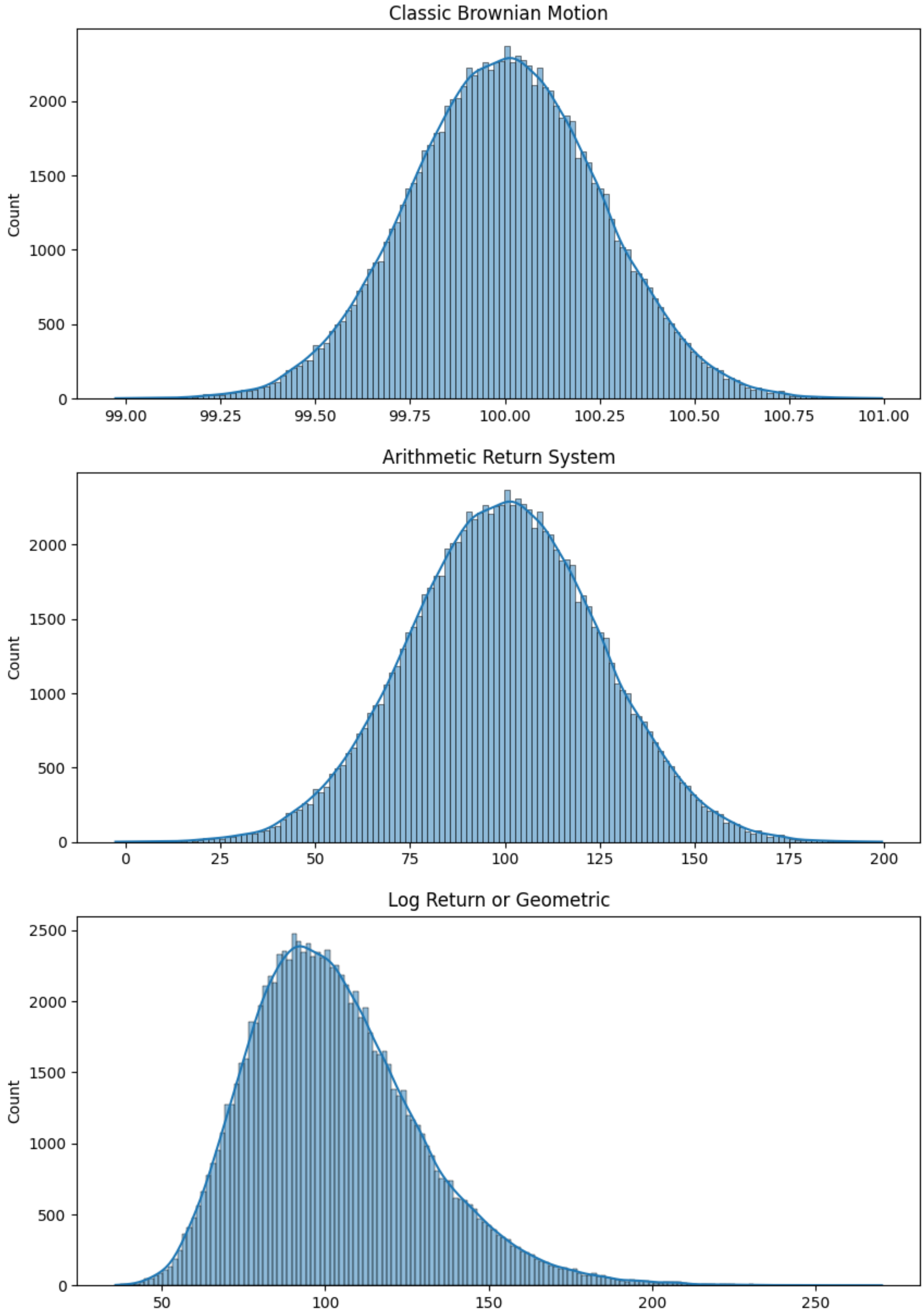
Log Return or Geometric Brownian Motion: P1 ~ LN(P0,σ^2)

mean: P0*e^(0.5*σ^2) , std: P0*e^(0.5*σ^2) *$\sqrt{-1 + e^{\sigma}}$.

In my code, I assumed the P0=100, sigma=0.25. The results of these three types are as follows:

```
mean of brownian: 99.99973 std of brownian: 0.249
Theoretical mean of brownian: 100.00000 Theoretical std of brownian: 0.250
mean of Arithmetic Return: 99.97321 std of Arithmetic Return: 24.936
Theoretical mean of Arithmetic Return: 100.00000 Theoretical std of Arithmetic
Return: 25.000
mean of log return : 103.12950 std of log return: 26.109
Theoretical mean of log return: 103.17434 Theoretical std of log return: 26.202
```

Method	Mean		Standard deviation	
	Expected	Simulated	Expected	Simulated
Classic Brownian Motion	100.0000	99.99973	0.250	0.249
Arithmetic Return System	100.0000	99.97321	25.000	24.936
Log Return	103.17434	103. 12950	26.202	26.109



We can see from the above data that the simulated data almost match the expected data.

Problem 2

Question:

Implement a function similar to the “return_calculate()” in this week’s code.Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

##Break the returns for META into 2 groups, a modeling group and a holdout sample.

Make the holdout sample be the last 60 observations.

Remove the mean from the series so that the mean(META)=0

Calculate VaR

1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ( )λ=0.94
3. Using a MLE fitted T distribution.
4. Using a fitted AR(1) model.
5. Using a Historical Simulation.

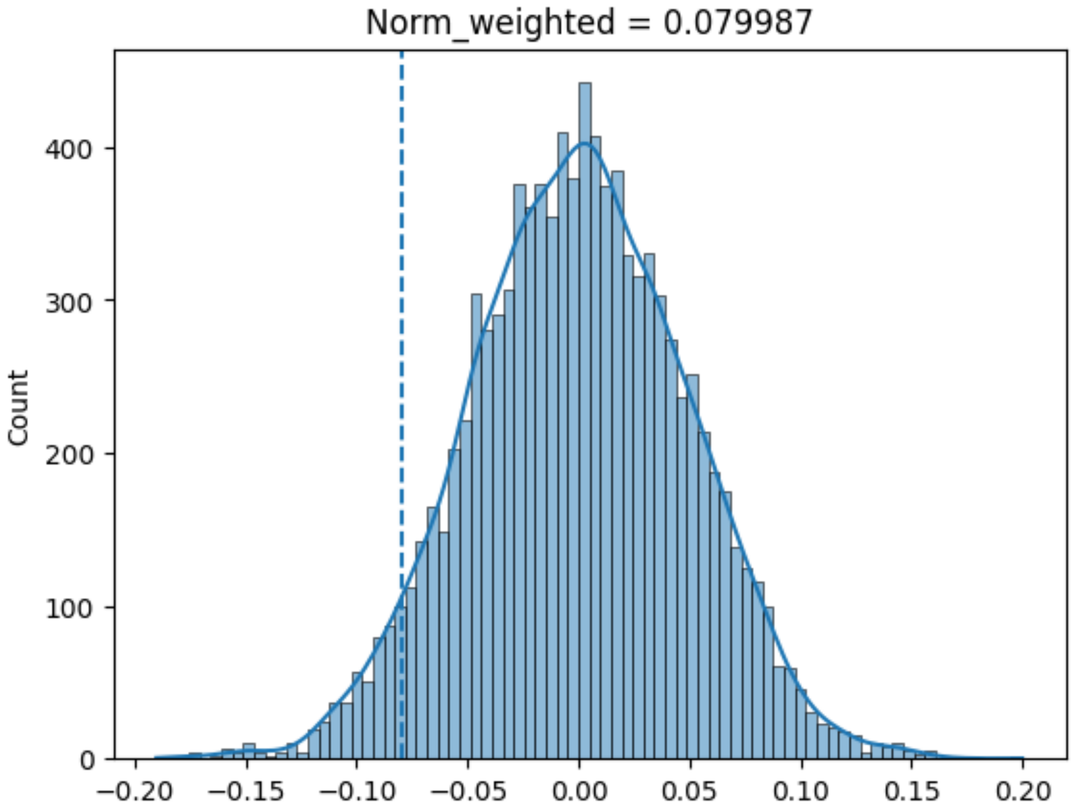
Compare the 5 values.

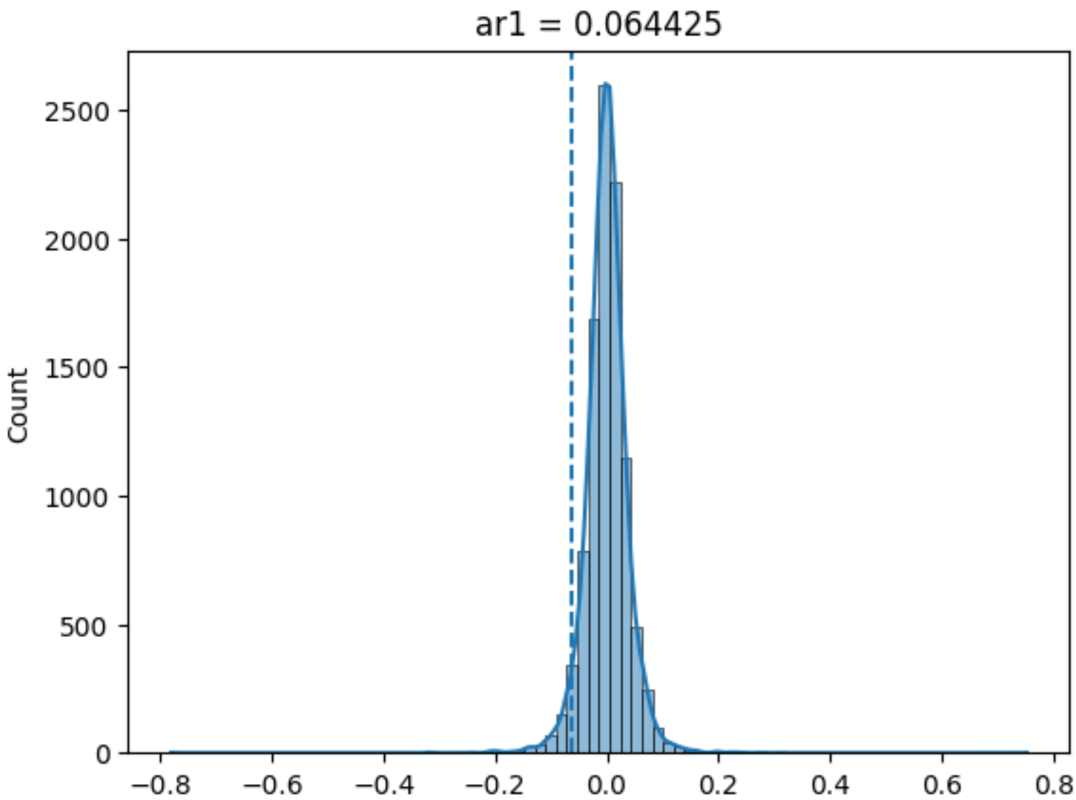
Solution:

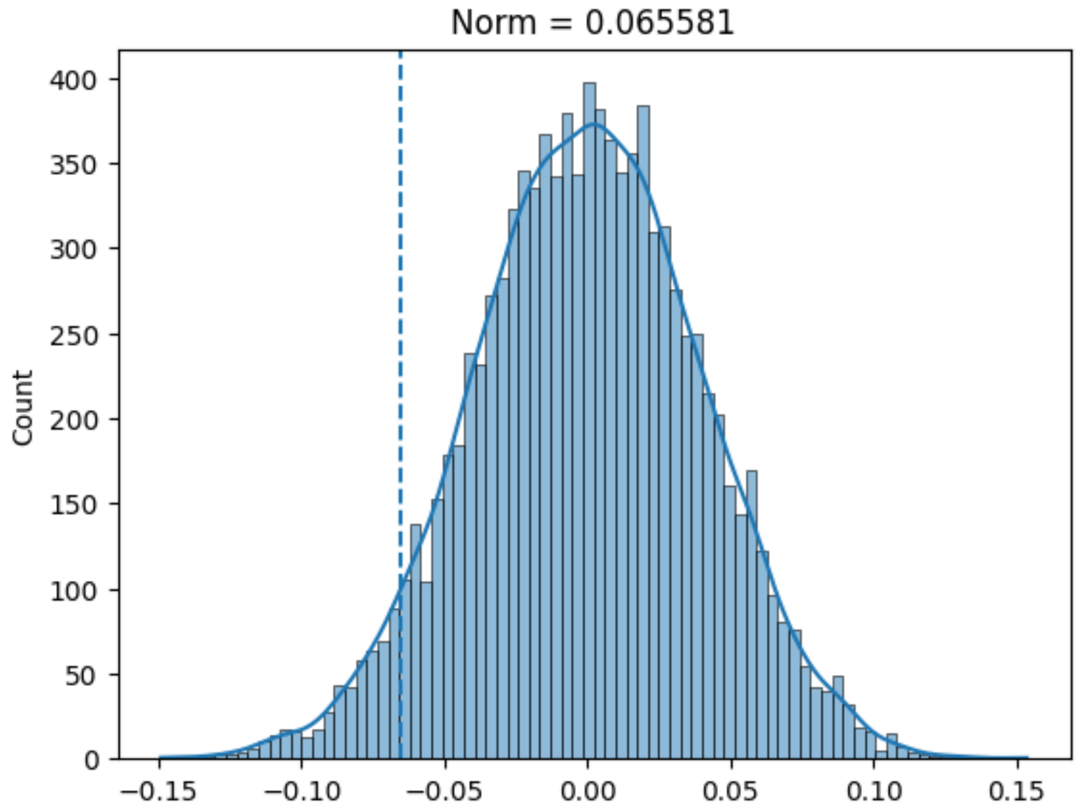
I used the DailyPrices.csv to process data and calculated VaR of five ways.

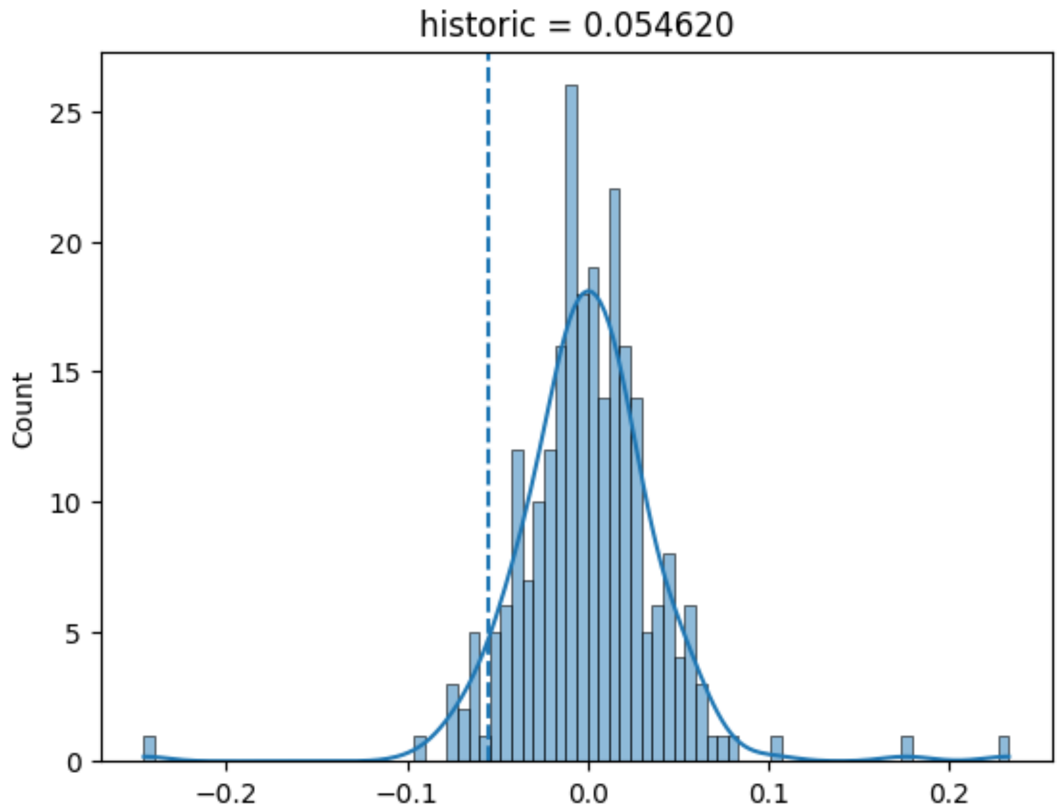
Method	**VAR**	**Dollar Loss**
Normal distribution	**6.55%**	**11.671128**
Normal distribution with EW=0.94	**8.00%**	**14.234133**
MLE fitted T distribution	**5.79%**	**10.305263**
AR(1) model	**6.44%**	**11.465331**
Historical Simulation	**5.46%**	**9.720905**

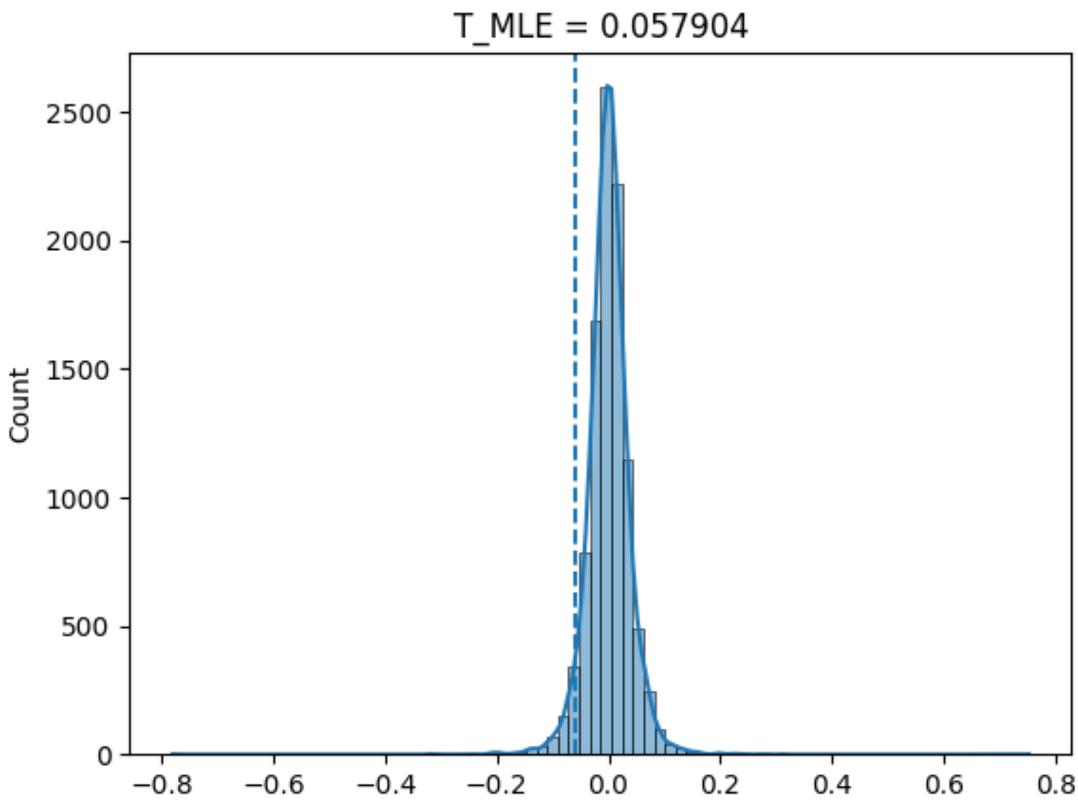
From the data above, we noticed that the value of historical simulation and MLE fitted T distribution are pretty similar, Fitted AR(1) model and Normal distribution are similar. The value of Normal distribution with EW=0.94 is the higest one because of META.


Norm_weighted = 0.079987
Count
400
300
200
100
0
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20


ar1 = 0.064425
Count
2500
2000
1500
1000
500
0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8


Norm = 0.065581
Count
400
350
300
250
200
150
100
50
0
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15


historic = 0.054620
Count
25
20
15
10
5
0
−0.2
−0.1
0.0
0.1
0.2



Problem 3

Question:

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.This file contains the stock holdings of 3 portfolios. You own each of these portfolios.

Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings).

Express VaR as a $.Discuss your methods and your results.

Choose a different model for returns and calculate VaR again.

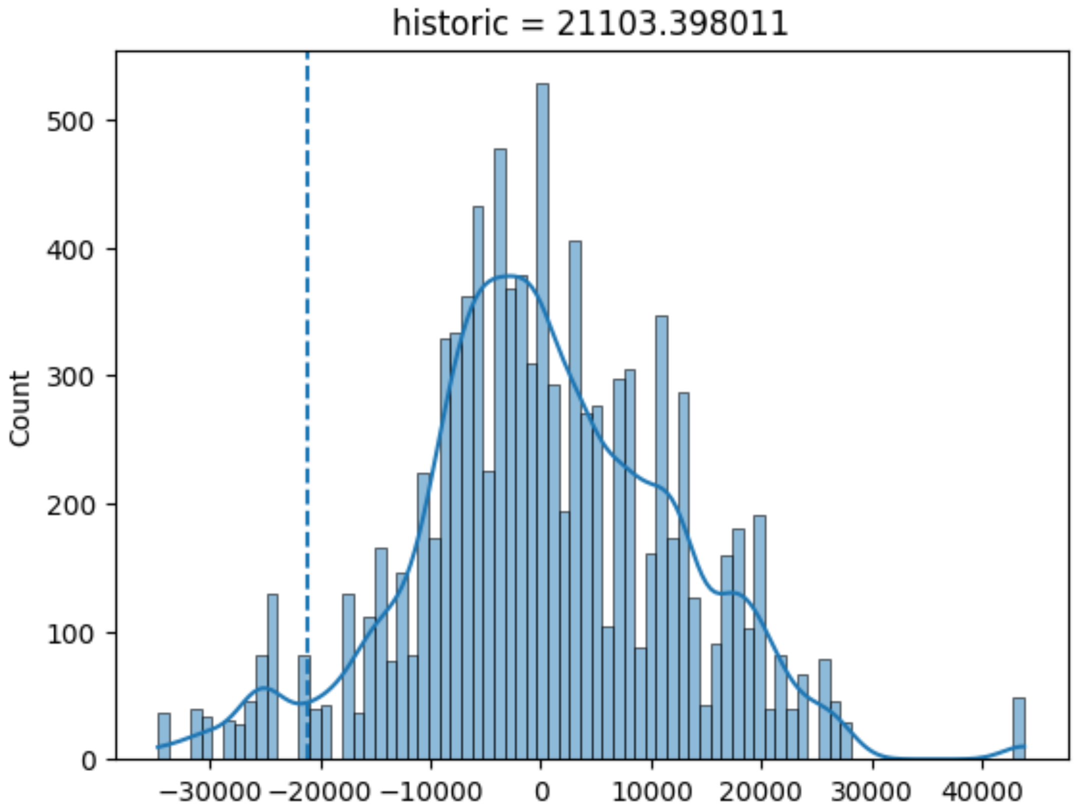
Why did you choose that model?How did the model change affect the results?

Solution:

I assumed that the returns were normality distributed and linear at first, and I calculated the VaR by Delta Normal. Then, I used Monte Carlo to recalculate the VaR. The results of these two simulations are almost the same .So, I choose historical simulation to calculate VaR again because the returns are generally not distributed or always linear. Historical simulation is based on past data, real data, which can emilite the problem of the previous two ways. We can see from the data in the following and find that the VaR from historical method are higher than other two ways.

Portfolio	Normal	Monte Carlo	Historic
Total	13577.0771	13308.9016	21103.3980
A	5670.2035	5614.9593	9005.0672
B	4494.5990	4475.6403	7273.7691
C	3786.5895	3732.2504	5773.4722

TOTAL


historic = 21103.398011
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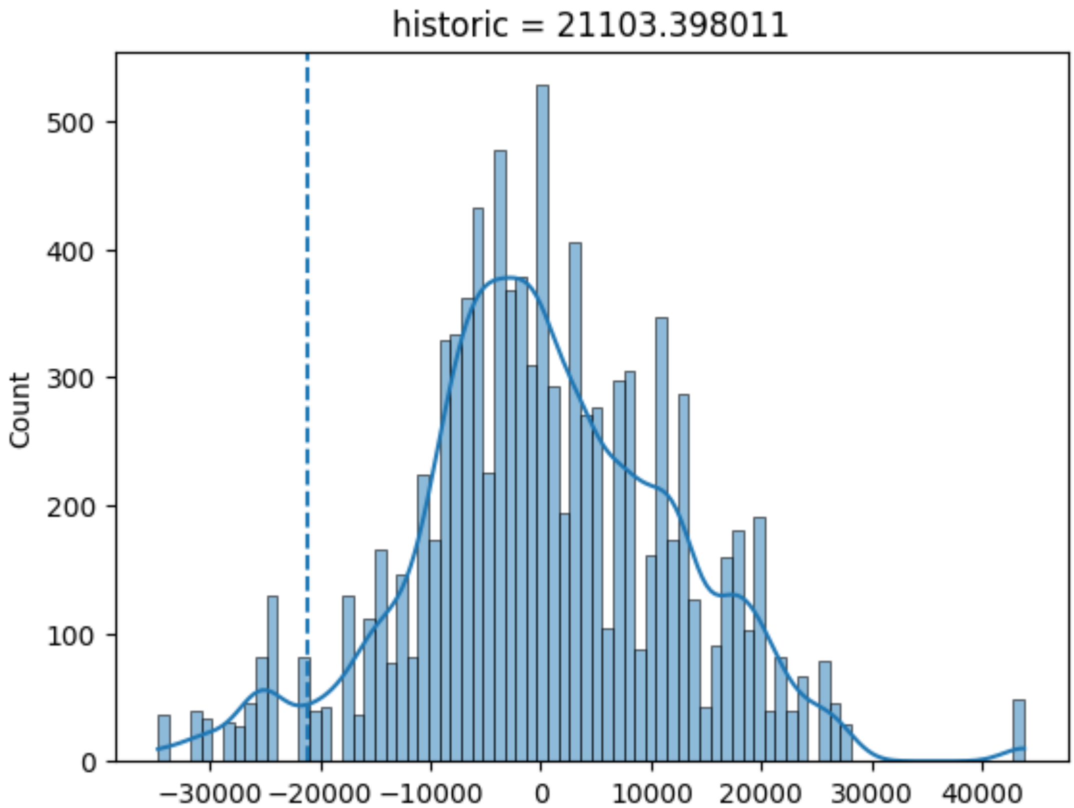
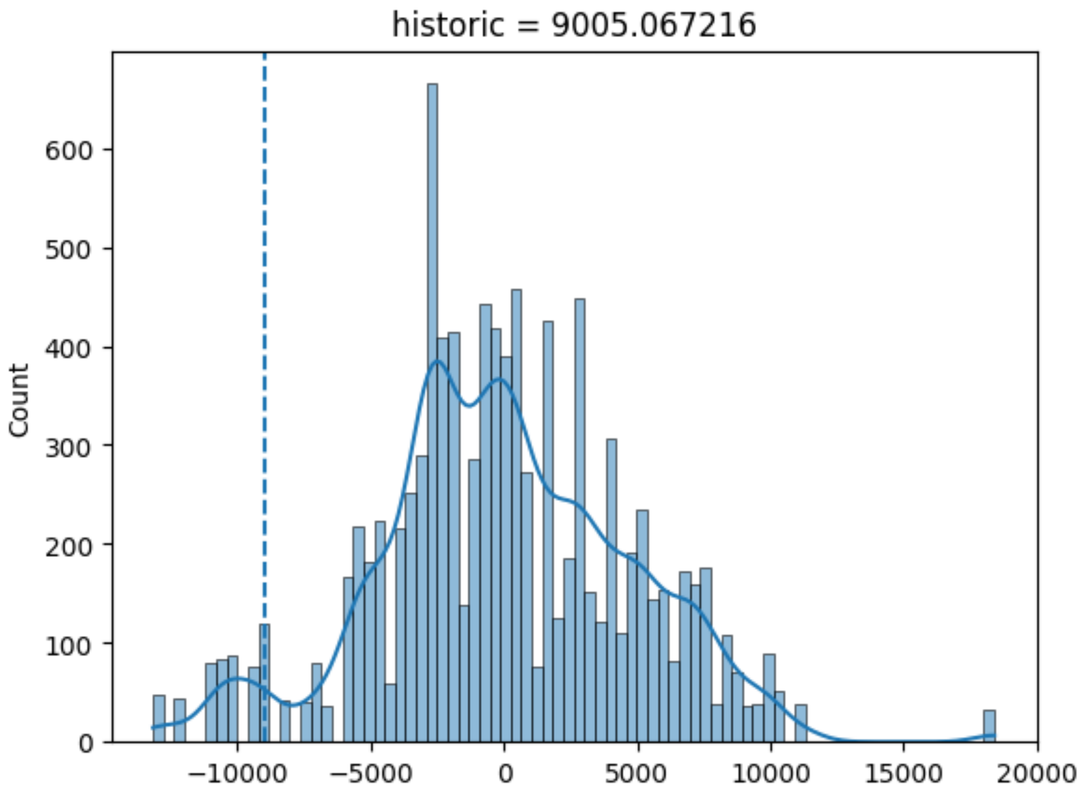
100
200
300
400
500
−30000
−20000
−10000
0
10000
20000
30000
40000


historic = 21103.398011
Count
0
100
200
300
400
500
−30000
−20000
−10000
0
10000
20000
30000
40000

A


historic = 9005.067216
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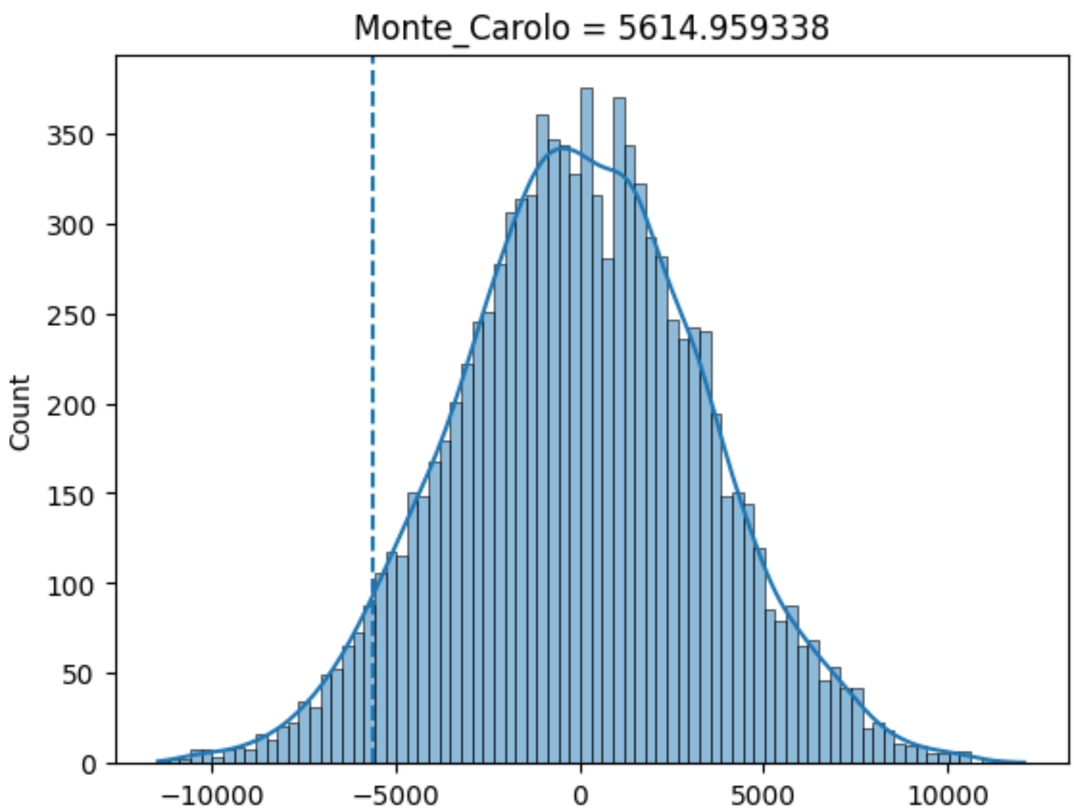
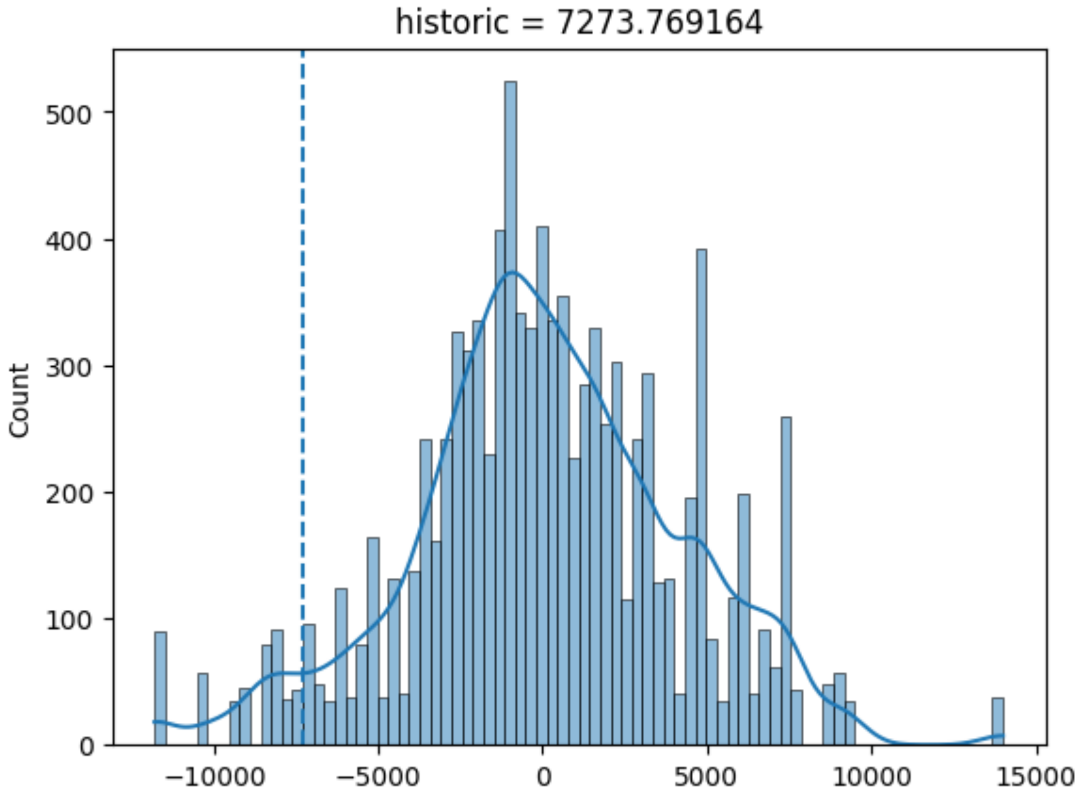
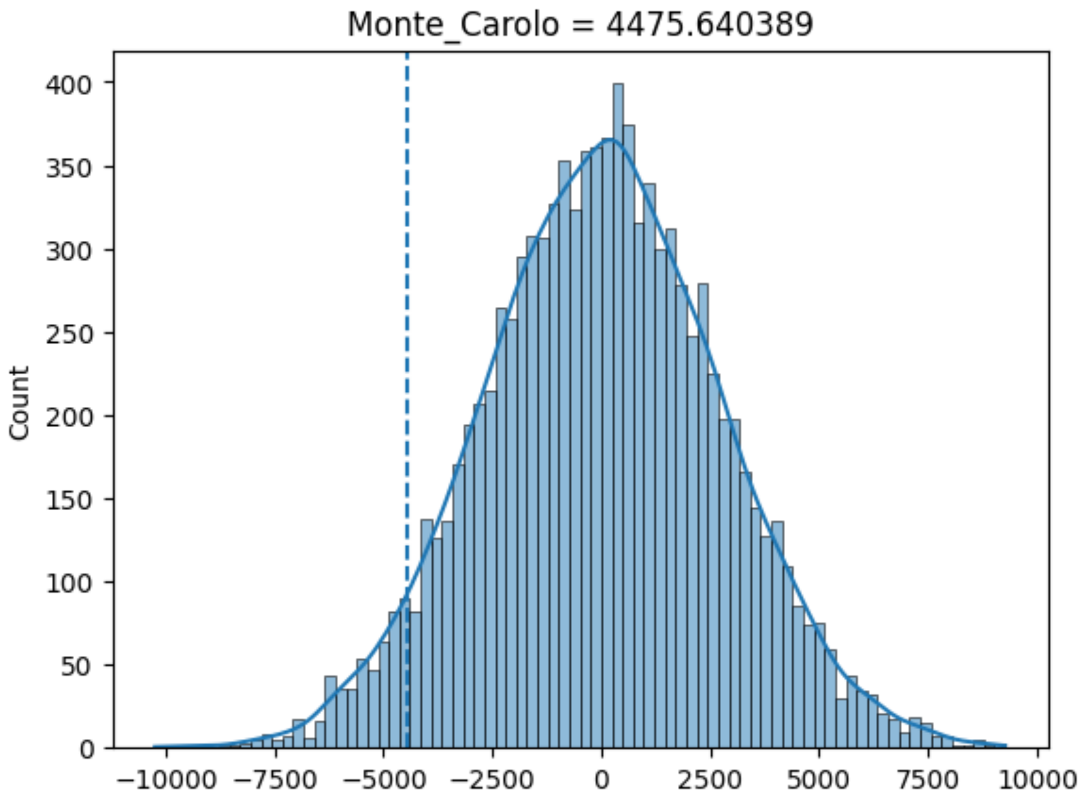
200
300
400
500
600
−10000
−5000
0
5000
10000
15000
20000

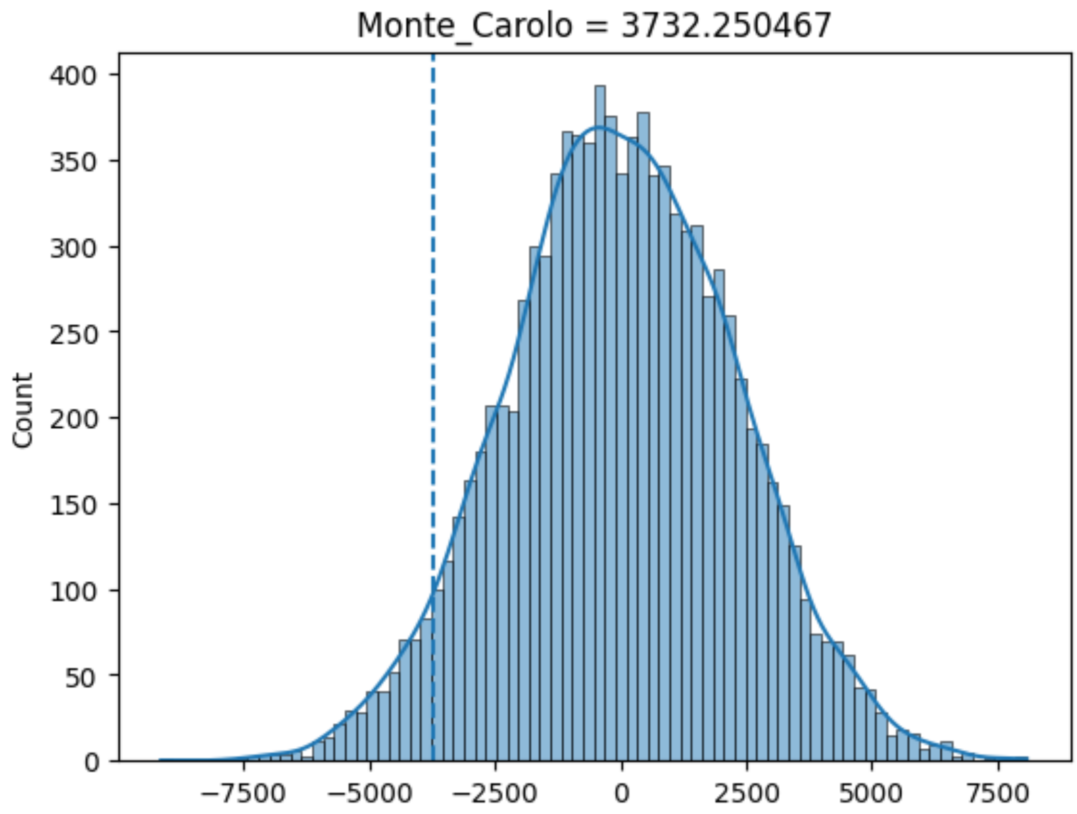

Monte_Carolo = 5614.959338
Count
0
50
100
150
200
250
300
350
−10000
−5000
0
5000
10000

B


historic = 7273.769164
Count
500
400
300
200
100
0
−10000
−5000
0
5000
10000
15000


Monte_Carolo = 4475.640389
Count
400
350
300
250
200
150
100
50
0
−10000
−7500
−5000
−2500
0
2500
5000
7500
10000

C


Monte_Carolo = 3732.250467
Count
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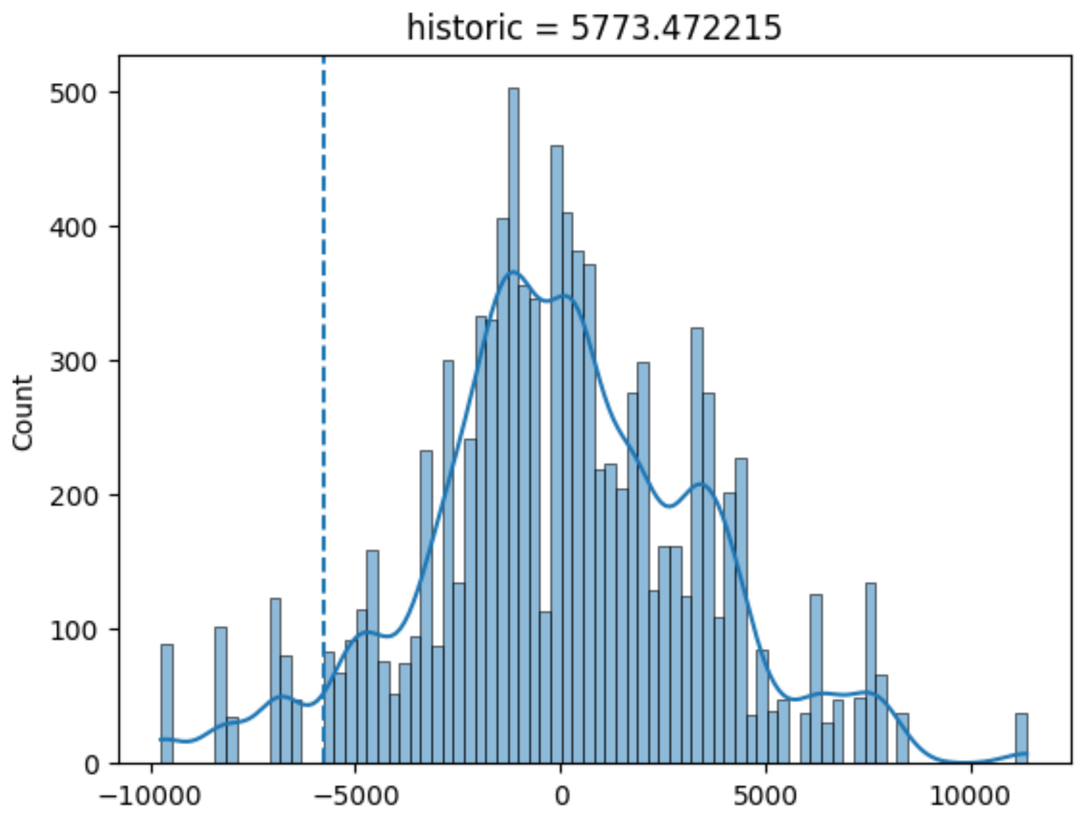
350
300
250
200
150
100
50
0
−7500
−5000
−2500
0
2500
5000
7500


historic = 5773.472215
Count
500
400
300
200
100
0
−10000
−5000
0
5000
10000