

2次元物体の定常状態の温度分布の解析に関して、ガウス・ザイデル法による数値解析プログラムをソースコード 1 に示す。また、分割数が 6 の場合と 12 の場合の実行結果を図 1(a)、図 1(b) に示す。

ソースコード 1: ガウス・ザイデル法による数値解析プログラム

```
1 #include <stdio.h>
2 #include <stdlib.h>
3 #include <math.h>
4
5 int N; // メッシュ数
6 float L; // 正方形の辺の長さ
7 #define Tm 50
8 #define dx (L / N) // メッシュの間隔
9 #define dy (L / N)
10 #define T0 0.0 // 境界条件 1
11 // #define T1 10.0 // 境界条件 2 → sin 関数
12 #define T2 0.0 // 境界条件 3
13 #define T3 0.0 // 境界条件 4
14 #define eps 1e-5 // 収束判定の閾値
15
16
17 int main()
18 {
19     printf("width? : ");
20     scanf("%f", &L);
21     printf("mesh? : ");
22     scanf("%d", &N);
23     int i, j;
24     int ITER = 0;
25     double T[N+1][N+1] = {0}; // 未知数の温度分布
26     double T_true[N+1][N+1] = {0};
27     double diff = eps + 1.0; // 収束判定の差分 (1回目のループ条件を満たすための初期化)
28
29     // 境界条件の設定
30     for (i = 0; i < N+1; i++) {
31         T[i][0] = T0; // 左辺
32         T[i][N] = T2; // 右辺
33     }
34     for (j = 0; j < N+1; j++) {
35         T[0][j] = Tm*sin((M_PI/L)*j*dx); // 上辺
36         T[N][j] = T3; // 下辺
37     }
38
39     // ガウスザイデル法の反復
40     while (diff > eps) {
41         diff = 0.0;
42         for (i = 1; i < N; i++) {
43             for (j = 1; j < N; j++) {
44                 double T_old = T[i][j];
45                 T[i][j] = 0.25*(T[i+1][j] + T[i-1][j] + T[i][j+1] + T[i][j-1]);
46                 diff += fabs(T[i][j] - T_old); // 1サイクル分の誤差
47             }
48         }
49     }
```

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48     }
49     ITER++;
50 }
51
52 // 厳密解
53 for (i = 0; i < N+1; i++) {
54     for (j = 0; j < N+1; j++) {
55         T_true[i][j] = Tm/sinh(M_PI)*sin(M_PI/L*j*dx)*sinh(M_PI/L*i*dy);
56     }
57 }
58
59 // 結果の出力
60 printf("Result: Gauss-Seidel Approximate Solution (Exact Solution)\n");
61 printf("Cycle: %d\n", ITER);
62 for(i = 0; i < N+1; i++){
63     for (j = 0; j < N+1; j++) {
64         printf("%f", T[i][j]);
65     }
66     printf("\n");
67     for (j = 0; j < N+1; j++){
68         printf("(%f)", T_true[N-i][N-j]);
69     }
70     printf("\n");
71 }
72 return 0;
73 }
```

```

width? [m]: 0.12
mesh?: 6
Result: Gauss-Seidel Approximate Solution (Exact Solution)
Cycle: 55
0.000000 25.000000 43.301270 50.000000 43.301270 25.000000 0.000000
(0.000000) (25.000000) (43.301270) (50.000000) (43.301270) (25.000000) (0.000000)
0.000000 14.924511 25.850012 29.849023 25.850012 14.924511 0.000000
(0.000000) (14.758371) (25.562249) (29.516742) (25.562249) (14.758371) (0.000000)
0.000000 8.848033 15.325244 17.696067 15.325244 8.848033 0.000000
(0.000000) (8.656121) (14.992841) (17.312242) (14.992841) (8.656121) (0.000000)
0.000000 5.142380 8.906863 10.284760 8.906864 5.142380 0.000000
(0.000000) (4.981710) (8.628575) (9.963420) (8.628575) (4.981710) (0.000000)
0.000000 2.814623 4.875071 5.629247 4.875071 2.814624 0.000000
(0.000000) (2.704553) (4.684423) (5.409106) (4.684423) (2.704553) (0.000000)
0.000000 1.241043 2.149550 2.482087 2.149550 1.241044 0.000000
(0.000000) (1.185959) (2.054142) (2.371919) (2.054142) (1.185959) (0.000000)
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
(0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000)

```

(a) 分割数:6

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width? [m]: 0.12
mesh?: 12
Result: Gauss-Seidel Approximate Solution (Exact Solution)
Cycle: 225
0.000000 12.940952 25.000000 35.355339 43.301270 48.296291 50.000000 48.296291 43.301270 35.355339 25.000000 12.940952 0.000000
(0.000000) (12.940952) (25.000000) (35.355339) (43.301270) (48.296291) (50.000000) (48.296291) (43.301270) (35.355339) (25.000000) (12.940952) (0.000000)
0.000000 9.961771 19.244663 27.216063 33.332734 37.177834 38.489326 37.177834 33.332734 27.216063 19.244663 9.961771 0.000000
(0.000000) (9.947381) (19.216864) (27.176750) (33.284585) (37.124131) (38.433728) (37.124131) (33.284585) (27.176750) (19.216864) (9.947381) (0.000000)
0.000000 7.661467 14.800818 20.931517 25.635768 28.592984 29.601636 28.592984 25.635768 20.931517 14.800818 7.661467 0.000000
(0.000000) (7.639495) (14.758371) (20.871489) (25.562249) (28.510984) (29.516742) (28.510984) (25.562249) (20.871489) (14.758371) (7.639495) (0.000000)
0.000000 5.883280 11.365624 16.073420 19.685839 21.956700 22.731248 21.956700 19.685839 16.073420 11.365624 5.883280 0.000000
(0.000000) (5.858210) (11.317192) (16.004926) (19.601951) (21.863136) (22.634384) (21.863136) (19.601951) (16.004926) (11.317192) (5.858210) (0.000000)
0.000000 4.506029 8.704979 12.310699 15.077466 16.816728 17.409958 16.816728 15.077466 12.310700 8.704979 4.506029 0.000000
(0.000000) (4.480738) (8.656121) (12.241604) (14.992841) (16.722341) (17.312242) (16.722341) (14.992841) (12.241604) (8.656121) (4.480738) (0.000000)
0.000000 3.435856 6.637564 9.386933 11.496598 12.822789 13.275129 12.822790 11.496599 9.386934 6.637564 3.435856 0.000000
(0.000000) (3.412129) (6.591727) (9.322110) (11.417207) (12.734240) (13.183455) (12.734240) (11.417207) (9.322110) (6.591727) (3.412129) (0.000000)
0.000000 2.599831 5.022488 7.102871 8.699205 9.702703 10.044977 9.702703 8.699205 7.102872 5.022489 2.599831 0.000000
(0.000000) (2.578723) (4.981710) (7.045202) (8.628575) (9.623925) (9.963420) (9.623925) (8.628575) (7.045202) (4.981710) (2.578723) (0.000000)
0.000000 1.940981 3.749687 5.302858 6.494649 7.243839 7.493375 7.243840 6.494649 5.302859 3.749687 1.940981 0.000000
(0.000000) (1.923071) (3.715089) (5.253929) (6.434722) (7.177000) (7.430177) (7.177000) (6.434722) (5.253929) (3.715089) (1.923071) (0.000000)
0.000000 1.414405 2.732421 3.864227 4.732692 5.278632 5.464842 5.278632 4.732692 3.864227 2.732421 1.414405 0.000000
(0.000000) (0.984219) (1.901385) (2.889336) (3.293261) (3.673155) (3.802730) (3.673155) (3.293261) (2.889336) (1.901385) (0.984219) (0.000000)
0.000000 0.973390 1.880445 2.689351 3.257027 3.632742 3.760681 3.632742 3.257027 2.689351 1.880445 0.973390 0.000000
(0.000000) (0.934219) (1.89884) (2.696892) (3.267260) (3.643798) (3.771738) (3.643798) (3.267260) (2.696892) (1.89884) (0.934219) (0.000000)
0.000000 0.621106 1.199884 1.696892 2.078260 2.317938 2.393768 2.317938 2.078260 1.696892 1.199884 0.621106 0.000000
(0.000000) (0.613898) (1.185959) (1.677200) (2.054142) (2.291098) (2.371919) (2.291098) (2.054142) (1.677200) (1.185959) (0.613898) (0.000000)
0.000000 0.300320 0.580173 0.820489 1.004889 1.120808 1.160346 1.120808 1.004889 0.820489 0.580173 0.300320 0.000000
(0.000000) (0.296722) (0.573223) (0.810660) (0.992852) (1.107382) (1.146446) (1.107382) (0.992852) (0.810660) (0.573223) (0.296722) (0.000000)
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000
(0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000) (0.000000)

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(b) 分割数:12

図 1: 実行結果