Exam 1 - extra materials
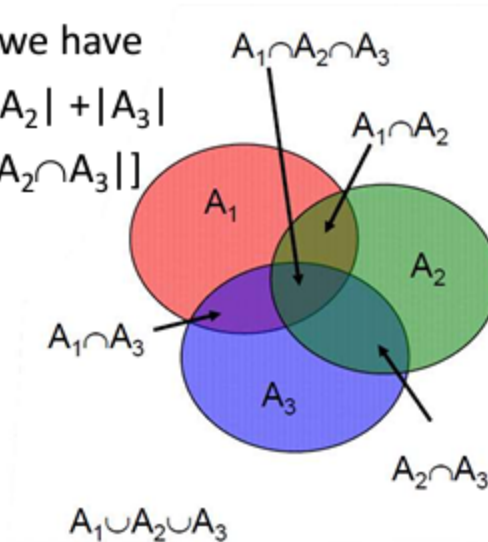
Inclusion-exclusion principle

- **Example:** when n=3, we have

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3|$$
$$- [|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|]$$
$$+ |A_1 \cap A_2 \cap A_3|$$

- **Example:** To illustrate, when n=4, we have

$$|A_1 \cup A_2 \cup A_3 \cup A_4| = |A_1| + |A_2| + |A_3| + |A_4|$$
$$- [|A_1 \cap A_2| + |A_1 \cap A_3| + |A_1 \cap A_4|$$
$$+ |A_2 \cap A_3| + |A_2 \cap A_4| + |A_3 \cap A_4|]$$
$$+ [|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4|$$
$$+ |A_1 \cap A_3 \cap A_4| + |A_2 \cap A_3 \cap A_4|]$$
$$- |A_1 \cap A_2 \cap A_3 \cap A_4|$$

Combinations with repetitions

The number of r-combinations with repetition allowed (multisets of size r) that can be selected from a set of n elements is

$$\binom{r + n - 1}{r}.$$

This equals the number of ways r objects can be selected from n categories of objects with repetition allowed.