Lecture 3 - Combinatorics Part 1

Counting

If a task can be done in n_1 ways and a second task in n_2 ways, and if these two tasks cannot be done at the same time, then there are $n_1 + n_2$ ways to do either task.

Suppose that a procedure can be broken down into two successive tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been down, then there are $n_1 n_2$ ways to do it.

Pigeonhole principle

If $(k+1)$ or more objects are placed into k boxes, then there is at least one box containing two or more objects.

Or more generally if you have N objects and k boxes there is at least one box containing $\lceil N/k \rceil$ objects.

Permutations

The number of r-permutations of a set with n distinct elements is denoted by $P(n, r)$. We can calculate it with this formula:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Combinations

The number of combinations $C(n, r)$ can be calculated with this formula:

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!}{r!(n-r)!}$$

Corollary

Let n and r be nonnegative integers with $r \leq n$. Then $C(n, r) = C(n, n-r)$.

Pascal's Identity

Let n and k be positive integers with $n \geq k$. Then $C(n+1, k) = C(n, k-1) + C(n, k)$.

Inclusion-Exclusion principle

The inclusion-exclusion principle applies when we are trying to count the number of cases when either the property nr. 1 is true, or the property nr. 2 is true.

The inclusion-exclusion principle then tells us to calculate the number of cases when the property nr. 1 is true, the cases when the property nr 2. is true, and the cases when both properties are true. Then the total number is the sum of the first two minus the third number. This can be expressed using set notation:

$$|A \cup B| = |A| + |B| - |A \cap B|$$