

Euler cycles and paths.

Complexity and algorithms

- Euler path is a trail (sequence of edges) that visits every edge exactly once.
- Euler cycle is an Euler path that starts and ends on the same vertex.
- A connected graph has an Euler cycle if and only if every vertex has even degree.
- A connected graph has an Euler path if and only if zero or two vertices have an odd degree and all the other vertices have an even degree.

Hierholzer's algorithm

- It is used for finding Euler cycles
- The procedure is:
 1. Check if the graph is connected and every node has an even degree
 2. Choose arbitrary vertex
 3. Follow the outgoing edges of the vertex that we haven't followed before and traverse vertices. We can choose any edge we want, except those that have been traversed before
 4. At some point we will visit a vertex and there will be no new edges to follow. This means we are back at the starting vertex.
 5. Push that vertex on to the stack that will hold the Euler cycle and backtrack from this vertex to the previous one.
 6. If there are other edges to follow, return to step 3.
- The algorithm takes $O(|E|)$ time.