Sorting algorithms

Time complexities

Algorithm	Worst case	Average	Best case
Quick sort (QS)	$O(n^2)$	$O(n \log n)$	$O(n \log n)$
Heap sort (HS)	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Merge sort (MS)	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Insertion sort (IS)	$O(n^2)$	$O(n^2)$	$O(n)$
Selection sort (SS)	$O(n^2)$	$O(n^2)$	$O(n^2)$
Bubble sort (BS)	$O(n^2)$	$O(n^2)$	$O(n)$
Shell sort (ShS)	$O(n^2)$	$O()$	$O(n \log n)$
Counting sort (CS)	$O(n+k)$	$O(n+k)$	$O(n+k)$

Where k is the value range of numbers.

Quick sort

This is a recursive algorithm. If the array is one element long, return as it is. Else pick a pivot element (Juan picks the last one) then partition the array so that all the elements that are smaller or equal than the pivot (with the pivot itself) are on one side of the array, and the rest are on the other side. Then just recursively sort two partitions, and leave the pivot as it is, because it is in the right position already.

```
def quick_sort(a):
    quick_sort_helper(a, 0, len(a)-1)
    return

def quick_sort_helper(a, low, high):
    if low >= high or low < 0: return
    p = partition(a, low, high)
    quick_sort_helper(a, low, p-1)
    quick_sort_helper(a, p+1, high)

def partition(a, low, high):
    pivot = a[high]
    # The index one before the correct position of the pivot
    i = low - 1

    for j in range(low, high):
        if a[j] <= pivot:
            i += 1
            a[j], a[i] = a[i], a[j]
    i += 1
    a[i], a[high] = a[high], a[i]
    return i
```

Heap sort

The heapsort algorithm begins by rearranging the array into a binary max-heap. A heap is an array/binary tree data structure where each array element is a tree node and its children are at indices 2i+1 and 2i+2 where i is the index of that node. A max heap is a heap where each node is greater or equal to its children. The algorithm then repeatedly swaps the root of the heap (the greatest element remaining in the heap) with its last element, which is then declared to be part of the sorted suffix. Then the heap, which was damaged by replacing the root, is repaired so that the greatest element is again at the root. This repeats until only one value remains in the heap.

`build_max_heap` function

The `build_max_heap` function runs the `max_heapify` function from the last non-leaf node in the heap. The `max_heapify` function itself works by repairing a damage max heap where the parent node is no longer greater than its children.

`max_heapify` function

`max_heapify` function take an index in the array and then checks whether the tree node represented by the elements of the array at that index is greater then its children. If not then it swaps the greatest node among {parent, left child, right child} nodes with the parent node. It then does the same with the child node that was swapped.

```
def heap_sort(a):
    build_max_heap(a)
    n = len(a)
    for i in range(n-1, 0, -1):
        a[0], a[i] = a[i], a[0]
        max_heapify(a, 0, i)

def build_max_heap(a):
    n = len(a)
    for i in range(n//2-1, -1, -1):
        max_heapify(a, i, n)

def max_heapify(a,i,n):
    largest = i
    left = 2*i+1
    right = 2*i+2
    if left < n and a[left] > a[largest]:
        largest = left
    if right < n and a[right] > a[largest]:
        largest = right
    if largest != i:
        a[i], a[largest] = a[largest], a[i]
        max_heapify(a, largest, n)
```

Merge sort

Split array into two and sort them recursively, then merge. In the simplest case, when the array is one element, just return it.

```
def merge_sort(a):
    if len(a) < 2:
        return a
    m = len(a) // 2
    left = merge_sort(a[:m])
    right = merge_sort(a[m:])
    return merge(left, right)

def merge(a,b):
    result = []
    while len(a) > 0 and len(b) > 0:
        if a[0] < b[0]:
            result.append(a[0])
            a.pop(0)
        else:
            result.append(b[0])
            b.pop(0)
    result.extend(a)
    result.extend(b)
    return result
```

Insertion sort

Go through the whole array, element by element, and look on the left to check at what position does this element belong in, and then insert it there.

```
def insertion(a):
    for i in range(1,len(a)):
        temp = a[i]
        j = i-1
        while a[j] > temp and j >= 0:
            a[j+1] = a[j]
            j -= 1
        a[j+1] = temp
    return a
```

Selection sort

Separate the array into two parts, unsorted and sorted. Go through the unsorted array and find the smallest element, and then append it to the sorted part.

```
def selection(a):
    for i in range(len(a)-1):
        x = i
        for j in range(i, len(a)):
            if a[j] < a[x]:
                x = j
        a[x], a[i] = a[i], a[x]
    return a
```

Bubble sort

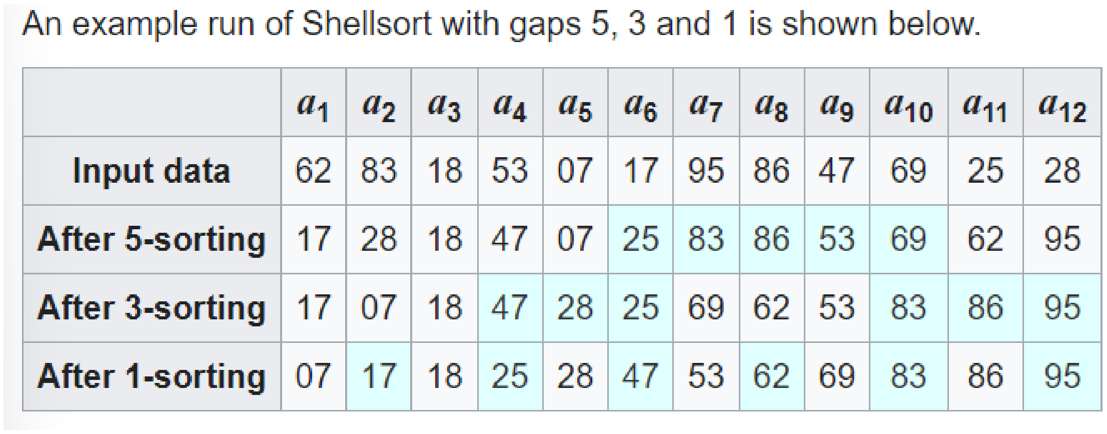
Go through the whole array and sort it by swapping two elements if they are in the wrong order, and keeping them if they are in the right order. Repeat this n-1 times, every time go to index one less than from the previous iteration.

```
def bubble_sort(a):
    n = len(a)
    for i in range(0, n-1):
        for j in range(0, n-1-i):
            if a[j] > a[j+1]:
                a[j], a[j+1] = a[j+1], a[j]
    return a
```

Shell sort

It is a variation of insertion sort, but this time it sorts the array in gaps (intervals). The general procedure is as follows:

- Find some decreasing sequence of integers (gaps) and then for each gap:
    1. Sort the subarray of our main array consisting of entries 0, 0+gap, 0+2gap,... etc.
    2. Do the same for 1,1+gap,1+2gap,... and for all starting positions up to gap-1.
    3. Do the same for the next integer in the gap sequence.


An example run of Shellsort with gaps 5, 3 and 1 is shown below.

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
Input data	62	83	18	53	07	17	95	86	47	69	25	28
After 5-sorting	17	28	18	47	07	25	83	86	53	69	62	95
After 3-sorting	17	07	18	47	28	25	69	62	53	83	86	95
After 1-sorting	07	17	18	25	28	47	53	62	69	83	86	95

```
def shell_sort(a):
    gap = len(a)//2
    while gap > 0:
        for i in range(gap, len(a)):
            temp = a[i]
            j = i - gap
            while j >= 0 and a[j] > temp:
                a[j+gap] = a[j]
                j -= gap
            a[j+gap] = temp
        gap //= 2
    return a
```

Counting sort

Counting sort goes through the array and records the count of every element. Then it adds the count of every value to the count of this value plus one, to get the correct position of each element. Then it creates another array and fills it up with the elements from the original array according to the order given by the count array.

```
def counting_sort(a):
    n = len(a)
    m = max(a)
    count = [0 for _ in range(m+1)]
    for e in a:
        count[e] += 1
    for i in range(1, m+1):
        count[i] += count[i-1]

    sorted = [0 for _ in range(n)]
    for i in range(n-1, -1, -1):
        sorted[count[a[i]]-1] = a[i]
        count[a[i]] -= 1
    return sorted
```