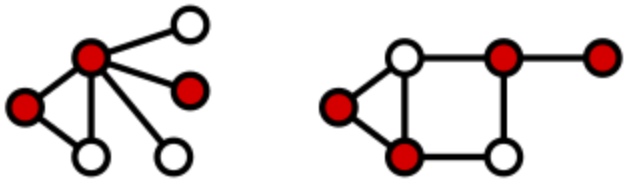
Formulation and representative examples of 5 NP-complete problems.

3-Dimensional Matching

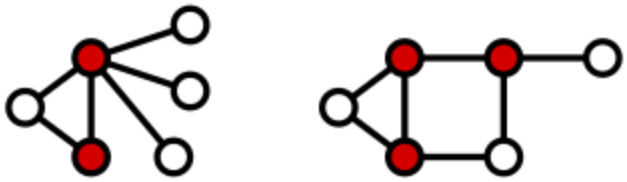
- Let X, Y, Z be finite sets.
- Let $T \subseteq X \times Y \times Z$, that is T consists of triples (x, y, z) where $x \in X$, $y \in Y$, $z \in Z$.
- $M \subseteq T$ is a 3-dimensional matching if and only if for any two elements of M: $(x_1, y_1, z_1) \in M$, $(x_2, y_2, z_2) \in M$ it holds $x_1 \neq x_2, y_1 \neq y_2, z_1 \neq z_2$.
- The problem is: given the set T and an integer k decide whether there exists a 3-dimensional matching $M \subseteq T$ with $|M| \geq k$.
- This problem is known to be strongly NP-complete.

Graph Vertex Cover

- Let $G = (V, E)$ be an undirected graph.
- Then $V' \subseteq V$ is a vertex cover if and only if for every edge $(u, v) \in E$ at least one of the endpoints is in V', i.e. $(u, v) \in E \implies v \in V' \vee v \in V'$.
- The vertex cover problem if formulated as follows: Given a graph $G = (V, E)$ and a positive integer k decide whether G has a vertex cover of size at most k .
- This problem is known to be strongly NP-complete



Example of a vertex cover



Example of minimum vertex cover

Clique

- Given an undirected graph $G = (V, E)$ a clique is a subset $K \subseteq V$ such that every pair of vertices in K is connected by some edge in E.
- The problem is formulated as follows: is it possible to find a clique K with size greater than some integer k?
- This problem is known to be strongly NP-complete.

Hamiltonian Circuit

- Now usually called Hamiltonian cycle problem.
- Given a directed or undirected graph G decide whether there exist a sequence of vertices connected by edges where the starting vertex is also the ending vertex and each vertex is used exactly once (except the first one).
- This is a special case of the traveling salesman problem.
- This problem is known to be strongly NP-complete.

Set partition

- Given a list of positive integers decide whether is can be partitioned into two sub lists S_1 and S_2 such that the sum of the numbers in S_1 equals the sum of the numbers in S_2.
- This problem is known to be WEAKLY NP-complete.