Lecture 1 - Sets, Logic, Mathematical Reasoning

Notes from the lecture:

Set theory

Natural number start at 0: $\mathbb{N} = \{0, 1, 2, 3, 4, 5, \ldots\}$

Positive integers are denoted with $\mathbb{Z}^+ = \{1, 2, 3, 4, 5, \ldots\}$

- The **alphabet** is finite and non empty set Σ, which elements are symbols (the **letters** of the alphabet Σ)
- The **word** of the alphabet Σ is arbitrary and finite string of the letters from Σ.
- Σ^* set of all the words built with the letters of Σ.
- The **language** is an arbitrary subset of Σ^*.

Example

- Σ = {A, C, G, T}.
- Σ^* contains all strings built with A, C, G, T and **empty word** λ.
- Set L = {A, AT, ATTT, ATTTTT, CGC} is one of the infinite set of languages over Σ.

Proper subsets:

$A \subset B$ "A is a proper subset of B"

$A \subset B \Leftrightarrow \forall x\, (x \in A \rightarrow x \in B) \wedge \exists x\, (x \in B \wedge x \notin A)$

or

$A \subset B \Leftrightarrow \forall x\, (x \in A \rightarrow x \in B) \wedge \neg\forall x\, (x \in B \rightarrow x \in A)$

The power set $P(\mathbb{A})$ is the set of all subset of a set $\mathbb{A}$ and has the cardinality $|P(\mathbb{A})| = 2^{|\mathbb{A}|}$.

The cartesian product of any set with an empty set is the empty set:

$$\mathbb{A} \times \varnothing = \varnothing$$

$$\varnothing \times \mathbb{A} = \varnothing$$

And also:

$$|\mathbb{A} \times \mathbb{B}| = |\mathbb{A}| \cdot |\mathbb{B}|$$

Two sets are called disjoint if their intersection is empty.

Logic

A proposition is a statement that is either true or false.

- 110 < 5 is a statement and a proposition.
- y < 25 is a statement but not a proposition.
- "Please do not fall asleep" is not a statement, so it cannot be a proposition.

A propositional function is a proposition dependent on some variable(s), like x < 25. It can be true or false and we usually denote it as P(x).

Mathematical Reasoning

The steps that connect statements in a proof are called rules of inference.

One important rule of inference is modus ponens:

$$(p \land (p \rightarrow q)) \rightarrow q$$

$$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore\ q \end{array}$$

The two **hypotheses** p and p → q are written in a column, and the **conclusion** below a bar, where ∴ means "therefore".

We say that an argument is valid if whenever all its hypotheses are true, its conclusion is also true.