

Problem 1 | r_j, d_j | -

- This is another optimal job scheduling problem.
- The notation means:
 - 1 — single machine
 - r_j — for each job a release time is given before which it cannot be scheduled, default is 0.
 - d_j — for each job a due date is given.
 - '-' — means there is no objective function, the task is to simply produce a feasible scheduling
- The problem description is: We have a single processor and a set of tasks, where the j -th task takes p_j time to complete and it needs to be completed in the interval $[r_j, d_j]$. Is it possible to perform all tasks within their intervals?
- This problem is NPH, because you can reduce set partition problem to it.

Reducing set partition

- Given a set of integers $A = \{a_1, a_2, \dots, a_n\}$ decide whether it can be partitioned into two subsets A_1, A_2 of equal sums.
- Let's create a job for each integer in A setting the $r_i = 0, d_i = 3T, p_i = 2a_i$, where $T = \sum_{i=1}^n a_i$ (total sum of A) and creating an extra job with $r_{n+1} = T, d_i = 2T, p_i = T$.
- If we can schedule all the jobs that means we partitioned the set A into two subsets of equal sums, since the extra task needs to be performed exactly at T until $2T$, and other tasks need to be performed in two intervals of equal length.
- That means we can reduce the set partition problem into our job scheduling problem.