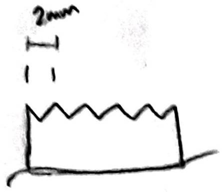
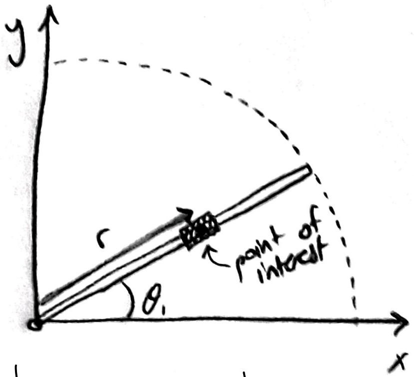


$$\frac{25.4\text{ mm}}{1\text{ in}} \quad \frac{1\text{ th}}{2\text{ mm}} = \frac{12.7\text{ th}}{\text{in}}$$

Setup

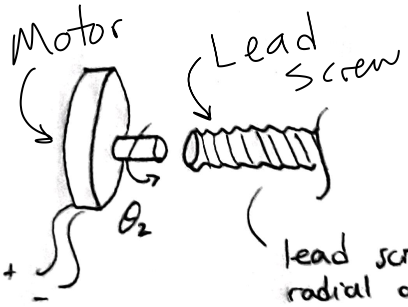


X - coord

$$x = r\cos\theta_1 \Rightarrow x = \frac{\theta_2}{25.4\pi}\cos\theta_1$$

y - coord

$$y = r\sin\theta_1 \Rightarrow y = \frac{\theta_2}{25.4\pi}\sin\theta_1$$



lead screw controls radial advancement

$$\frac{\theta_2\ (\text{rad})}{1} \quad \frac{2\text{mm}}{2\pi\text{ rad}} \quad \frac{1\text{in}}{25.4\text{mm}} = \frac{\theta_2}{\pi 25.4}\text{ in}$$

☆ $\pi 25.4 = \alpha$ Constant for Lead Screw Dimensions

In matrix form

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \theta_2/\alpha \ \cos\theta_1 \\ \theta_2/\alpha \ \sin\theta_1 \end{bmatrix} \begin{matrix} \leftarrow f_1 \\ \leftarrow f_2 \end{matrix} \Rightarrow \underline{x} = f(\underline{\theta})$$

Jacobian matrix

$$\frac{\partial f}{\partial \theta} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial \theta_1} = -\frac{\theta_2}{\alpha}\sin\theta_1 \qquad \frac{\partial f_1}{\partial \theta_2} = \frac{1}{\alpha}\cos\theta_1$$

$$\frac{\partial f_2}{\partial \theta_1} = \frac{\theta_2}{\alpha}\cos\theta_1 \qquad \frac{\partial f_2}{\partial \theta_2} = \frac{1}{\alpha}\sin\theta_1$$

Differentiating $\underline{x} = f(\theta)$ w/respect to time for velocity kinematics

$$\underline{\dot{x}} = \frac{d}{dt}\left(\underline{f}(\underline{\theta})\right)$$

$$\underline{\dot{x}} = \frac{\partial f}{\partial \theta}\dot{\underline{\theta}} \longrightarrow f(\theta)\frac{d\theta}{dt}$$

↑ from Jacobian

$$\frac{df_1}{dt} = \frac{d}{dt}\left[\frac{\theta_2}{\alpha}\cos\theta_1\right] = \frac{1}{\alpha}\cos\theta_1 - \frac{\theta_2}{\alpha}\sin\theta_1$$

$$\frac{df_2}{dt} = \frac{d}{dt}\left[\frac{\theta_2}{\alpha}\sin\theta_1\right] = \frac{1}{\alpha}\sin\theta_1 + \frac{\theta_2}{\alpha}\cos\theta_1$$

$$\underline{\dot{x}} = \underbrace{\begin{bmatrix} -\frac{\theta_2}{\alpha}\sin\theta_1 & \frac{1}{\alpha}\cos\theta_1 \\ \frac{\theta_2}{\alpha}\cos\theta_1 & \frac{1}{\alpha}\sin\theta_1 \end{bmatrix}}_{\frac{\partial f}{\partial \theta}} \underbrace{\begin{bmatrix} \frac{1}{\alpha}(\cos\theta_1 - \theta_2\sin\theta_1) \\ \frac{1}{\alpha}(\sin\theta_1 + \theta_2\cos\theta_1) \end{bmatrix}}_{\dot{\theta}}$$

$$\underline{\dot{x}} = \begin{bmatrix} -\frac{\theta_2}{\alpha}\sin\theta_1\left[\frac{1}{\alpha}(\cos\theta_1 - \theta_2\sin\theta_1)\right] + \frac{1}{\alpha}\cos\theta_1\left[\frac{1}{\alpha}(\sin\theta_1 + \theta_2\cos\theta_1)\right] \\ \frac{\theta_2}{\alpha}\cos\theta_1\left[\frac{1}{\alpha}\cos\theta_1 - \theta_2\sin\theta_1\right] + \frac{1}{\alpha}\sin\theta_1\left[\frac{1}{\alpha}(\sin\theta_1 + \theta_2\cos\theta_1\right] \end{bmatrix}$$

Inverse Kinematics

$$\underline{y} = \underline{x} - \underline{f}(\underline{\theta}) = g(\underline{\theta})$$

constant

$$\frac{\partial g(\underline{\theta})}{\partial \underline{\theta}} = \frac{\partial}{\partial \theta}\left[\underset{0}{\cancel{x}} - f(\theta)\right]$$

$$\frac{\partial g(\underline{\theta})}{\partial \underline{\theta}} = -\frac{\partial}{\partial \theta}f(\theta)$$