Inverse Kinematics for EEzyBot Arm

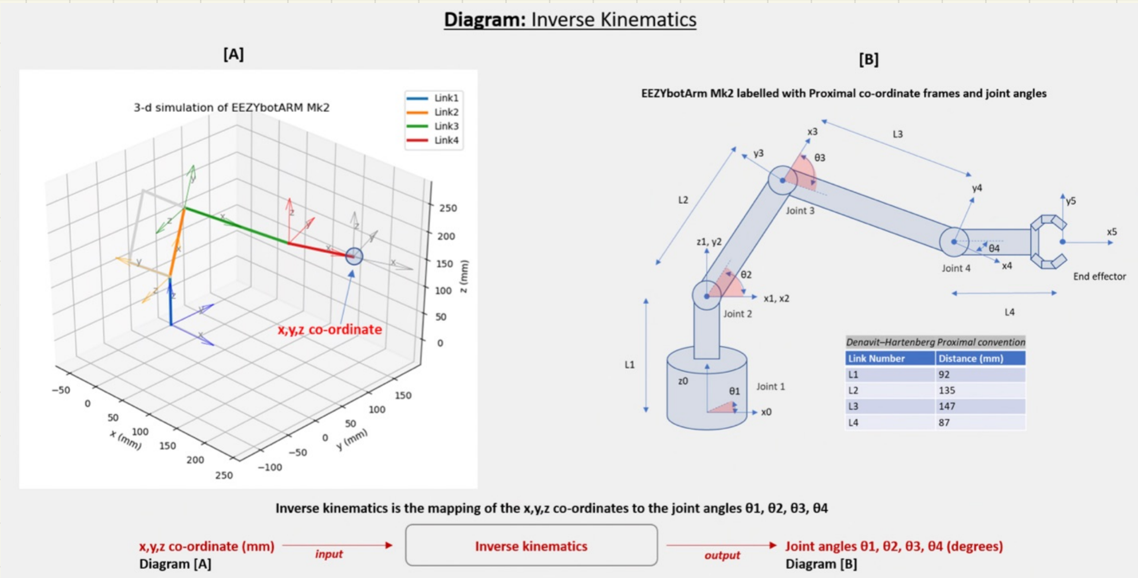
27th Sept 2023
Ben Money-Coomes.

Method: Closed form solution using geometry method

$$
{}^0_4T = \begin{bmatrix} c_1 c_{234} & -c_1 s_{234} & s_1 & c_1 (c_{23} L_3 + c_2 L_2) \\ s_1 c_{234} & -s_1 s_{234} & -c_1 & s_1 (c_{23} L_3 + c_2 L_2) \\ s_{234} & c_{234} & 0 & L_1 + s_2 L_2 + s_{23} L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

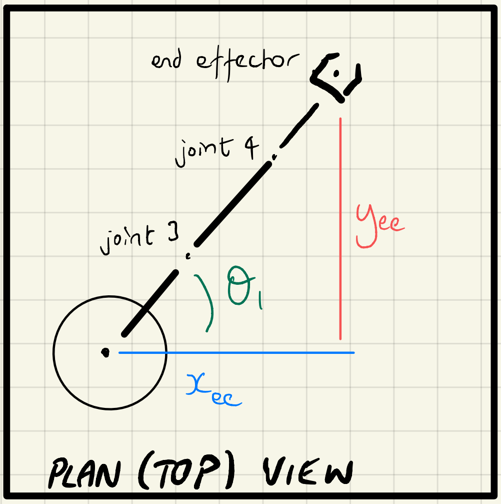
End effector position.

So we could try and solve using simultaneous equations. However there is a way to solve using geometry $\Rightarrow$ that is what we will use here.



Finding θ_1

θ_1 is the simplest of the angles to find. If we consider the plan view (top down) of the robot arm:



Then we see that

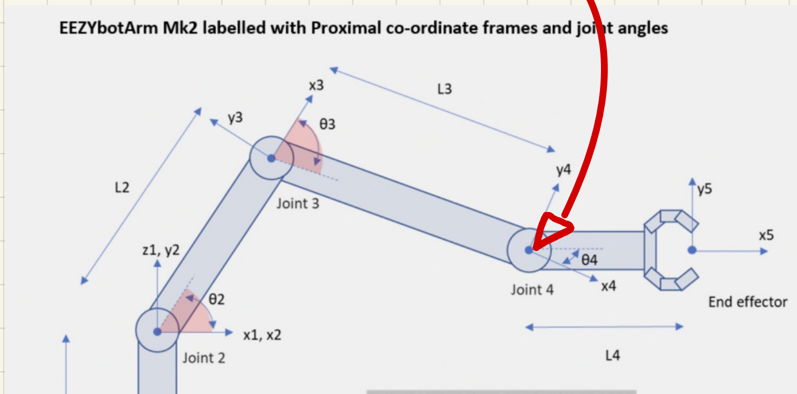
$$\theta_1 = \tan^{-1}\left(\frac{y_{ee}}{x_{ee}}\right)$$

We can use the function atan2 in python to give use the correct angle in different quadrants

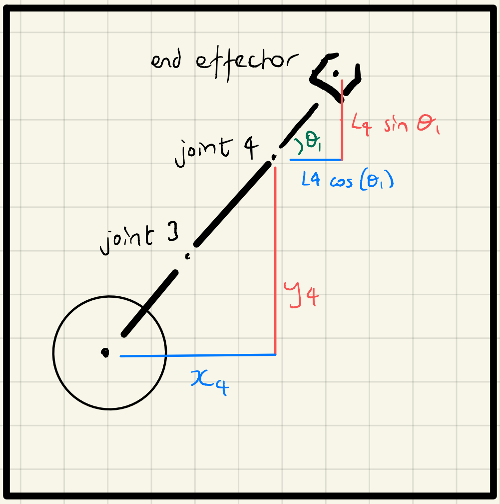
so $\Rightarrow$ $\theta_1 = \text{atan2}(y\text{-}EE, x\text{-}EE)$

Finding $\begin{bmatrix} x_4 \\ y_4 \\ z_4 \end{bmatrix}$

We know L_4 is always parallel to the world plane x-y because of the robot arm construction.



Denavit–Hartenberg Proximal convention	
Link Number	Distance (mm)
L1	92
L2	135
L3	147
L4	87

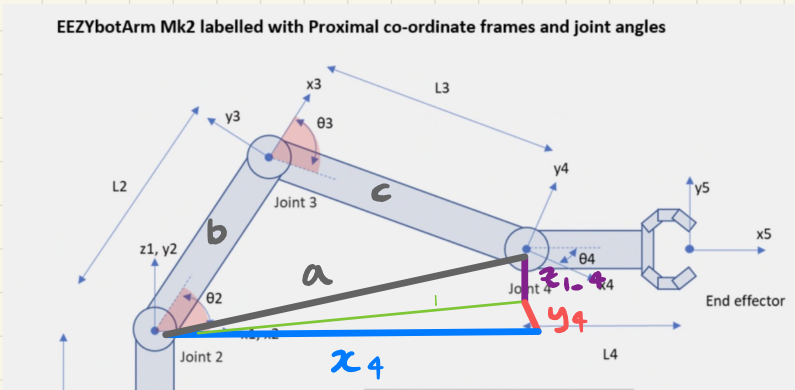


So...

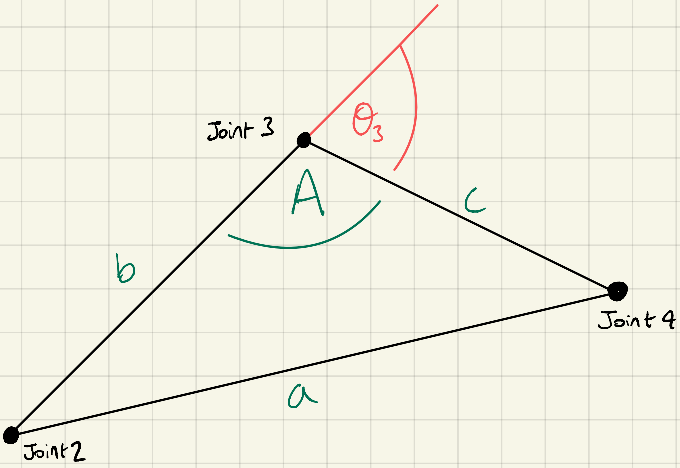
We can see $\begin{bmatrix} x_4 \\ y_4 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_{ee} - L_4 \cos(\theta_1) \\ y_{ee} - L_4 \sin(\theta_1) \\ z_{ee} \end{bmatrix}$

Finding θ_3

To find θ_3 we will use the cosine rule



Denavit–Hartenberg Proximal convention	
Link Number	Distance (mm)
L1	92
L2	135
L3	147
L4	87



From inspection we can see that

$b = L_2$

$c = L_3$

$a = \sqrt{z_{1,4}^2 + \sqrt{x_4^2 + y_4^2}}$

Where $\beta = \sqrt{x_4^2 + y_4^2}$

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos(A)$

$A = \cos^{-1}\left(\dfrac{b^2 + c^2 - a^2}{2bc}\right)$

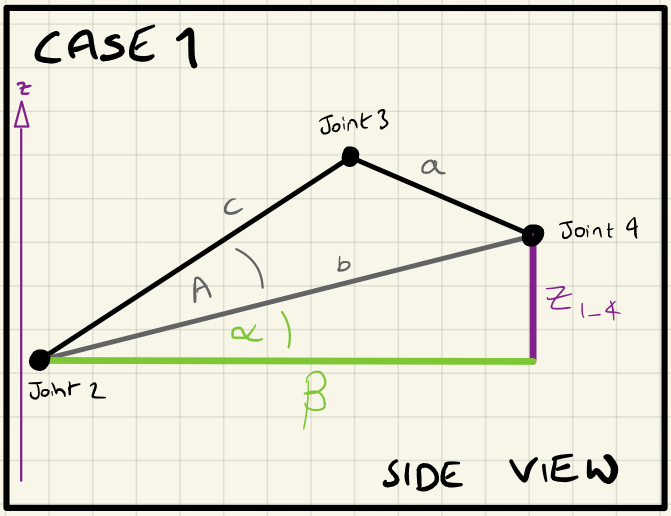
So

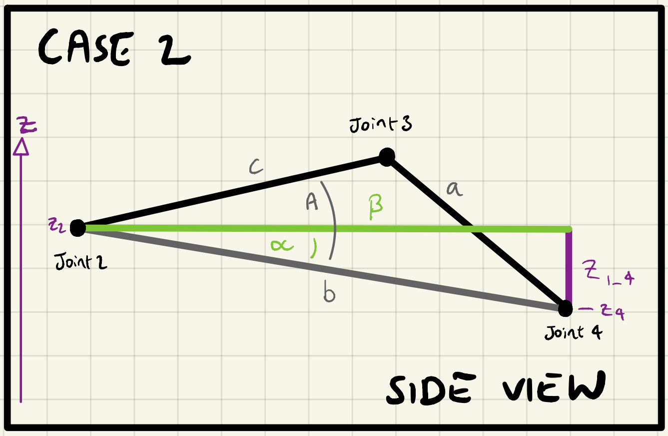
$$A = \cos^{-1}\left(\frac{L_2^2 + L_3^2 - a^2}{2(L_2)(L_3)}\right)$$

and $\theta_3 = -(\pi - A)$

we need the negative sign here due to the rotation direction and the right hand rule.

Finding θ_2





Again we use the cosine rule.

We note that the robot arm position can be categorised into two cases: Case 1, and case 2.

We can see from inspection that

$$\theta_3 = A + \alpha$$

This holds for both cases when we use atan2

$$\alpha = \operatorname{atan2}(z_{1-4}, \beta)$$

where $\beta = \sqrt{x_4^2 + y_4^2}$

$$z_{1-4} = z_4 - z_2$$

And applying the cosine rule

$a = L_3$
$c = L_2$
$b = \sqrt{z_{1-4}^2 + \beta}$

$$A = \cos^{-1}\left(\frac{b^2 + L_2^2 - L_3^2}{2 b L_2}\right)$$

So

$$\theta_3 = \cos^{-1}\left(\frac{b^2 + L_2^2 - L_3^2}{2 b L_2}\right) + \operatorname{atan2}(z_{1-4}, \beta)$$

We now know $\theta_1, \theta_2, \theta_3$. So we have completed the inverse kinematic calculation

N.B. θ_4 is not an actuated joint