Forward Kinematics for EEzyBot Arm

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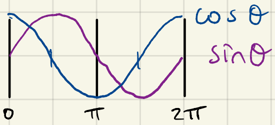
Method: Devanit Hartenberg | Proximal Method

N.B. The calculations presented here use the labelled parameters given in the Github repo easy EEZYbot ARM

Useful graphs →

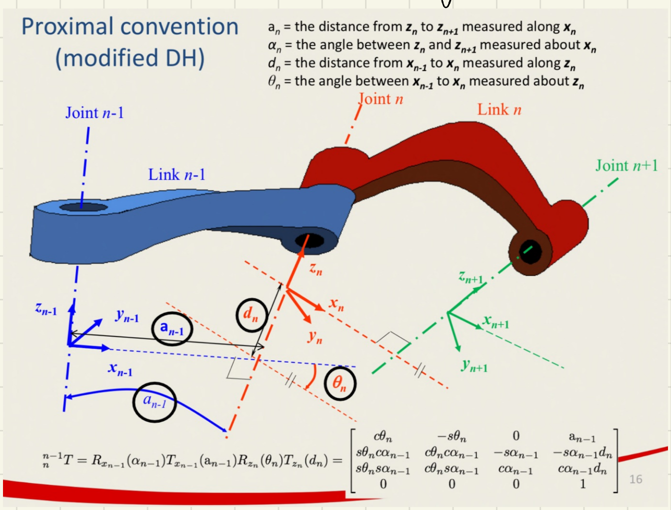


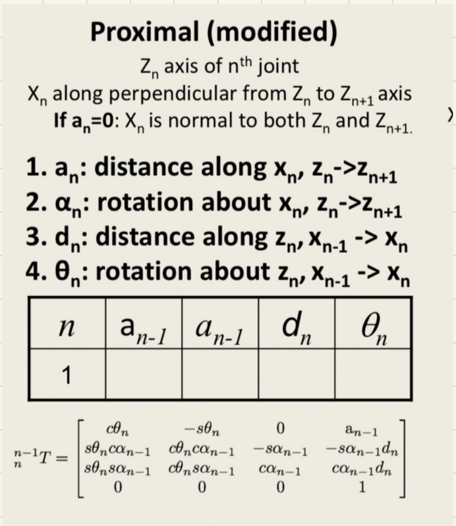
Notation → $c_1 c_2$ means $\cos\theta_1 \cos\theta_2$
c_{23} means $\cos(\theta_2+\theta_3)$

Useful trigonomic identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

i A quick recap of how the Devanit Hartenberg method works





n	a_{n-1}	a_{n-1}	d_n	θ_n
1				

I find it helpful when analysing the geometry of the robot arm to remember that:

$$^{n-1}_{\ \ n}T = \underbrace{R_{x_{n-1}}(\alpha_{n-1})}_{\text{Rotation about the x-axis}} \underbrace{T_{x_{n-1}}(a_{n-1})}_{\text{Translation along the x-axis}} \underbrace{R_{z_n}(\theta_n)}_{\text{Rotation about the z-axis}} \underbrace{T_{z_n}(d_n)}_{\text{Translation along the z-axis}} = \begin{bmatrix} c\theta_n & -s\theta_n & 0 & a_{n-1} \\ s\theta_n\, c\alpha_{n-1} & c\theta_n\, c\alpha_{n-1} & -s\alpha_{n-1} & -s\alpha_{n-1}\, d_n \\ s\theta_n\, s\alpha_{n-1} & c\theta_n\, s\alpha_{n-1} & c\alpha_{n-1} & c\alpha_{n-1}\, d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $s\theta_n\, c\alpha_{n-1}$ means $(\sin(\theta_n))(\cos(\alpha_{n-1}))$

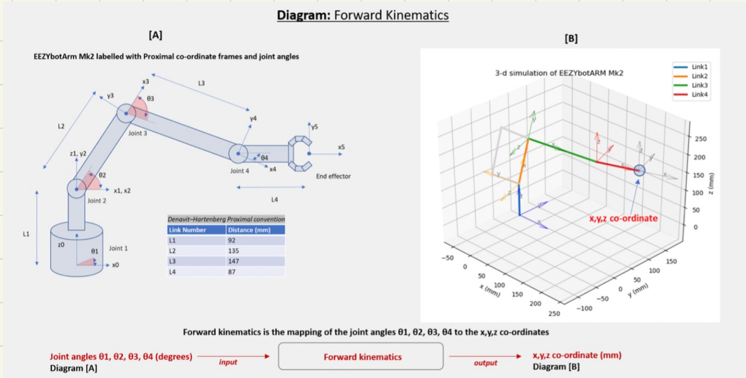
And so I proceed with analysis from the base frame of the robot arm by imagining the coordinate frame moving through cartesian space in the in the following order

(1) Rotation about the x-axis
(2) Translation along the x-axis
(3) Rotation about the z-axis
(4) Translation along the z-axis

⟹ $R(x)\ T(x)\ R(z)\ T(z)$

Applying this afore described method we arrive at these Devanit Hartenberg (DH) parameters in the proximal table form

n	a_{n-1}	α_{n-1}	d_n	θ_n
1	0	0	L_1	θ_1
2	0	$\pi/2$	0	θ_2
3	L_2	0	0	θ_3
4	L_3	0	0	θ_4



And we can use this table to create the following Homogeneous Transformation Matricies

$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^3_4T = \begin{bmatrix} c_4 & -s_4 & 0 & L_3 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_2T = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & 0 \\ s_1c_2 & -s_1s_2 & -c_1 & 0 \\ s_2 & c_2 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_3T = {}^0_2T\,{}^2_3T = \begin{bmatrix} c_1c_2c_3 - c_1s_2s_3 & -c_1c_2s_3 - c_1s_2c_3 & s_1 & c_1c_2L_2 \\ s_1c_2c_3 - s_1s_2s_3 & -s_1c_2s_3 - s_1s_2c_3 & -c_1 & s_1c_2L_2 \\ s_2c_3 + c_2s_3 & -s_2s_3 + c_2c_3 & 0 & s_2L_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1c_2L_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & s_1c_2L_2 \\ s_{23} & c_{23} & 0 & s_2L_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_3T\,{}^3_4T = {}^0_4T = \begin{bmatrix} c_1c_{23}c_4 - c_1s_{23}s_4 & -c_1c_{23}s_4 - c_1c_{23}s_4 & s_1 & c_1c_{23}L_3 + c_1c_2L_2 \\ s_1c_{23}c_4 - s_1s_{23}s_4 & -s_1c_{23}s_4 - s_1s_{23}c_4 & -c_1 & s_1c_{23}L_3 + s_1c_2L_2 \\ s_{23}c_4 + c_{23}s_4 & -s_{23}s_4 + c_{23}c_4 & 0 & s_{23}L_3 + s_2L_2 + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_4T = \begin{bmatrix} \boxed{\begin{matrix} c_1c_{234} & -c_1s_{234} & s_1 \\ s_1c_{234} & -s_1s_{234} & -c_1 \\ s_{234} & c_{234} & 0 \end{matrix}} & \boxed{\begin{matrix} c_1(c_{23}L_3 + c_2L_2) \\ s_1(c_{23}L_3 + c_2L_2) \\ L_1 + s_2L_2 + s_{23}L_3 \end{matrix}} \\ \begin{matrix} 0 & \quad 0 & \quad 0 \end{matrix} & 1 \end{bmatrix}
\Rightarrow \text{where} \begin{bmatrix} \boxed{\text{Rotation Matrix } {}^0_4R} & \boxed{\text{Translation}} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}
$$

Homogeneous Transformation Matrices take this form

This is the result. It allows us to calculate the rotation and translation of the end effector, given the joint angles.

$$x_{ee} = \cos(\theta_1)\left(\cos(\theta_2+\theta_3)L_3 + \cos(\theta_2)L_2\right)$$

x co-ordinate at the end effector

$$y_{ee} = \sin(\theta_1)\left(\cos(\theta_2+\theta_3)L_3 + \cos(\theta_2)L_2\right)$$

$$z_{ee} = L_1 + \sin(\theta_2)L_2 + \sin(\theta_2+\theta_3)L_3$$