

**BNF grammar**

$N = \{ \langle \text{term} \rangle, \langle \text{doll} \rangle, \langle \text{sha} \rangle, \langle \text{expr} \rangle, \langle \text{ned} \rangle, \langle \text{factor} \rangle \}$

$T = \{ \$, *, @, \# \}$

$P = \{$   
      $\langle \text{expr} \rangle ::= \langle \text{term} \rangle \langle \text{doll} \rangle$   
      $\langle \text{doll} \rangle ::= \$ \langle \text{term} \rangle \langle \text{doll} \rangle \mid \epsilon$   
      $\langle \text{term} \rangle ::= \langle \text{factor} \rangle \langle \text{sha} \rangle$   
      $\langle \text{sha} \rangle ::= \# \langle \text{factor} \rangle \langle \text{sha} \rangle \mid \epsilon$   
      $\langle \text{factor} \rangle ::= \langle \text{ned} \rangle \mid * \langle \text{ned} \rangle \mid @ \langle \text{ned} \rangle$   
      $\langle \text{ned} \rangle ::= X \mid \langle \text{expr} \rangle \}$

$S = \langle \text{expr} \rangle$

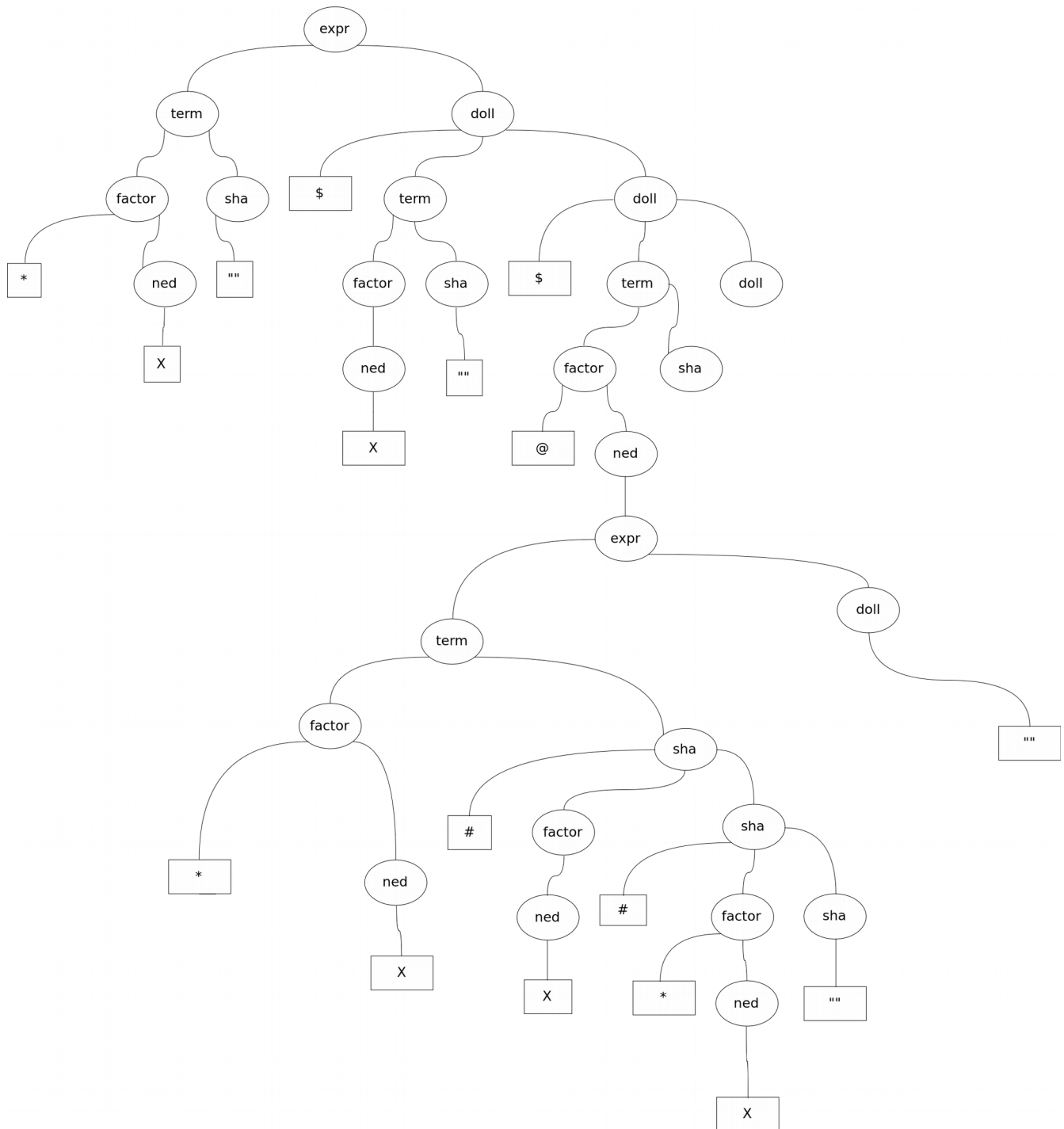
**Associativity**

both \$ and # are left and right associative.

**Precedence table**

|        |
|--------|
| @<br>* |
| #      |
| \$     |

### \*X\$X\$@\*X#X#\*X Parse Tree



## Ambiguity proof of BNF

```

<expr>
<term><doll>
<term>$<term><doll>
<factor><sha>$<term><doll>
*<ned><sha>$<term><doll>
*X<sha>$<term><doll>
*X$<factor><sha><doll>
*X$<ned><sha><doll>
*X$X<sha><doll>
*X$X$<term><doll>
*X$X$<factor><sha><doll>
*X$X$@<ned><sha><doll>
*X$X$@<expr><sha><doll>
*X$X$@<term><doll><sha><doll>
*X$X$@<factor><sha><doll><sha><doll>
*X$X$@*<ned><sha><doll><sha><doll>
*X$X$@*X<sha><doll><sha><doll>
*X$X$@*X#<factor><sha><doll><sha><doll>
*X$X$@*X#<ned><sha><doll><sha><doll>
*X$X$@*X#X<sha><doll><sha><doll>
*X$X$@*X#X#<factor><sha><doll><sha><doll>
*X$X$@*X#X#*<ned><sha><doll><sha><doll>
*X$X$@*X#X#*X<sha><doll><sha><doll>
*X$X$@*X#X#*X<doll><sha><doll>
*X$X$@*X#X#*X<sha><doll>
*X$X$@*X#X#*X<doll>
*X$X$@*X#X#*X

```

and (next page)

```

<expr>
<term><doll>
<factor><sha><doll>
*<ned><sha><doll>
*X<sha><doll>
*X<doll>
*X$<term><doll>
*X$<factor><sha><doll>
*X$<ned><sha><doll>

```

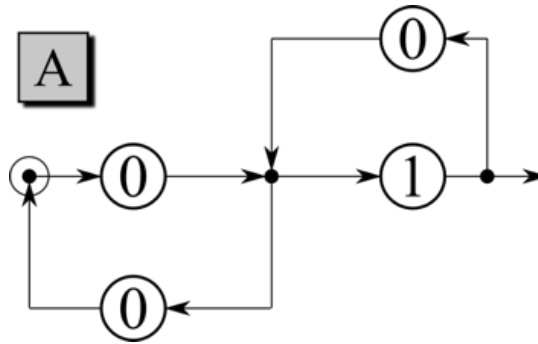
```

*X$X<sha><doll>
*X$X$<term><doll>
*X$X$<factor><sha><doll>
*X$X$@<ned><sha><doll>
*X$X$@<expr><sha><doll>
*X$X$@<term><doll><sha><doll>
*X$X$@<factor><sha><doll><sha><doll>
*X$X$@*<ned><sha><doll><sha><doll>
*X$X$@*X<sha><doll><sha><doll>
*X$X$@*X#<factor><sha><doll><sha><doll>
*X$X$@*X#<ned><sha><doll><sha><doll>
*X$X$@*X#X<sha><doll><sha><doll>
*X$X$@*X#X#<factor><sha><doll><sha><doll>
*X$X$@*X#X#*<ned><sha><doll><sha><doll>
*X$X$@*X#X#*X<sha><doll><sha><doll>
*X$X$@*X#X#*X<doll><sha><doll>
*X$X$@*X#X#*X<sha><doll>
*X$X$@*X#X#*X<doll>
*X$X$@*X#X#*X

```

are different derivations for the same string, thus the BNF is ambiguous.

**EBNF for Syntax chart A**

$$A ::= 0 (0 A \mid 1 0 0 A \mid \{1 0\}) 1$$
**Justification**

A can be interpreted as all combinations of the following recursive loops:

(1)

$$Start \rightarrow 0 \rightarrow Branch_1 \rightarrow 0 \rightarrow Start$$

*Production*

$$Path_1 ::= 0 0$$

In English: follow the arrows to the first 0, then to Branch<sub>1</sub>, follow the downward arrow to the second 0, then to Start.

(2)

$$Start \rightarrow 0 \rightarrow Branch_1 \rightarrow 1 \rightarrow Branch_2 \rightarrow 0 \rightarrow Branch_1 \rightarrow 0 \rightarrow Start$$

*Production*

$$Path_2 ::= 0 1 0 0$$

in English: follow the arrows to the first 0, then to Branch<sub>1</sub>, follow the forward arrow to the 1, then to Branch<sub>2</sub>, follow the up arrow towards the 0, then to Branch<sub>1</sub>, follow the downward arrow to the 0, then to the Start.

(3)

$$Branch_1 \rightarrow 1 \rightarrow Branch_2 \rightarrow 0 \rightarrow Branch_1$$

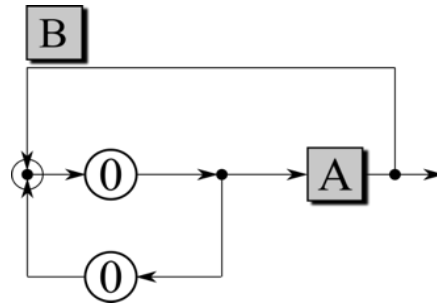
*Production*

$$InnerPath ::= \{1 0\} 1 End$$

In English: following any combination of the above recursive paths, whenever you reach Branch<sub>1</sub> you can loop through the 1 0 path, 0 or more times.

With tracing the combinations of these paths, and understanding the options given at each branch, I used the tool of EBNF to come up with the production of A.

**EBNF for Syntax Chart B**

$$B ::= \emptyset (\{A B\} \mid \emptyset B) A$$
**Justification**

B can be interpreted as all combinations of the following recursive loops:

(1)

$$Start \rightarrow \emptyset \rightarrow Branch \rightarrow \emptyset \rightarrow Start$$

*Production*

$$Path_1 ::= \emptyset \emptyset$$

This is identical to the first path in A's chart: Following the arrows back to the beginning, encountering only 0s.

(2)

$$Start \rightarrow \emptyset \rightarrow Branch \rightarrow A \rightarrow Start$$

*Production*

$$Path_2 ::= \emptyset A$$

This path actually contains the syntax of A, so the paths inside A will be defined using A's EBNF

Using these paths you can traverse every combination of the syntax, as long as you end with A, with the power of EBNF this can be represented as

$$B ::= \emptyset (\{A B\} \mid \emptyset B) A$$

## Ambiguity proof for BNF

$N = \{ \langle S \rangle, \langle A \rangle, \langle I \rangle \}$   
 $T = \{ a, b, c, x \}$   
 $P = \{ \langle S \rangle ::= \langle A \rangle$   
 $\quad \langle A \rangle ::= \langle A \rangle x \langle A \rangle$   
 $\quad \langle A \rangle ::= \langle I \rangle$   
 $\quad \langle I \rangle ::= a$   
 $\quad \langle I \rangle ::= b$   
 $\quad \langle I \rangle ::= c \}$   
 $S = \langle S \rangle$

### axaxa **derivations**

$\langle S \rangle$   
 $\langle A \rangle$   
 $\langle A \rangle x \langle A \rangle$   
 $\langle A \rangle x \langle I \rangle$   
 $\langle A \rangle x a$   
 $\langle A \rangle x \langle A \rangle x a$   
 $\langle I \rangle x \langle A \rangle x a$   
 $a x \langle A \rangle x a$   
 $a x \langle I \rangle x a$   
 $a x a x a$

and

$\langle S \rangle$   
 $\langle A \rangle$   
 $\langle A \rangle x \langle A \rangle$   
 $\langle I \rangle x \langle A \rangle$   
 $a x \langle A \rangle$   
 $a x \langle A \rangle x \langle A \rangle$   
 $a x \langle I \rangle x \langle A \rangle$   
 $a x a x \langle A \rangle$   
 $a x a x \langle I \rangle$   
 $a x a x a$

are different derivations for the same string, thus the BNF is ambiguous.