

Optimal number of firms with land:

$$\max_k k f\left(\frac{h}{k}, \frac{l}{k}\right) \quad \text{with } f(h, l) = A(h-c)^\alpha l^\beta \quad \text{no fixed cost for land}$$

$$\frac{d(kf(\frac{h}{k}, \frac{l}{k}))}{dk} = f(\frac{h}{k}, \frac{l}{k}) + k \cdot A \cdot \alpha (\frac{h}{k}-c)^{\alpha-1} \left(\frac{l}{k}\right)^\beta \left(\frac{-h}{k^2}\right) + k A (\frac{h}{k}-c)^\alpha \beta \left(\frac{l}{k}\right)^{\beta-1} \left(\frac{-l}{k^2}\right)$$

$$= \underbrace{A \left(\frac{h}{k}-c\right)^\alpha \left(\frac{l}{k}\right)^\beta}_{=0 \text{ if production zero}} \underbrace{\left(1 + k \alpha \left(\frac{h}{k}-c\right)^{-1} \left(\frac{-h}{k^2}\right) + k \beta \left(\frac{l}{k}\right)^{-1} \left(\frac{-l}{k^2}\right)\right)}_{=0} \stackrel{!}{=} 0$$

$$1 = k \left(\alpha \frac{h}{k^2 \left(\frac{h}{k}-c\right)} + \beta \frac{l}{k} \right)$$

$$1 = \alpha \frac{h}{h-kc} + \beta$$

$$\frac{1-\beta}{\alpha} = \frac{h}{h-kc}$$

$$\frac{\alpha}{1-\beta} = \frac{h-kc}{h} = 1 - \frac{kc}{h}$$

$$\Rightarrow \frac{kc}{h} = \frac{1-\beta}{1-\beta} - \frac{\alpha}{1-\beta} = \frac{1-\beta-\alpha}{1-\beta}$$

$$k = \frac{h}{c} \left(\frac{1-\beta-\alpha}{1-\beta} \right)$$