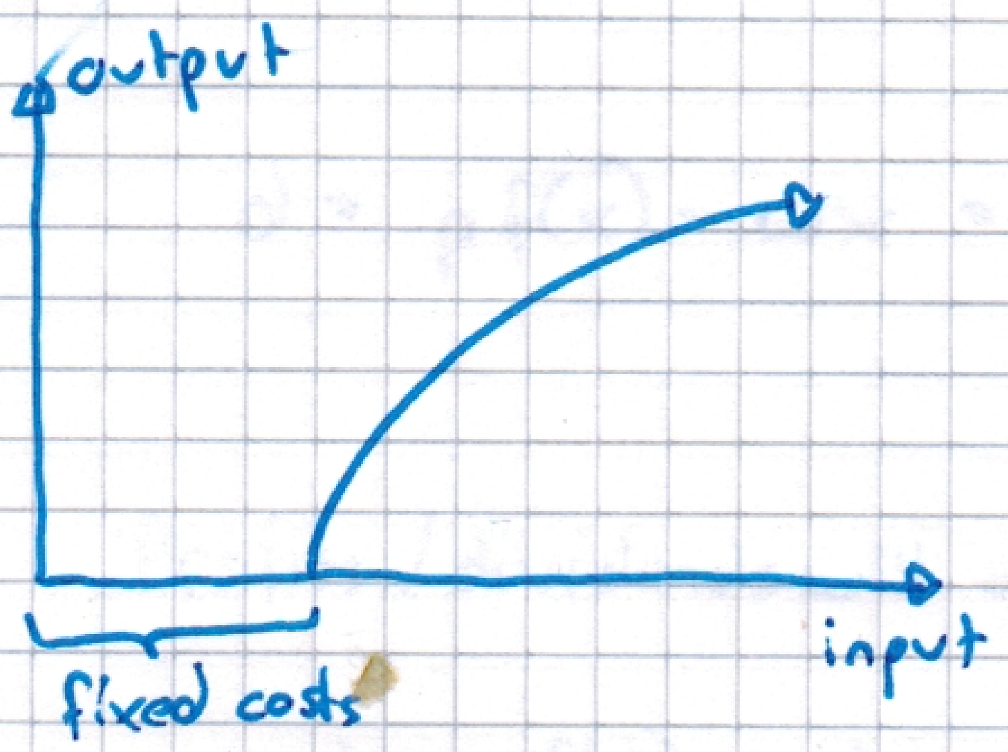
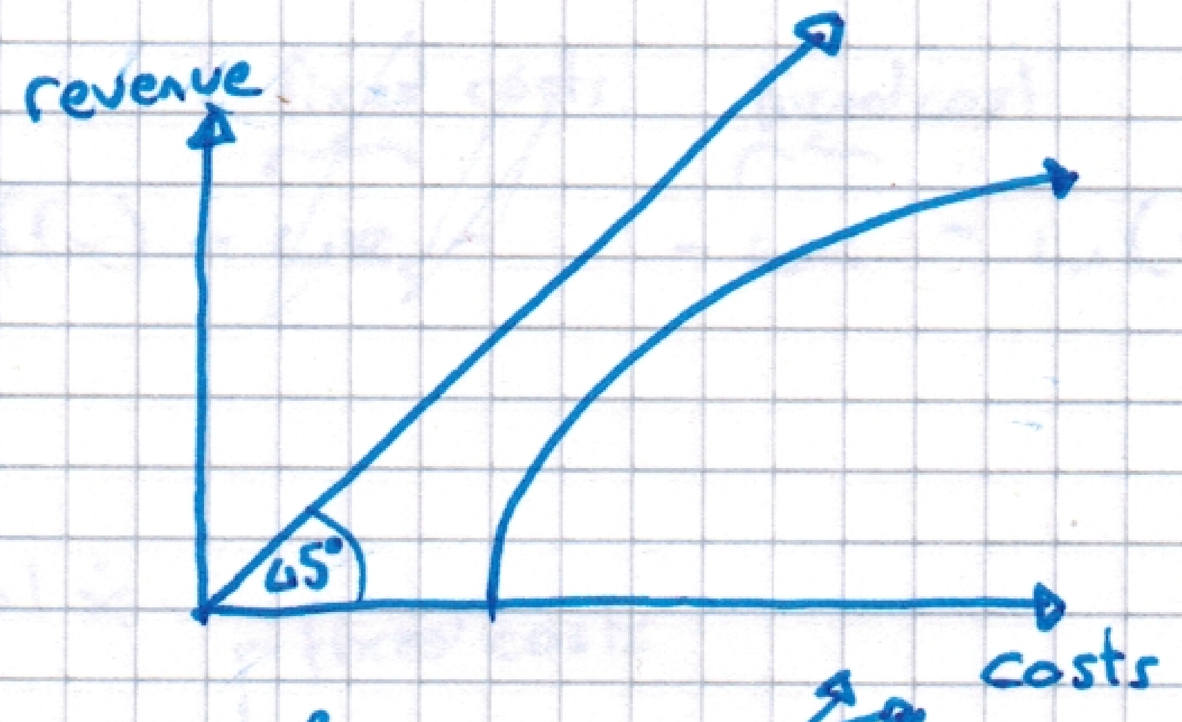
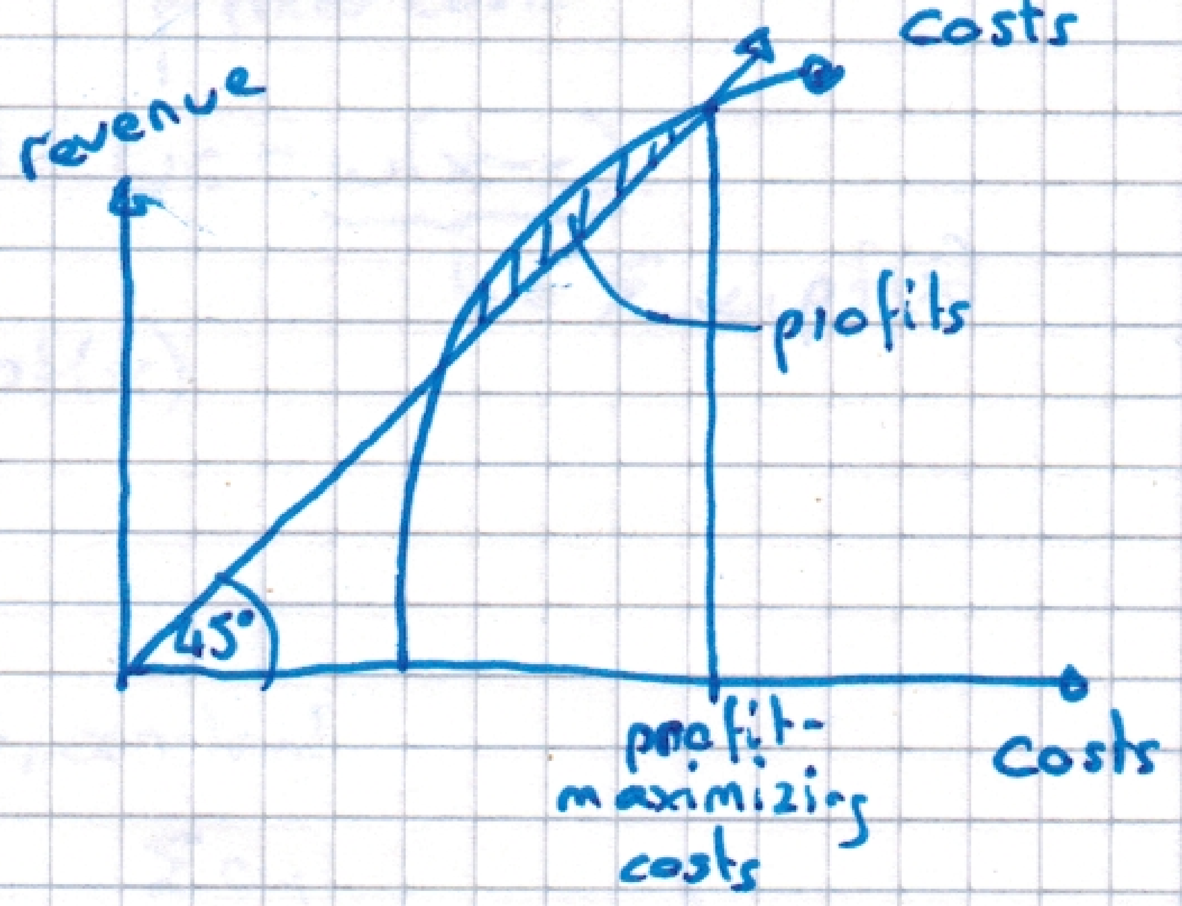
Cobb-Douglas with fixed costs

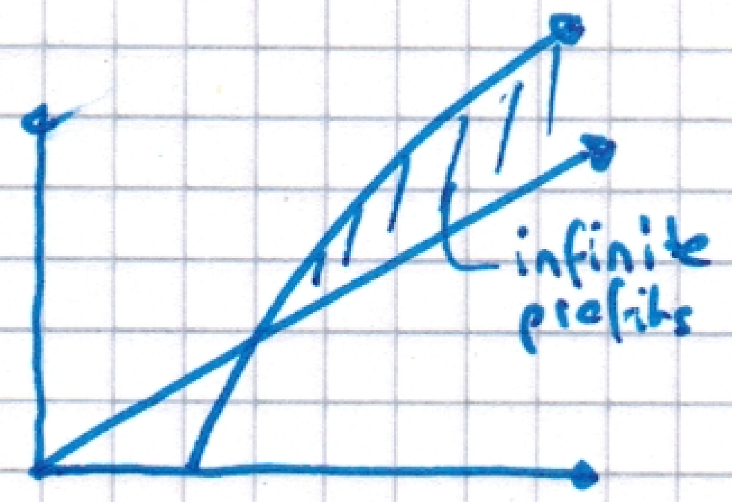


Production function



no profitable production plan
→ optimal is 0





increasing returns to scale

Maximization problem

Production function: $f(x) = A\left(\prod_{i=1}^{k}(x_i - c_i)^{\alpha_i}\right)$

constant (A); i-th input (x_i); fixed cost of i-th input (c_i); x: vector; α_i: share of i-th input

Note $\sum_{i=1}^{k}\alpha_i = 1$ implies constant returns to scale

> 1 implies increasing returns to scale

< 1 implies decreasing returns to scale with profits $= \underbrace{(1-\sum\alpha_i)}_{\text{"profit share, but not quite"}} \cdot \underbrace{\text{revenue}}_{= pf(x)}$

Profits: $\text{profit}(x) = \text{revenue}(x) - \text{cost}(x)$

$= p\,f(x) - wx$

p: output price; w: wage vector

Profit maximization: $\dfrac{d\,\text{profits}(x)}{dx_i} = 0$ for all i

$$\Rightarrow pA\alpha_i(x_i-c_i)^{\alpha_i-1}\prod_{j\neq i}(x_j-c_j)^{\alpha_j} = w_i$$

$$\Rightarrow \frac{w_k}{w_l} = \frac{pA}{pA}\cdot\frac{\alpha_k(x_k-c_k)^{\alpha_k-1}}{\alpha_l(x_l-c_l)^{\alpha_l-1}}\cdot\frac{\prod_{j\neq k}(x_j-c_j)^{\alpha_j}}{\prod_{j\neq l}(x_j-c_j)^{\alpha_j}}$$

$$= \frac{\alpha_k}{\alpha_l}\,\frac{(x_k-c_k)^{-1}}{(x_l-c_l)^{-1}}$$

$$\Rightarrow \frac{(x_k-c_k)w_k}{(x_l-c_l)w_l} = \frac{\alpha_k}{\alpha_l} \qquad \Rightarrow (x_k-c_k)w_k = \alpha_k\cdot p\cdot f(x)$$

⇒ What is spent on each input factor after having paid the fixed costs is proportional to the input weights.